A case study of lee waves over the Lake District in northern England

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SUMMARY

The structure and intensity of stationary, orographically-forced lee waves over the Lake District and northern Pennines on the afternoon of 26 November 1991 is determined through a combination of field observations and high-resolution numerical modelling. Observations were provided by the Meteorological Office’s C130 aircraft flying at four different levels and by five radiosondes (of differing ascent rates) released in rapid succession from an observing station located on the Cumbrian coast. Vertical velocity was inferred from both of these data sources and, with the help of satellite imagery, it was possible to build up a comprehensive picture of the lee wave structure.

In addition, a non-hydrostatic primitive equation model was run with a 2-km grid, covering a 180-km square region of the northern Pennines, in an attempt to simulate the gravity wave motion observed. The model was initialized with an upstream profile formed by merging a descent profile obtained from the aircraft with one from the sondes. The model orography was defined using a terrain height data-set with 500-m resolution and with some smoothing to the model grid. After about four hours of integration the model achieves a quasi-steady state with strong lower-tropospheric lee waves.

Verification of the model response was sought by comparing the vertical velocity in the model along the paths of the aircraft and sondes with that actually deduced from the observations. The overall agreement of the simulated amplitude and phase of the lee waves with that observed is very encouraging—as found in a different case study (Shutts 1992). The success of the model in predicting lee wave motion gives strong support for the use of these models for the development of gravity-wave-drag parametrization schemes.

The domain-averaged surface frictional stress was found to be much larger than the wave drag, and consistent with the high roughness lengths likely in the case of mountainous terrain.

1. INTRODUCTION

Research into atmospheric gravity waves has been spurred on in the last decade by the recognition of the important role played by wave drag in the global momentum budget. In the mesosphere, gravity wave stress is a dominating dynamical influence, whilst in the troposphere and lower stratosphere it plays a secondary role in the momentum budget of the zonally-averaged flow. The incorporation of gravity-wave-drag parametrization schemes into operational forecast and climate models (e.g. Boer et al. 1984; Palmer et al. 1986; Macfarlane 1987) was shown to eliminate much of the systematic westerly-wind model-bias in middle latitudes of the northern hemisphere, and their use is now standard. However, many aspects of these schemes are over-simplified; some completely ignore wave trapping and assume free upward radiation of wave energy until some condition for wave breaking is satisfied. There is therefore a practical need to study and observe trapped lee wave motion.

In this context, it is important to know whether or not the assumed level of gravity wave activity implied in parametrization schemes is realistic for a wide range of orographic types (e.g. rolling hills or rugged mountainous terrain). Furthermore, the widespread

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interest in gravity-wave-drag parametrization may have unfairly directed attention away from the role of the boundary layer in complex terrain—particularly amongst the numerical-modelling community. The size of the net aerodynamic form drag relative to the gravity wave stress remains unknown. Field experiments continue to play an essential role in providing information about these orographically-forced flows, and the data collected are being used increasingly to verify direct model simulations of actual cases.

One of the earliest attempts to use a model in conjunction with field data was by Vergeiner and Lilly (1970). They describe case studies of lee wave events, downstream of the Colorado Rocky mountains, using a combination of radar-tracked constant-volume balloons and research aircraft. The linear operational lee-wave model of Vergeiner (1971) was used to compute wave properties, given the upstream profile of temperature and wind, and these were compared with the observations. When the observed wave motion was of large amplitude and steady, the agreement with the model was good. The severe down-slope windstorm (in Boulder, Colorado on 11 January 1972) described by Lilly and Zipser (1972) and Lilly (1978) has been the subject of many numerical modelling studies, (e.g. Klemp and Lilly 1975, 1978; Peltier and Clark 1979; Clark and Farley 1984; Hoinka 1985). Klemp and Lilly (1975) used a three-layer linear model to study the resonant excitation of lee waves, and used it to explain some aspects of the Boulder windstorm. They went on to use a fully nonlinear, hydrostatic model to simulate the same wave event (Klemp and Lilly 1978), which together with the non-hydrostatic model simulation of Peltier and Clark (1979) achieved a remarkable degree of realism.

Clark and Gall (1982) used a three-dimensional, non-hydrostatic model to simulate trapped lee waves downstream of the Elk mountain in Wyoming, U.S.A. With a 1-km grid and 32 vertical levels spaced 500 m apart they were able to simulate correctly the 10-km wavelength lee wave train that was observed. Even the amplitude and phase of the simulated wave appeared to be in reasonable agreement with the observations. Apart from this study, few attempts have been made to use a real-terrain height specification in three-dimensional numerical modelling studies of orographic gravity waves (some notable exceptions are Smolarkiewicz et al. 1988; Hoinka and Clark 1991; Clark and Miller 1991; Shutts 1992): two-dimensional simulations using idealized orographic profiles are much more common (e.g. Hoinka 1985; Klemp and Durran 1987; Pitts and Lyons 1990). Even when a real-terrain height specification is used, the orography is often rather simple in form, e.g. the single island of Hawaii in the study by Smolarkiewicz et al. Alternatively, real isolated hills or mountains have been approximated by an analytic function, as in the three-dimensional study of island-induced ship waves by Peltier and Clark (1983).

Shutts (1992) used a non-hydrostatic model with a terrain height field representing Wales taken from a data-set with 500 m resolution and smoothed to a 3-km grid. In spite of the rather complex distribution of hills, mountains and valleys in Wales, the simulated vertical-momentum field agreed well with that inferred by examining rate-of-ascent fluctuations from radiosonde ascents. The fact that the model appears to cope with complex orographic specifications is reassuring and suggests that such an approach can be used to deduce wave drag and vertical momentum flux above real mountain ranges. However, Hoinka and Clark (1991) found, in a study of a strong Alpine Föhn event, that the computed upper-tropospheric vertical momentum flux was a factor of six larger than that deduced from aircraft data—a deficiency they attributed in part to poorly represented dissipative processes. Clearly, the modelling of flow over and around such steep and high mountain ranges is a very severe test for a numerical model, and the sharp transition zone between the blocked flow which passes around the mountain range and the flow above that passes over the mountain range may be difficult to handle. The Froude
number is greater than 1.5 in Shutts's (1992) case study, and similarly high in the case presented here: flow blocking is not an issue and the wave response may be much closer to that of a linearized wave problem.

Our aim is to provide further evidence, through a detailed case study, that high-resolution non-hydrostatic numerical models are capable of simulating lee wave fields over complex terrain. The simulated vertical velocity field is verified by direct comparison with radiosonde and aircraft data along their respective paths.

2. Observations of the Lee Wave Structure

(a) Synoptic overview

The lee wave event which forms the subject of this paper occurred on the afternoon of 26 November 1991 over the Lake District and northern Pennines of England. The sea-level-pressure analysis at 14 GMT (Fig. 1) shows a south-westerly airstream ahead of a minor polar airstream trough. At this time the trough is less than 100 km from Eskmeals, the sonde release station (location shown in Fig. 1), and is rather weak. Light showers were reported ahead of the trough at a number of stations (including Eskmeals). The surface geostrophic wind speed ahead of the trough is about 17 m s\(^{-1}\), which implies

![Figure 1. Sea-level pressure analysis at 14 GMT. The Lake District is shaded and the location of Eskmeals is identified by the letter E.](image-url)
Figure 2. Vertical profiles of (a) wind speed, (b) wind direction and (c) temperature on 26 November 1991 obtained from the aircraft descent profile below height 7.3 km and from sonde E above.

substantial orographic forcing of vertical motion. Vertical profiles of wind speed, wind direction and temperature are shown in Fig. 2(a–c). They are derived from the composite of an upstream aircraft profile and observations from a radiosonde released from Eskmeals (further details will be given in section 2(b)). The wind speed increases to approximately 41 m s\(^{-1}\) at a height of about 9 km—close to the height of the tropopause. The wind direction is south-westerly throughout most of the troposphere, but like the wind speed, it oscillates in the stratosphere owing to the presence of gravity waves. The
temperature variation below a height of 1.3 km is close to the dry adiabatic and is rather stable above that, up to a height of 4 km.

A satellite photograph at 1422 GMT (Fig. 3) shows extensive low-level cloud over the Irish Sea and near the Cumbrian coast, but very little cirrus. Cloud bands with a range of orientations about NW–SE can be seen over the Lake District and further downstream: these we take to be the visible manifestation of lee waves. The spacing between the bands is also rather variable, but generally around 20 km.

As will be seen later, the trough lies outside of the horizontal domain of the model at this time, and it is unlikely that the horizontal inhomogeneity expected near it invalidates the use of a single vertical profile to initialize the model.

(b) Radiosonde observations

The lee-wave case under study was just one of many which occurred during a field experiment held at Eskmeals between 11 and 28 November 1991. The primary purpose of the experiment was to exploit the use of five radiosondes released in very rapid succession to determine the structure of gravity waves over the Lake District mountains and beyond. By giving the balloons differing amounts of helium gas, the rates of ascent could be varied so that the sondes followed different trajectories in the vertical plane. In each group of five, consecutive sondes (labelled A to E in alphabetical order of release) were given progressively more gas. Therefore, in the mean, the sondes followed trajectories of progressively greater elevation angle, with sonde E attaining the maximum. With practice, it was possible to get a range of mean ascent rates between 2 and 5 m s\(^{-1}\), ensuring wide data coverage in the plane of the mean wind. Provided that the closest approach of neighbouring sondes is not greater than a horizontal wavelength, it is possible to build up a picture of the wave structure, assuming stationarity. It may also be possible to deduce a phase speed, in addition to the horizontal and vertical wavelengths, by making use of the time of the observations and the dispersion relation. In most of our multiple-sonde studies of lee waves there is far greater coherence in tropospheric ascent-
rate fluctuations if the ascent rates of the sondes are plotted versus downstream distance rather than versus height: this implies that the wave motion is usually trapped and has near-vertical phase lines.

Pressure, temperature and humidity are received on approximately 1.5 second cycle, and the wind is determined by NAVAID; LORAN signals are retransmitted by the
sonde and the arrival-time differences, calculated from five LORAN stations, can be used to determine the horizontal coordinates of the sonde. The rate of horizontal movement of the sonde is always very close to the horizontal wind velocity and, with the current system, a 30-second mean-wind measurement has a root-mean-square error of about 0.2 m s\(^{-1}\). Height is found by integrating the hydrostatic equation, and ascent rates are computed as averages over about 150-metre vertical intervals. As noted by Shutts (1992), the error in the ascent rate is likely to be smaller than 0.2 m s\(^{-1}\).

Figure 4. Vertical velocity deduced for each sonde, plotted above the orography in the order of increasing mean ascent rate. The numbers entered beside each curve give the height of the sonde in kilometres.

On the afternoon of 26 November 1991, five sondes, A to E, were released from Eskmeals at times 1359, 1407, 1412, 1419 and 1426 GMT, respectively. Figure 4 shows the vertical velocity measured by each of the five sondes, plotted against distance downstream. The lee wave is most noticeable in sonde ascents A and B which stayed at low levels for longer because of the smaller mean ascent rates. A peak vertical-velocity amplitude of at least 3 m s\(^{-1}\) is evident with pronounced downdraughts at ranges of 20 and 40 km. Further aspects of the radiosonde data will be discussed later.
(c) Aircraft observations

The Meteorological Office's C130 aircraft was available on two days of the ground-based field experiment, one of which was on 26 November—the day of the lee wave event featured here (since this was the biggest lee wave event of the field experiment, a considerable degree of good fortune was implied). The forecast conditions were not expected to be conducive to lee waves, principally because the minor trough which strengthened low-level winds over the Lake District had not been handled well by the Meteorological Office's operational limited-area forecast model.

The instrumentation relevant to the measurements used in this study is described elsewhere (Nicholls et al. 1983; Brown 1983). Unlike those earlier studies, however, the position in the horizontal was obtained using the Global Positioning System (GPS). Horizontal wind fluctuations of period less than one minute are, essentially, derived from the Inertial Navigation System (INS), whilst longer-period variations are corrected for drift and oscillations using the GPS. Vertical velocities are derived from accelerations measured using the INS for time periods less than ten minutes, and are corrected over longer times using pressure altitude. True airspeed calibration errors are the main limitation on the accuracy of the horizontal wind speed, but they are likely to be less than 0.5 m s⁻¹.

The aircraft experiment consisted of four level flight legs of length approximately 160 km oriented essentially NE to SW and at heights of 2.35, 3.69, 5.52 and 7.35 km (runs 1 to 4, respectively). Run 1 began at 1320 GMT and run 4 finished at 1525 GMT. Before these level runs a descent profile was obtained over the Irish Sea from a height of 7.35 km down to 16 m above sea level to serve as an 'undisturbed' upstream profile.

![Figure 5. Map showing the horizontal paths of the aircraft on the descent flight leg over the sea (solid line), and sonde A (dashed line).](image)
This was obtained between 1243 GMT and 1308 GMT—about one hour before the first sonde was released. Figure 5 shows the location of the descent-profile run which begins just north of Anglesey, flying north-east to arrive near Eskmeals at a height of 3 km and then looping back in descent towards sea level. Also shown is the trajectory of sonde A. The horizontal paths of runs 1 to 4 are omitted since they, together with all five sondes, lay within a corridor about 10 km wide. The closeness of sonde and aircraft paths provides some assurance that a composite of the data can be used to analyse (in vertical cross-section) the wave motion.

Figure 6 shows the vertical velocity inferred from the four level flight legs with the mean removed. The first thing to notice is that the greatest wave amplitudes are to be found on the lowest two flight legs and, with 5 m s\(^{-1}\) amplitude, are similar to the sonde ascent rate fluctuations about the mean. Vertical coherence in the vertical velocity is readily detectable on the lowest two flight levels, but is harder to see higher up. At low levels, around 70 to 100 km downstream of Eskmeals, the vertical coherence also seems to be lost. We presume that this is primarily due to transience, since the aircraft takes about half an hour to get from this region on run 1 to the same region on run 2.

![Vertical velocity measured along the four aircraft flight legs. The mean on each leg was removed.](image)

Vertical velocity deduced from the five sondes and all four aircraft legs has been plotted on a vertical cross-section along a line extending north-eastwards from Eskmeals (positions are projected normally on to this line). The information from the sondes is found to complement successfully the aircraft-derived vertical velocities; the resulting hand analysis is presented in Fig. 7. This clearly shows the existence of a trapped lee
wave pattern with considerable amplitude modulation in sympathy with the underlying orography. The dominant horizontal wavelength is approximately 20 km and the amplitude of the wave has a maximum at a height of about 3 km, with vertical velocities of up to 3.5 m s\(^{-1}\). Above 5 km the wave amplitude becomes small and the phase lines have an upstream tilt with height, consistent with upward energy leakage to the stratosphere.

Analysis of the wave field for heights above 10 km (with only sonde data) was possible but resulted in some apparent inconsistency between the horizontal wind perturbation and the vertical velocity. The problem stems from attempting to continue the wave field upwards as a pattern with constant-slope phase lines. This would require the horizontal wind component in the plane of the cross-section to be 180 degrees out of phase with the vertical velocity, yet, in the height range 15 to 20 km, sonde D shows a definite quadrature relation (Fig. 8). One possible explanation for this is that the wave field has standing wave (nodal) structure due to local reflection. A sonde passing through such a wave field can show different phase relationships between horizontal wind fluctuations and vertical velocity, depending on the path taken. Similar phase relationships between horizontal wind and vertical velocity can be found in the data from sondes C and E. Therefore, in view of the uncertainty in the details of the wave field in this region, this wave field was left out of the hand analysis (Fig. 7).

3. A NON-HYDROSTATIC PRIMITIVE EQUATION MODEL SIMULATION

(a) Model configuration

Numerical experiments with an adapted version of the Meteorological Office's operational mesoscale model (see Golding 1987) were carried out in an attempt to
simulate the observed lee wave motion using very high horizontal resolution. The model, developed originally by Tapp and White (1976), solves the fully compressible, non-hydrostatic equations of motion on a regular Cartesian, Arakawa C grid with vertical velocity held on intermediate levels. A semi-implicit treatment of acoustic modes avoids timestep dependence on the speed of sound. After Carpenter (1979), the vertical coordinate, \( \eta \), is given by

\[
\eta = z - E(x, y)
\]  

(1)

where \( E(x, y) \) is the terrain height and \( z \) is the actual height above mean sea level. Whilst this choice of terrain-following coordinate is not ideal it does have the merit of preserving the Helmholtz form of the diagnostic equation for the pressure, and of permitting an efficient ADI technique for its solution (Tapp and White 1976).

One potential difficulty with the coordinate is associated with the specification of the upper-boundary condition. In our experiments, the model enforces the condition \( w = 0 \), on the top \( \eta \)-surface (where \( w \) is the vertical velocity), and since the uppermost 6 km of the model is used as a damping layer, and the wind speed within this layer is allowed to tend linearly to zero at the model top, there is little possibility of spurious gravity wave activity being forced.

Another potential problem of all terrain-following coordinates is that the Eulerian description of purely horizontal advection involves the near cancellation of a term representing advection along the (non-horizontal) coordinate surfaces by a pseudo-vertical advection term. These individual terms may be large, and imperfect cancellation in a finite difference scheme could cause noise. In the prognostic equation for \( \eta \), the problem is further exacerbated by advecting \( \eta \) rather than \( w \), resulting in the appearance of centrifugal (or in tensor terminology, Christoffel symbol) terms. To be specific, the
vertical-momentum equation can be written as

$$\frac{\partial w}{\partial t} = -\mathbf{V} \cdot \nabla w + g \frac{\theta'}{\theta_0} - c_p \frac{\partial \pi'}{\partial \eta}$$

(2)

where $\theta'$ and $\pi'$ are the perturbation potential temperature and Exner function, $\theta$ is the potential temperature with constant reference value $\theta_0$ and $g$ the acceleration due to gravity.

Since

$$\dot{\eta} = w - \mathbf{V} \cdot \nabla E$$

(3)

Eq. (2) may be written as

$$\frac{\partial \dot{\eta}}{\partial t} = -\mathbf{V} \cdot \nabla \eta + g \frac{\theta'}{\theta_0} - c_p \frac{\partial \pi'}{\partial \eta} - \frac{\partial \mathbf{V}}{\partial t} \cdot \nabla E - \nabla \cdot (\mathbf{V} \cdot \nabla E).$$

(4)

This form of the vertical momentum equation was given by Carpenter (1979) in his reformulation of the Tapp and White (1976) mesoscale model to include orography. The last term on the right-hand side of Eq. (4) acts like a centrifugal force and contains individual components such as $u^2(\partial^2 E/\partial z^2)$. Such a term could be very large in jetstreams, irrespective of whether there was any orographically-induced ascent at the surface. For instance, if the orographic profile is sinusoidal with amplitude 500 m and wavelength 2 km, a 50 m s$^{-1}$ jetstream will cause this term to be greater than the acceleration due to gravity. Imperfect numerical cancellation of this term with $\nabla \cdot \mathbf{E}$ is inevitable and would cause spurious excitation of gridscale noise. This effect is unlikely to have had any impact on the model in its operational implementation (with 15 km grid), and was not detectable in the study of Welsh lee waves (Shutts 1992) which used a 3 km grid. However, if the gridpoint spacing is reduced to 2 km or less, unacceptable computational noise results.

To avoid the problem we now effectively use Eq. (2) and convert forecast values of $w$ to $\dot{\eta}$.

In the experiments to be described, all surface energy transfer was suppressed (by setting the surface heat exchange coefficient to zero) and no radiative forcing was allowed. Surface frictional stress and boundary-layer momentum transport were retained however. The model has a 1½ order turbulence closure scheme of the type developed by Mellor and Yamada (1982), which involves a prognostic equation for turbulent kinetic energy, to compute vertical eddy transfer coefficients $K_h$ and $K_m$ for heat and momentum, respectively. Essentially these coefficients are the product of the turbulent kinetic energy (TKE) and an eddy timescale dependent on the squares of the buoyancy frequency and vertical wind shear. The prognostic equation for the TKE is

$$\frac{D(TKE)}{Dt} = K_m S^2 + K_h N^2 - \frac{1}{\rho} \frac{\partial}{\partial z} \left( K_h \frac{\partial (TKE)}{\partial z} \right) - \frac{TKE}{\tau}$$

(5)

where the dissipation timescale, $\tau$, is given by the semi-empirical relation

$$\tau = \frac{c_0 L}{(TKE + F_L L^2 N^2)^{1/2}}$$

(6)

and where the length scale, $L$, is given by the Blackadar formula:

$$\frac{1}{L} = \frac{1}{kz} + \frac{1}{\lambda(z)}$$

(7)

and the non-dimensional empirical constants $c_0$ and $F_L$ have the values 5.524 and 7.098,
respectively. The quantity \( \lambda \), the asymptotic value of \( L \) as \( z \to \infty \), is set equal to one third of a nominal turbulent layer depth, \( h(z) \), defined, at any height, to be the thickness of the layer containing height \( z \) for which the \( TKE > 10^{-3} \text{m}^2\text{s}^{-2} \). If \( TKE < 10^{-3} \text{m}^2\text{s}^{-2} \), \( h(z) \) is set to the model layer thickness, except at the surface, where the combined thickness of the bottom two model layers is used. Further detail on the \( TKE \) scheme and other model parametrizations can be found in the paper by Ballard et al. (1991).

The roughness length, \( z_0 \), was taken to be proportional to the local gridpoint terrain height over land, with a peak value of 3 m occurring over the highest mountain in the model domain: over the sea \( z_0 \) was taken to be \( 10^{-4} \text{m} \). This somewhat arbitrary choice was made on the basis that the roughness is largely due to subgridscale orography, and that this would increase with mountain height. The peak value of 3 m is not inconsistent with the size of \( z_0 \) quoted by Mason (1985) for Wales, although his estimates were for ten-kilometre squares rather than the two-kilometre squares used here. A comparison model run using a constant land value of \( z_0 = 0.1 \text{m} \) is described in section 3(c).

The main model integration of this paper was forced to be dry and cloudless by setting the relative humidity of the initial state (and the boundary values during the simulation) to 1\%. A moist integration with cloud was carried out as a sensitivity experiment and this, together with some problems in the modelling of cloud, will be discussed later.

A uniform grid length of 2 km in the \( x \) and \( y \) directions was chosen so that 90 \( \times \) 90 points covered a large area of northern England, including the Lake District. A 500-m resolution terrain height data set was smoothed to the 2-km grid with a normal-distribution weighting function broad enough to remove gridscale roughness. The terrain heights were augmented by 0.4 of a standard deviation of the orographic height within a 2-km grid square to offset the local reduction of mountain heights caused by smoothing. The resulting terrain height map is shown in Fig. 9. We note in advance of the main

![Figure 9. Model terrain height field with the location of Eskmeals marked by the letter E. Areas shaded are above 500 m. Contour interval: 100 m. The maximum height is 891 m.](image)
simulation that the model lee wave motion is not sensitive to the method used to prepare the model orography. This is primarily because the main mountain ranges are well resolved at the scale of the lee waves. Also the orographic Froude number \( U/Nh \), where \( U \) is a typical surface geostrophic wind, \( N \) is the buoyancy frequency and \( h \) is a typical mountain height) is greater than about 2, implying that nonlinear effects are not dominant. In this regime (quasi-linear) one would not expect extreme sensitivity in the flow response to the mountain height.

The 43 model levels used in these experiments have a uniform spacing of 600 m within the height range 2.11 to 23.71 km: the lowest seven model levels are located at heights (in metres) given by the formula

\[
z_n = 10 + 50n(n - 1) \quad (n = 1, \ldots, 7).
\]  

(8)

Artificial damping was introduced above a height of 18 km to reduce spurious wave reflection from the upper lid. This has the form of a linear increase in the explicit horizontal diffusion coefficient from a background value of 500 m\(^2\)s\(^{-1}\) at and below 18.31 km to 25 000 m\(^2\)s\(^{-1}\) at 23.71 km. In addition, a Newtonian damping term was added to all prognostic equations for heights above 18 km, relaxing fields to their initial state. The time constant, \( \tau \), of this damping term is given by

\[
\tau = \tau_* \left\{ \exp \left( \frac{z - z_*}{z_d} \right) - 1 \right\}^{-1}
\]

(9)

where \( z_* \) is the bottom of the damping layer (equal to 18.31 km here); \( \tau_* = 9000 \) s and \( z_d = 1.6 \) km. The net rate of damping in the uppermost level is about as large as is permitted without violating timestep stability constraints imposed by the leapfrog scheme (with dissipative terms lagged by one timestep). In order to further increase the effectiveness of the damping layer, the wind speed above 18 km was allowed to decrease linearly to zero at the model top to slow down the upward gravity-wave propagation in the damping layer. In effect, the intention was to create an artificial critical layer near the model top (S. Mobbs, personal communication).

The boundary conditions enforced on the wind components depend on whether the wind component is normal or tangential to that boundary. All tangential components are held constant on all boundary faces. Boundary conditions on the normal velocity component at lateral walls depend on whether there is inflow or outflow (see Shutts 1992).

At the uppermost level the vertical velocity rather than \( \eta \) is effectively zero: at ground level \( (\eta = 0) \) \( \dot{\eta} = 0 \). The potential temperature is held constant at lateral boundaries.

The model was initialized by interpolating a smoothed vertical profile of the wind and temperature (obtained from the composite upstream profile) to the model levels, and then by computing the height-dependent part of the pressure field from the hydrostatic relation. A linear horizontal perturbation component of temperature and pressure was then added to ensure that thermal wind balance and geostrophy were satisfied. On the lowest three model levels, the wind used for initialization and for the lateral boundaries was chosen to be the one on level 3, corresponding to a height of 310 m above the surface. This effectively imposes the geostrophic wind on the inflow boundary, avoiding a possible ‘double-counting’ of the effects of surface friction.

(b) Model results

A four-hour model integration was carried out, by which time the resulting wave field was virtually steady (i.e. differences between plotted vertical velocity fields for
integrations of four and six hours were negligible). The vertical velocity at a height of 3 km (Fig. 10) shows a fairly complex banded pattern with a dominant wavelength of around 20 km over much of the land in the model domain. Vertical velocities range between −3.6 and +4.7 m s⁻¹, and are largest over the Lake District mountains.

It is interesting to attempt to relate this pattern of vertical motion to the distribution of cloud apparent in the satellite image (Fig. 3), bearing in mind that the cloud is likely to be located where upward displacement of parcels is greatest, i.e. at the downwind edge of the positive w-bands. The main cloud band lies from NW to SE through the centre of the box collocated with the model domain. Such a feature is discernible in the model w-field, with a change in orientation, from NW–SE in the northern half of the domain, to WNW–ESE further south. There is some evidence of the same change of orientation in the imagery, though perhaps not so pronounced. The first cloud band downwind of Eskmeals also seems to be captured well by the model, with its reduced north–south extent. In the north-eastern corner of the domain two lee wavelains are visible with a quiet zone between them: the satellite imagery reveals these very clearly, and the model has represented their orientation well. Our less successful attempts to simulate the cloud field itself will be described later.

A vertical cross-section taken on a line parallel to the aircraft flight path and through the model position of Eskmeals is shown in Fig. 11. Comparing this with the hand-analysis of the vertical velocity (Fig. 7) reveals a high level of agreement; the positions of individual updraughts or downdraughts, with respect to the orography, can be seen to correspond (e.g. the first two descent regions downwind of Eskmeals occur just to the lee side of the Scafell and Helvellyn mountains, respectively). The dominant wavelength, as seen in the w-map at a height of 3 km, is about 20 km, and the extreme values +3.7 and −3.2 m s⁻¹ occur close to this level. Above this, a pattern of waves of
roughly constant amplitude tilts upstream with height. Furthermore, the wave field shows nodal structure consistent with the inference made earlier regarding the positive correlation of horizontal wind speed and $w$ in the stratosphere. We shall return to this point later when discussing linear solutions of the vertical-structure equation.

A more exacting test of the success of the simulation is to make a direct comparison between the model vertical velocity and that derived from the aircraft and sondes along their respective paths within the model (e.g. Shutts 1992). Fig. 12(a–d) shows the comparison between the model’s and the aircraft-derived vertical velocities at the four flight levels. The agreement, particularly on run 2, is very good, and clearly demonstrates that the model is capable of reproducing the correct wave response, in spite of its near-resonant nature and the complexity of the underlying terrain.

A similar comparison for sondes A and C (Fig. 13(a, b)) shows good agreement in the lower troposphere, but phase errors become apparent at higher levels. Sonde C shows a shortening of the apparent wavelength beyond a range of 100 km (corresponding to heights above 12 km), which is suggested by the model but not to the right degree; this probably results from there being insufficient vertical resolution to handle the shortening of the vertical wavelength as the wind speed decreases.

Figure 14(a) shows the horizontal wind component perturbation (resolved north-eastwards, along the aircraft path, run 2), comparing the model with that observed. Again the correspondence seems to be very good; although the observed quantity has somewhat greater amplitude, and there is a significant departure between the two curves beyond 140 km.

Similarly, Fig. 14(b) shows the temperature perturbation (defined as the departure from the flight-leg mean) for run 1 compared to the corresponding model variation (this level has the largest temperature perturbations). The agreement between model and
Figure 12. Vertical velocity along the aircraft flight legs compared to that found along the equivalent paths in the model. (a) to (d) correspond to runs 1 to 4, respectively.
Figure 12. Continued
Figure 13. Vertical velocity along the trajectories of the sondes compared to that found along the equivalent paths within the model: (a) sonde A; (b) sonde C.
Figure 14.  (a) Comparison between the wind component resolved along a bearing of 042 degrees found on C130 run 2 (thin line) and that found within the model (thick line). (b) Comparison between temperature perturbation in the model (thick line) and that observed on run 1 (thin line).
observations is good, except for the absence of a pronounced minimum near 60 km, which the model fails to capture. The temperature perturbation is dependent on the vertical displacement of air parcels passing through the wave field, and this is effectively proportional to the integral of the vertical velocity along the aircraft path (assuming the horizontal wind speed is essentially constant). Figure 12(a) shows that the area of positive vertical velocity just upstream of the temperature minimum near 60 km is considerably greater for the aircraft curve than for the model curve, consistent with the more pronounced temperature minimum in the aircraft data. Other differences between the model and aircraft temperatures may be ascribed to transience or fine structure in the vertical stratification omitted from the model, which causes a different temperature perturbation for the same vertical displacement.

In order to further clarify the nature of the model's response, linear theory was used to find the horizontal wavelength and form of the near-resonant lee wave. The vertical amplitude structure of a single plane harmonic wave is governed by the well-known equation (e.g. Sawyer 1960)

$$\frac{d^2 W}{dz^2} + (l_s^2 - K^2)W = 0$$

(10)

where \( W(z) \) is the complex vertical-structure function in the following expression for the vertical velocity, \( w(x, y, z) \):

$$w(x, y, z) = \text{Re} \left[ W(z) \exp \left( i(kx + ly) + \frac{z}{2H_0} \right) \right]$$

(11)

where \( K^2 = k^2 + l_s^2 \), \( H_0 \) is the density scale height, and \( l_s \) is the Scorer parameter defined by

$$l_s^2 = \frac{N^2}{U_n^2} - \frac{1}{U_n} \frac{d^2 U_n}{dz^2}$$

(12)

where \( N \) is the buoyancy frequency and \( U_n \) the wind component in the direction of the

![Figure 15. Smoothed Scorer parameter \( l_s^2 \) derived from the upwind profile.](image)
wavenumber vector \((k, l)\). This definition ignores a contribution from terms involving \(H_0\) (see Smith 1979; p. 95). Figure 15 shows the profile of \(l^2;\) before its calculation the wind (resolved along a bearing of 042 degrees) and the temperature profiles were smoothed. Values of \(l^2\) are large below 2 km and again above 15 km, whilst in the middle and upper troposphere they are close to zero. Equation (10) was solved repeatedly for different horizontal wavelengths (using the method given by Lindzen and Kuo 1969) until the maximum amplitude response was found. This occurred at a horizontal wavelength of 18.8 km and resulted in the amplitude and phase profiles shown in Fig. 16(a, b). The 'surface' amplitude is set equal to unity and a radiation boundary condition is employed at a height of 22 km.

![Amplitude and Phase Profiles](image)

**Figure 16.** (a) Amplitude and (b) phase (degrees) of the near-resonant response found for a horizontal wavelength of 18.8 km using linear theory. The amplitude units are arbitrary.
The amplitude attains a maximum at a height of about 3 km, where the phase lines are almost vertical. Above 6 km the phase lines tilt upstream with height, and partial internal reflection causes short vertical wavelength amplitude oscillations. A nodal response could result only if wave reflection near the model top was total; in spite of the rapid increase with height of $l_2$ above 15 km, wave energy still escapes through the upper boundary. The calculation is consistent with the gross aspects of the numerical simulation but suggests that the vertical wavelength of the lower-stratospheric model response is too long. Since the model has a mean layer separation of 600 m near 16 km, it is unable to handle correctly the 1.5 to 2 km vertical wavelengths implied by the linear calculation. As noted earlier, this might explain the discrepancy between sonde C and the model in the lower stratosphere.

(c) Some additional numerical experiments

The numerical model used in these studies has a comprehensive physical parameterization package, yet we have chosen to render it largely ineffective for the purposes of the main simulation: the only irreversible physical processes allowed were boundary-layer friction and the turbulent energy production/decay through the TKE equation. Radiative transfer is only likely to affect the lee-wave response through its direct effect on the vertical profile of temperature and consequent effect on the boundary-layer wind profile. Given that the satellite imagery shows considerable cloud cover and that, at this time of the year, solar insolation is rather weak, it was felt that the boundary layer would not be greatly changed by surface energy fluxes in its passage over land. Potentially greater uncertainty in the model response results from the difficulty in defining an appropriate field of roughness length. Herein lie deeper conceptual and modelling issues concerning the meaning of $z_0$ in partially-resolved hilly terrain and the realization of its influence through the implementation of turbulent momentum-transfer parametrization schemes (Mason 1985; Grant and Mason 1990). It is advisable, therefore, to treat the actual values of $z_0$, used here, with some caution. The forcing and dissipation of trapped lee waves in the presence of a realistic boundary layer has received little attention in the literature. The main effect of increasing the roughness length is to deepen the layer with a logarithmic wind profile, thereby reducing the effective surface wind and hence the wave forcing. In some situations, boundary-layer separation will limit the amplitude of trapped lee waves, as streamlines are prevented from following the orographic profile. Also, boundary-layer dissipation of kinetic energy in the trapped lee wave will cause downstream decay so that the decelerating effect of surface wave drag will be felt within the boundary layer.

In view of the uncertainty attached to the prescribed field of roughness length and its effect, another run was carried out using $z_0 = 0.1$ m to assess the sensitivity of the wave response. This value would normally be considered typical of lowland England, with a mixture of open fields, scattered trees and hedgerows (Smith and Carson 1977). The model wave field which results after four hours of integration time is very similar to that in the main run, as can be seen in Fig. 17. The differences in the wave response are only really noticeable in the downstream half of the domain where the differences in boundary wind structure are greatest.

In order to ascertain what effect the larger roughness lengths are having on the boundary-layer structure, Fig. 18(a,b) shows vertical cross-sections of the TKE along the path of the aircraft for the two cases. As expected, with the higher $z_0$ values, the low-level TKE is both more intense and more extensive, with peak values of 7 m$^2$s$^{-2}$ in the case with the height-dependent roughness, compared to 4.7 m$^2$s$^{-2}$ for the run with
$z_0 = 0.1$ m. The spatial distribution of TKE is interesting, with elevated maxima occurring at ranges of 60 and 100 km—beneath maxima in the vertical velocity. The Richardson number was found to be less than $\frac{1}{4}$ in these regions, suggesting that this effect was due to wave breaking rather than to surface-layer shearing instability. The wind component along the cross-section line in the case with high $z_0$ (Fig. 19(a)) shows marked lee-wave-related variation over the mountains with lee enhancement noticeable downwind of the main peaks. The near-surface wind component varies from about 10 m s$^{-1}$ at about 45 km to 4 m s$^{-1}$ at 62 km downstream of Eskmeals in the valley of the river Eden. The difference in this wind component between the main run (height-dependent $z_0$) and the $z_0 = 0.1$ m run is given in Fig. 19(b). Generally speaking, the boundary layer up to a height of 1 km has smaller wind speeds in the height-dependent, high roughness-length run—in places by up to 7 m s$^{-1}$. In spite of these boundary-layer wind changes, the net effect on the gravity wave field is rather small.

Another important concern with the main simulation is the absence of moisture and cloud. Durran and Klemp (1982) showed that the existence of moist layers in the upstream profile could have a potentially large impact on the form and amplitude of orographic lee waves, compared to a dry airstream with the same wind and temperature profile. The effective reduction of stability accompanying latent-heat release in the moist zone could reduce the effective Scorer parameter (typically in the lower troposphere). This in turn could lead to lee waves being substantially 'leaky', or even untrapped when the dry $f$ profile suggests trapping.

The distribution of relative humidity in the vertical found in the aircraft descent profile is given in Fig. 20. Cloud was found not only in the region between 1.1 and 1.6 km, where the relative humidity was greater than 95%, but also higher up. The intention had been to carry out run 1 at 6000 feet (about 1.8 km), but this was found to
be the top of a stratocumulus cloud layer where vertical motions from smaller-scale entrainment might have detracted from the analysis of lee waves. The flight level was therefore increased to 7500 feet (about 2.3 km). A moist layer also appears at around 6.3 km, related to the presence of cirrus (fallstreaks were encountered on run 4).

In order to gauge the sensitivity of the response of the model to moist effects, a number of integrations were carried out with a variety of slightly different specifications of the humidity and cloud liquid-water mixing ratio. If saturation was assumed to exist wherever the relative humidity was found to be greater than 95%, and the cloud liquid-water content was initialized, it was found that an unreasonable level of convective instability resulted over most of the region upstream of the mountains. Although light showers were reported in this region, it was clear that the specified inflow profile was too unstable. Since cloud was patchy over the Irish sea, it was decided merely to specify the relative humidity profile alone and ignore any cloud liquid water initially, and also on the inflow boundary.

After four hours of integration the resulting steady state was very similar to the dry run, confirming, as in the Welsh study of Shutts (1992), that moisture is not a crucial
Figure 19. (a) Wind component along the line of cross-section for the main simulation. Contour interval 2 m s$^{-1}$. (b) Wind-component difference (main run minus the $z_0 = 0.1$ m run). Contour interval 1 m s$^{-1}$.

Figure 20. Relative humidity from the aircraft descent profile.
factor in the determination of lee wave structure. Orographic cloud (defined to exist when the cloud liquid-water mixing ratio exceeds 0.1 g kg\(^{-1}\)) was produced however, and its distribution (shown in Fig. 21) should be compared with the satellite image (Fig. 3). The correspondence is generally good, though there is a tendency for the high-humidity values to be lost by diffusion, so that cloud bands are less evident downwind of the Lake District.

Figure 21. Model cloud distribution at \(z = 1.6\) km in the moist run.

4. DISCUSSION

It has been demonstrated that a dry, non-hydrostatic primitive equation model using a real terrain height specification can provide a fairly accurate description of the gravity wave motion forced by hills and mountains—even when their spatial distribution is complex. Such a conclusion was suggested by Shutts (1992), but in the case study described here a better observational data-set was available. In particular, the threesonde experiment by Shutts suffered from insufficient dispersion of the balloons, resulting in each sonde giving effectively the same profile, though at different times. Here five sondes were deployed with sufficiently different trajectories to make it possible to infer the structure of the wave field. More importantly, aircraft data were available on four levels within the troposphere and this provided the backbone of the hand-analysis shown in Fig. 7. Also satellite imagery was available at the time of the experiment; it showed the positions of lee-wave cloud bands over the model domain area, unlike that available in Shutts's study.

The practical motivation for this work comes from the need to parametrize correctly gravity wave drag in numerical weather prediction and climate models (e.g. Palmer et al. 1986). The good agreement between simulated and observed vertical velocity encourages us to use the model to compute vertical momentum fluxes, though there is no
guarantee that they will be accurate. The main discrepancy encountered in the simulation was in the lower-stratospheric part of the solution. The vertical momentum flux depends critically on (i) the ability of the model to resolve the vertical structure of the wave in regions of large values of the Scorer parameter, (ii) to represent any wave breaking that should occur and (iii) to provide a physically appropriate upper-boundary condition. On this last point one should recall that the use of a damping layer is appropriate only if downward reflection of wave energy from above the model top is unlikely. In simulations of real cases it is difficult to be certain that this will not occur, and one is forced to suppose that waves are very effectively dissipated at some height below any potentially reflecting layers (where, for instance, $I_s < k^2$).

With these uncertainties in mind, we now show the vertical profiles of domain-averaged vertical momentum-flux components (Fig. 22(a,b)). Also shown on the plots is the pressure force exerted on the orography, resolved in the appropriate directions; some difficulty is attached to the computation of this quantity since the orographic heights are non-zero at the lateral boundary of the domain. The procedure adopted was to calculate the net pressure force on the orography in the initial state and subtract this from the resulting steady-state value. For the high Rossby and Froude numbers, associated with the orographically-forced aspects of the flow, it seems safe to suppose that this additional pressure force should be due to gravity wave drag.

Fig. 22(a,b) shows very similar profiles of momentum flux in the eastward and northward directions, implying a drag vector pointing south-west—opposite to the wind direction in the troposphere. The magnitude of the wave stress decreases from 0.35 N m$^{-2}$ at the surface to 0.16 N m$^{-2}$ at a height of 2 km, and then decreases linearly to zero up to a height of about 20 km. Care is required in the interpretation of these domain-averaged vertical momentum fluxes when, as for trapped lee waves, a substantial proportion of the wave energy passes through the downstream lateral boundary. Steady mean-flow deceleration by the gravity wave field requires wave dissipation (e.g. Bretherton 1969), yet a decrease of $\tilde{V}'w'$ with height does not guarantee this since some wave energy passes through the lateral boundary. The linear decrease of vertical momentum flux with height between 2 and 18 km is due partly to this and other weak dissipative influences in the model.

Attempts to compute vertical momentum fluxes from the aircraft data were unsuccessful. Even after removing the mean and linear trend in both $u'$ and $w'$, the product $u'w'$ was found to be sensitive to the averaging length. Fig. 14(a) shows a pronounced divergence beyond 140 km between the model and aircraft $u'$: so this region was excluded from the average, giving $u'w' = -0.38$ N m$^{-2}$. However, successive averages taken out to 125, 130 and 135 km give $u'w' = 0.63$, 0.05 and $-0.65$ N m$^{-2}$, respectively. On the other hand, if the model's wind components are used, $u'w'$ is found to be insensitive to averaging length, varying between $-0.28$ and $-0.30$ N m$^{-2}$ in the same range. An inspection of plots of $u'$ and $w'$ shows that in both aircraft and model data these quantities are close to being in quadrature: $w'$ passes through zero where $u'$ has a maximum or minimum. The negative correlation between $u'$ and $w'$ in the model data is barely discernible to the eye. This suggests that the computation of the vertical momentum flux will be sensitive to small errors such as the imperfect correction of ‘drift’ in the winds derived from the inertial navigation system. It might also be caused by real horizontal fine structure in the wind field. Further re-processing of raw wind data is planned in the hope that, by so doing, better wind estimates will reduce some of the sensitivity to averaging length.

From the viewpoint of wave-drag parametrization in numerical weather prediction models, the size of the simulated wave drag is fairly typical, yet the domain-averaged
Fig. 22. Profile of vertical flux of horizontal momentum: (a) eastward component; (b) northward component. The dashed line extending to $z = 0$ connects the pressure-drag component to the momentum flux at the height of the highest model mountain.
frictional stress was found to be 1.65 N·m⁻², oriented at 25 degrees—nearly five times larger than surface wave stress. Using the estimated geostrophic wind speed of 17 m·s⁻¹, the domain-averaged geostrophic drag coefficient is found to be about 4.2 × 10⁻³—somewhat larger than values normally quoted for mountainous regions of this type (e.g., Haltiner and Williams 1980, p. 283, suggest 2 to 3 × 10⁻³ for ‘low relief topography–0.5 to 1 km’). Nevertheless, the size of surface frictional drag found here is in accord with measurements made in the Welsh field experiment reported by Grant and Mason (1990). They found friction velocities, ɯₙ, greater than 1 m·s⁻¹, and geostrophic drag coefficients of 10⁻² under similar conditions. It is interesting to note that even the low-roughness-length run produced a net surface frictional stress exceeding that of gravity wave drag (viz. 0.82 N·m⁻²) reinforcing the view that frictional stress dominates the total force exerted on the surface. Although some uncertainty surrounds the appropriate roughness-length specification, the ₀ = 0.1 m simulation showed a very similar gravity wave field, in spite of large TKE and boundary-layer wind-speed differences. This would seem to imply that the effective lower boundary for the lee waves is at the top of the well-mixed layer, where wind speeds are close to their geostrophic value.

The importance of correctly modelling the effects of roughness in numerical weather prediction and climate models has been emphasized by this simulation; in most situations gravity wave drag probably plays a secondary role in the exchange of momentum with the surface. This is not to discount the importance of wave drag, but it serves as a reminder that the impact of parametrized gravity wave drag must be assessed in conjunction with a boundary-layer frictional stress formulation that captures the correct level of aerodynamic drag associated with high effective roughness lengths in mountainous terrain.

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