Reply to comments by L. Levi and F. Prodi: Another look at the dependence of ice accretion density on non-dimensional parameters

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SUMMARY

In response to the comments of Levi and Prodi I have made a comparison between their rime density data and a similar subset of my data. The choice of air temperature rather than accretion temperature to characterize droplet freezing is further explained, and the convective-heat-flux $\pi$-product is modified by using the droplet diameter rather than the cylinder diameter as the length scale. A limitation to the regression analysis associated with the form of the function relating ice density to non-dimensional parameters associated with ice accretion is discussed.

1. INTRODUCTION

I thank Levi and Prodi for their comments, which would seem to comprise two main points:

1) They do not believe that the density formula of Jones (1990) (henceforth J1990) can be extended to values of icing parameters in the range that was in use for their experiments (Prodi et al. 1991, henceforth PLNL).

2) They are not in favour of the $\pi_c$ parameter, and believe that the accretion temperature should be incorporated in any parameter that represents droplet freezing. In my reply I have compared their experiment (PLNL) with my measurements (J1990). A subset of the J1990 data that best matches their PLNL data is chosen and the measured and predicted densities are compared. I then discuss the basis for my choice of variables in $\pi_c$ to represent the initial stage of droplet freezing. Finally, I make suggestions for possible improvements to the density formula based on non-dimensional parameters.

2. COMPARISON BETWEEN J1990 AND PLNL

I first compare the experiment by PLNL with the measurements used by J1990 and point out the differences between them. The nine density measurements by PLNL were done in a cold room, accreting rime on a 1-cm-diameter ice-coated plastic cylinder rotating at 4 Hz. The medians of windspeed, $V$, and air temperature, $T$, are shown in Table 1. The median volume droplet diameter, $d$, for all the experiments was 20 ± 2 $\mu$m. The liquid-water content, $W$, was not measured but was

<table>
<thead>
<tr>
<th>TABLE 1. COMPARISON OF PLNL ICING CONDITIONS WITH J1990 ICING CONDITIONS ON MT WASHINGTON, AND A SUBSET OF THE J1990 DATA SATISFYING CONSTRAINTS ON $X$, $K$ AND $\phi$</th>
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</thead>
<tbody>
<tr>
<td>Number of cases</td>
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<tr>
<td>Cylinder diameter (cm)</td>
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<tr>
<td>Accretion thickness (cm)</td>
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<tr>
<td>Rotation rate (Hz)</td>
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<td>Atmospheric pressure (mb)</td>
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<td>Median $D$ (cm)</td>
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<td>Median $d$ ($\mu$m)</td>
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<td>Median $V$ (m s$^{-1}$)</td>
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<td>Median $W$ (g m$^{-2}$)</td>
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<td>Median $T$ ($^\circ$C)</td>
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<td>Median $X$ ($\mu$m m/s$^\circ$C)</td>
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<td>Median $K$</td>
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<td>Median $\phi$</td>
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estimated to be between 1 and 2 g m\(^{-3}\), and apparently varied along the cylinder axis. Levi (personal communication) indicated that \(W\) may have been higher than 2 g m\(^{-3}\) in some cases. PLNL measured the density, \(\rho\), of the first approximately 1-mm-thick layer of 5 mm of rime that, typically, accreted, using an optical densitometer to analyse the x-ray micrographs of 1-mm-thick sections of the deposit.

In contrast, in the 2665 cases that were used by J1990 from multicylinder runs in natural icing conditions at the Mt Washington Observatory the ice was accreted on six brass or aluminum cylinders ranging in diameter from 0.158 cm to 7.62 cm, rotating at 0.017 or 0.033 Hz. The median values for the icing variables are given in Table 1. The densities were calculated using the records of the measurements of mass, length, and diameter of the accreted ice on each cylinder that were used to determine \(W\) and \(d\) of the icing fog.

Overall, the icing conditions on Mt Washington were warmer and windier and were associated with droplet sizes and liquid-water contents that were substantially smaller than those used by PLNL. However, these differences do not necessarily mean that the accretion conditions differ significantly. If the values of the parameters that determine density are in the same range, then densities associated with these different conditions should be comparable. A more significant difference between PLNL and J1990 may be the method of determining density. Because J1990 determined a bulk density and PLNL measured the density in lobes of the accretion, I would expect the PLNL method to give a higher value for density if both methods were used on the same rime sample. In their comments Levi and Prodi have noted my disagreement with the \(\rho_\alpha\) values that they show in their Fig. 1 and state correctly that this does not significantly change the disagreement between the two density predictions. The \(\rho_\alpha\) values in their Fig. 2 are also incorrect, but more importantly, they have determined densities for accretion conditions outside the range in their experiments. Not only did they not accrete ice on cylinders larger than 1 cm in diameter, the value of \(K\) for cylinders with diameters between 2 and 7 cm, for \(d = 20 \mu\text{m}\) and \(V = 3 \text{ m s}^{-1}\), ranges from 0.1 to 0.4, which is well below the range of \(K\) in their experiments. Furthermore, there are no multicylinder data from J1990 that match these accretion conditions. The combination of extrapolating both density formulae beyond the data used to obtain them, compounded by incorrectly calculating \(\rho_\alpha\), makes this comparison invalid.

Although there were no cases from J1990 that matched values of \(V, d, W,\) and \(T\) with those used by PLNL, there were a number that fitted the constraints of PLNL, namely the cases for 0.5 < \(X\) < 7 and 0.6 < \(K\) < 3.1; \(X\) is Macklin's parameter (\(\mu\text{m m/s °C}\)) and \(K = \rho_w d^2 V / 9 \mu_a D\) is the inertial coefficient or Stokes number (\(\rho_w\) is the density of water and \(\mu_a\) the air dynamic viscosity). This range of \(K\) was calculated from the PLNL data using air temperatures and wind speeds provided by Levi (personal communication) for each of their nine runs, and putting \(d = 20 \mu\text{m}\) and \(D = 1.1\) cm (the mean iced-cylinder diameter). It is slightly different from that given by PLNL, which may be explained by their using the bare cylinder diameter, or perhaps a different value for dynamic viscosity. I should comment here that Levi and Prodi's statement, that for the three smallest cylinders in the J1990 data \(K\) varied between 1.5 and 3, is incorrect. The actual range was 0.4 to about 15 with a median value of 3.0. The median value of \(K\) for the three largest cylinders was 0.64 ranging from 0.11 to 3.

To compare data that are similar in the parameters used to determine density both by J1990 and PLNL, I calculated the range of \(\phi\) where

\[
\phi = 9 \rho_w^2 D V / \rho_a \mu_a
\]

(Langmuir and Blodgett 1946; \(\pi_\alpha\) in J1990) for PLNL's cases, with the requirement that the chosen J1990 cases also satisfy these limits, namely 30 < \(\phi\) < 155. In addition to \(K\) and \(\phi\), a heat-flux parameter

\[
\pi_c = k_a (1-T) / W V L_f D,
\]

where \(k_a\) is the conductivity of air and \(L_f\) is the latent heat of fusion of water, is also used in the density calculation of J1990. However, because of the uncertainty in \(W\) of PLNL, this parameter was not used to limit the cases chosen from J1990. Of the 2665 cases in J1990, 202 of them satisfied the constraints on \(X, K,\) and \(\phi\). The median icing conditions for this \((K, X, \phi)\) subset are given in the third column of Table 1. The \((K, X, \phi)\) median wind speed and air temperature are closer to PLNL's, but the droplet size and liquid-water content are even smaller than those for the entire data-set.

The measured densities are plotted in Fig. 1 as a function of the predicted densities using the
Figure 1. Comparison of measured and predicted rime densities (g cm\(^{-3}\)) from J1990 using Eq. (1). The cases satisfying the constraints on \(X\), \(K\) and \(\phi\) are indicated by triangles (\(\triangle\)).

Figure 2. Comparison of measured and predicted rime densities (g cm\(^{-3}\)) using Eq. (2). PLNL cases are indicated by squares (\(\square\)), and the subset from J1990, satisfying the constraints on \(X\), \(K\), and \(\phi\), by triangles (\(\triangle\)).
J1990 formula:
\[
\rho = 0.249 - 0.0840 \ln \pi_C - 0.00624 (\ln \pi_C)^2 + 0.135 \ln \pi_K
+ 0.0185 \ln \pi_K \ln \pi_\phi - 0.0339 (\ln \pi_K)^2
\]
for the entire data-set (shown as dots) and for the 202 cases in \((K, X, \phi)\) (shown as triangles). In Fig 2 are plotted the measured and predicted densities for the \((K, X, \phi)\) and PLNL cases with the predicted density calculated from PLNL’s formula:
\[
\rho = 0.2 X^{0.5}.
\]
In both figures the 1:1 line, representing perfect agreement between measured and predicted densities, is drawn. Neither looks very good, so a choice between the two formulations based on a better fit to this particular subset of the data is not warranted. The J1990 formula tends to predict higher densities than were measured for the \((K, X, \phi)\) cases, and PLNL’s formula underpredicts the density. The trend shown in Fig. 1 for the entire data-set is for the J1990 formulation to underpredict density for high-density accretions, say those with \(\rho > 0.6 \text{ g cm}^{-3}\), and to overpredict density for low-density accretions. This will be discussed further after dealing with Levi and Prodi’s comments on the heat-flux \(\pi\)-product.

3. Droplet Freezing

The quantity \(\pi_c\) is intended to be a measure of the speed of initial droplet freezing. All else being equal, droplets that freeze faster will tend to spread less before freezing and result in a lower-density accretion. Macklin (1962) argued that, as far as the rime density is concerned, a droplet can be considered to be frozen when an outer shell of ice forms, which in a typical case will occur in less than 0.1 s; although it may take the entire droplet a minute or two to freeze. During this time-period, \(\Delta t\), of about 0.1 s, very little of the accretion surface is covered with droplets in their initial stage of freezing. The number of droplets that accrete per unit length of cylinder is given by the expression
\[
N = 6WVD\Delta t/\pi d^3 \rho_w
\]
covering a total accretion area of \(\pi D\theta/180\) per unit length, where \(E\) is the collection efficiency and \(\theta\) is the maximum accretion angle. The area of the accreted droplets is \(\pi d^2/4\), supposing that they remain spherical, so the ratio of droplet area to the accretion area is \((3WVE\Delta t/2\pi d\rho_w) \times (180/\theta)\). Using the median values for J1990 in Table 1 results in an area ratio of 4%, indicating that accreted droplets are well separated. As a just-accreted droplet begins to freeze, the latent heat of fusion that it releases is more than sufficient to warm the droplet up from the air temperature \(T\) to 0°C. The droplet will then freeze completely when the remaining heat of fusion is dissipated by convective or evaporative cooling; or as Levi and Prodi point out, by conduction to the surrounding ice. During this initial freezing stage the difference between the temperatures of the droplet and the air is \((0 - T)\) deg C, which is why I used \(-T\) in \(\pi_c\) rather than the ice temperature or the difference between the air and ice temperatures as Levi and Prodi suggest. However, rather than using a length scale \(D\), the iced-cylinder diameter, I should have used \(d\), the droplet diameter. While \(D\) is appropriate for heat-flow calculations for the whole accretion, the temperature gradient for individual droplets scales as \(- T/d\). Thus a better choice for \(\pi_c\) is
\[
\pi_c = k_{a}(-T)/dWVL_t.
\]
A \(\pi\)-product describing conduction into the ice could also be significant. If the droplets accrete as rime, the ice at the very surface of the accretion is at or very near the air temperature, rather than at the temperature that PLNL measured underneath the 1 mm of clear ice coating their plastic cylinder. PLNL’s measured temperature, characterizing the bulk of the ice accretion, is appropriate for use in Macklin’s parameter \(X\) but not in the temperature gradient between a newly accreted droplet and its immediate surroundings. Thus \(-T\) is again the characteristic temperature difference and \(d\) is the appropriate length scale in a \(\pi\)-product, \(\pi_1\), describing heat conduction from the droplet into the ice:
\[
\pi_1 = k_i(-T)/dWVL_t
\]
(where \(k_i\) is the thermal conductivity of ice). \(\pi_1\) has the same dependence on the basic icing
variables as $\pi_C$, so its inclusion is redundant. If the ice accretes as glaze, the surface of the accretion is at 0°C which implies that there is no temperature gradient between the freezing droplet and the nearby ice; so heat loss by conduction is not significant. My initial analysis using the modified $\pi_C$ indicates that it correlates better with rime density than the former $\pi_C$. However, the overall fit to the density data of J1990 is not improved.

4. FUTURE WORK

I believe that this non-dimensional parameter formulation is the best approach to developing a comprehensive predictive density formula. The non-dimensional parameters I propose are all based on standard meteorological and icing variables and can be readily calculated. However, the goodness of fit that was obtained by J1990 (80% of the variability in density explained) is, I believe, limited by the form of the relationship that was postulated between density and the non-dimensional parameters. The scatter in the data (Fig. 1) is unavoidable because of the difficulty in measuring the accretion thickness; but the tendency of the formula to underpredict high densities and overpredict low densities should be corrected. A judicious choice of a functional relationship between density and $K$, $\phi$, and the modified $\pi_C$, which are probably the three crucial parameters, is needed. A possible candidate for the density dependence on $K$ and $\phi$ is the function used by Finstad et al. (1988) for collection efficiency, maximum impingement angle, and stagnation-line impact velocity. A product of this type of function of $K$ and $\phi$ and a function of $\pi_C$ may be the best formulation. This work has not yet been done but will be undertaken in the near future.

REFERENCES