Importance of diabatic processes on ageostrophic circulations observed during the FRONTS 87 experiment

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(Received 17 August 1992; revised 31 March 1993)

SUMMARY

The impact of condensation and evaporation on real cross-frontal circulations is studied, using data from the FRONTS 87 field experiment. The Sawyer–Ellissén equation is solved numerically in its primitive equation form, for one intensive observing period of FRONTS 87. Three different schemes of parametrization of condensation heating are used. Comparison between diagnosed and observed ageostrophic circulations shows that latent heat release plays an important role in enhancing the ascending branch of the ageostrophic circulation ahead of the frontal zone. In particular, when a parametrization scheme for the low-level moisture convergence is applied, a cell of strong ascending motion is formed ahead of the frontal zone, in good agreement with the observations. Further, a simple parametrization of evaporative cooling below the frontal discontinuity leads to a reinforcement of downdraughts in the cold sector of the front, as observed in regions where precipitation originating from the strong line convection evaporates. In comparison with their adiabatic counterparts, the current solutions show strong similarities with the observed circulation and indicate the predominance of diabatic effects in the frontal zone.

1. INTRODUCTION

The impact of non-conservative processes on frontogenesis was raised in the very early stages of frontal research. When Sawyer (1956) demonstrated that frontogenesis is necessarily accompanied by a vertical circulation system (i.e. the frontal secondary circulation), he also underlined the insufficiency of the calculated upward motions to account for the observed vertical velocities ahead of a frontal zone. According to him, latent heat release has to be taken into account within the saturated prefrontal atmosphere. Later on, the influence of the condensation of water vapour was studied thoroughly by several authors, using either time-dependent models (Hoskins and Bretherton 1972; Ross and Orlanski 1978; Williams et al. 1981; Mak and Bannon 1984; Thorpe and Emanuel 1985; Knight and Hobbs 1988; Chan and Cho 1991) or diagnostic formulations (Thorpe 1984; Thorpe and Nash 1984). Comparison with adiabatic solutions in these studies revealed that latent heat release strengthens frontogenesis and can account for the intense updraughts observed ahead of a frontal discontinuity. The magnitude and the extent of the unstable conditions ahead of the frontal zone were found to be of major importance in these numerical studies. Consequently, evaluation of the relative importance of water vapour condensation varies, from points of view which consider the contribution of latent heat release to be secondary to the frontal formation (Hoskins and Bretherton 1972) to results that demonstrated that convection-induced circulation can completely overshadow the original frontal circulation (Ross and Orlanski 1978). In a similar manner, Huang and Emanuel (1991) underlined the effect of evaporation of rain on frontal circulations, which was found to be responsible for the appearance of a concentrated sloping downdraught beneath the frontal zone. On the other hand, Clough and Franks (1991) underlined the impact of snow evaporation at the rear of a cold front and claimed that this provides a very sufficient mechanism for the maintenance of saturated descent, above or in the melting layer, in the cold sector of the front.

Lagouvardos et al. 1992 (hereafter referred to as LLS) compared observed frontal

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circulations of FRONTS 87 with solutions obtained by applying the diagnostic Sawyer–Eliassen equation (SE equation hereafter) on real data from two intensive observational periods of FRONTS 87. The discussion was limited to adiabatic forms of the diagnostic equation, whose exact form depends on the validity of the along-front geostrophy assumption. The results showed clearly the insufficiency of adiabatic solutions to account for the observed strong upright motions ahead of the fronts, or for the concentrated downdraughts behind. However, the adiabatic solutions were capable of reproducing the overall structure of the observed ageostrophic circulations around a positive (frontogenetic) forcing, with weak ascending motions in the warm sector and subsiding motions in the cold. In the same paper, friction at the surface was found to be responsible for the generation of vertical velocity cells near the ground, in good agreement with earlier numerical investigations (Keyser and Anthes 1982; Thorpe and Nash 1984; Reeder 1986).

The natural continuation of this work would be to attempt to understand the influence of diabatic processes on frontal circulations. Our present aim is not to do numerical simulation of moist frontogenesis, nor to propose a new parametrization technique of moist processes, but simply to attempt (following the conclusions of LLS) to examine the impact of condensation and evaporation upon observed frontal circulations and to identify the parametrization scheme that can create features that are closer to the observations. After LLS, the high resolution of the upper-air data allows a detailed comparison of the observed frontal circulations with the circulations diagnosed through the resolution of the SE equation.

A brief account of the FRONTS 87 experiment has been given by Lagouvardos et al. (1993); a discussion on two of its scientific objectives relative to frontal circulations are given here in section 2. The formulation of the problem (the Sawyer–Eliassen equation in its primitive equation form) and the parametrization techniques of condensation and evaporation are discussed in section 3, where also a derived version of the SE equation suitable for use with real data is proposed. Section 4 contains the results from the comparison between observations and parametrizations and also a discussion concerning the physical consistency of these parametrizations. Finally, in section 5, conclusions are drawn on the impact of each parametrization scheme on the observed ageostrophic circulations.

2. THE FRONTS 87 FIELD EXPERIMENT

For the purpose of this study we use the data obtained by an enhanced rawinsonde network installed in Brittany, France. The three mobile sounding stations at Brest, Lannion and Lorient were operating during the eight intensive observing periods of the FRONTS 87 experiment. Their relative position was chosen so as to form a nearly equilateral triangle with 80 km sides, permitting an accurate evaluation of cross and along-front gradients within the triangle. Among the theoretical objectives of the scientific program (Thorpe et al. 1987; Thorpe and Clough 1991), our interest has been focused on two main hypotheses:

(1) Frontal motion is balanced on horizontal scales down to about 50 km.
(2) Precipitation and diabatic processes have a substantial influence upon mesoscale circulations at cold fronts.

Concerning the first of these, a detailed study of the frontogenetic forcing and the associated ageostrophic circulations for two case-studies of FRONTS 87 has been given by LLS who resolved the adiabatic Sawyer–Eliassen equation in both forms (under the geostrophic momentum assumption and in the primitive equation form, Keyser and
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Pecnick 1985) using real data from Fronts 87. The main results of this study can be summarized as follows:

(1) The geostrophic momentum approximation (based on the assumption of along-front geostrophy) was able to reproduce the main features of the ageostrophic circulation developed around a well-marked frontal zone (cold front observed during the IOP 7 of Fronts 87).

(2) The primitive equation approach was required for a frontal case characterized by weak temperature gradients and strong winds (IOP 8 of Fronts 87).

(3) For the IOP 7 cold front, surface friction was responsible for the intensification of upright motions ahead of the front at the surface. Friction was apparently of minor effect in the IOP 8 case-study.

(4) For both frontal cases studied, the neglect of diabatic phenomena was apparently responsible for the discrepancy between the observed updraught's intensity ahead of the frontal surface and vertical motions diagnosed by the Sawyer–Eliassen equation.

In this paper our interest is focused on the second hypothesis, which we test by including condensation heating and/or evaporative cooling in the SE equation. Comparison with rawinsonde measurements will be used again in order to validate the SE diagnoses.

3. FORMULATION OF THE PROBLEM: PARAMETRIZATIONS

The Sawyer–Eliassen diagnostic equation in an adiabatic inviscid atmosphere, with the Boussinesq approximation, is written in its primitive equation form as (Keyser and Pecnick 1985)

\[
N^2 \psi_{xx} - S^2 \psi_{xz} + F^2 \psi_{zz} = \frac{2}{\theta_0} \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial x} \right) + \frac{2}{\theta_0} \frac{\partial}{\partial x} \left( \frac{\partial u_a}{\partial x} \right) - \frac{Q_s}{Q_a}
\]

\[
- f \frac{\partial v_k}{\partial y} + 2 \frac{g}{\theta_0} \frac{\partial v_k}{\partial y} - f \frac{D}{Dt} \left( \frac{\partial v_k}{\partial z} \right)
\]


(1)

where \( \psi \) is the ageostrophic circulation stream function, \( g \) the acceleration of gravity, \( f \) the Coriolis parameter, \( \theta_0 \) a reference value for the potential temperature \( \theta \), \( v \) and \( u \) the along-front and the cross-front wind components respectively, and the subscripts 'g' and 'a' refer to the geostrophic and ageostrophic part of the wind. In addition \( Q_s \) and \( Q_a \) denote the geostrophic and the ageostrophic forcing. The ageostrophic circulation (\( u_a \) and \( w \)) is defined through the ageostrophic streamfunction, \( \psi \), by the expressions

\[
w = -\frac{\partial \psi}{\partial x}, \quad u_a = \frac{\partial \psi}{\partial z}.
\]

(2)

The coefficients \( N^2 \), \( F^2 \) and \( S^2 \) on the left-hand side of Eq. 1 have the following meanings:

\[
S^2 = f \left( \frac{\partial v}{\partial z} + \frac{\partial v_k}{\partial z} \right) \quad \text{measures baroclinicity},
\]

\[
F^2 = f \left( f + \frac{\partial v}{\partial x} \right) \quad \text{measures inertial stability},
\]
\[ N^2 = \frac{g}{\theta_0} \frac{\partial \theta}{\partial z} \] measures static stability,

where \( v \) is the total along-front wind component \( (v = v_e + v_a) \). The primitive equation approach (PE hereafter) is not based on the along-front geostrophic balance assumption. Although an along-front ageostrophic wind is allowed, it is non-divergent so that the ageostrophic streamfunction, \( \psi \), can be defined in a way similar to that in the standard form of the Sawyer–Elionassen equation. Therefore, the only constraint retained in this approach is that the secondary ageostrophic circulation be confined to the frontal transverse plane.

Keyser and Pecnick (1985) identified the first two terms of \( Q_s \) (Eq. (1)) as the ageostrophic counterparts of the confluence and shear term of the geostrophic forcing, \( Q_g \). The third term is related to unbalanced effects such as inertia-gravity waves or symmetric overturnings (Orlanski and Ross 1977).

(a) Application of the Sawyer–Elionassen equation to real data

Before proceeding to the numerical solution of the SE equation it would be appropriate to derive a form of the equation suitable for real data. Since available measurements cannot provide accurate values for the geostrophic components of \( u \) and \( v \), the terms involved in Eq. (1) cannot be easily estimated. Our aim is to obtain a form of the Sawyer–Elionassen equation in terms of the total wind components—that is, the observed wind components. Manipulation of the momentum and thermodynamic equations leads to a modified form of the SE equation, viz.

\[
N^2 \psi_{xx} + f^2 \psi_{zz} = - \left( -\frac{g}{\theta_0} \frac{\partial \theta}{\partial x} \frac{\partial u}{\partial x} - \frac{g}{\theta_0} \frac{\partial \theta}{\partial y} \frac{\partial v}{\partial y} - f \frac{\partial u}{\partial x} \frac{\partial v}{\partial z} \right) = - Q_{\text{tot}}
\]

(3)

where wind components without subscripts are the observed values. This derivation was based on the assumption that the evolution with time of the vertical variation of \( v_a \) can be neglected (last term in Eq. (1)). This assumption is invalid within a region of active conditional symmetric instability (CSI). In the present case, this assumption is valid since the region of active CSI is located far away from the surface front where the frontogenetic forcing is concentrated (Lagouvardos et al. 1993).

The modified form of the Sawyer–Elionassen equation (Eq. (3)) has two advantages when dealing with real data, namely, all the remaining terms can be directly evaluated from standard sounding data since all the wind components appearing on both sides of the last equation are the observed ones, and the ellipticity condition of Eq. (3) is now less restrictive. From a mathematical point of view, Eq. (1) is an elliptical differential equation which has a unique solution, provided that the difference \( F^2 N^2 - S^4 \) is positive. Physically, this restriction corresponds to there being positive potential vorticity and implies that a solution exists only in the absence of inertial, convective or symmetric instability. On the other hand, the ellipticity condition of Eq. (3) is less restrictive; the product \( F^2 N^2 \) is necessarily positive, which means that a unique solution can be obtained within a region susceptible to symmetric overturnings.

(b) Resolution of the Sawyer–Elionassen equation

The procedure used in resolving the Sawyer–Elionassen equation numerically is given in detail by LLS; here we give only an outline of the method. The computational domain consists of a rectangular grid constructed from the sounding data covering, horizontally, 29 hours of observations. If we suppose that the mean frontal velocity is 6 m s\(^{-1}\), then this interval corresponds to approximately 620 km. The time-to-space conversion used
here assumes stationarity of the phenomenon. However, the similarity that exists between the cross-front fields, constructed by means of data provided from dropsondes released several hours earlier above the Atlantic Ocean (Thorpe and Clough 1991) and rawinsonde fields, seems to confirm the stationarity of the system. In the vertical, the domain extends up to a height of 5 km.

Thirteen rawinsondings were carried out within this time interval, providing vertical profiles of temperature and humidity every 25 to 30 m, and of wind speed and direction every 150 m. A cubic-spline interpolation scheme was applied horizontally to obtain values on a regular horizontal grid. The two coefficients $N^2$ and $F^2$, and also the total forcing $Q_{\text{tot}}$, were calculated, using centred differences, for every gridpoint $(x, z)$ of the computational domain. Since one of the three mobile sounding stations (at Lorient) did not provide wind measurements during this frontal event, gradients were calculated using data from the other two stations, taking into account the advection of the front to ensure coincidence of timing. LLS pointed out that this kind of calculation might entail making a bad estimation of gradients in the $y$-direction (i.e. parallel to the front). However, tests with data from the IOP 7 case-study, for which all three stations were operating, have shown that the along-front gradients can be evaluated accurately with data from only two stations.

At the gridpoints at which the ellipticity condition is violated (regions of inertial or static instability), the coefficients are replaced by values interpolated from the neighbouring gridpoints; for the adiabatic form of the SE equation the number of these points is very limited. However, as will be seen in the next subsection, incorporation of diabatic processes has a more serious effect on the ellipticity criterion.

The SE equation is then solved numerically, using a scheme of successive over-relaxation. The lateral and vertical boundary conditions for the ageostrophic streamfunction are $\psi = 0$, which implies (see Eq. (2)) that $u_y = 0$ along the lateral boundaries and that $w = 0$ along the lower and upper boundaries. As demonstrated in LLS, the applied boundary conditions for the resolution of the SE equation at the upper boundary do not affect the results at the low levels. In the following sections it will be seen that when diabatic processes are taken into account, the scheme that is applied for the resolution of the SE equation is iterative.

(c) *Parametrization of diabatic processes*

Inside a diabatic atmosphere the equation for the conservation of entropy is written as

$$\frac{D\theta}{Dt} = P$$

where $P$ denotes diabatic heat sources (sinks) related to condensation (evaporation). This additional parameter modifies the adiabatic form of the Sawyer-Eliassen equation which now becomes

$$N^2 \psi_{xx} + F^2 \psi_{zz} = -Q_{\text{tot}} - \frac{g}{\theta_0} P_x.$$  

In the following, we shall give three different parametrization schemes for condensation; a more thorough discussion of these schemes can be found in the paper by Thorpe (1984), together with an application to a diagnostic model of frontogenesis. In what follows, our interest will be focused on their physical realism, their ability to reproduce the observed features and on their part in the violation of the ellipticity criterion. An attempt will also
be made at formulating a crude parametrization of evaporation. The total heating (or cooling) induced by these schemes can then be easily estimated. In section 4 these schemes are applied to the real data, and the diagnosed heating or cooling is compared with the observations, so to allow a test to be carried out, a posteriori, on the realism of the obtained solutions.

(i) *Latent heat on the entire domain.* The simplest parametrization scheme of latent heating corresponds to an idealized situation in which the diabatic term $P$ (Eq. (4)) is proportional to the local vertical velocity. In this case, term $P$ can be expressed as

$$ P = \gamma w $$

(6)

where $\gamma = -(L/c_p)(\partial q_{\text{sat}}/\partial z)$, with $L$ the heat of condensation, $c_p$ the heat capacity of dry air at constant pressure and $q_{\text{sat}}$ the saturated mixing ratio (see also Thorpe 1984). Since $q_{\text{sat}}$ diminishes with height, $\gamma$ is positive and therefore the term $P$ represents a heat source. The diabatic form of the SE equation (Eq. 5) is written in a way similar to its adiabatic counterpart (Eq. (3)), except that the Brunt–Väisälä frequency, $N_w^2$, is now based on the lapse rate of the wet-bulb potential temperature, $\theta_w$, viz.

$$ N_w^2 \psi_{xx} + F^2 \psi_{zz} = -Q_{\text{tot}} $$

(7)

The above formulation accounts not only for condensation heating in the updraughts but also for an evaporation cooling that maintains saturation in the downdraughts. Therefore it is supposed that the entire frontal atmosphere has reached saturation. This supposition is clearly not consistent with frontal observations where the extent of cloud and/or precipitation is limited in the horizontal, especially in the rear part of the frontal system. Note also that the replacement of $N^2$ by $N_w^2$ results in a violation of the ellipticity criterion within some regions of the computational domain. At the end of section 4, a discussion has been included about the regions for which real data violate the ellipticity condition.

(ii) *Latent heat proportional to ascending motions.* According to Sawyer (1956), the diabatic term $P$ of Eq. (4) can be set proportional to the vertical velocity, $w$, for ascending motions only. Thus, we can write

$$ P \begin{cases} \gamma w & (w > 0) \\ 0 & (w < 0) \end{cases} $$

(8)

Therefore, evaporation processes are not taken into consideration and the SE equation now takes the form

$$ N_w^2 \psi_{xx} + F^2 \psi_{zz} = -Q_{\text{tot}} $$

(9)

where

$$ N_w^2 \begin{cases} N_w^2 & (w > 0) \\ N^2 & (w < 0) \end{cases} $$

(10)

One advantage of this scheme is that the regions in which the ellipticity criterion is violated are substantially reduced. In Sawyer's paper it was clearly indicated that incorporation of this parametrization enhanced ascending motions ahead of a frontal surface (see his Fig. 7(b)). Thorpe (1984), using a diagnostic model of surface frontogenesis, also demonstrated that diabatic heating which was proportional to the local vertical velocity on ascent resulted in a more upright and intense updraught, compared with the adiabatic solution.

(iii) *CIK-type parametrization of condensation.* The aforementioned parametrization schemes relate the latent heat release with the ascending motions resulting from the
weak inclined ageostrophic circulation that develops around the frontal discontinuity. This circulation undoubtedly results in heating ahead of the frontal zone, but observations of frontal events clearly show that strong forced frontal convection occurs mainly within a narrow cold-frontal rainband (Browning and Pardoe 1973; Browning 1985; Lemaître et al. 1989; Roux et al. 1993). Consequently, latent heat release is generated by ascending motions that are steeper than the frontal slope and stronger than the vertical component of the ageostrophic circulation. As Emanuel (1982) pointed out for mid-latitude squall lines, and Thorpe and Nash (1984) for cold fronts, a CISK-type parametrization connecting mesoscale low-level convergence with moist upright convection could be a more realistic mechanism on the mesoscale. This parametrization of diabatic processes differs from the two aforementioned schemes; in particular this difference depends on a scale separation between intense convective elements and the frontal horizontal scale. It can also be seen as an attempt to parametrize the heating effect of the narrow cold-frontal rainband. Although this parametrization scheme is often referred to as a CISK-type approach, it would be more consistent with mesoscale dynamics if it were referred to hereafter as ‘convective parametrization’—a term first used by Thorpe (1984).

Emanuel (1982) suggested that the CISK description of cumulus convection can also be applied to the study of baroclinic flow and that the CISK mathematical formulation can be extrapolated to mid-latitude convective systems. According to Emanuel, the intensity of convection depends on the degree of conditional instability and the rate at which the mesoscale flow supplies the convective element with moisture. This rate can be set proportional to the vertical velocity at the top of a low-level convergence layer, and the heating \( P \) can be written as

\[
P = \frac{\theta_0}{g} N^2 G(z) w_{z=z_c} \tag{11}
\]

where \( G(z) \) is a function giving the vertical distribution of heating and \( z_c \) is the height at the top of the convergence layer. This form of diabatic parametrization can also account for cooling (or rather, a ‘negative heating’) in the regions where the vertical velocity, \( w \), is directed downwards at \( z = z_c \); that is at the rear of the frontal zone. When only warming is considered, Eq. (11) is written as (Thorpe and Nash 1984)

\[
P = \frac{1}{2} \frac{\theta_0}{g} N^2 G(z) (w + |w|)_{z=z_c} \tag{12}
\]

The corresponding Sawyer–Eliassen equation then takes the form:

\[
N^2 \psi_{xx} + F^2 \psi_{zz} = -Q_{\text{tot}} \frac{\frac{\partial w}{\partial x} \bigg|_{z=z_c}}{Q_{\text{diab}}} \tag{13}
\]

where \( Q_{\text{diab}} \) represents the diabatic forcing due to low-level moisture convergence.

The next level of realism could be a scheme combining the isolated heat source described by Eq. (12) and Sawyer’s parametrization scheme. Since vertical motion in frontal environments can result from ageostrophic circulation and from low-level convergence, it would be physically more accurate to consider heating provided by both mechanisms. Thorpe and Nashes’ results did not consider this synergy which is expected to enhance the direct ageostrophic cell (although it was suggested in their conclusion). In section 4, this approach will be applied to real data.

(iv) Evaporation. In Thorpe and Nashes’ (1984) paper an attempt was made to par-
ametrize evaporating cooling. It was based on the same formulation as was used in the moisture convergence approach, but the function $G(z)$ in Eq. (12) was defined so as to produce a cooling effect (term $P$ is negative). They did not comment on the influence on the downdraughts, but they did mention that the maximum of the ascending motions is decreased as a result of enhanced low-level divergence due to evaporative cooling. Section 4 contains an application to FRONTS 87 real data.

On the other hand, Huang and Emanuel (1991) introduced rain evaporating processes in their time-dependent semi-geostrophic model of frontogenesis. In their case, the cooling $P$ was set proportional to the rate of evaporation of rain, based on the supposition that all the condensed water evaporates. Incorporation of this scheme yielded an increased downdraught beneath the zone of concentrated ascent whose maximum was located at a height slightly lower than the updraught's maximum. As will be seen in section 4, this feature is confirmed by the observations. For our purposes a diagnostic formulation for evaporation will be used in which the parametrization constants are fixed by the observations.

4. Diagnostics of moist circulations

In this paper, our interest is focused on the last frontal event of FRONTS 87, hereafter referred to as the IOP 8 case. A detailed description of the thermodynamic and dynamic structure of this cold front was given by Lagouvardos et al. (1992, 1993). However, a brief summary of its main characteristics is also given here.

Four parametrization schemes (three for condensation and one for evaporation) are applied to the IOP 8 real data. Table 1 gives a summary of the experiments, together with the form of the corresponding Sawyer–Eliasen equation. Since all four experiments involve the vertical velocity, the $w$-field obtained from the adiabatic form of the SE equation is used as a first guess.

(a) Brief description of the front

The cold front passed over Brest at 2300 GMT, on 12 January 1988. The frontal speed was estimated as to be about 6 m s$^{-1}$, coming from the 300° sector. Heavy rain (maximum 12 mm h$^{-1}$) accompanied the frontal passage. The radar PPI imagery (Fig. 1) reveals evidence of precipitation activity related to the front, especially the strong echoes coming from the narrow cold-frontal rainband. Maximum values in the distinct reflectivity cores were over 40 dBZ. This line convection was among the strongest features observed during FRONTS 87. Note also the sharp edge of the cloudiness in the rear of the cold front, presumably due to evaporative downdraughts, evidenced in the vertical velocity pattern shown in Fig. 4, which led to a strong decrease of precipitating particles.

The time–height cross-section of the along-front wind component, shown in Fig. 2(a), is constructed from thirteen rawinsondes released from Brest airport in a time interval of 29 hours (about 620 km if a time-to-space conversion is made). The along-front wind pattern reveals the presence of strong cross-front gradients along the frontal discontinuity. A low-level jet with a horizontal extent of about 100 km and a maximum of 32 m s$^{-1}$ is evident in the warm sector of the front at a height of 1100 m above the surface. This field compares well with the dropsonde pattern presented by Thorpe and Clough (see their Fig. 4 where cross-sections of about 800 km length are shown). Figure 2(b) presents the wet-bulb potential temperature field, showing a sharp contrast within the frontal zone. Note also, the vertical orientation of the $\theta_w$-surfaces in the vicinity of the frontal discontinuity at heights below 2 km, which is indicative of the adjustment to neutral convective conditions within the narrow cold-frontal rainband. In contrast, the
<table>
<thead>
<tr>
<th>Experiment</th>
<th>Heat sources or sinks</th>
<th>Sawyer–Eliassen equation</th>
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<tbody>
<tr>
<td>Expt. M1: Saturated ascent and</td>
<td>$P = \gamma w$ over entire domain</td>
<td>$N^2_\psi \psi_{xx} + F^2 \psi_{zz} = -Q_{ext}$</td>
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<td>descent</td>
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<td>Expt. M2: saturated ascent only</td>
<td>$P = \gamma w$ ($w &gt; 0$)</td>
<td>$N^2_\psi \psi_{xx} + F^2 \psi_{zz} = -Q_{tot}$ ($w &gt; 0$)</td>
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<td></td>
<td>$P = 0$ ($w &lt; 0$)</td>
<td>$N^2_\psi \psi_{xx} + F^2 \psi_{zz} = -Q_{tot}$ ($w &lt; 0$)</td>
</tr>
<tr>
<td>Expt. M3: Convective parameterization plus</td>
<td>$P = \frac{1}{2} \theta_0 N^2 G(z) (w +</td>
<td>w</td>
</tr>
<tr>
<td>Expt. M2</td>
<td>where $G(z)$ is a function giving maximum heating ahead of the frontal surface at $z = 1400$--$1600$ m.</td>
<td>$N^2_\psi \psi_{xx} + F^2 \psi_{zz} = -Q_{tot} - N^2 G(z) \left( \frac{\partial w}{\partial x} \right)_{z=z_0}$ ($w &lt; 0$)</td>
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<tr>
<td>Expt. M4: Evaporation plus Expt. M3</td>
<td>$P = \frac{\theta_0}{g} N^2 (G(z) + G'(z))w_{z=z_0} + \left[ \begin{array}{c} \gamma w \ 0 \end{array} \right] (w &gt; 0)$</td>
<td>$N^2_\psi \psi_{xx} + F^2 \psi_{zz} = -Q_{tot} - N^2 (G(z) + G'(z)) \left( \frac{\partial w}{\partial x} \right)_{z=z_0}$ ($w &gt; 0$)</td>
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<tr>
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<td>where $G'(z)$ is a function giving maximum cooling below the frontal surface at $z = 1200$ m.</td>
<td>$N^2_\psi \psi_{xx} + F^2 \psi_{zz} = -Q_{tot} - N^2 (G(z) + G'(z)) \left( \frac{\partial w}{\partial x} \right)_{z=z_0}$ ($w &lt; 0$)</td>
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potential temperature pattern at the same location (not shown) was characterized by very weak cross-frontal thermal gradients (see Fig. 11 of LLS).

A first view of the frontal circulation can be obtained from the cross-front and the vertical velocity cross-sections. Figure 3 presents the cross-front wind component, in a reference frame moving with the propagation speed of the front (6 m s⁻¹). The overall shape of the cross-front wind reveals a circulation that is not consistent with the concept of a large-scale direct ageostrophic circulation. It is however consistent with the conclusions of LLS where the direct secondary circulation was found to be confined to the low levels. A strong flow towards the front is evident in the prefrontal boundary layer, with a maximum exceeding 16 m s⁻¹, just ahead of the frontal surface. On the other hand, there is a return flow within the warm sector (shaded areas). As will be shown later, this return flow is related to an indirect ageostrophic cell. The strong flow towards the front in the prefrontal boundary layer and the return flow just above it have also been observed by means of dropsondes (see Fig. 10 of Thorpe and Clough 1991).

Since the vertical velocity is not directly measured by the soundings, a vertical integration scheme of the continuity equation was used, putting $w = 0$ at the ground and at $z = 10$ km, to avoid accumulation of errors in the divergence of the measured horizontal winds at each height level (O'Brien 1970). The resulting $w$-field is shown in Fig. 4. Strong ascending motions are found ahead of the frontal surface, with a maximum of 30 cm s⁻¹
Figure 2. (a) Time–height cross-section, constructed from thirteen soundings released from Brest airport, of along-front wind component $v$ (at $3\text{ m s}^{-1}$ intervals). The hours of the timescale are referred to the time of frontal passage over Brest (2330 UT). The arrows at the top of the cross-section indicate the times at which the rawinsondes were released from Brest. (b) As in Fig. 2(a), but for the wet-bulb potential temperature, $\theta_w$, (at $0.5\text{ K}$ intervals).
Figure 3. Vertical cross-section of the cross-front wind component, $u$, (at 2 m s$^{-1}$ intervals), in the frame of reference moving with the front. Shading denotes forward airflow towards the warm air.

Figure 4. Vertical cross-section of the vertical velocity (at 5 cm s$^{-1}$ intervals), obtained through vertical integration of the continuity equation. Negative values (subsiding motions) are shaded; the maximum positive value is observed just ahead of the cold front (approx. 30 cm s$^{-1}$).
at a height of 1400 m, related to the strong updraughts within the narrow cold-frontal rainband. Subsiding motions are evident below the frontal surface, with a maximum of 10 cm s\(^{-1}\) at height 1200 m. This pattern of concentrated updraughts and downdraughts is similar to Huang and Emanuel's (1991) numerical modelled feature, (see their Fig. 4(c)). It is important to note also the relatively weak downdraughts found in the warm sector of the front (between 7 and 10 hours prior to the frontal passage). These subsiding motions are related to the same indirect cell that generates the return flow noted earlier in the cross-front wind vertical section (Fig. 3).

(b) Adiabatic solutions

To facilitate comparison with the adiabatic solutions, the results by LLS concerning the IOP 8 frontal event, using the adiabatic form of the Sawyer–Eliassen equation with the primitive equation approach, are shown in Fig. 5. The total frontogenetic forcing \((Q_{\text{tot}}, \text{Eq. (3)})\) is shown in Fig. 5(a). Frontogenetic (positive) values mark the frontal position from the surface up to a height of 2 km, with the maximum located in the low levels. Consequently, the direct cell of the ageostrophic streamfunction, \(\psi\), is confined to the layer below a height of 1500 m, with a maximum of 2500 m\(^2\) s\(^{-1}\) (Fig. 5(b)). A weaker but well-developed indirect cell is evident at mid-tropospheric levels. Letter B denotes the part of this indirect cell which seems to be on the origin of the return flow depicted in the cross-front wind pattern (Fig. 3). The overall shape of the corresponding vertical velocity pattern (Fig. 5(c)) is in agreement with the observations (Fig. 4), with upward motions ahead of the front and downward motions below, but is clearly not consistent with the strong updraughts and downdraughts observed in the warm and cold sectors of the frontal discontinuity, respectively. As was suggested by LLS, the neglect of diabatic processes could be responsible for the discrepancy between the circulation diagnosed from the Sawyer–Eliassen equation and the observations.

(c) Solutions within a saturated domain (M1 experiment)

In the first place, convective processes are considered to occur in an entirely saturated atmosphere. As was pointed out in section 3(c(i)), the Sawyer–Eliassen equation takes the same form as its adiabatic counterpart, except that the Brunt–Väisälä frequency is now a function of the wet-bulb potential temperature, \(\theta_w\), (Eq. (7)). The resulting ageostrophic circulation (Fig. 6(a)) is now intensified and more concentrated compared with the adiabatic solutions, especially in the low levels, where a maximum of 4000 m\(^2\) s\(^{-1}\) can be seen. In addition, the slope of the direct cell is steeper than the slope of its adiabatic counterpart. The associated vertical velocity is shown in Fig. 6(b). Condensation heating ahead of the front resulted in an enhanced \(w\)-cell near the ground, with a maximum of 16 cm s\(^{-1}\). However, this maximum is clearly mislocated with respect to the maximum estimated from the rawinsondings (Fig. 4), which is found at a height of 1400 m. On the other hand, subsiding motions behind the frontal surface are also enhanced. The heating induced by this parametrization scheme (not shown) is about 1 K h\(^{-1}\), in the low levels; a cooling of about 0.5 K h\(^{-1}\) is present below the front. As the heating is set proportional to the local vertical velocity, the heating inside the narrow cold-frontal rainband is not represented. Therefore, the corresponding strong updraughts can not be reproduced by the current form of the SE equation. Another undesirable effect of this particular parametrization scheme is that evaporative cooling is allowed in the whole post-frontal atmosphere, even in regions that are free of cloud and/or precipitation.
Figure 5. (a) Time–height cross-section of the total frontogenetic forcing (at $2 \times 10^{-10} \text{s}^{-1}$ intervals), corresponding to the adiabatic solution of the SE equation. Shading denotes positive (frontogenetic) values. (b) As for (a), but for the ageostrophic streamfunction, $\psi$, (at $10^9 \text{m}^2\text{s}^{-1}$ intervals). Arrows indicate the circulation orientation. Letter B denotes an indirect ageostrophic cell located above the low-level jet. (c) As for (a), but for the vertical component of the ageostrophic circulation (at $2 \text{cm} \text{s}^{-1}$ intervals). Negative values (subsiding motions) are shaded. The maximum value in the low levels is about $5 \text{cm} \text{s}^{-1}$. 
(d) Heating proportional to ascending motions (M2 experiment)

When heating occurs only on ascent, the solution is quite different. The secondary circulation (Fig. 7(a)) is less intense than in Expt. M1, but more intense than that of the adiabatic solution. Furthermore, the ascending branch of the direct cell is less steep and less concentrated than before (Fig. 6(a)). The corresponding vertical velocity is shown in Fig. 7(b). The low-level maximum ahead of the front is about 7 cm s\(^{-1}\), instead of 5 cm s\(^{-1}\) in the adiabatic solution. In the cold sector of the front the downward motions are of the same order as in the adiabatic case since, now, evaporative cooling is not taken into account. It is also interesting to note how evaporation and the increased downward motions (M1 experiment) can account, through the mass conservation, for the strengthening of ascending motions (16 cm s\(^{-1}\) in Expt. M1 versus 7 cm s\(^{-1}\) in Expt. M2, where evaporation is turned off). It is therefore clear that the strong subsiding motions enhance convergence around the frontal discontinuity and help to reinforce the narrow cold-frontal rainband.

The corresponding rate of heating (not shown) is about 1.2 K h\(^{-1}\) and, as for the M1 experiment, its maximum is located below a height of 500 m. These results support the reasoning developed in section 3(c)(iii), concerning the origin of the heat sources. Since the vertical velocities associated with the secondary circulation are weak (in this particular case they are also very shallow), parametrization schemes based on a form proportional to these vertical velocities can not account for the strong updraughts observed ahead of the front. Therefore, a scheme combining low-level moisture convergence and latent heat relaxation is crucial, at least for this particular case-study.

(e) Convective parametrization (M3 experiment)

Thorpe and Nash (1984) and Thorpe (1984) included such a parametrization scheme in their diagnostic models of frontogenesis. For their purpose they used the following
Figure 6. (a) Time–height cross-section of the ageostrophic streamfunction, $\psi$, (at $10^3$ m$^3$s$^{-1}$ intervals) for Expt. M1. Arrows indicate the circulation orientation. (b) As for (a), except for the vertical component of the ageostrophic circulation (at 2 cm$^{-1}$ intervals). Negative values (subsiding motions) are shaded.
Figure 7. As for Fig. 6 but for the Expt. M2 results. The maximum value in the low levels is about 7 cm s\(^{-1}\).
form for function $G(z)$, which governs the vertical distribution of heating:

$$G(z) = A_0 \sin \left( \frac{\pi(z - z_0)}{z_1 - z_0} \right) \quad (z_0 < z < z_1) \quad (14)$$

where $A_0$ is a constant that controls the imposed heating, and the height levels $z_0$ and $z_1$ delimit the layer into which the heating is imposed. Consequently, level $z_1$ represents the depth of convection typical of a cold front. Outside this layer, $G(z)$ is set to zero. The effect of this scheme on the model’s output can be summarized in the following way, namely that strong updraughts were obtained ahead of the frontal surface, and that a disruption of the ageostrophic circulation occurred on the warm sector of the front, where descending motions appeared.

The same scheme was applied to the IOP 8 data, except that the moist Brunt–Väisälä frequency was used on ascent. This additional constraint permits us to parametrize the effect of latent heat release by the weaker ageostrophic circulation. On the basis of the observations, the height where strong low-level convergence occurred (see Fig. 3) was taken to be $z = 700$ m. Likewise, from the Doppler radar (Roux et al. 1993) and sounding data, level $z_1$ was chosen to be at $2300$ m. This value is in good agreement with the typical depth of line convection generally observed in mid latitudes (see Table I of James and Browning 1979). Consequently, the maximum of the heating occurs in the 1400–1600 m layer, and the maximum of the diabatic forcing $Q_{\text{diab}}$ is added at this level (Eq. (13)). The constant $A_0$ was taken equal to 2.5; the physical consistency of all trials concerning $A_0$ was checked a posteriori by calculating the induced total heating and comparing it with radar estimations. In section 4(g) there is a further discussion about the choice of $A_0$.

As in the case of the experiments discussed above, the vertical velocity corresponding to the adiabatic solution was used as a first guess (see Eq. (13)). The ageostrophic streamfunction obtained in this way is shown in Fig. 8(a). The direct cell has about the same size and magnitude as in the two other experiments, but is steeper than before, especially at the height where the maximum heating occurs (i.e. approx. 1100–1300 m). In addition, the indirect cell B is intensified. As Thorpe and Nash (1984) pointed out, this parametrization scheme splits the ageostrophic circulation into two parts, thus creating a dipole. As anticipated, the vertical velocity is strongly enhanced at the transition point between these two opposite cells, reaching a maximum of 22 cm s$^{-1}$ (Fig. 8(b)). The subsiding motions on the warm sector are also enhanced, with a maximum of 4 cm s$^{-1}$ at a height of 1500 m, in good agreement with the vertical-velocity values estimated from the soundings (Fig. 4). Below the frontal discontinuity, subsiding motions are comparable with the adiabatic solutions. Since evaporation is switched off, the maximum of 10 cm s$^{-1}$ depicted at a height of 1200 m (Fig. 4) is not reproduced.

This circulation compares well with the results of Thorpe and Nash (1984) mentioned earlier. However, the origin of cell B cannot be attributed only to the moisture convergence scheme. As a slight return flow is evident also in the adiabatic solution (letter B, Fig. 5(b)), it is likely to be due to an independent circulation that develops around the low-level jet axis (Fig. 2(a)). Indeed, Mak (1972) showed that the inertial acceleration can generate a clockwise circulation around a low-level jet maximum. Such a circulation would create a reinforcement of flow towards the front in the boundary layer and a return flow above the low-level jet maximum. Another possible explanation could be the direct circulation associated with the warm baroclinic zone observed in the $\theta_w$ pattern about 6 hours before the frontal passage (Fig. 2(b)). However, the circulation around the jet’s maximum has also been observed by Thorpe and Clough (1991), although such a warm baroclinic zone was not present (see their Fig. 4). Thus, the return flow seems to be
Figure 8. As for Fig. 6 but for Expt. M3 results. The maximum value of about 22 cm s$^{-1}$ is found at a height of about 1300 m.
generated initially by the circulation around the jet and then intensified by the vigorous upright convection.

(f) Evaporation (M4 experiment)

The vertical distribution of evaporative cooling in the cold air underneath the frontal surface can be represented in a form similar to Eq. (14), where now the coefficient $A_0$ is negative. Here again observations helped to localize the cooling in the layer between 300 and 2100 m within a narrow region just below the frontal surface. As was pointed out in section 4(a), in the discussion about the reflectivity pattern (Fig. 1), evaporation was very active, resulting in a sharp cut-off of the cloudiness behind the front. This experiment is run using the moisture convergence scheme for condensation, thus permitting an overall estimation of the diabatic processes, especially in the vicinity of the frontal surface.

The forcing distribution corresponding to the M4 experiment is shown in Fig. 9(a). Comparison with its adiabatic counterpart (Fig. 5(a)) reveals the importance of diabatic forcing ahead of the front at a height of about 1.2 km. The corresponding vertical velocity is shown in Fig. 9(b). As anticipated, the negative values behind the front are enhanced, the isoline of $-4$ cm s$^{-1}$ extends up to 2 km, giving a maximum of about $-5$ cm s$^{-1}$, at a height of about 1200 m. However, the corresponding values from the soundings were estimated to be approximately $-10$ cm s$^{-1}$. This difference could be attributed to an underestimation of the cooling, especially of that due to the neglect of snow evaporation (Clough and Franks 1991). Indeed, microphysical retrieval carried out by Marécal et al. (1993) for the IOP 8 cold front, show that snow evaporation is active below the frontal zone. Nevertheless, this parametrization scheme proves that it is capable of enhancing the downdraughts and for reproducing a vertical velocity pattern closer to reality.

(g) Estimation of heat sources and sinks

By way of providing a better illustration of the physical consistency of the M3 and M4 experiments, Fig. 10 shows the corresponding heat sources and sinks. For the given $A_0$ values (2.5 for heating and $-1$ for cooling), the heat source reaches its maximum value of 5.6 K h$^{-1}$ at a height of about 1400 m, ahead of the frontal discontinuity, while the cooling pattern depicts two maxima, one of $-0.6$ K h$^{-1}$ at height about 1300 m and a weaker one near the surface ($-0.3$ K h$^{-1}$). These values are consistent with those retrieved from COPLANE radar observations within the narrow cold-frontal rainband of the IOP 8 cold front (Roux et al. 1993) which revealed updraughts of 2–3 m s$^{-1}$ over a horizontal extension of about 3 km, and a heating maximum of 50 K h$^{-1}$. For a horizontal extension of about 40 km, these values correspond to means of 20–30 cm s$^{-1}$ for the vertical velocity and an average heating of about 5 K h$^{-1}$, in good agreement with ours (22 cm s$^{-1}$, 5.6 K h$^{-1}$). Below the frontal zone, Roux et al. (1993) reported finding a maximum cooling of $-5$ K h$^{-1}$.

It would be interesting at this point to conduct a discussion about the violation of the ellipticity criterion when real data are used to solve the SE equation. As already mentioned in section 3(a), incorporation of diabatic effects violates ellipticity in some regions. Figure 10 gives a graphical representation of the regions where the ellipticity criterion was violated for the M4 experiment. As previously explained, within these regions $N^2_0$ is replaced by interpolated values of the neighbouring gridpoints. These regions appear relatively concentrated and not developed enough to seriously affect the solutions. However, this procedure leads to a stabilization of the prefrontal boundary layer and thus to a decrease of upward motions deduced from the resolution of the SE
Figure 9. (a) Time–height cross-section of the total frontogenetic forcing (at $2 \times 10^{-10} \text{s}^{-2}$ intervals), for the M4 experiment. Shading denotes positive (frontogenetic) values. (b) As for (a), but for the vertical component of the ageostrophic circulation (at $-2 \text{ cm s}^{-1}$ intervals). The contour of $4 \text{ cm s}^{-1}$ below the frontal zone is shown with a heavy line. The maximum value of subsiding motions in the cold air is $-7 \text{ cm s}^{-1}$.
equation. Thus, the derived vertical velocity pattern (Fig. 9(b)) underestimates the real vertical velocity ahead of the cold front.

5. CONCLUSION

The impact of diabatic processes on the ageostrophic circulation has been studied, using real upper-air data obtained during the FRONTS 87 field experiment. A modified form of the diagnostic Sawyer–Eliassen equation in its primitive equation form, was applied to the data obtained during the last frontal event of the experiment. The importance of latent heat release in the observed frontal circulation was assessed by comparing the results with the adiabatic solutions (after LLS) and with the observations. Friction was not taken into account here since it is not significant in the present case and since incorporation of an Ekman pumping parametrization scheme seems inappropriate under highly ageostrophic conditions in the along-front direction.

To examine the sensitivity of the ageostrophic circulation, diagnosed by the SE equation, to condensation and/or evaporation, four different experiments were planned. In experiments M1 and M2 the diabatic term $P$ was taken proportional to the local vertical velocities. In M1 the diabatic effect of saturated descent is also included. The results, compared to the adiabatic solutions, show enhanced updraughts and downdraughts, but they clearly fail to represent the details of the observed vertical motions near the surface front, especially the distribution of vertical velocity inside the line convection ahead of the front.

Since this particular frontal event was characterized by a very active narrow cold-frontal rainband, most of the condensation heating is expected to be contained within a narrow zone ahead of the frontal surface, with its maximum centred in the layer between
1200 and 1500 m. As Emanuel (1982) and Thorpe and Nash (1984) pointed out, a parametrization scheme representing the low-level moisture convergence which leads to the narrow frontal rainband formation seems to be more appropriate, at least for the present case-study. Indeed, this scheme strengthens the ascending branch of the direct ageostrophic circulation, where values comparable with the observed ones were obtained. In agreement with earlier numerical simulations, the combined effect of this direct cell and of the indirect cell that appears in the warm sector provides a much more realistic vertical velocity field in terms of magnitude and distribution. In a similar manner, a parametrization of evaporative cooling behind the frontal surface enhanced subsiding motions but, compared with the observed pattern, the retrieved downdraughts were not so strong.

These parametrization schemes provided results that fit well with the observations and gave evidence of the importance of diabatic processes in the adiabatic solutions. However, certain observed features which may influence the distribution of the vertical velocity field were not taken into account. In this respect, the presence of active conditional symmetric instability (CSI) at the leading edge of the frontal zone aloft and the impact of moisture flux and latent heat release in the roll-like circulations that normally result from CSI also need to be studied. An observational approach to the problem of the CSI effects presents certain difficulties, as measurements include these zones that are unstable with respect to CSI. For this reason, numerical modelling of a conceptual frontal zone including, or not, CSI regions is particularly demanding.

ACKNOWLEDGEMENTS

The authors are grateful to Lily Ioanidou and Vassiliki Kotroni for many valuable comments in early versions of the manuscript. The FRONTS 87 experiment was the result of a cooperative work involving several French and British laboratories, in particular the Centre National des Recherches Météorologiques (CNRM/Météo France) which provided the radiosounding data used in this study. In addition to the contributions of the participating institutes, the main financial support was provided by the Institut National des Sciences de l'Univers (INSU).

APPENDIX

List of symbols

- $A_0$: parameter controlling the maximum intensity of heating or cooling
- $a$: subscript denoting 'ageostrophic'
- CSI: abbreviation for 'conditional symmetric instability'
- $c_p$: specific heat capacity of dry air at constant pressure
- $F^2$: squared frequency proportion to the $x$-gradient of the along-front wind component
- $f$: Coriolis parameter
- $G(z)$, $G'(z)$: functions giving the vertical distribution of heating or cooling
- $g$: gravitational acceleration
- $g$: subscript denoting 'geostrophic'
- $L$: heat of condensation
- $N^2$: squared Brunt–Väisälä frequency
- $N_w^2$: squared wet-bulb Brunt–Väisälä frequency
- $P$: heat sources or sinks
\( p \) air pressure
\( \text{PE} \) abbreviation for ‘primitive equation’
\( Q_a \) ageostrophic frontogenetic forcing
\( Q_g \) geostrophic frontogenetic forcing
\( Q_{\text{tot}} \) total adiabatic frontogenetic forcing \( (Q_{\text{tot}} = Q_g + Q_a) \)
\( Q_{\text{diab}} \) diabatic frontogenetic forcing
\( q_{\text{sat}} \) saturation mixing ratio
\( \text{SE} \) abbreviation for Sawyer–Eliassen
\( S^2, S^{\prime 2} \) squared frequencies related to the vertical shear of the along-front wind component
\( u \) total cross-front wind component \( (u = u_g + u_a) \)
\( v \) total along-front wind component \( (v = v_g + v_a) \)
\( w \) vertical velocity
\( x \) cross-front horizontal coordinate
\( y \) along-front horizontal coordinate
\( z \) vertical coordinate increasing upwards; physical height
\( z_c \) top of the low-level convergence layer
\( z_0 \) bottom of the layer where heating or cooling occurs
\( z_1 \) top of the layer where heating or cooling occurs
\( \gamma \) coefficient related to the vertical shear of the saturation mixing ratio
\( \theta \) potential temperature
\( \theta_0 \) reference potential temperature
\( \theta_w \) wet-bulb potential temperature
\( \psi \) ageostrophic streamfunction

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