Studies of the radiative properties of ice and mixed-phase clouds

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SUMMARY
Radiative parametrizations for both ice and water clouds are developed in terms of liquid/ice water content, based on Mie scattering theory. For ice crystals the application of Mie theory is guided by the hexagonal-crystal/equivalent-spheres comparison of Takano and Liou. These parametrizations are extensively tested against measurements from aircraft and are shown to perform satisfactorily, although corrections for unobserved small crystals and the effect of crystal shape are large and not currently well defined. The parametrizations are then used to investigate the effect of mixed-phase clouds on radiative transfer. It is found that, because the radiative properties of ice crystals and liquid droplets are significantly different, the radiative properties of mixed-phase clouds cannot be simulated successfully if the ice in clouds is converted into liquid water. Both the albedo and the rate of change of albedo with ice fraction are significantly dependent on the method by which the phases are mixed; these factors may be of especial importance in climate-sensitivity experiments that incorporate mixed-phase clouds. The presence of ice in clouds below the cirrus level is often ignored in climate-model and radiation-budget studies. The calculations presented here indicate that this neglect may lead to a serious bias in cloud albedo for a given path of condensed water.

1. INTRODUCTION

Many studies have confirmed that clouds play a fundamental role in the earth's radiation budget and climate change (e.g. Ramanathan 1987; Ramanathan et al. 1989; Mitchell et al. 1989; etc.). Clouds influence both solar and terrestrial radiation by absorption, scattering and emission. The intensity of these processes depends on cloud thickness, cover, altitude, geometry and microphysical properties.

Studies of the interaction of clouds and radiation often assume that low- and middle-level clouds are in the liquid phase and that high-level clouds are in the ice phase. In reality, however, lower-level clouds are often glaciated or are of mixed phase. Matveev (1984) presented results from a large amount of data on the phase of clouds collected from aircraft soundings throughout the former Soviet Union between 1957 and 1964. Although few details are given on the measurement techniques or the actual liquid or ice proportions observed, this data set (which consists of many thousands of observations) indicates that over a temperature range of −8 to −26°C, more than 30% of the clouds sampled were of mixed phase. Even amongst low stratus clouds, more than 20% were of mixed phase over this range. Using lidar and aircraft observation, Platt (1977) and Heymsfield et al. (1991) reported evidence of mixed phase in middle-level clouds. Platt found that the liquid droplets and ice crystals in altostratus clouds were in stratified layers. Heymsfield et al., however, found significant amounts of ice crystals embedded in the liquid layer in supercooled altostratus. Cirrus clouds may also not always be completely in the ice phase. Several investigators have reported extremely supercooled liquid droplets in high cirrus cloud layers with temperature as low as −40 to −50°C (e.g. Sassen et al. 1985; Heymsfield et al. 1990; Imasu and Iwasaka 1992; Sassen 1992).

An illustration of the importance of the cloud phase below the cirrus level can be found in the study of Rango and Hobbs (1991) who investigated small polar maritime clouds with cloud tops generally below 4 km. These clouds were often found to be completely glaciated and the phase of the cloud was related more to the age of the cloud.

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than its temperature. In addition, the drop size distribution was found to be a more useful determinant of ice concentration than the cloud-top temperature (see Mason (1971) for a review of earlier measurements of the dependence of freezing on drop size). On some occasions clouds with temperatures as low as \(-24.5^\circ\text{C}\) were found to be virtually unglaciated, probably on account of small droplet sizes.

Rango and Hobbs's (1991) study, which shows different states of glaciation among an ensemble of clouds, points to a second definition of 'mixed-phase clouds'. It is conventionally taken to be a single cloud with liquid and ice co-existing. However, on the scale of general-circulation-model grids and the spatial resolution of Earth Radiation Budget observations, clouds within that scale, which might be subject to the same macroscale meteorological fields, may exist in a mixture of phases. Thus while individual clouds may not be 'mixed phase', the cloud field as a whole may be. Therefore, we widen our definition of mixed phase to include an ensemble of clouds with a mixture of phases, as well as genuine mixed-phase clouds.

A final illustration of the potential importance of mixed-phase clouds can be taken from the modelling study of Mitchell et al. (1989). They employed a cloud prediction scheme of Smith (1990) in which the presence of mixed-phase clouds was modelled on the basis of simple, rather speculative, functions of cloud-phase probability on cloud temperature. Mitchell et al. (1989) found the sensitivity of their doubled-\(\text{CO}_2\) climate to be much reduced using the Smith scheme than when using a relative humidity cloud-prediction scheme. A substantial part of the difference was due to the decreased probability of ice-phase clouds in a warmer climate which leads to longer lived water clouds. A simple characterization of the radiative properties of mixed-phase clouds was incorporated which, as will be discussed in section 4(b), is similar to assuming the ensemble of single-phase clouds definition described above.

Because of the difference in the refractive indices, and in the sizes and shapes of water and ice particles, the radiative properties of clouds for the same water path (whether liquid or ice) will differ appreciably; accordingly, the radiative properties of ice and mixed-phase clouds will not be satisfactorily modelled if they are assumed to consist entirely of liquid water.

The radiative properties of clouds in either the water or ice phase have been investigated by many authors (e.g. Stephens 1978; Slingo and Schrecker 1982; Stephens et al. 1990; Liou et al. 1991; Ebert and Curry 1992). However, very few studies that consider the radiative properties of mixed-phase clouds are known to the authors. Sassen and Liou (1979 a, b) presented the angular scattering, depolarizing and multiple-scattering behaviour of water, ice and mixed-phase clouds using laboratory measurements and theoretical computations. Significant differences in these properties for the mixed-phase cloud were found. Rockel et al. (1991) ran the European Centre for Medium-range Weather Forecasts model for 10 days in two experiments; in one, clouds were assumed to be all liquid water, whilst in the other the clouds were partitioned into ice and liquid water using an expression derived from Matveev's (1984) data. In this limited experiment significant differences were found in various global-mean quantities.

In this paper the principal aim is to indicate the potential importance of mixed-phase clouds on both the radiation budget of individual clouds and the atmosphere as a whole. However, to achieve this aim it is necessary to develop and verify the radiative parametrizations of ice and water clouds. The studies are conducted using the Slingo and Schrecker (1982) 24-band short-wave delta-Eddington radiation scheme and a 10 cm\(^{-1}\) narrow-band thermal infrared scheme described by Shine (1991). The radiative parametrizations of clouds are first developed and introduced in section 2. These parametrizations are tested against aircraft observations and the results are presented in
section 3. The effects of mixed-phase clouds on radiation are then shown in section 4. Conclusions are given in section 5.

2. CLOUD RADIATIVE PARAMETRIZATIONS

(a) Shape assumptions for ice crystals

A detailed parametrization of the radiative effect of ice crystals would require information on crystal size, shape and orientation over the whole size range that contributes significantly to the scattering and absorption. Such information is not generally available from observations, particularly for small (<20 µm) crystals, so some simpler assumptions are desirable. Although ice-crystal shapes are not uniform even in the same portion of a cloud, the frequently observed optical phenomena indicate that column shapes of ice crystals are common. Therefore, ice crystals are assumed to be hexagonal columns in shape and their optical parameters are calculated using Mie theory by converting these hexagonal crystals into equivalent spheres. Takano and Liou (1989) compared the optical parameters of hexagonal crystals with those from the assumption of equivalent ice spheres with either the same surface area or the same volume as those of the hexagons. They found that the extinction cross-sections for randomly oriented hexagons computed from ray optics are close to those for surface-equivalent spheres, whereas the single-scattering albedos for volume-equivalent spheres are closer to those of hexagons. Based on Takano and Liou's study, the extinction coefficient and the asymmetry factor are calculated here using Mie theory, assuming equivalent spheres with the same surface area as those of hexagons; the single-scattering albedo is derived from the volume-equivalent spheres.

Based on aircraft observations, Heymsfield and Platt (1984) parametrized the ice-crystal size distribution, \( n(L) \), in terms of ambient temperature and ice water content. They used an expression

\[
    n(L) = A(T)L^{B(T)} IWC
\]

where \( A(T) \) and \( B(T) \) are the coefficients for the average size distribution within each 5 degC temperature interval over the range -20 to -60°C and \( IWC \) is the mean ice water content within each temperature interval. Eight crystal spectra are presented by Heymsfield and Platt using Eq. (1); these are adopted in this study. In addition, another crystal size spectrum for a cirrus uncinus presented by Heymsfield (1975) is also used. These data cover a relatively wide range of ice-crystal sizes and ice water contents; these data were also used by Ebert and Curry (1992) in developing their ice-cloud parametrization. The Mie scattering calculations were performed using the code of Wiscombe (1979) with refractive indices taken from Warren (1984).

Table 1 shows the comparison of the results determined using the Mie routine for five single crystals with those presented by Takano and Liou (1989) using the hexagonal model. It is seen that the extinction cross-section for a surface-equivalent sphere and the single scattering albedo for a volume-equivalent sphere are generally closer to the corresponding values for a hexagon. The asymmetry factor, \( g \), determined from the volume-equivalent spheres is almost the same as that determined from the surface-equivalent spheres, but they are all systematically greater than the values from the hexagonal model. Note that there is a slight difference between the present results and those of the equivalent sphere given by Takano and Liou due to the different spectral intervals used.
<table>
<thead>
<tr>
<th>L/W</th>
<th>λ (μm)</th>
<th>Extinction cross-section (cm²)</th>
<th>Single-scattering co-albedo/1 - ω</th>
<th>Asymmetry factor</th>
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<tr>
<td></td>
<td></td>
<td>Hexagon</td>
<td>Sphere (area)</td>
<td>Sphere (volume)</td>
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<td>0.55</td>
<td>8.598-6†</td>
<td>8.942-6</td>
<td>7.337-6</td>
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<td>2.2</td>
<td>8.598-6</td>
<td>1.043-6</td>
<td>8.600-6</td>
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<td>50/40</td>
<td>0.55</td>
<td>4.039-5</td>
<td>4.132-5</td>
<td>3.480-5</td>
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<td>2.2</td>
<td>4.039-5</td>
<td>4.186-5</td>
<td>3.641-5</td>
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<td>1.314-4</td>
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The values for hexagonal crystals were taken from Takano and Liou (1989). L and W are the length and width, respectively, of the ice crystal. † 8.598-6 means 8.598 × 10⁻⁶.

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In order to obtain the radius of equivalent spheres we need to know the length, $L$, and width, $W$, (defined as being measured from apex to apex) of the hexagonal columns. With these two parameters, the radius of an equivalent-surface sphere is given by

$$r_e^2 = \frac{3}{4\pi} \left( \frac{W}{2} \right)^2 \left[ \sqrt{3} + 4 \left( \frac{L}{W} \right) \right]$$

and the radius of an equivalent-volume sphere is given by

$$r_e^3 = \frac{9\sqrt{3}}{4\pi} \left( \frac{W}{2} \right)^3 \left( \frac{L}{W} \right).$$

Since the ice crystals were measured against their maximum dimension (Heymsfield and Platt 1984) the length can be given by the size spectra. It is assumed that the width can be computed using the expressions given by Auer and Veal (1970)

$$W = -8.479 + 1.002L - 0.0023L^2 \quad L \leq 200 \mu m$$

$$W = 11.3L^{0.414} \quad L > 200 \mu m.$$  

(b) **Parametrizations**

To facilitate calculations of radiative transfer the size distribution of the cloud particles must be described with the minimum number of parameters. The variables commonly used in prediction of the cloud optical parameters are liquid/ice water content and effective particle radius (e.g. Slingo and Schrecker 1982; Liou et al. 1991; Ebert and Curry 1992). One of the major difficulties in using these types of the parametrizations in climate models is that the only variable normally available in climate models for describing the absorption and scattering processes in clouds is the liquid/ice water content. Under many other circumstances the cloud droplet size distribution is not available. Therefore, it may be more convenient to parametrize the optical parameters of various kinds of droplet (or crystal) size distributions in terms of liquid/ice water content only (e.g. Stephens 1978; Rockel et al. 1991).

Although cloud particle sizes are important in the determination of the optical parameters, especially for the single scattering albedo and the asymmetry factor of clouds, the effect of particle size variations can be implicitly related to the liquid/ice water content changes. Observations within stratocumulus provide evidence that the liquid water content increases with the effective droplet radius (e.g. see Stephens and Platt (1987), Fig. 6). The theoretical studies performed by Jonas (1991) also support these observations. Essentially, the droplet concentration is approximately defined by the number of cloud condensation nuclei that are activated close to the cloud base; any change in water content is then manifested in the growth or decay of these droplets. For ice clouds the ice water content increases as the effective crystal length increases; this can be demonstrated using the data provided by Heymsfield and Platt (1984).

It should be pointed out that the relationship between liquid water content and effective radius may only be valid for horizontally homogeneous clouds. Blyth and Latham (1991) presented observational results obtained in small continental cumulus clouds, which show that the effective droplet radius is relatively independent of the liquid water content. This phenomenon can be explained by turbulent mixing between the cloud and entrained unsaturated environmental air, which causes evaporation of cloud droplets. Since the entrainment rate is inversely proportional to the cloud radius (or horizontal size) (Simpson and Wiggert 1969), the turbulent mixing has a larger effect for small cumulus than for horizontally homogeneous clouds. Nevertheless, since the
stratocumulus cloud sheets greatly affect the flow of solar radiation into the earth–
atmosphere system owing to their large spatial coverage, the relationship between water
content and effective radius is still used in the present study.

To illustrate the possibility of the ice-cloud parametrization in terms of ice water
content, the optical parameters of the nine ice-crystal size distributions calculated using
Mie scattering theory as described in section 2(a) are plotted against the ice water
contents as the symbols in Fig. 1. It is seen that these parameters systematically change
as the ice water content varies, and can be fitted using the following expressions,

\[
\beta_e = a(4 + \log_{10}(IWC))^b
\]

\[
1 - \omega = c(4 + \log_{10}(IWC))^d
\]

\[
g = a_0 + a_1 \log_{10}(IWC) + a_2[\log_{10}(IWC)]^2
\]

for short wave and

\[
\beta_a = e(4 + \log_{10}(IWC))^f
\]

for long wave, where \(\beta_e\) and \(\beta_a\) are the extinction and absorption coefficient, respectively,
\(\omega\) the single scattering albedo and \(g\) the asymmetry factor. In the above equations the
ice water content (g m\(^{-3}\)) is in the range 0.0009 to 0.24. Note that this range of ice water
content, taken from Heymsfield and Platt (1984), includes an estimate of the contribution

Figure 1. Optical parameters of ice cloud as a function of ice water content for selected wavelengths. The
symbols represent the values calculated from Mie theory, the lines denote the least-squares fit. (a) Extinction
coefficient (note that the parametrization is almost identical at all three wavelengths); (b) \(1 - \omega\); (c) asymmetry
factor, \(g\); and (d) absorption coefficient at 11 \(\mu\)m.
of sub-20 μm particles; thus the correction to the extinction coefficient discussed later is mainly to correct for the scattering effect of these particles rather than their contribution to the total mass of the cloud. Equations (5) to (7) were fitted for 24 spectral bands in the short wave. The values of the coefficients for each of the 24 bands are given in Table 2. Equation (8) was fitted for 250 spectral bands in the long-wave. The coefficients e and f at 11 μm are 0.0120 and 3.7521, respectively, which may be used in any other broad-band model because the long-wave radiative characteristics of clouds tend to be dominated by the radiative interactions in the atmospheric-window spectral region between 8 and 12 μm.

![Image](image-url)

**TABLE 2. VALUES OF THE COEFFICIENTS FOR ICE-CLOUD RADIATIVE PARAMETERS IN EOS. (5) TO (7)**

<table>
<thead>
<tr>
<th>Spectral limits</th>
<th>Band i</th>
<th>μm</th>
<th>a</th>
<th>x10^-2 km^-1</th>
<th>b</th>
<th>c</th>
<th>x10^-3</th>
<th>d</th>
<th>a0</th>
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<td>0.25-0.30</td>
<td>2.220</td>
<td>3.908</td>
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<td>0</td>
<td>0.883</td>
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<tr>
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<tr>
<td>24</td>
<td>3.42-4.00</td>
<td>2.484</td>
<td>3.839</td>
<td>346.81</td>
<td>0.251</td>
<td>0.961</td>
<td>−7.26</td>
<td>−6.613</td>
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The solid lines in Fig. 1 represent the results of the parametrization which are close to the results of Mie scattering calculations; note that the fits to extinction coefficient in Fig. 1(a) are almost coincident for all three wavelengths shown. It can be seen that the single point at high ice water content (which corresponds to the cirrus uncinus case reported by Heymsfield (1975)) strongly influences the fit for the extinction and absorption coefficients; it would clearly be desirable in the future to repeat these fits with more data points better distributed in ice water content.

For liquid clouds the original Slingo and Schrecker (1982) parametrization in terms of liquid water content, LWC, and effective radius, re, could have been retained. However, for consistency with the ice parametrization, and to avoid the necessity of specifying re, the scattering properties are rederived using LWC as the only variable. The optical parameters are first calculated using the Mie code with the cloud-droplet spectra presented in Welch et al. (1980). The results are then fitted using the following expressions,
\[ \beta_{cw} = a (LWC)^b \]  
\[ \omega_w = c (LWC)^d \]  
\[ g_w = e (LWC)^f \]  
for short wave, and
\[ \beta_{sw} = h + k \{ \log_{10} (1 + LWC) \}. \]
for long wave.

In the above equations the liquid water content \((g \cdot m^{-3})\) ranges from 0.114 to 1.034. The subscript \(w\) denotes water. All coefficients are a function of spectral bands used by the radiation models. The coefficients in Eqs. (9) to (11) are given in Table 3. The values of \(h\) and \(k\) at 11 \(\mu m\) in Eq. (12) are 3.099 and 169.1, respectively. Figure 2 shows the selected results of the least-squares fitting.

<table>
<thead>
<tr>
<th>Spectral limits</th>
<th>Band (i)</th>
<th>(\mu m)</th>
<th>(a) (\text{km}^{-1})</th>
<th>(b)</th>
<th>(c)</th>
<th>(d) (\times 10^{-3})</th>
<th>(e)</th>
<th>(f) (\times 10^{-2})</th>
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<td>1</td>
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<td>98.8</td>
<td>0.656</td>
<td>1.0</td>
<td>0.0</td>
<td>0.869</td>
<td>0.36</td>
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<td>1.0</td>
<td>0.0</td>
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<td>0.42</td>
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<td>0.652</td>
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<tr>
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<td>1.05</td>
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<td>0.999</td>
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<td>0.859</td>
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<tr>
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<td>0.639</td>
<td>0.956</td>
<td>-10.4</td>
<td>0.865</td>
<td>2.10</td>
<td></td>
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<tr>
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<td>0.508</td>
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<tr>
<td>23</td>
<td>2.91-3.42</td>
<td>106.0</td>
<td>0.636</td>
<td>0.517</td>
<td>20.3</td>
<td>0.953</td>
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<tr>
<td>24</td>
<td>3.42-4.00</td>
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<td>0.787</td>
<td>-50.1</td>
<td>0.864</td>
<td>5.32</td>
<td></td>
</tr>
</tbody>
</table>

(c) Adjustments of optical parameters for ice clouds

(i) Effect of small ice crystals. It has been pointed out by many investigators (e.g. Heymsfield and Platt 1984; Dowling and Radke 1990) that current optical probes cannot correctly detect crystals that are smaller than a certain size (normally 20 \(\mu m\)). This often leads to the observations of ice-crystal size distribution being biased in favour of large particles. The crystal size distributions used in the present study are restricted to sizes greater than 20 \(\mu m\). Since the concentration of small ice crystals is usually high and the single scattering albedo of these small crystals is also larger, the contribution to the extinction of the solar radiation from the small particles can be appreciable. This
MIXED-PHASE CLOUDS

![Graphs showing extinction and absorption coefficients as functions of liquid water content.](image)

Figure 2. Same as Fig. 1 but for water clouds.

The contribution has been estimated by Heymsfield and Platt (1984), using the measurements from a forward scattering spectrometer probe (FSSP) sizing in the range 2 to 30 μm in 2 μm bins. They assumed that the extinction coefficient was equal to twice the geometric cross-section for all sizes, and that the particles were cylindrical in shape. The results are shown in Table 4. The extinction coefficient from the 1 to 20 μm size range is a substantial

<table>
<thead>
<tr>
<th>Temperature range</th>
<th>1−20 μm (×10^{-6})</th>
<th>20 μm − ∞ (×10^{-4})</th>
<th>Total (×10^{-4})</th>
<th>( \beta_{1-20\mu m}/\beta_{total} ) (%)</th>
<th>( \beta_{20\mu m-\infty}/\beta_{total} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-25 to -40°C</td>
<td>3.58</td>
<td>6.64</td>
<td>10.22</td>
<td>35</td>
<td>65</td>
</tr>
<tr>
<td>-40 to -50°C</td>
<td>1.64</td>
<td>1.46</td>
<td>3.10</td>
<td>53</td>
<td>47</td>
</tr>
<tr>
<td>&lt; -50°C</td>
<td>1.20</td>
<td>1.06</td>
<td>2.26</td>
<td>53</td>
<td>47</td>
</tr>
</tbody>
</table>

fraction of the total extinction, particularly at the lower temperatures. The extinction coefficient for particles whose lengths are less than 20 μm is about 35% of the total.
extinction for temperatures between $-25$ and $-40^\circ$C and about 53% for the temperatures below $-40^\circ$C. Since methods to estimate with greater accuracy the effect of small particles are not available, the results estimated above are adopted to adjust the extinction coefficient. From Table 4 we have, therefore,

$$\beta_{\text{total}} = \beta_{\text{20}\mu m-\epsilon}/0.65 = 1.54\beta_e$$  \hspace{1cm} (13)

for temperatures between $-25$ and $-40^\circ$C and

$$\beta_{\text{total}} = \beta_{\text{20}\mu m-\epsilon}/0.47 = 2.13\beta_e$$  \hspace{1cm} (14)

for temperatures below $-40^\circ$C. These uniform adjustments are reasonable because the extinction cross-section of ice crystals in the short-wave spectrum is independent of wavelength at the geometric optics limit. Note that the extra mass of these small crystals makes only a small contribution to the added extinction (see Stackhouse and Stephens 1991); it is the increase in cross-sectional area of the scatterers that is most important for this correction.

In the infrared window region (8–12 $\mu$m) the contribution to the extinction due to the small crystals will be less because of the reduction in extinction efficiency for the small particles with length less than about 10 $\mu$m. Heymsfield and Platt (1984) estimated that the contribution of the infrared cross-section in the sub-20 $\mu$m region would be about half that at the solar wavelengths. However, comparison of the modelled long-wave irradiance for a cirrus cloud with observations indicates that the application of the same adjustment described above to the absorption coefficient $\beta_a$ in the long-wave spectrum produces a better result. For this reason the same adjustment was applied to the absorption coefficient $\beta_a$ in the long-wave spectrum as in the solar region, although this area is clearly in need of further study.

Table 1 shows that the single-scattering albedo for a hexagonal ice crystal with the aspect ratio of 750 $\mu$m/160 $\mu$m is 0.75, whereas the single-scattering albedo for a small crystal (20 $\mu$m/20 $\mu$m) is 0.96. Hence it was necessary to consider whether the single-scattering albedo should also be corrected. In order to estimate this effect it is necessary to have size spectra of these small particles. Since these were not available in this study the following simple method was used to estimate the effect of small crystals on the single-scattering albedo. A modified gamma distribution is added onto the lower end of the crystal size distribution of Heymsfield and Platt (1984) for the temperature range from $-35$ to $-40^\circ$C such that the extinction coefficient obtained using the new distribution is equal to the adjusted value. The adjusted single-scattering albedo can then be computed using this size spectrum. Figure 3(a) shows the adjusted result. It is seen that small crystals lead to an increase in the single-scattering albedo of up to about 0.10 at wavelengths greater than 1.5 $\mu$m, except around 3.0 $\mu$m where the single-scattering albedo decreases. The influence on the solar-irradiance profiles due to this change was then investigated. The solar irradiiances were computed using the McClatchey et al. (1972) mid-latitude summer atmosphere, a surface albedo of zero, and a 100 mb thick cloud layer centred at 550 mb with an ice water path of 100 g m$^{-2}$. The results are shown in Fig. 3(b). The correction for the single-scattering albedo has a small effect on irradiance profiles, with changes of less than 2%. Since this difference is far less than the differences caused by the uncertainty of ice water content (see section 3), the effect of small crystals on the single-scattering albedo is neglected in this paper. However, this effect should be taken into account if more reliable observations become available. No account was taken of changes in the asymmetry factor for the small crystals. In view of the gross changes to this property described in the next sub-section, such changes were not felt to be warranted given current understanding.
(ii) Adjustment of the asymmetry factor. Takano and Liou (1989) show that the asymmetry factor determined for equivalent spheres is uniformly larger than for hexagonal ice crystals (see also Table 1). This implies that using the assumption of the equivalent spheres in cirrus clouds would over-estimate the proportion of the solar radiation scattered into the forward direction. Nevertheless, the variation with the particle aspect ratio of the asymmetry factor for the equivalent spheres is similar to that for hexagons. Therefore, if the asymmetry factors from the equivalent spheres are appropriately adjusted, they may match those from the hexagonal particles. In addition, numerical simulations show that, if the asymmetry factor is taken to be 0.7, the albedos of ice clouds calculated from radiative-transfer models are very close to those of in situ aircraft measurements during the FIRE (First ISCCP* Regional Experiment) IFO (Intensive Field Observations) (Stephens et al. 1990; Wielicki et al. 1990). For these reasons, the asymmetry factor is adjusted such that

$$g' = 0.85 \, g$$

where \( g' \) is the adjusted asymmetry factor and \( g \) is that calculated for the spheres using Eq. (7). Although this adjustment is more or less arbitrary, it can produce a good estimate of irradiance and albedo of cirrus clouds (see below).

It should be pointed out that the above simple corrections for both the extinction coefficient and the asymmetry factor may present considerable uncertainty, and should be further improved with more observations in the future.

\[(d) \quad \text{Sensitivity study}\]

The observations of Heymsfield and Platt (1984) show that at individual temperatures there are a large range of possible ice water contents. The parametrization presented in this paper assumes an implicit relationship between ice water content and crystal size distribution. Since there are insufficient data available to ensure that such a relationship is robust, it is important to assess the dependence of the derived optical parameters when alternative assumptions are made. Two cases are investigated here. One assumes the size distribution to be independent of IWC; the other examines the effect of variations of size distribution at fixed IWC.

In the first case the relationship between the optical properties and the ice water content is investigated using Eq. (1), with the coefficients \( A \) and \( B \) fixed and the ice water content changed. This is equivalent to changing the ice water content by changing the concentration of particles for the same size spectrum. The single-scattering albedo and the asymmetry factor are unchanged in this process. In order to keep the ice water content equal to the value calculated using Eq. (1), the size distribution is normalized such that

\[
\frac{4}{3} \pi \rho_i \int r_e^2 A'(T)n(r_e)B(T) \, dr_e = 1
\]

(16)

where \( r_e \) represents a radius of volume equivalent sphere, \( A' = A/C \), where \( C \) is the normalization constant, and \( \rho_i \) is the density of ice. Two pairs of the coefficients \( A \) and \( B \) were used corresponding to the temperature ranges from \(-20\) to \(-25^\circ C\) and from \(-40\) to \(-45^\circ C\). A linear relationship between the extinction coefficient and the ice water content is obtained using the above size spectrum. We refer to this as the linear scheme. A nonlinear relationship is produced by the parametrization presented in section 2(b) and 2(c) which assumes the variation of size spectrum with ice water content. Figure 4 shows the relationship between the albedo of cirrus clouds situated between 250 and 300 mb and the ice water content, determined using both the linear and the parametrization schemes. The calculations were performed using the mid-latitude summer atmosphere, a solar zenith angle of 61.3\(^\circ\), and a surface albedo of 0.072. The reason for using this configuration is to allow comparison with observations (see next section). The corrections to both the extinction coefficient and the asymmetry factor were also applied to the linear schemes. It is seen that within the range of ice water path shown in Fig. 4 the linear schemes produce cloud albedos lower than the parametrization scheme. It will be seen in next section that the cloud albedo determined using the parametrization is closer to observations. Nevertheless the albedo curve from the linear scheme for a size distribution of cold cirrus (-40 to -45°C) seems to be close to the solid curve from the parametrization; this implies that a linear scheme could produce good radiative properties if the coefficients in Eq. (1) were selected appropriately. However, this curve deviates from the solid curve as the ice water path further increases beyond the values of the ice water path shown in the figure; it seems unlikely that such a linear scheme could satisfactorily model albedos over a wide range of ice water paths.
In the second case study the ice water content is fixed while the coefficient $B$ in Eq. (1) is changed. This is equivalent to changing the crystal size distribution for the same ice water content. The calculations were performed using the size distribution in the temperature range of $-40$ to $-45^\circ C$ with coefficient $B$ changing to $-2.9$, $-3.3$ and $-3.7$. The original value of $B$ for this distribution is $-3.23$. Again the normalization is applied to each size distribution. Figure 5 shows the sensitivity of scattering parameters to such changes. It is seen from Fig. 5(a) that the concentration of small crystals represented by the coefficient $B = -2.9$ is less than that represented by $B = -3.7$. The effective particle radii for the three size spectra (for particles converted into surface-equivalent spheres) are 80.8, 53.3 and 36.6 $\mu$m, respectively. The scattering properties are quite sensitive to different size distributions that have the same ice water content. The extinction coefficient and single-scattering albedo increase if large particles are converted into small ones. Both of these case studies indicate a significant source of uncertainty in our parametrization, and further observations of the general variation of size spectra are needed to reduce this uncertainty.

3. Validation of the Parametrizations

(a) Ice clouds

The parametrization for ice clouds is tested against published observations of cirrus clouds obtained from the National Center for Atmospheric Research Sabreliner jet aircraft on the 28 October 1986 FIRE IFO (Smith et al. 1990). In addition, the code has been compared with observations from the Meteorological Office Research Flight C-130 experiment (R. Saunders, private communication); the comparison shows that the model agrees with the observations within the error bars of the observations.

The profiles of the ice water content and the radiative properties observed during FIRE were taken from Smith et al. (1990) and Stackhouse and Stephens (1991). For these observations the cirrus clouds were found near the western shore of Lake Michigan, close to Green Bay, Wisconsin on 28 October 1986. The cirrus cloud base was estimated to vary from 9.0 to 9.4 km with the cloud top estimated at 10.9 to 11.5 km. Figure 6
Figure 5. Effect on the optical properties of ice clouds of changing crystal size spectra without changing the ice water content. The parameter $B$ is that used in Eq. (1). Three size spectra which have the same ice water content are shown in (a), while (b), (c) and (d) show the extinction coefficient, single-scattering albedo and asymmetry factor, respectively, as a function of wavelength for the three size distributions shown in (a).

Figure 6. The mean profile of ice water content observed by the NCAR Subreliner jet aircraft during the FIRE IFO (from Smith et al. 1990).
shows the profile of mean IWC and uncertainty derived from 2D-C cloud probes, where cloud sample 1 corresponds to the south region and cloud sample 2 to the north. The observations for cloud sample 1 and sample 2 were performed between 1545 and 1620 GMT and between 1553 and 1630 GMT, respectively. The mean solar zenith angle during the observations was 61.3° (Stackhouse and Stephens 1991). Since atmospheric profiles are not readily available, for convenience atmospheric profiles derived by the Meteorological Office Research Flight (ICE217, R. Saunders, private communication) on 20 October 1989 over the south-west coast of England were used. The water vapour mixing ratios were increased to saturated values with respect to ice at the cloud-layer temperature. A surface albedo of 0.072 is used as given by Stackhouse and Stephens (1991). Figure 7 shows the comparison of observed and modelled downwelling short-wave irradiances. The horizontal lines through the circles represent the standard deviations of the observations. The modelled irradiances are displayed using the mean, lower and upper limits of the ice water content shown in Fig. 6. It is seen that the downward irradiances at the cloud boundaries are well modelled, although the irradiances at the top are slightly over-estimated. The errors in the calculations due to the uncertainty of the ice water content are generally larger than the standard deviation of the observation, particularly at the cloud base. The modelled irradiances from the mean ice water content are generally within the standard deviation of the observations except at the middle level of the clouds.

Figure 7. A comparison of modelled downward solar irradiances (solid and dotted lines) with the observations (circles) presented by Stackhouse and Stephens (1991). The dotted lines represent the modelled results using the lower and upper limits of ice water content shown in Fig. 6; the solid line shows the modelled results using the mean ice water content.

In order to investigate the effect of uncertainty in the water vapour profile, the calculations were repeated using the tropical, mid-latitude summer and sub-arctic winter atmospheres of McClatchey et al. (1972). The results show that, within the cloud layer, the maximum difference of irradiances between these profiles is less than 10 Wm\(^{-2}\). These differences are much less than the standard deviation of the observations. Therefore, the discrepancy between the modelled results and the observations is more likely to be due to the uncertainty of ice water content.

Figure 8 shows both the modelled and observed cloud albedo determined following the definition of Paltridge and Platt (1981). The current scheme without any correction
significantly underestimates the cloud albedo. The correction for the extinction coefficient to account for the effects of small particles brings the albedo curve towards the observations, but the discrepancies are still quite large. The further correction for the asymmetry factor to account for the scattering effects of nonspherical particles results in the albedo curve being much closer to the observations. These comparisons, which were also performed by Stackhouse and Stephens (1991), clearly show the important effects of both small particles and particle shape.

![Graph showing cloud albedo as a function of ice water path](image)

*Figure 8.* Albedo of ice cloud as a function of ice water path (g m⁻²). The symbols represent observations taken from Stackhouse and Stephens (1991). The dashed line shows results from the current scheme without adjustments to the extinction coefficient (to account for small crystals) and asymmetry factor (to account for crystal shape). The dotted and solid lines show the effect of first adjusting the extinction coefficient and then adjusting the asymmetry factor, as described in the text. The dot-dashed line denotes the results of Stackhouse and Stephens (1991).

Given the somewhat crude method of the adjustments, the agreement between our model and the observations may be somewhat coincidental; however, within the parameter space of currently plausible corrections, it is clearly possible to reproduce these irradiance observations.

Stackhouse and Stephens (1991) calculated the optical properties of ice crystals using Mie theory and the observed size distribution of ice particles. The effect of small particles was taken into account by adding a standard size distribution which covers the particle sizes less than 30 μm to the observed size distribution. The dash-dotted line in Fig. 8 shows their result, which includes the correction for both small particles and asymmetry factor (their C1 curve). As they stated, this curve is not close enough to explain the observations. The reason may be due to the uncertainty in the specification of small particles. Although they used the standard size distribution to estimate the effect of small particles, the concentration of small particles they obtained is still about two orders of magnitude less than that presented by Platt et al. (1989). Stackhouse and Stephens also mentioned that the existence of small particles might be substantially larger than assumed in their simulations. The current calculations use the observed ice water content and amplify the extinction coefficient using the suggested adjustment from Heymsfield and Platt (1984) based on their estimate of the number of small particles. The good agreement between the modelled and the observed results suggests that the Heymsfield and Platt method might be more appropriate.
(b) Water clouds

Slingo and Schrecker (1982) developed parametrizations of the Mie scattering parameters in terms of the liquid water content and the effective droplet radius of water clouds. An important feature of this scheme is the separation of the dependence of the cloud radiative properties on the liquid water path and the effective droplet radius. This makes the parametrization suitable for investigations of the effect of independent changes in these two parameters.

In order to compare the present parametrization of water cloud with the Slingo and Schrecker scheme, the radiation model was run using the two cloud schemes with the cloud data from the Joint Air–Sea Interaction Experiment (JASIN) (Slingo et al. 1982). The measurements were performed using three aircraft (C-130, Electra and Falcon) through a stratocumulus cloud centred at 940 mb. The profiles of temperature and humidity as well as the cloud liquid water content and effective droplet radius were kindly provided by Dr A. Slingo. Figure 9 shows the results. The symbols represent the averaged short-wave irradiance measured by the three aircraft on the horizontal traverses (see Slingo et al. (1982) for details). The modelled irradiances from the present scheme are given by the dashed lines which show an excellent agreement with both the observations and those derived using the Slingo and Schrecker scheme. The overestimated irradiances at the cloud top are mainly due to the effect of a medium cloud layer at about 700 mb, which was identified as a thin and broken layer of altostratus, and also due to the neglect of aerosol scattering which might reduce the irradiance by about 20 W m$^{-2}$ at the cloud-top region, as discussed by Slingo et al. (1982). This comparison suggests that the present parametrization for water clouds might be treated as a complement to the Slingo and Schrecker scheme if cloud droplet radius data are not available.

The long-wave parametrization of water cloud is also compared with the measurements from JASIN. Figure 10 depicts the modelled long-wave irradiance determined using the current radiation scheme and the water cloud parametrization as well as the observed irradiance from aircraft during the JASIN. The symbols represent the long-wave irradiances measured by three aircraft on the horizontal traverses, the solid lines
represent the irradiance profiles measured by the Meteorological Office C-130 and the dashed lines are the theoretical irradiances determined from the present cloud long-wave parametrization scheme. The theoretical irradiances at the cloud boundaries from the present parametrization are in reasonable agreement with the observations. The discrepancy between the downward theoretical irradiance and the observed irradiance at the cloud top is also due to the extra medium-level cloud at about 700 mb as discussed earlier. The net irradiance is close to zero in the cloud itself, which indicates that the cloud is calculated to be close to being a black body. This is in disagreement with the observations but in agreement with the modelling studies reported by Slingo et al. (1982), who attribute the difference to calibration problems in the observations.

4. Radiative properties of ice and mixed-phase clouds

(a) Difference of radiative properties between ice and water

The extinction coefficient and single-scattering albedo determined from three liquid droplets and ice crystal size distributions are presented in Fig. 11. The effective radius and liquid (or ice) water content are shown in the diagram. It is seen from Figs. 11(a) and (b) that the extinction coefficients for all droplet size distributions are about one or two orders of magnitude greater than for the ice crystal size distributions. Even for the drop size distribution of stratus base whose liquid water content is 0.114 g m⁻³, the extinction coefficient is about one order of magnitude greater than that for the ice crystal size distribution in the cirrus uncinus whose ice water content is 0.24 g m⁻³. The reason for this can be explained by the geometric optics limit. When particles are large relative to the wavelength, the extinction coefficient can be approximated by (e.g. Stephens 1978)

\[ \beta_c = \frac{3WC}{2\rho r_c} \]  

where \( WC \) can be either the liquid or ice water content and \( \rho \) is the density of either liquid water or ice. For the same value of \( WC \), \( r_c \) in ice clouds is at least one order of magnitude greater than that in liquid clouds. Physically, the number density of particles
in the ice cloud is much lower than in the liquid cloud for the same water content, the chances of multiple scattering, which increases extinction, are greatly reduced.

The plots of single-scattering albedo (Figs. 11(c) and (d)) represent the absorption spectra of different particles. In those absorption bands in the near infrared, the absorption of radiation by ice crystals is significantly greater than by liquid droplets. The reasons for this difference could be due to differences in the size, the shape and the refractive index of particles. In order to investigate these effects the single-scattering albedos for the three water clouds were calculated using the refractive index of ice, and the results are compared with those using the refractive index of water. The differences due to the refractive index change were found to be generally small except in the spectral band between 2.5 and 3.0 \( \mu m \). It is shown in Table 1 that the differences of the single-scattering albedo due to the difference in particle shape are relatively small. Therefore, it can be concluded that the differences of the single-scattering albedo between water droplets and ice crystals shown in Figs. 11(c) and 11(d) are mainly due to the difference in particle sizes.

Figure 12 shows the reflectivity and absorptivity of ice and water clouds as a function of liquid/ice water path and solar zenith angle. These variables were calculated for a cloud layer positioned between 500 and 550 mb in the mid-latitude summer atmosphere of McClatchey et al. (1972) with zero surface albedo. The reflectivities (left-hand column of Fig. 12) of ice cloud show the same variation with both solar zenith angle and water path as that of the liquid water cloud; however the values are smaller. The absorptivity of the liquid water cloud decreases as the solar zenith angle increases for almost all values
Figure 12. Reflectivity (left-hand column) and absorptivity (right-hand column) of ice and water clouds as a function of solar zenith angle and liquid/ice water path. The bottom panels show the ratio of these two parameters for water to those for ice.
of the water path; by contrast the absorptivity of the ice cloud increases as the solar zenith angle increases for most values of the water path as has also been found by, for example, Ramaswamy and Ramanathan (1989).

The difference in properties between the water and ice clouds is dependent on a number of factors. The generally lower albedo of ice cloud is mainly due to a lower optical thickness for a given water path, although this effect will be slightly offset by the lower asymmetry factor for ice. The dependence of the absorption on zenith angle indicates that for a water cloud the increase in effective optical depth with zenith angle mainly contributes to a higher probability of a photon being scattered out of the cloud; for the ice cloud, with its generally lower single-scattering albedo, the same increase in optical depth increases the probability of absorption. The bottom panels in Fig. 12 show the ratios of these two parameters for water to those for ice. Over the whole domain the difference is substantial; for most water paths the water cloud albedo can be 2–4 times greater than that of the ice cloud. The difference of the absorptivity depends more on the solar zenith angle than on the water path. For solar zenith angles less than 40° the absorptivity of water cloud is greater than that of ice cloud for the same water path. The reverse is true for solar zenith angles greater than 40°.

(b) Effects of mixed-phase clouds

As mentioned earlier the observations of significant ice cloud layers embedded in supercooled altocumulus were reported by Heymsfield et al. (1991). It should be noted that they were unable to rule out whether these ice crystals were generated by the aircraft; this is a persistent problem in aircraft studies of cloud microphysics (e.g. Kelly and Vali 1991). Nevertheless the information presented in their paper is still used because it is the most extensively reported example of a mixed-phase cloud in the literature so far. Their observations on 25 October 1986 are used in the present study. The lidar backscatter measurements show a strongly scattering cloud situated between 7.4 and 7.9 km, generally due to the presence of liquid water. Evidence of cirrus clouds above and below this region is also indicated (see Heymsfield et al. (1991) Fig. 4). The liquid droplets and the ice crystals were detected through this cloud layer using the FSSP probe and a two-dimensional imaging probe. The profiles of ice and liquid water content between 7.4 and 7.9 km are given in their paper. The observations above 7.9 km were not available, so the information on the cirrus cloud above the altocumulus is unknown. To consider the effect of this cirrus cloud the profile of the ice water content in the cloud-top region from the ICE217 (see section 3(a)) is added to the top of the observed profile of ice water content. The temperature and humidity profiles within the cloud are taken from Fig. 6 of Heymsfield et al. (1991), the data above and below the cloud are taken from the ICE217.

In the mixed-cloud layer the optical properties are determined from the optical parameters for water (index w) and ice (index i) clouds in the following way (e.g. Rockel et al. 1991):

\[ \tau_{\text{mix}} = \tau_i + \tau_w \]  
\[ \omega_{\text{mix}} = \frac{\omega_i \tau_i + \omega_w \tau_w}{\tau_{\text{mix}}} \]  
\[ g_{\text{mix}} = \frac{\omega_i \tau_i g_i + \omega_w \tau_w g_w}{\omega_i \tau_i + \omega_w \tau_w} \]  
\[ T_{\tau_{\text{mix}}} = T_{\tau_i} T_{\tau_w} \]
where $T_i$ is the cloud thermal infrared transmissivity. If both ice and water are present within a cloud, they are assumed to be mixed uniformly together, although the effect of this assumption will be examined below. These expressions also assume that the size spectra for the ice crystals and the water droplets in the mixed-phase clouds are the same as those assumed for the single-phase clouds discussed in section 2; further observations are necessary to assess the validity of this assumption. The simulations were performed assuming liquid water only, liquid water and ice, and liquid water only but with the ice water content added to the liquid water content. The results of simulations are given in Fig. 13. The net irradiances obtained for the mixed cloud are generally closer to the observed values, although this is not always true for the individual components of the irradiances, particularly the downward thermal infrared irradiance. Assuming the ice to be in the liquid form in the simulations commits a greater error than if the ice is totally ignored. Physically, the large ice particles, when 'converted' to liquid water, increase the number of small drops, increasing the albedo and the emissivity of cloud.

Figure 12 showed the variation of reflectivity with both solar zenith angle and water path for an ice cloud and a water cloud in an otherwise identical atmosphere. The actual albedo for a mixed-phase cloud could lie anywhere between the ice-only or water-only values depending on the precise mixture of phases.

The dependence of radiative properties on the fraction of ice and water in the cloud depends on how the phases are mixed within the cloud. The effect of mixing in three different ways, shown schematically in Fig. 14, is investigated here, although there are, of course, many different ways that such mixing could occur. The first, which we refer to as 'uniform', considers the ice and water co-existing throughout the cloud. The second, which we refer to as 'stratified', considers the ice layers to lie above the liquid water layers. The final way ('adjacent'), following the method used by Mitchell et al. (1989) (W. J. Ingram, personal communication) considers the mixed-phase cloud to be made up of two separate vertically homogeneous ice and liquid water clouds. This case may also be appropriate to our alternative definition of mixed-phase clouds, described in the introduction, in which individual clouds are of single phase, but the ensemble of clouds as a whole is mixed phase. Figure 15 shows the variation of cloud albedo with the fraction of ice in the cloud; the optical depth at 2.6 $\mu$m is also shown. The calculation assumed a mid-latitude summer atmosphere with a cloud positioned between 480 and 580 mb split into ten layers with equal water path in each. The total water path is 100 g m$^{-2}$, the solar zenith angle is 45° and surface albedo is zero.

The optical depth varies from 28.2 to 2.9 as the cloud changes from totally liquid to totally ice. The variation is more or less a linear function of the ice fraction. The corresponding change in albedo is from 72.6% to 29.8%. The albedo of 'adjacent' mixed-phase cloud would vary linearly with ice fraction between these two values. For the other two methods of mixing the ice and water the variation is much more nonlinear, especially for the 'uniform' case. For the uniform case the albedo increases sharply as the fraction of ice decreases from 1 to 0.8. This large change in albedo is consistent with models of dependence of albedo on optical depth. The figure emphasizes that even a relatively small amount of liquid water (<10% of the condensed water) can make a substantive impact on the cloud properties. It also emphasizes that substantive differences in albedo can occur, depending on the way the phases are mixed. A further point which is important for climate-sensitivity experiments, such as those of Mitchell et al. (1989), is that the rate of change of albedo with fraction depends not only on the mixing method but also, for the uniform and stratified cases, on the unperturbed ice fraction. It would require careful experimentation with a general circulation model to assess whether the mixing method significantly alters climate sensitivity.
Figure 13. Effect of mixed-phase clouds on profiles of both solar and infrared irradiances. (a) Upward L↑ and downward L↓ thermal infrared irradiances, (b) net thermal infrared irradiance, (c) upward S↑ and downward S↓ solar irradiances, and (d) net solar irradiance. The symbols represent observations taken from Heymsfield et al. (1991). The solid line assumes mixed-phase cloud, the dashed line neglects the contribution of the ice, and the dotted line assumes the cloud is in the liquid phase with the ice water content added to the liquid water content.

Figure 14. Schematic diagram of three possible configurations of ice (represented by asterisks) and water (represented by circles) in a mixed-phase cloud. In the text these are referred to as (a) uniform, (b) stratified and (c) adjacent.
Figure 15. Cloud albedo and 2.6 μm optical depth as a function of ice fraction for the uniform, stratified and adjacent assumptions shown in Fig. 14. The cloud has a water path of 100 g m⁻², located between 480 and 580 mb; calculations assume a solar zenith angle of 45° and a surface albedo of zero.

This simulation shows that cloud albedo is very sensitive to the change in the mixture of cloud water and ice. A small part of supercooled liquid water embedded in a high cirrus cloud as indicated by Sassen (1992) would greatly increase cloud albedo. A glaciated portion in the middle- and low-level clouds as reported by Heymsfield et al. (1991), on the other hand, would reduce cloud albedo. In a study of the radiation budget of the Tibetan Plateau to be reported elsewhere (see also Sun 1992), the impact on the planetary albedo when mid-level cloud was changed from water to mixed phase (using the uniform assumption) was to decrease it considerably in March; for example, the area mean planetary albedo decreased from 40% to 36.5%.

5. CONCLUSIONS AND DISCUSSIONS

In this paper a set of cloud radiative parametrizations is developed based on Mie scattering calculations. The optical properties for ice clouds are parametrized in terms of ice water content, and account is taken of departures in the shape of ice crystals from spheres. The effect of small ice crystals is taken into account by scaling the extinction coefficient by a factor dependent on cloud temperature. The effect of crystal shape on asymmetry factor requires a further correction. Comparison with aircraft observations shows that the ice-cloud parametrizations perform well. The precise roles of the small crystals and crystal shapes are, as yet, poorly determined and represent a very considerable uncertainty in modelling of cirrus cloud properties.

The optical parameters of water cloud are made a function of liquid water content. This simple scheme can produce the radiative properties of stratocumulus cloud which are almost the same as those determined using the Slingo and Schrecker scheme and, therefore, it may be used as an alternative method if the cloud droplet size distribution is not available.

The radiative properties of ice crystals differ significantly from those of liquid droplets. For the same water path and solar zenith angle, the reflectivity of water cloud can be 2–4 times greater than that of ice cloud.
The most important result in this study concerns the need to consider mixed-phase clouds in modelling both the radiative properties of an individual cloud and the radiative budget of the atmosphere as a whole. In the specific mixed-phase case studied, the assumption that all the condensed liquid and ice water present was only in the liquid phase led to a greater error in the calculation of cloud properties than if the ice phase was neglected completely.

The dependence of cloud properties on the fraction of ice and water depends on the details by which they are mixed; in climate-sensitivity studies the variation of albedo with ice fraction as atmospheric temperature changes would also depend on these details. There is a clear need for refined observation of morphology and microphysical details of mixed-phase clouds.

The necessity of accounting for cloud phase for clouds at all levels has been illustrated. Determining cloud phase is probably as important as determining the cloud water content. Comparisons of modelled and observed radiation budgets, which have ignored lower-level ice in clouds, may have achieved reasonable agreement by compensating errors, perhaps in cloud thickness or assumptions about overlap. However, before models can include mixed-phase clouds, account needs to be taken as to whether the source of cloud thickness data already implicitly assumes the phase of clouds. An important example is the International Satellite Cloud Climatology Project (ISCCP) (e.g. Rossow and Schiffer 1991). There the optical depth is determined by comparison with a radiative-transfer model on the assumption that the cloud equivalent droplet radius is 10 μm; the optical depth is then converted into liquid water path using the same assumption. Use of this liquid water path or optical depth in a radiation scheme with any other droplet size or phase would be inconsistent, even though the optical depth derived from observation is incorrect, because of its assumption about the nature of the cloud particles.

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REFERENCES


Ramaswamy, V. and Ramanathan, V. 1989 Solar absorption by cirrus clouds and the maintenance of the tropical upper troposphere thermal structure. J. Atmos. Sci., 46, 2293-2310


Sassen, K and Liou, K.-N. 1979b Scattering of polarized laser light by water droplet, mixed-phase and ice crystal clouds. Part II: Angular depolarizing and multiple-scattering behaviour. J. Atmos. Sci., 36, 852-861


Shine, K. P. 1991 On the cause of the relative greenhouse strength of gases such as the halocarbons. J. Atmos. Sci., 48, 1513–1518


Sun, Z. 1992 ‘Radiation budget over the Tibetan Plateau’. PhD thesis University of Reading


