Thermal compression waves. II: Mass adjustment and vertical transfer of total energy

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SUMMARY

A fully compressible model is used to simulate the mass adjustment that occurs in response to a prescribed heat source. Results illustrate the role that thermal compression waves have in this process. The vertical mass transport associated with compression waves decreases rapidly with height. Most of the mass transport occurs in the horizontal, with the vertical structure of the disturbance similar to that of a Lamb wave. The vertical transfer of total energy in a thermally driven mixed layer is also examined. It is shown that the upward transport of total energy is accomplished by a compression effect rather than by the exchange of warm and cold air by buoyant thermals. Model results are analysed to determine budgets of total energy, mass and entropy. It is demonstrated that buoyant thermals are predominantly responsible for a transfer of entropy, rather than total energy. In the light of these results the notion of 'heat transport' in a fluid is discussed.

1. INTRODUCTION

In Part I (Nicholls and Pielke 1994) simulations were carried out with a rigid lid. In Part II this restriction is removed and the transport properties of thermal compression waves are investigated in more detail. It is well known that if sound waves are eliminated from the governing equations changes in pressure propagate instantaneously (Lamb 1932). This paradox is resolved when the governing equations are fully compressible. Linear analytic solutions to fully compressible equations for a pulse of mass outflow from a simple source have been discussed in standard texts (see, for instance, Lamb (1932) and Lighthill (1978)), which to some extent are similar to the compression waves that are the subject of this investigation. The resulting pressure perturbation propagates at the speed of sound and takes a different shape in one, two, and three dimensions. The role that compression waves, produced by heat sources and sinks, have in mass adjustment in the atmosphere has been mentioned briefly by Tripoli and Cotton (1982), Anderson et al. (1985), Droegemeir and Wilhelmson (1987) and Nicholls (1987). However, the details of this process have received little attention. In these previous studies the term 'sound waves' is used to describe this compressional effect. In this investigation the term 'thermal compression waves' is used in order to emphasize the role that heating has in their generation, and that they are not high-frequency periodic waves which might be inferred from the use of the word 'sound'. To refer to these phenomena as waves is stretching the definition of a wave in certain situations, particularly in the case of vertical propagation, as shall be discussed. However, for want of a better terminology, we continue to use the term 'wave' to discuss this compressional effect.

Systems of equations that eliminate thermal compression waves, or Lamb waves in the case of hydrostatic models, may not conserve mass. For instance, consider the solution to the Boussinesq equations for a prescribed heat source such as discussed by Nicholls et al. (1991). A heat source representing the release of latent heat in a thunderstorm leads to a pressure fall at the surface (Fig. 3(c) of that paper). Such surface-pressure falls have been observed in the early stages of the development of thunderstorms (Cunning and DeMaria 1986). This means that mass has been removed from the column of air above the surface low-pressure region. However, the analytic solution to the Boussinesq

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equations shows no corresponding increase in surface pressure elsewhere. If gravity waves were responsible for the mass adjustment there would be a compensating increase in surface pressure adjacent to the surface low. The results of a simulation with a fully compressible model (Figs. 8 and 10 of Part I) show that outside the surface low, the surface pressure is increased within a broad region expanding at the speed of sound. Another example is sensible heating over land that is surrounded by water. The pressure decreases over the land, and sea breezes develop. As in the previous example, a lowering of the surface pressure means mass has been removed from the air column over the land. Models which eliminate thermal compression waves from the governing system of equations cannot simulate the details of this mass-adjustment process (this is also discussed by Feliks and Huss (1982)). In this article this latter example is simulated using the fully compressible model described in Part I.

The fully compressible model is also used to investigate the vertical transport of mass and total energy within a thermally driven mixed layer. The hypothesis is tested that the vertical transfer of total energy is due to compression of the air aloft, rather than to the exchange of warm and cold air masses by buoyant thermals. This raises the question as to what conserved quantity the vertical eddy sensible-heat flux refers to, and why it should be a good measure of the total-energy flux? An entropy budget is carried out to determine if the eddy sensible-heat flux (when put in units of entropy) can be considered a flux of this quantity, and the relationship with the total-energy flux is explored. In the light of these results the notion of 'heat transport' in a fluid is discussed.

In section 2 some theoretical results are presented, which are relevant to mass and energy transfer in fluids and to the interpretation of the numerical experiments. The first part of this section presents an analytic solution for a vertically propagating compression wave, which gives an indication of how the vertical mass flux varies with height. Next, Lamb waves are discussed, which is relevant to the vertical structure of the thermal compression waves produced by a heat source of finite horizontal extent. A conceptual model is then presented which illustrates how a change in internal energy per unit volume can be produced in a region by compression, and contrasts this process with how temperature changes in a region are produced by eddy motions. Finally in this section the definition and derivation of the turbulent heat flux are discussed and connections are drawn with the flux of entropy. In section 3 the design of the numerical experiments is described and in section 4 results are presented. Conclusions are drawn in section 5.

2. Mass and Energy Transfer

(a) Analytic solution for vertically propagating compression waves

Wave solutions for small perturbations from the rest state for a compressible fluid are discussed in many standard texts (see, for example, Gill 1982). For purposes of this discussion attention is limited to the one-dimensional equations for vertically propagating waves in an isothermal atmosphere. In the absence of heat sources or sinks the one-dimensional linearized momentum, thermodynamic and continuity equations are

$$\frac{\partial w}{\partial t} + \frac{1}{\rho_0} \frac{\partial p'}{\partial z} + \frac{\rho'}{\rho_0} g = 0$$  \hspace{1cm} (1)

$$\frac{1}{c^2} \frac{\partial p'}{\partial t} - \frac{\partial p'}{\partial t} + \frac{\rho_0 N^2}{g} w = 0$$  \hspace{1cm} (2)

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial z} (\rho_0 w) = 0$$  \hspace{1cm} (3)
where $\rho'$ and $p'$ are small perturbations of density and pressure, respectively, from a hydrostatic reference state. The reference state density $\rho_0$ is a function of height and is given by $\rho_0(z) = \rho_0(0) \exp(-z/H_s)$. $H_s = g/(RT)$ is the scale height. The buoyancy frequency $N^2 = -g(1/H_s + g/c^2)$. $c = \sqrt{(\gamma RT)}$ is the speed of sound and $\gamma = c_p/c_v$, where $c_p$ and $c_v$ are, respectively, the specific heats at constant pressure and volume, $R$ is the gas constant and $T$ is temperature. Wave solutions of the form,

$$w = w^* \exp(z/2H_s) \exp(i(kz - \sigma t))$$

$$\rho' = \rho^* \exp(-z/2H_s) \exp(i(kz - \sigma t))$$

$$p' = p^* \exp(-z/2H_s) \exp(i(kz - \sigma t))$$

where $w^*$, $\rho^*$ and $p^*$ are constant coefficients with dimensions of the respective variables, give the dispersion relation

$$\sigma^2 = c^2\{k^2 + 1/(2H_s)^2\}$$

for non-stationary solutions. The solutions are referred to as acoustic waves for $\sigma > N_A$, where $N_A = c/(2H_s)$ is called the acoustic cut-off frequency ($N_A = 1.11N$ for a perfect diatomic gas (Gill 1982)). The vertical velocity associated with these waves increases as $\exp(z/2H_s)$. On the other hand, the mass flux will change with height as $\rho_0 w \propto \exp(-z/H_s) \exp(z/2H_s) = \exp(-z/2H_s)$. This result suggests that the e-folding length for the vertical mass flux is $2H_s$. Therefore, for a low-level heat source, the vertical mass flux is expected to decrease quite rapidly with height.

Although the terminology ‘wave’ is used to refer to these solutions (Lamb 1932) they are not ‘simple’ wave solutions due to the exponential dependence on $z/2H_s$. For the purposes of this study the important point is that this system is hyperbolic, so that information propagates at a finite speed that is of the order of the speed of sound. A low-level heat source will result in a compression of the air aloft and hence a mass flux and a total-energy transfer. In this study it is emphasized that this compressional effect needs to be taken into account when describing the transfer of mass and total energy.

(b) Lamb waves

For the first experiment a heat source is prescribed at the surface in a finite region to represent the sensible-heat flux from a land surface with ocean on either side. For this case there are both vertical and horizontal mass fluxes associated with compression waves. In discussing the horizontal mass flux it is helpful to draw on the results of analytic studies of Lamb waves. These waves have no vertical velocity and travel at the speed of sound. Their properties have been discussed by numerous researchers (see, for example, Lamb 1910, 1932; Taylor 1936; Lindzen and Blake 1972).

The vertical structure of Lamb waves for an isothermal atmosphere is given by:

$$u' \propto \exp\left(\frac{(\gamma - 1)z}{\gamma H_s}\right)$$

$$p' \propto \exp\left(-\frac{z}{\gamma H_s}\right)$$

$$\rho' \propto \exp\left(-\frac{z}{\gamma H_s}\right).$$

The perturbation horizontal velocity increases with height, whereas the perturbation pressure and density decrease with height.
Rapidly propagating pressure pulses produced by volcanic eruptions, meteorite impacts and nuclear explosions have been identified as Lamb waves (see, for example, the discussion by Gill (1982)). Lindzen and Blake (1972) investigated dissipative effects and estimated decay times of 10–15 days. The thermal compression waves discussed in this study are similar, but produced by much weaker forcing. The response to a net heat input within some region is to produce a compression wave that results in a net mass transfer away from the source. These thermal compression waves are not periodic waves and hence differ in this respect from what are normally considered Lamb waves, which are usually assumed to be periodic in theoretical studies.

(c) Conceptual model

The energy per unit volume in a region can be changed by a distinctly different process than that responsible for producing a significant temperature change, as can be illustrated by a simple conceptual model. Consider uniformly heating gas 1 on one side of an insulated container (see Fig. 1) which is separated from gas 2 on the other side by a movable frictionless partition. As heat is added the pressure in gas 1 will increase. This

![Diagram](image)

Figure 1. Schematic of a thermally insulated gas divided by a movable partition. (a) Initial state. (b) After heat input.

will cause gas 1 to expand and gas 2 to be compressed. If the heating is discontinued the partition will come to an equilibrium position, as shown in Fig. 1(b), such that the pressure in gas 2 equals the pressure in gas 1. The temperature of gas 2 will have increased but will be considerably less than that of gas 1. Since the internal energy per unit volume \((c_v p/R)\) is only a function of pressure, it will be the same in gas 2 as in gas 1.

The compression of gas 2 is essentially a sound-wave response, since the process is described by the sound-wave equation derived from Eqs. (25)–(27) of Part I, for
$Q_m = 0$ (where $Q_m$ is the heating rate per unit mass) and suitable boundary conditions. As the partition moves from left to right it imparts momentum to the adjacent molecules in gas 2, which causes a wave of compression to travel through gas 2 at the speed of sound. This reflects backwards and forwards off the lateral boundaries. At some fixed point in gas 2 the pressure increases in a step-wise manner every time the wave front passes; but reflections occur so frequently that for all intents and purposes the pressure increases uniformly throughout gas 2. The energy is increased within a fixed volume of gas 2 mainly because there is an increase in the number of molecules within the volume. In gas 1 the number of molecules decreases in a fixed volume, but this is more than compensated for by the increase in the mean energy of the molecules.

Now consider removing the partition. The gases could be allowed to reach thermal equilibrium by a slow conductive process. Alternatively, this process could be speeded up by mixing with a stirrer, as long as this does not impart significant energy to the gas. In either case the final temperature will be uniform and the pressure will be unchanged. The gas on the right side will now be warmer than before conduction or mixing took place, but this will not have resulted in a total-energy transfer from left to right since $c_p\rho/\mathcal{R}$ is unchanged. Therefore, if a fixed volume on the right side of the container is considered, an increase of the total energy in that volume occurred during the first stage, when gas 2 was compressed. In the second stage the temperature of the gas in this fixed volume increased significantly, but this was not associated with any change in the total energy within that volume. In experiment 2 it will be shown that this is an analogue of what happens in a thermally driven boundary layer.

If one is interested in a heat-transfer problem where the left wall of the container is heated and energy is extracted at the right wall, it is important to consider the mean molecular energy. Heat transfer from the gas to the right wall of the container is dependent on the molecules of the gas adjacent to the wall having more energy (i.e. the gas having a higher temperature) than the molecules composing the wall. The rate at which energy can be extracted depends on the thermal conduction of the gas, or the degree of mixing. However, the transport of the quantity total energy takes place much more quickly, at the speed of sound. The net energy within a fixed volume of a gas depends on both the temperature and the number of molecules. Both effects need to be taken into account when determining energy budgets.

\[(d) \quad \text{Turbulent heat flux}\]

The rate at which heat is transferred across a horizontal area has been stated to be $\int c_p\rho w T dx dy$ by Taylor (1915), Montgomery (1948), Fleagle and Businger (1963), and others. Near the surface this expression is almost identical to the vertical flux appearing in the total-energy equation (Eq. (5) of Part I, for a dry atmosphere). The flux per unit area can be written $c_p\bar{\rho}w\bar{T}$, where the overbar denotes a horizontal average and $c_p$ has been assumed constant. In this paper, $c_p\bar{\rho}w\bar{T}$ will be referred to as the sensible-energy flux, and denoted as $\mathcal{F}_s$. This expression has been used as the starting point for the derivation of a turbulent eddy transfer of heat in a thermally driven mixed layer (see, for example, Fleagle and Businger 1963). Variables are expanded into the sum of a horizontal average and deviations from this average (denoted by double primes), giving:

$$\mathcal{F}_s = c_p\bar{\rho}w\bar{T} = c_p[\bar{\rho}w + (\rho w)'][\bar{T} + T'] = c_p[\bar{\rho}w\bar{T} + (\rho w)'\bar{T}']$$

(11)

where the Reynolds averaging assumption has been made (i.e. $\langle \rho w \rangle'' = 0$; $\bar{T}'' = 0$). The first term of this expression is neglected based on the assumption that $\bar{\rho}w = 0$. The
remaining term $c_p (\rho w)''T''$ has been referred to as the turbulent heat flux, or the eddy sensible-heat flux. Further expanding, noting that if $\bar{w} = 0$, then $(\rho w)'' = \rho \bar{w} + \rho' \bar{w} + \rho'' w''$, gives

$$c_p (\rho w)''T'' = c_p \rho \bar{w}''T'' + c_p \rho \omega''T'' + c_p \rho'' w''T''.$$  \hspace{1cm} (12)

The second and third terms are usually ignored (Fleagle and Businger 1980), giving

$$H_t = c_p \rho \bar{w}''T''$$  \hspace{1cm} (13)

as an expression for the turbulent heat flux. Often in observational studies time averages rather than spatial averages are considered. A point we wish to emphasize in this study is that although the turbulent heat flux $H_t$, may be numerically equivalent to the sensible-energy flux $F_s$, in the case of a thermally driven mixed layer, they are not synonymous and represent the fluxes of distinctly different quantities. The main assumptions made in the derivation of Eq. (13) is that the physical mechanism responsible for the sensible-energy flux $c_p \rho \bar{w}''T''$ is turbulent eddies and that there is no net mass transfer across the averaging surface. However, the conceptual model discussed in the last section indicates that the flux $c_p \rho \bar{w}''T''$ occurs when the heated air expands and the unheated air is compressed, and it is not identical with the turbulent flux $c_p \rho \bar{w}''T''$ which occurs when the gas is stirred. Iribane and Godson (1973, section 8.15) discuss the change in gravitational potential energy that occurs as the gas in a vertical column is heated (see also Johnson 1970). They state that, as gas expands it lifts the mass centre of a vertical column, which implies an increase in potential energy. They also discuss heating the lower part of the column, and the resultant change in potential energy of the upper part of the column. Hence, they allude to a mechanism for the vertical transfer of total energy which is distinctly different from a turbulent eddy flux and involves expansion and compression of air in different regions of the column. On the other hand, Oort and Peixoto (1983) discuss the vertical flux $\rho \bar{w}(c_p T + gz)$ (Eq. (4.14b) of that paper) and state that the main mechanisms for the vertical transport of total energy are organized convection, cumulus activity and small-scale turbulence. Therefore, two different mechanisms have been proposed for the vertical transfer of total energy, one involving compression waves and the other involving an eddy transfer. If these transfers are numerically equivalent, then in practice this may not be a problem, as long as the conditions for which they are equivalent are clarified. These considerations point out that there is an ambiguity in the use of the term 'heat flux' in a fluid, since it is used to describe the flux $c_p \rho \bar{w}''T''$, which can be associated with a transfer of total energy at the speed of sound, as well as the flux $c_p \rho \bar{w}''T''$, which is a transfer associated with the advective motions of turbulent eddies (similar considerations apply to the horizontal fluxes).

The equation for total energy is of a flux conservative form. From the previous discussion the vertical turbulent heat flux $c_p \rho \bar{w}''T''$ does not appear to be identical to the sensible-energy flux $c_p \rho \bar{w}''T''$ which appears in the total-energy equation. The question arises as to whether it is a measure of the flux of a conserved quantity and why it is numerically equivalent to the upward total-energy transfer in a thermally driven mixed layer? Consider the thermodynamic equation,

$$\frac{Q_m}{T} = c_p \frac{dT}{dt} - \frac{R}{p} \frac{dp}{dt}$$  \hspace{1cm} (14)

For a reversible process the heating rate can be expressed in terms of the entropy per unit mass, $Q_m = T dS/dt$. Entropy, $S$, has an absolute value and is a measure of the number of accessible quantum states. However, in many applications only entropy differences are of importance. In meteorology the potential temperature is defined as
\[ \theta = T \left( \frac{p_r}{p} \right)^{\alpha / p} \]  

(15)

where \( p_r \) is a reference pressure, usually taken to be 10^5 Pa.

From Eq. (14) the potential temperature is related to the heating rate by,

\[ \frac{d \ln \theta}{dt} = \frac{Q_m}{c_p T} \]  

(16)

and to the entropy by,

\[ S = c_p \ln \theta + \text{constant}. \]  

(17)

Consider the function

\[ r = c_p \ln \left( \frac{\theta}{\theta_s} \right) \]  

(18)

where \( \theta_s \) is a constant. Then

\[ \frac{dr}{dt} = \frac{dS}{dt} = \frac{Q_m}{T}. \]  

(19)

Hence, \( r \) is a thermodynamic function which differs from the absolute entropy by a constant. For convenience, the quantity \( r \) will be referred to as the reduced entropy. Multiplying Eq. (19) by the density and using the continuity equation gives the flux conservative equation:

\[ \frac{\partial}{\partial t} (\rho r) + \nabla \cdot (\rho u r) = \frac{\rho Q_m}{T}. \]  

(20)

Let \( \theta_s \) be the base-state value of \( \theta \) at the surface and consider small perturbations \( \theta' \) such that \( \ln(\theta/\theta_s) = \ln(1 + \theta'/\theta_s) \approx \theta'/\theta_s \). Also, let \( \rho = \rho_0(z) + \rho' \), then

\[ \frac{\partial}{\partial t} \left( c_p \rho_0 \frac{\theta'}{\theta_s} \right) + \nabla \cdot \left( c_p u \rho_0 \frac{\theta'}{\theta_s} \right) \approx \frac{\rho_0 Q_m}{T} \]  

(21)

where second-order terms \( \rho' \theta' \) have been neglected, which is a good approximation for sufficiently small perturbations. The expression \( u c_p \rho_0 \theta'/\theta_s \) is an approximation to the reduced entropy flux, and for a typical atmospheric sounding should be fairly accurate at low levels where \( \theta \approx \theta_s \). Near the surface where \( p \approx p_r, T \approx \theta_s \), from the definition of \( \theta \) (Eq. (15)). Also, \( \theta' \approx T' \) near the surface, which can be justified from the linearized form of the equation of state, \( \theta'/\theta_0 = -R \rho'/\rho_0 + c_o T'/(c_p T_0) \), and the substitution of \( \rho'/\rho_0 \approx -T'/T_0 \), valid for shallow convection (Dutton and Fitch 1969). Therefore, close to the surface,

\[ \frac{\partial}{\partial t} (c_p \rho_0 T') + \nabla \cdot (u c_p \rho_0 T') \approx \rho_0 Q_m. \]  

(22)

Horizontally averaging, the vertical flux at some level near the surface is:

\[ h_i = c_p \rho_0 w T. \]  

(23)

The quantity \( h_i \) is similar to the turbulent heat flux \( H_i \), except perturbations from a base-state density \( \rho_0(z) \) and temperature \( T_0(z) \) are considered, rather than from a horizontal average (the vertical velocity has not been expanded into a base state and perturbation,
since a zero base-state vertical velocity would be the obvious choice, leaving the expression unchanged). For the purpose of analysing the results of the numerical simulation of a thermally driven mixed layer (section 4(b)) it is more convenient to use base-state values that are equal to the initial model values, rather than horizontal averages which have a slight time dependence. Comparison of $h_i$ with $H_i$ for this simulation showed differences of only a few per cent. Although the quantity $h_i$ has been put in units of energy, it does not necessarily represent the physical mechanism for total-energy transport since it was derived from the equation for conservation of entropy. It is a flux which changes the mean molecular energy, or temperature, in a volume. This can be compared with the vertical sensible-energy flux $c_p \rho w T$ which, when $\rho$ and $T$ are decomposed into base state and perturbed quantities ($\rho = \rho_0(z) + \rho'$; $T = T_0(z) + T'$) and $\rho' T'$ is neglected, is

$$F_s = c_p \rho w (\rho_0 T_0 + \rho' T_0 + \rho_0 T').$$  \hspace{1cm} (24)

It is tempting to combine the first two terms, using the condition of no net mass flux,

$$\overline{\rho w} = \rho_0 \overline{w} + \rho' \overline{w} = 0$$  \hspace{1cm} (25)

which suggests that $F_s \sim h_i$. This was the condition used in the derivation of Eq. (13) for the turbulent heat flux $H_i$. However, as shall be seen, this obscures the physical mechanism for the transfer of total energy. For the case of the conceptual model presented in the previous section, the mass flux is not equal to zero when the initial expansion of gas 1 and compression of gas 2 takes place. Neither is it zero when conduction or mixing occurs. The final state is one of no net mass transfer, but mass transport did occur during the different stages, only one of which resulted in a total-energy transfer.

In the second experiment (section 4(b)) the terms in Eq. (24) are determined from a numerical simulation, and it will be shown that compression waves produce a large flux $c_p \overline{w} \rho_0 T_0$ (the first term of Eq. (24)). On the other hand, mixing by buoyant thermals does not produce a large contribution to $F_s$ since the second and third terms, $c_p \overline{w} \rho_0 T'$ and $c_p \rho_0 \overline{w} T'$, respectively, tend to cancel, because $\rho'/\rho_0 = -T'/T_0$ within updraughts and downdraughts.

3. DESCRIPTION OF EXPERIMENTS

A radiative upper-boundary condition for sound waves has been used by Tripoli and Cotton (1982). A slightly different radiative boundary condition is used in this investigation which is based on the analytic solution discussed in section 2(a). According to Eq. (4) the local rate of change of vertical velocity for a propagating wave in an isothermal atmosphere is given by:

$$\frac{\partial w}{\partial t} = -c \frac{\partial w}{\partial z} + \frac{cw}{2H_s}. \hspace{1cm} (26)$$

This can be verified by direct substitution and where the approximation has been made that $c \sim \sigma/k$. This approximation is valid for wavelengths that are small compared with $4\pi H_s \sim 100$ km, as can be seen from the dispersion relation obtained in section 2(a) (Eq. (7)). For a simple wave, which does not change its shape as it propagates, the second term on the right-hand side of Eq. (26) would not be included. However, for vertically propagating waves the vertical velocity increases with height. This equation is applied at the upper boundary of the model, in addition to a Rayleigh friction layer applied at the upper ten levels, which is included to damp internal gravity waves. This radiative boundary condition appears to become unstable eventually to internal gravity waves, so
it is necessary to use it in conjunction with the Rayleigh friction layer. A radiative boundary condition also allows some mass transfer at the top boundary. For comparison, simulations were also run with a rigid lid and with a deep Rayleigh friction layer with a very small relaxation time-scale to damp compression waves.

The temperature profile used in this study is the US standard profile shown in Fig. 2. The first experiment has 150 horizontal grid points and 25 vertical grid points. The horizontal grid increment is 50 km and the vertical grid increment 1 km. A heat source of magnitude 0.001 K s\(^{-1}\) is prescribed at the central 20 grid points and at the lowest level above the surface. It is applied for 1 h after which it is turned off. The simulation is then run for a further 1 h. A comparison is made with a simulation using the standard version of the Regional Atmospheric Modeling System (RAMS) of the Colorado State University.

![Temperature profile](image.png)

Figure 2. Initial temperature profile (US Standard Atmosphere).

The second experiment is a two-dimensional large-eddy simulation with horizontally uniform heating prescribed at the lowest level above the surface. The horizontal and vertical grid increments are 400 m. There are 30 horizontal grid points and 60 vertical levels. Lateral boundaries are periodic. The magnitude of the prescribed heating is 0.002 K s\(^{-1}\). This heating rate is quite large so that the mixed layer will develop rapidly. The simulation is run for 2 h. It should be kept in mind that this is an idealized experiment to illustrate the basic physical processes taking place, and that compromises have been made because the time step used in the model (0.25 s) is very short.

4. Results

(a) Experiment 1

Figures 3(a), (b), (c), (d) and (e), show the horizontal mass flux, vertical mass flux, perturbation temperature, perturbation pressure and perturbation density, respectively, at \(t = 900\) s for experiment 1. Sea-breeze circulations have begun to develop at the coasts.
Figure 3. Results for the heated land surface at $t = 900$ s for experiment 1. (a) Horizontal mass flux (contour interval is 0.02 kg m$^{-2}$ s$^{-1}$). (b) Vertical mass flux (contour interval is 0.002 kg m$^{-2}$ s$^{-1}$). (c) Perturbation temperature (contour interval is 0.2 K). (d) Perturbation pressure (contour interval is 8 Pa). (e) Perturbation density (contour interval is $60 \times 10^{-6}$ kg m$^{-3}$). The label scale is 10$^6$. 
(in this and subsequent figures the land is indicated by the broad solid line). However, broader deeper regions of off-shore flow are evident which are propagating at the speed of sound. These regions are propagating inland as well as off shore. The vertical mass flux is strongest within the sea-breeze circulations. However, there is a deep region of weak upward motion above the land surface associated with thermal compression waves. The upward mass flux due to thermal compression waves is much weaker than the horizontal flux (note that different contour intervals are used). The surface temperature over land has increased uniformly. The perturbation pressure is positive everywhere; however, relative surface lows occur just on shore. The positive pressure perturbations extend off shore and delineate the position of the compression-wave front. The perturbation density has increased exterior to the heat source. The contour interval used is too small to show the variation within the heat source region, but it is similar, though of opposite sign, to the perturbation temperature. Although the density has decreased over the centre of the land owing to an upward mass flux, the surface-pressure perturbations are still positive at this time.

Figures 4(a), (b), (c), (d) and (e) show the horizontal mass flux, vertical mass flux, perturbation temperature, perturbation pressure and perturbation density, respectively, at \( t = 3600 \) s. Note that the contour intervals have been increased from those used at \( t = 900 \) s. The sea-breeze circulations have strengthened considerably, the surface temperature has continued to increase, and the surface pressure has decreased over the whole of the land surface. The perturbation density shows relatively strong positive perturbations at the top of the sea-breeze circulation updraughts.

After an hour the heating is terminated. Figures 5(a), (b) and (c) show the perturbation temperature, perturbation pressure and perturbation density, respectively at \( t = 7200 \) s. The perturbation temperature field is very similar to that an hour earlier. The perturbation pressure field shows that two oppositely moving compression waves have formed, leaving an enhanced region of low pressure at the centre of the domain compared with an hour earlier. The on-shore directed pressure gradient continues to drive sea-breeze circulations (not shown). The perturbation density within the compression waves is largest near the surface. The strongest perturbations occur within the heated region and in the sea-breeze circulations.
Figure 4. Results for the heated land surface at $t = 3600$ s for experiment 1. (a) Horizontal mass flux (contour interval is $0.04$ kg m$^{-2}$ s$^{-1}$). (b) Vertical mass flux (contour interval is $0.006$ kg m$^{-2}$ s$^{-1}$). The label scale is $10^6$. (c) Perturbation temperature (contour interval is $0.4$ K). (d) Perturbation pressure (contour interval is $16$ Pa). (e) Perturbation density (contour interval is $8.0 \times 10^{-6}$ kg m$^{-3}$). The label scale is $10^6$. 

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In section 2(b) the vertical structure of Lamb waves was discussed. For an isothermal atmosphere the perturbation pressure and density decrease with height as \( \exp(-z/(\gamma H)) \). This is, approximately, also the case for the horizontally propagating thermal compression waves generated in the numerical simulation. The analytic solution gives the magnitude of the horizontal velocity increasing with height as \( \exp((\gamma - 1)z/(\gamma H)) \), and this is approximately the case for the numerical simulation (not shown). Hence, the vertical structure of these thermally forced compression waves is similar to that of Lamb waves. Lamb waves are normally considered to be periodic in theoretical studies, unlike the compression waves generated in this numerical simulation.

A simulation using a rigid lid at \( z = 50 \) km, and a 25 km deep Rayleigh friction layer to damp compression waves gave similar results to that using the radiative boundary condition. It is also of interest to compare the results of this simulation with an identical one carried out with the standard version of the RAMS. Figure 6 shows the perturbation pressure for this case at \( t = 900 \) s. There is a uniform decrease in pressure over the land surface which is quite different from the solution for the fully compressible RAMS model at this time (Fig. 3(d)). However, the sea breezes develop in a similar way, and at \( t = 7200 \) s there is no discernible difference in the circulation over land. When thermal compression waves are explicitly simulated, the lowering of surface pressure over the land is matched by compensating increases in surface pressure elsewhere. However, when they are eliminated from the governing equations there are no compensating increases in surface pressure. Therefore, if the density is diagnosed from the fields of pressure and temperature for the standard version of the RAMS, it will be found that mass is not conserved.

(b) Experiment 2

In experiment 2 the mechanism for the vertical transfer of total energy in a thermally driven mixed layer was investigated. This experiment was run with both a radiative upper-boundary condition and a rigid lid. Since the radiative boundary condition leads to some energy loss from the domain, results are presented for the rigid-lid case without a Rayleigh friction layer. For the rigid-lid case there is continual reflection of the compression-wave front at the upper and lower boundaries; however, the development of the mixed layer is very similar to the radiative boundary-condition simulation.
Figure 5. Results for the heated land surface at $t = 7200$ s for experiment 1. (a) Perturbation temperature (contour interval is 0.4 K). (b) Perturbation pressure (contour interval is 16 Pa). (c) Perturbation density (contour interval is $80 \times 10^{-8}$ kg m$^{-3}$). The label scale is 10$^6$. 
During the first 4500 s the perturbed fields are horizontally homogeneous. Vertical profiles of perturbation temperature, perturbation pressure and perturbation density, at $t = 4500$ s, are shown in Figs. 7(a), (b) and (c) respectively. The lowest model level above the surface ($z = 200$ m) is 9 K warmer than the initial state. Large temperature changes are confined to the lowest level. The perturbation pressure increases rapidly with height at low levels, peaking at 600 m and then slowly decreasing. The perturbation density is large and negative at low levels and weakly positive aloft.

Figure 8 shows the vertical-velocity field at $t = 5400$ s. Weak thermals have developed in the lowest kilometre. Soon after this the strongest thermals develop (maximum vertical velocities of around 8 m s$^{-1}$) and the mixed layer grows rapidly. Figures 9(a), (b), (c) and (d) show the vertical velocity, perturbation temperature, perturbation pressure and perturbation density, respectively, at $t = 7200$ s. Maximum vertical velocities in the thermals are around 4 m s$^{-1}$ and they penetrate above $z = 2$ km. The magnitude of the perturbation temperature is weaker than at $t = 4500$ s, but warming occurs through a much deeper layer. Weak negative perturbations occur above the strongest updraughts. The perturbation pressure gradient is also much weaker at low levels than at $t = 4500$ s. The maximum pressure perturbations occur at the top of the mixed layer. The density has decreased throughout the depth of the mixed layer with lowest values at the surface. Above the mixed layer the density changes are weakly positive.

The mixed layer has developed in a way which might be expected. The lowest level warms until the lapse rate becomes superadiabatic and unstable to small perturbations. There is then a sudden transition as thermals develop, resulting in strong eddy vertical transports. However, if the real atmosphere had this temperature profile, surface heating would lead to small thermals developing earlier on and a more gradually growing boundary layer. For this simulation only relatively large thermals can be resolved, which results in the onset of convection being delayed and perhaps a more rapidly growing mixed layer than would actually occur. Nevertheless, rapidly growing mixed layers do occur when low-level inversions are broken, so this is not an altogether unusual situation.

The sudden transition to a rapidly growing mixed layer enables the investigation of whether the vertical transports associated with the thermals are responsible for an upward
Figure 7. Vertical profiles for the thermally driven mixed layer at $t = 4500$ s for experiment 2. (a) Perturbation temperature. (b) Perturbation pressure. (c) Perturbation density.
total-energy transfer. Figures 10(a), (b) and (c) show vertical profiles of the horizontally averaged perturbation total energy (internal + potential + kinetic), perturbation internal energy and perturbation potential energy, respectively, at $t = 4500$ s, $5400$ s, $6300$ s and $7200$ s. It is evident from the profile at $t = 4500$ s that total energy has been transported to upper levels even before thermals develop. This is due to compression of the air aloft, which results in an increase in internal and potential energies. The maximum in perturbation total energy occurs at the top of the boundary layer, where the pressure perturbation, and hence internal energy change, is largest.

The perturbation potential energy is negative at low levels where expansion of air has occurred. This layer of negative perturbation potential energy deepens with time as the mixed layer grows. The potential energy increases aloft and peaks at a height of 12.5 km approx. The kinetic energy (not shown) is largest in the mixed layer, but it is negligible compared with the changes in internal and potential energies.

The total energy input into a 1 m wide column at $t = 7200$ s is $6.55 \times 10^6$ J m$^{-1}$, whereas the average energy in a 1 m wide column in the domain at this time is $6.45 \times 10^6$ J m$^{-1}$. Exact agreement would not be expected, since model diffusion will lead to some energy dissipation through the $\varepsilon$ term of Eq. (3) of Part I. This total energy is composed of $4.792 \times 10^6$ J m$^{-1}$ of internal energy and $1.647 \times 10^6$ J m$^{-1}$ of potential energy. Using the hydrostatic approximation it can be shown that the potential energy $E_p$ in a column should be related to the internal energy $E_i$, by

$$E_p = -H p'(H) + \frac{R}{c_o} E_i$$  (27)

where $H$ is the height of the domain. At the top of the domain $p'(H) = 16$ Pa. Substituting $E_i = 4.792 \times 10^6$ J m$^{-1}$ into Eq. (27) gives $E_p = 1.55 \times 10^6$ J m$^{-1}$, which is in fair agreement with the simulated value for the potential energy.

In analogy to the energy budget, perturbations of the reduced entropy per unit volume from the initial state are considered. Vertical profiles of the horizontally averaged perturbation reduced entropy are shown in Fig. 11, at $t = 4500$ s, $5400$ s, $6300$ s and $7200$ s. At $t = 4500$ s the perturbation reduced entropy at the lowest level is large owing
Figure 9. Simulated fields for the thermally driven mixed layer at $t = 7200\,\text{s}$ for experiment 2. (a) Vertical velocity (contour interval is 0.8 m s$^{-1}$). (b) Perturbation pressure (contour interval is 20 Pa). (c) Perturbation temperature (contour interval is 0.4 K). The label scale is 10$^3$. 

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Figure 10. Horizontally averaged energy profiles for the thermally driven mixed layer at $t = 4500$ s, $5400$ s, $6300$ s and $7200$ s for experiment 2. (a) Perturbation total energy. (b) Perturbation internal energy. (c) Perturbation potential energy.
to warming. It is larger still at $t = 5400$ s. As the mixed layer grows, the perturbation reduced entropy decreases at the lowest level and increases above, becoming negative just above the mixed layer.

Figure 12 shows the approximation to the reduced-entropy flux $\rho_0 c_p w\theta'/\theta_s$ (the vertical component of the flux term in Eq. (21)) at $t = 5400$ s, 6300 s and 7200 s (at $t = 4500$ s there is no discernible flux). At $t = 5400$ s the vertical gradient is positive near the surface and negative just above, which is consistent with the reduced entropy changes at $t = 6300$ s (Fig. 11). At $t = 6300$ s and 7200 s large oscillations occur above the mixed layer; these are internal gravity waves. Within the mixed layer and the region occupied by internal gravity waves, comparison with the actual reduced entropy flux shows that this approximation to the reduced entropy flux is a good one.

The average reduced-entropy change in a 1 m wide column at $t = 7200$ s is $2.22 \times 10^4$ J m$^{-1}$, whereas the entropy input is $2.19 \times 10^4$ J m$^{-1}$. These are in reasonable agreement. The reduced-entropy equation (Eq. (20)) neglects a diffusion term, which may account for the slight discrepancy.

In section 2(d) an equation was derived for the vertical sensible-energy flux (Eq. (24)). If the horizontally averaged mass flux was zero then the first two terms of Eq. (24) would cancel, giving $c_p \rho_0 w\theta'$, which is $h_i$ (Eq. (23)), a good approximation to the usual expression for the turbulent heat flux $H_i$ (Eq. (13)). Figures 13(a), (b), (c) and (d), show the individual terms in Eq. (24) at $t = 4500$ s, 5400 s, 6300 s and 7200 s respectively. The sum of these is the sensible-energy flux. At $t = 4500$ s the term $c_p \rho_0 T_{0} \tilde{w}$ is the only one making a significant contribution, consequently the other terms are not shown in this figure. An upward transfer of mass is occurring as the heated air is expanding and the air above is compressed.

At $t = 5400$ s there is a net downward sensible-energy flux at low levels (Fig. 13(b)).
This is coincident with the strong development of thermals and a net downward mass flux at low levels as less-dense air near the surface is replaced by relatively cooler heavier air from aloft. Notice that the second and third terms of Eq. (24) very nearly cancel one another, and this is also true at later times. At these later times it can be seen that there is a net upward flux of total energy in the mixed layer by virtue of the first term of Eq. (24). The third term, which is associated with buoyant thermals and has been suggested as the physical mechanism for the total-energy transport, is offset by the second term, which is also associated with buoyant thermals. This second term is large because buoyant thermals result in a downward eddy mass flux.

Comparing the approximate entropy flux (Fig. 12) with the $\rho_0 c_p w T'$ term in Fig. 13, it is evident that they are almost identical (within, of course, a factor of $\theta_z$). Therefore, these results suggest that buoyant thermals are mainly responsible for an entropy transfer rather than a net transfer of total energy. It is emphasized that this is not a measure of the absolute entropy transfer. Rather it has been shown that the entropy transfer due to buoyant thermals is one which results in changes in the temperature or mean molecular energy within a volume. Consideration of the flux conservative form of the equation governing the entropy per unit volume $\rho S$ (analogous to Eq. (20) for reduced entropy) suggests that the transfer associated with buoyant thermals is partially counteracting the upward transfer that occurs as the low-level air expands and the air above is compressed. When the thermals first developed there was actually a net downward transfer of total energy at low levels, even though the reduced-entropy flux (or eddy sensible-heat flux) was upwards. This was only a short-lived episode.

At first sight it appears that the eddy sensible-heat flux may not be a good measure of the total-energy flux. However, the eddy sensible-heat flux is usually measured very close to the surface, at $z \sim 10$ m. This is a small fraction of the mixed-layer depth.
Figure 13. Comparison of terms in the sensible-energy flux for the thermally driven mixed layer. (a) $t = 4500$ s, (b) $t = 5400$ s, (c) $t = 6300$ s and (d) $t = 7200$ s for experiment 2.
Therefore, there cannot be a large storage of reduced entropy within this very shallow layer and most of the incoming flux from the surface must exit the top of the layer. If the heat flux due to molecular conduction from the surface to the air is $F_Q$, then the surface entropy flux is $F_Q/T$. Therefore, the reduced-entropy flux due to turbulent eddies at the top of the shallow layer is approximately given by,

$$\int_A \left( wc_p \rho_0 \frac{\theta'}{\theta_s} \right) d \theta \sim \int_A \frac{F_Q}{T} d \theta$$  \hspace{1cm} (28)

where $A$ is the horizontal averaging area, $d$ is the depth of the layer (e.g. \(~10\) m), and it has been assumed that horizontal fluxes can be neglected. Since $T(z=0) \sim \theta_s$, then

$$\int_A \left( wc_p \rho_0 \theta' \right) d \theta \sim \int_A F_Q d \theta$$  \hspace{1cm} (29)

which shows that the eddy sensible-heat flux at $z = d$ is approximately equal to the surface heat flux. As far as the total energy is concerned, most of it is transported across the 10 m level as the heated air expands and the air above is compressed. Therefore, the total-energy flux at $z = d$ is also a good approximation to the surface heat flux. Consequently, Eq. (29) shows that the eddy sensible-heat flux measured at 10 m above the surface should be a good approximation to the total-energy flux.

5. Conclusions

The first experiment focused on the role thermal compression waves have in mass adjustment in the atmosphere. The vertical mass flux resulting from a low-level heat source decreases rapidly with height. Most of the mass flux occurs in the horizontal. The vertical structure of the compression wave produced in experiment 1 is similar to that of a Lamb wave. As mass is removed from the heated region, buoyancy effects lead to the development of sea breezes. There is no discernible difference between the sea-breeze circulation for the fully compressible model and the standard version of the RAMS. It can be shown that for the system of equations used in the standard version of the RAMS, which eliminate thermal compression waves, a heat source implies that mass is removed from the heated region, without there being a corresponding increase in mass elsewhere. Results of this investigation support the view that this is a good approximation for the earth's atmosphere when describing meteorologically significant motions and, therefore, helps to verify the approximations used to derive the anelastic system of equations (Ogura and Philips 1962; Dutton and Fitchl 1969). A comparison of analytic solutions for incompressible versus compressible systems can be found in Gill (1982). However, the role that compression waves have in the transport of mass and total energy away from a heat source does not appear to have been fully discussed. This role should at least be recognized, if correct physical interpretations of the transfer of these quantities are to be made. Hydrostatic models that retain Lamb waves should produce similar results for the first experiment. The Lamb waves produced by diabatic heating in a hydrostatic model can be considered horizontally propagating thermal compression waves. Since the fully compressible continuity equation is used, mass should be conserved. In the appendix, a perturbation form of the total-energy equation is derived from the linearized equations. This equation is of a flux conservative form, indicating that total energy should also be conserved reasonably well by hydrostatic models. Vertically propagating compression waves are eliminated from this system; however, their effects are approximated by an instantaneous hydrostatic adjustment (Johnson 1970).
Results of the second experiment indicate that compression waves are responsible for the vertical transport of total energy. A quantity called the reduced entropy was introduced and results suggest that the eddy sensible-heat flux (when put in units of entropy) can be considered to produce a transfer of this quantity rather than total energy. The following picture is suggested. At the surface, collisions of air molecules with the more energetic radiatively heated surface molecules result in a transfer of energy and entropy. The average energy of the molecules next to the surface increases. A mass adjustment occurs as the mean distance between the warm molecules increases and the air above is compressed. As the warm air near the surface becomes buoyant with respect to the air above, macroscopic overturning turbulent thermals develop which result in a downward mass transport opposing the upward mass transport that results from the expanding heated air. Lighter, warmer air near the surface is replaced by heavier, colder air from aloft. In the absence of buoyant thermals or turbulent mixing the density of the air would only decrease in a very thin layer of air (a few millimetres) next to the surface. Turbulent mixing distributes this density decrease over a much deeper layer. The total-energy input is distributed over a very deep region owing to compression of the air aloft. The buoyant thermals distribute the reduced entropy over a shallower layer (about 1–2 km), but one which is still very deep compared with the height at which the eddy sensible-heat flux is usually measured (about 10 m). Therefore, most of the surface input of total energy and reduced entropy is transported above 10 m. Since the reduced-entropy input is related to the energy input by \(1/T\), and the fractional change of temperature near the surface is small, the fluxes of these quantities are related by this factor.

The following is a summary of the main conclusions of this two-part paper:

1. In this study we have investigated the mechanism for the transfer of total energy from a small localized source, such as a thunderstorm. Results illustrate how this quantity can be effectively transferred at the speed of sound. The result that only a small fraction of the input of total energy is converted into motions that are meteorologically significant is consistent with the concept of available potential energy put forward by Margules (1903) and Lorenz (1955). However, the result that the perturbation of the total-energy field propagates at the speed of sound, rather than remaining localized in the region of the heat source, does not appear to have been fully discussed previously, although it may well have been recognized by researchers who use the classical hydrostatic system.

2. The physical significance of total-energy transport in the atmosphere may be brought into question by these results, since it appears that total energy can be transferred without significant temperature changes or atmospheric motions taking place.

3. The results of this investigation indicate that the term 'heat flux' has been used to describe the transfer of two fundamentally different quantities. The vertical sensible-energy flux \((c_p \rho Tw)\) has been referred to as a 'heat flux' (Taylor 1915; Montgomery 1948; Fleagle and Businger 1963) and in this study appears to be associated with thermal compression waves which lead to changes in the total energy per unit volume. On the other hand, the term 'heat flux' is also used to describe a flux resulting from turbulent eddies. It is suggested that the eddy sensible-heat flux can be regarded as an approximation to the flux of reduced entropy which has been put into units of energy. The horizontal flux of sensible energy \((c_p \rho T v_h)\), where \(v_h\) is the horizontal component of velocity, is also referred to as a heat flux (Priestly 1945; White 1951a, b; Oort 1971). For the simulations presented in this study, this horizontal flux is also associated with thermal compression waves.

4. Aircraft measurements of the eddy sensible-heat flux are sometimes made at a level that is a considerable fraction of the mixed-layer depth, rather than near the surface.
Results suggest that when this is the case the eddy sensible-heat flux is no longer such a good approximation to the vertical flux of total energy.

(5) This study provides an explanation as to how pressure at the surface can change even at large distances from the boundary of a region of surface heating or cooling. It also describes a mechanism to explain how thickness adjustment occurs due to diabatic heating or cooling in the free atmosphere.

(6) Results of this study indicate that numerical models that eliminate thermal compression waves cannot be expected to conserve mass and total energy in the presence of heat sources or sinks. Nevertheless, models that neglect thermal compression waves are capable of describing the generation of available potential energy that leads to meteorologically significant motions.

(7) Although the thermal compression waves investigated in this study would appear to be difficult to measure, there are observations of low-frequency infrasound waves emanating from thunderstorms for which lightning is an unlikely source mechanism (Beasley et al. 1976; Bedard et al. 1986). It is possible that this study provides an explanation for the source of this infrasound. The amplitude of a thermal compression wave produced by a thunderstorm should be directly related to the latent-heat release.

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APPENDIX

Perturbation form of the total-energy equation for the linearized hydrostatic equations

The two-dimensional linearized horizontal momentum, hydrostatic, thermodynamic and continuity equations for small perturbations from a base-state atmosphere, which is at rest, are:

\[
\frac{\partial u}{\partial t} + \frac{1}{\rho_0} \frac{\partial p'}{\partial x} = 0 \tag{A.1}
\]

\[
\frac{\partial p'}{\partial z} + \rho' g = 0 \tag{A.2}
\]

\[
\frac{\partial}{\partial t} \left( \frac{c_s p'}{R} \right) - c_p T \frac{\partial p'}{\partial t} + \frac{c_p T N^2}{g} \rho_0 w = Q_v \tag{A.3}
\]

\[
\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x} (\rho_0 u) + \frac{\partial}{\partial z} (\rho_0 w) = 0 \tag{A.4}
\]

where \(Q_v\) is the heating rate per unit volume. This is a linearized version of the equations used in hydrostatic models which retain Lamb waves. From Eqs. (A.3) and (A.4), the ideal gas law for the base-state variables \((p_0 = \rho_0 RT_0)\), and the definition \(N^2 = -g(d \ln \rho_0)/dz + g/c^2\), the following equation can be obtained,

\[
\frac{\partial}{\partial t} \left( \frac{c_s p'}{R} + \rho' g z \right) + \frac{\partial}{\partial x} (c_p T_0 \rho_0 u + g z \rho_0 u) + \frac{\partial}{\partial z} (c_p T_0 \rho_0 w + g z \rho_0 w) = Q_v. \tag{A.5}
\]
This is an equation of flux conservative form for the sum of the perturbation internal and gravitational potential energies, which is a good approximation to the perturbation total energy since the kinetic energy contribution is small. Hydrostatic models which retain Lamb waves utilize a surface-pressure-tendency equation. The linearized version of this surface-pressure-tendency equation is obtained by substituting for $\rho'$ from Eq. (A.2) into Eq. (A.4) and integrating from the surface to the top of the domain, giving

$$\frac{\partial p_{st}}{\partial t} = -g \int_{0}^{z_t} \frac{\partial}{\partial x} (\rho_0 u_t) \, dz$$

(A.6)

where it has been assumed that at the surface and at $z_t$ the vertical mass flux is zero. Hydrostatic models are often formulated in pressure or isentropic coordinates (see, for instance, Haltiner and Williams 1980). Finite-speed vertically propagating compression waves are not admitted by this system of equations. Therefore, a low-level heat source will instantaneously produce a hydrostatic adjustment throughout the depth of a column of air, rather than leading to an upward propagating compression wave (Johnson 1970). Equation (A.5) indicates that total energy should be fairly well conserved. The change in the sum of internal and gravitational potential energies in the domain is equal to the heat input, as long as there are no boundary fluxes, and this energy can be transferred in the horizontal by Lamb waves in a conservative manner. The vertical adjustment of internal and gravitational potential energies which would result from vertically propagating compression waves are approximated by an instantaneous hydrostatic adjustment.

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