Localization of optimal perturbations using a projection operator

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SUMMARY

Ensemble prediction is an attempt to estimate the probability distribution of forecast states through a finite sample of nonlinear deterministic integrations of a numerical weather-prediction model. At the European Centre for Medium-range Weather Forecasts (ECMWF) it is based on 32+1 (control) model integrations at horizontal spectral triangular truncation T63, with 19 vertical levels. The initial conditions of the perturbed forecasts are generated from optimal perturbations, which identify the directions in the phase space of the system that guarantee the maximum growth of the total energy of the perturbation over a fixed time interval. Since the ECMWF is mainly interested in predicting the atmospheric flow over the northern hemisphere, in particular over the European region, optimal perturbations are chosen to give different forecasts in this region.

One of the problems faced during the first months of ensemble prediction was that, on some occasions, the spread between the perturbed and the control forecasts appeared to be small. One case, for 14 February 1993, which represents an extreme among these cases, is analysed in detail. For that period, for three consecutive days, the two unperturbed forecasts (the control T63L19 and the high-resolution operational forecast) and all the perturbed forecasts of the ensemble system were very similar over the European region.

A second problem, closely related to the first, that had to be prevented, was the inability of the system to identify optimal perturbations which amplify over the northern hemisphere during the warm seasons, when the relative instability of the northern hemisphere is smaller than the instability of the southern hemisphere.

It is shown how the introduction of a local projection operator, which confines the region over which the optimal perturbation growth is maximized, can improve the spread among the ensemble members. Moreover, it is proven that its application avoids the occurrence of the second problem.

1. INTRODUCTION

During December 1992 two of the more important numerical weather-prediction centres started ensemble forecasting. On 7 December 1992 the National Meteorological Center (NMC) replaced the single 10-day global medium-range forecast by an ensemble of four 12-day forecasts, plus an extension to 12 days of the aviation 3-day forecast, to reach the operational configuration of 14 forecasts, originating from analysis within the most recent 48 hours, that verify over the same 10-day period (Toth and Kalnay 1993).

On 19 December 1992, at the European Centre for Medium-range Weather Forecasts (ECMWF), the new Ensemble Prediction System (hereafter EPS) started (Molteni and Palmer 1993; Mureau et al. 1993). It is based on the execution of an ensemble of 32+1 (control) nonlinear deterministic integrations of a numerical weather-prediction model (Epstein 1969; Leith 1974). The EPS is run three days a week, from each Saturday, Sunday and Monday 12 GMT analysis. Each member of the ensemble system is a 10-day time integration of a version of the ECMWF model at horizontal spectral triangular truncation T63, with 19 vertical levels. So, every week, for each of these days, 34 10-day forecasts are generated: the high-resolution ECMWF operational forecast, run at horizontal truncation T213 with 31 vertical levels, the unperturbed (control) and 32 perturbed T63L19 members of the EPS.

To create realistic perturbations that could represent the errors actually present in the analysis cycle, at the NMC they have developed a method denoted ‘breeding of growing modes’. It consists of the following steps: (a) add a small arbitrary perturbation to the atmospheric analysis, (b) integrate the model for 6 hours from both the unperturbed and the perturbed initial conditions, (c) subtract the 6-hour control forecast from the perturbed forecast, and (d) scale down the difference field to have a similar amplitude

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to the initial perturbation. Readers are referred to Toth and Kalnay (1993) for a detailed description of the NMC ensemble system.

The ECMWF approach is, instead, based on optimal perturbations, i.e. structures that grow fastest over a finite time interval. The 32 initial conditions (ICs) of the perturbed forecasts are generated adding to the unperturbed IC a linear combination of 16 optimal perturbations, automatically selected among the most unstable ones. For a comparison between the two methods of computation of the perturbations see Toth and Kalnay (1993).

The importance of optimal transient instabilities was first suggested by Lorenz (1965). Subsequently, Farrell (1982) investigated their structures in simple models, and concluded that they are likely to identify the directions in the phase space of the systems that are more active dynamically (Farrell 1988). Calculations of these finite time-interval instabilities have been performed by Borges and Hartmann (1992) using a barotropic model, and by Molteni and Palmer (1993) using T21 barotropic and T21 three-level quasi-geostrophic (QG) models. Buizza et al. (1993) computed optimal perturbations with a 19-level primitive-equation (PE) model. Comparing the effectiveness of the PE versus the QG perturbations, they found that, when added to an analysis field to define the IC for a perturbed forecast, the PE optimal perturbations were able to give a larger deviation from the unperturbed trajectory than the QG three-level optimal structures.

Note that since the fastest-growing perturbations are the singular vectors (SVs) of the propagator of the forward tangent model with the largest singular values (Buizza et al. 1993), hereafter we will use the abbreviation SVs to identify them.

Optimal perturbations can be computed, in the linear approximation, applying the adjoint technique, which uses the tangent forward and adjoint versions of the full nonlinear model. The adjoint of a dynamical model was first used for sensitivity studies by Kontarev (1980) and Hall and Cacuci (1983). Later on, Le Dimet and Talagrand (1986) proposed an algorithm, based on an appropriate use of an adjoint dynamical equation, for solving constrained minimization problems in the context of analysis and assimilation of meteorological observations. More recently, Lacarra and Talagrand (1988) applied the adjoint technique to determine optimal perturbations using a simple numerical model. Following Urban (1985) they use a Lanczos algorithm in order to solve the related eigenvalue problem (for Lanczos algorithm theory see, for example, Strang (1986)).

Since the systems studied by Borges and Hartmann (1992), and by Molteni and Palmer (1993), were not too large, they could identify the optimal perturbations using a conventional matrix algorithm. Instead of this their identification needs sophisticated algorithms when the analysed system is a PE model with a large number of degrees of freedom. Buizza (1992) and Buizza et al. (1993) applied a Lanczos algorithm to compute optimal perturbations using a T21 19-level adiabatic PE system. In their studies they applied the adjoint technique defining the norm of a perturbation as its total energy.

As regards SV sensitivity to model parametrizations, Buizza et al. (1993) studied the impact of a nonlinear normal mode (NNMI) procedure on the optimal perturbations, and they concluded that, during the SV computation, it should be restricted to not more than the five gravest modes. They also concluded that a planetary boundary-layer parametrization is essential to eliminate 'non-meteorological' low-level perturbations which arise in the adiabatic computation, but do not correspond to unstable modes of the real atmosphere. In a later paper Buizza (1994) studied this problem, and showed that the implementation of a simple vertical-diffusion and surface-drag scheme can solve it.

A crucial parameter to define when computing optimal perturbations is the optimization time interval (hereafter OTI), i.e. the time over which the growth is maximized.
Buizza (1994) concluded that time intervals shorter than 12 hours should be avoided, and suggested that the choice of time intervals longer than 36 hours could give unstable perturbations with characteristic time long enough to give large divergence among the trajectories also after the nonlinear effects become important, i.e. after 2–3 days.

The following define the ECMWF configuration used to compute the SVs, which is also used for the experiments described in this paper (exceptions will be clearly pointed out):

(i) total energy norm;
(ii) NNMI applied to the five gravest modes;
(iii) adiabatic model plus vertical-diffusion and surface-drag parametrization; and
(iv) 36-hour OTI.

At the ECMWF the computation of the optimal perturbations uses the Integrated Forecasting System (IFS), developed by the ECMWF and Metéo-France for, inter alia, the application of four-dimensional variational data assimilation (Courtier et al. 1991; Rabier and Courtier 1992). Readers are referred to Buizza (1992) for details on the computation of the SVs using a Lanczos algorithm.

Toth and Kalnay (1993) re-phrased the goal of ensemble forecasting in the following way: ‘the time evolution of the atmosphere should be a plausible member of the ensemble’. Since at the ECMWF we are mainly interested in predicting the atmospheric flow over the northern hemisphere (NH), and in particular over Europe, we would like this to be true especially for this region. To verify this we can compare the mean spread and the mean error of an ensemble experiment, computed averaging the spread (defined as the distance between a perturbed and the unperturbed integrations), and the error of all the ensemble members (in terms of root-mean-square distance). The mean spread can be taken as indicative of the typical size of the ensemble cloud, while the mean error gives an indication of the typical distance between one member and reality. Since forecast errors are due to initial-condition errors and model errors, and since we do not want to compensate for model errors perturbing the IC, we do not expect the two mean curves to have the same values, but at least to have similar behaviour.

Once an ensemble system satisfies this requirement, if forecast errors are essentially related to errors in the IC, one can expect some correspondence between the spread among the ensemble members and the skill score of the unperturbed forecast. A small spread among the EPS trajectories can indicate that the situation is very easily predictable, and so that we can expect a very skilful unperturbed forecast. On the contrary, it can indicate that the perturbations are too small, or are located in regions that do not influence the prediction over Europe. At the ECMWF we want to investigate if we can improve the EPS performances by introducing a constraint to the physical space where the perturbations can maximize their growth. Following Barkmeijer (1992), who defined a local projection operator to compute errors growing in restricted geographical areas with a barotropic model, a local projection operator (LPO) is defined to identify SVs with the maximum growth over a specified region of the physical space.

In fact, the analysis of the EPS performance during the 1992/93 winter season indicates that, on some occasions, the spread among the EPS members seemed to be too small. The first part of this work deals with this problem. Three case studies are analysed to verify the impact of the LPO on the EPS performances. In particular, attention is focused on a case study for 14 February 1993, which represents an extreme among the small-spread cases, since all the perturbed forecasts up to forecast day (hereafter fc-day) 7 seemed to be small 'variazioni sul tema' of the unperturbed forecast. The analysis of this case study will also point out the usefulness of the LPO as a diagnostic tool.
In the second part of this work a problem very closely related to the one discussed above is studied. As already mentioned, at the ECMWF we would like that, in the NH, the true state of the atmosphere is included among the forecast states, or at least is very close to one ensemble member. This explains why, when selecting 16 among the 30–35 perturbations that the IFS system provides to generate the perturbed IC, we discard the ones growing in the southern hemisphere (SH). Optimal perturbations are iteratively computed at the ECMWF, and 100 iterations are usually needed to find around 35 SVs with an acceptable precision (Buizza et al, 1993), with the number of iterations limited by computer power availability.

When the relative instability of the NH is smaller than the instability of the SH, the first 30–35 SVs identified by the IFS system appear to be characterized by maximum growth over the SH. This makes it impossible to satisfy the requirement of having forecasts differing in the NH. Later it will be shown how this problem starts being present in April–May, and becomes very important during the summer, and how the use of the LPO can avoid this problem completely.

Section 2 describes the atmospheric situation for 14 February 1993 and the following days. In section 3 the LPO is introduced. In section 4 the impact of the LPO on the SVs definition and on the forecast spread is studied. In this section some details on the automatic algorithm that selects the 16 optimal perturbations to be used to construct the perturbed IC are also given. Section 5 shows the impact of the LPO when computing the SVs for spring and summer cases. Some conclusions are drawn in section 6.

2. The 14 February 1993 Case

Figure 1 shows the 500 hPa geopotential height analysis for 14 and 21 February 1993, corresponding to fc-day 7 of the integrations started on the 14th. On the 14th a region of high pressure dominated the European area, with a south-westerly flow north of the British Isles and a trough over Russia; a zonal flow characterized the circulation over the Asian continent and western Pacific, with a deep ridge north-west of the Rockies and a region of low pressure over eastern Canada. At fc-day 2 the ridge north-west of the Rockies evolved into a blocking structure, which lasted till fc-day 7. In the mean time an anticyclonic circulation developed in the eastern Atlantic, and at fc-day 7 a strong

![Figure 1. 500 hPa geopotential height field at 12 GMT on (a) 14 February 1993 and (b) 21 February 1993. Contour intervals are drawn every 160 m.](image-url)
north, north-westerly flow characterized the flow over the British Isles. The trough over central Europe deepened during the following days, and a cut-off low developed at fc-day 9 over the Adriatic sea.

To evaluate the skill of the forecasts prepared on 14 February 1993 the anomaly correlation (AC) between the unperturbed and perturbed forecasts and the analysis, for different areas and fc-days, can be computed. In particular, the forecast skill at fc-day 7 over the European region, defined by the following coordinates: latitude between 30°N and 75°N, and longitude between 20°W and 45°E, is focused upon. Figure 2(a) shows the spread between each perturbed forecast and the control, while Fig. 2(b) shows the skill of the perturbed and unperturbed forecasts. The high-resolution T213L31 model forecast is very skillful up to fc-day 5. The control forecast is slightly worse than the high-resolution model, especially between fc-day 7 and 9 when AC values of 44% are reached. The perturbed forecasts start diverging from the control at fc-day 2, all of them characterized by worse skill scores during the following 3 days.

The mean spread and errors were computed and then compared to verify if the spread among the ensemble trajectories can be considered large enough. Figure 2(c) shows the mean spread and the mean error curves, computed averaging the ensemble AC scores. It can be clearly seen that the two curves are not very similar, especially in the 5-8 fc-day range, confirming the impression given by Figs. 2(a) and 2(b) that the spread is too small compared with the error.

Figure 3 shows the 500 hPa geopotential height forecasts over Europe, predicted by the two unperturbed and by the EPS members run with starting time 12 GMT 14 February 1993 and verifying date 21 February 1993 (t + 168 h). The first impression is that the forecasts are very similar, all characterized by a trough over northern Europe, with a north-eastern tilt, and a north-westerly wind over the British Isles. A closer inspection reveals that two EPS members (numbers 29 and 31) have a very high skill score at fc-day 7 (79% AC skill score). At fc-day 7, if compared with the unperturbed forecasts, two of the EPS members have an AC skill score equal and six better than the T213L31 forecast, and one of the EPS members has an AC skill score equal and 17 have better than the control (see Table 1). If compared with the analysis, we can say that the latter is outside the range of the ensemble forecasts.

| TABLE 1. SKILL SCORES OF THE ENSEMBLE PREDICTION SCHEME FOR 14 FEBRUARY 1993, 21 MARCH 1993, AND 28 MARCH 1993 AT FC-DAY 7 AND 10, RUN IN CONFIGURATIONS OPE AND e03 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | 14 February 1993 | 21 March 1993  | 28 March 1993  |                 |                 |
|                 | 7 day | 10 day | 7 day | 10 day | 7 day | 10 day |
| OPE             |       |        |       |        |       |        |
| AC > 70%        | 9     | 1      |       |        |       |        |
| 70% > AC > 60%  | 15    | 8      |       |        |       |        |
| Perturbed better than control | 18| 4| 25| 4| 7| 14|
| Perturbed better than T213  | 8    | 13     | 19   | 24    | 7    | 2     |
| e03             |       |        |       |        |       |        |
| AC > 70%        | 11    | 3      |       |        |       |        |
| 70% > AC > 60%  | 9     | 3      |       |        |       |        |
| Perturbed better than control | 17| 4| 25| 7| 13| 15|
| Perturbed better than T213  | 10   | 15     | 17   | 22    | 16   | 1     |
| Control AC      |       |        |       |        |       |        |
| T213 AC         | 63%   | 66%    | -40% | 49%   | 37%  | -27%  |
|                 | 71%   | 52%    | -24% | 5%    | 35%  | 2%    |

For each date and each experiment the 1st and 2nd lines of the Table show the number of perturbed forecasts with the anomaly correlation (AC) inside a specified AC interval, the 3rd and the 4th line the number of perturbed forecast better than the control and the T213 forecasts respectively. The last two lines of the Table show the AC of the control and of the T213.
Figure 2. Spread and skill scores of the OPE Ensemble Prediction Scheme 500 hPa geopotential height field for 14 February 1993. (a) Spread between each member and the control, measured as anomaly correlation; (b) skill of the T63 control (solid line with full dots), of the T213 forecast (dashed line with full squares), and of the 32 perturbed members (dotted lines); and (c) mean root-mean-square spread (dotted line) and error (solid-line) curves. The numbers on the right-hand side of (a) and (b) refer to the member number.
In conclusion, the analysis of the spread between the perturbed and the control forecasts, shown in Fig. 2, confirms that the trajectories of the EPS forecasts in the phase space of the system are very close at fc-day 7, with none of them able to develop a flow closer to the analysis (see Fig. 3).

If we consider the EPS forecasts of the integrations started on the 13th and on the 15th and verifying at the same date, the same picture can be drawn. Very low spread between the forecasts, with only three of the more recent forecasts started on the 15th giving a trough with a more correct tilt over Europe. The fact that, for the same verification date, 99 different forecasts with different ICs and starting dates gave very similar predictions suggests that the situation over Europe for the weekend of 14 February 1993 was very stable and very predictable. This was really the case, and both the control and the T213 forecasts were characterized by AC skill scores higher than average. Nevertheless, we would have liked the two mean spread and skill curves to have values that are more similar and, maybe, at least a few of the EPS forecasts to diverge more from the control and possibly to be more skillful up to fc-day 10.

The low spread between the EPS forecasts at fc-day 7, and the fact that very skillful forecasts with an AC of over 70% up to fc-day 10 are missing, can be related to the following reasons:

(i) the directions in the phase space of the system identified by the perturbations chosen to generate the perturbed initial conditions were not amplifying over Europe at fc-day 7;
(ii) the directions identified were amplifying over Europe, but their initial amplitudes were too small to generate enough spread; and
(iii) adding small, although optimal, perturbations to the unperturbed initial conditions cannot compensate for model errors.

The perturbation amplitudes are defined after comparing them with an estimate of the analysis error field, and they seem to have amplitudes similar to an estimate of the analysis error field given by the optimal interpolation (OI) procedure (see subsection 4(a)). So we will not study the sensitivity of the EPS results to the perturbation amplitude in this paper (sensitivity studies on the impact of the perturbation amplitude on the EPS performances will be presented in a forthcoming paper). The first point will be investigated in section 4, with comments on the third point in the conclusions.

3. The local projection operator

The SVs are the direction in the phase space of the system which maximize the norm of a perturbation after a time interval \( t \), called the optimization time interval:

\[
\|x(t)\| = \langle x(t); E x(t) \rangle
\]  

where the matrix \( E \) is a matrix of weight factors that in our case defines the total energy norm (see Eq. (5.1) in Buizza et al. (1993)), and where the vector \( x \) represents a perturbation which satisfies the linearized model equations:

\[
\frac{dx}{dt} = A_1 x
\]

where \( A_1 = (\partial A/\partial x)|_{x(t)} \) is the tangent operator that corresponds to the nonlinear model.
operator $A(x)$. If we indicate the resolvent of Eq. (2) by $L(t_0;t)$, the norm at time $t$ can be computed as:

$$\|x(t)\|^2 = \langle Lx_0; ELx_0 \rangle$$  \hspace{1cm} (3)

where $x_0$ is the IC. Let us suppose that the state vector of the system is defined in the
Figure 3. Continued.

spectral space, as it is the case for the IFS system. We can define the local projection operator $T$ as:

$$ T = S^{-1} GS $$

(4)

where $S$, $S^{-1}$ represent the spectral to grid point and the inverse transformations, and
where \( G \) represents the multiplication of the state vector in grid-point space by a weighting function defined to be 1 inside a localized area, and zero outside. If we apply the operator \( T \) to an input state vector, then we have as the output a vector that, in grid-point space, is equal to the input one inside the defined local area and is zero outside.

Applying the operator \( T \) after the resolvent operator, we can confine the perturbation over the local area defined by the weighting function \( G \):

\[
\|x(t)\|^2 = \langle TLx_0; ETLx_0 \rangle.
\]  

(5)

Applying the adjoint technique, the norm of the state vector can be computed as:

\[
\|x(t)\|^2 = \langle x_0; L^*TETLx_0 \rangle
\]

(6)

where \( L^* \) is the adjoint of the propagator \( L \) with respect to the canonical Euclidean norm (\( T \) is self-adjoint, since \( G \) is diagonal and \( S \) is orthogonal). The optimal perturbations constrained to grow over a localized area are SVs of the operator \( TL \) with respect to \( E \), and they are computed applying a Lanczos algorithm (see Buizza et al. (1993) for more details).

4. Optimal perturbations generated applying the LPO

The 32 perturbations added to the control IC to generate the perturbed IC are constructed using 16 selected SVs. The selection of 16 from among the 30–35 orthogonal SVs computed by the IFS system is based on the following criteria:

(i) the SVs with maximum amplitude in the SH are neglected;
(ii) the SVs are selected so that to minimize a cost function that compares the SVs with the analysis error; and
(iii) a test is made so that the selected SVs do not overlap over large areas (not more than four perturbations are allowed to have more than half of their total energy concentrated in the same region).

Since the 16 selected SVs are very localized in space, a phase-space rotation is applied to generate 16 less-localized fields. These 16 perturbations are also re-scaled in order to have local maxima comparable with the local analysis error. The coefficients of this phase-space rotation are computed applying a minimization procedure to the ratio between the perturbation amplitude and an estimate of the initial analysis error given by the OE procedure.

Table 2 lists the configurations used to compute the SVs. OPE identifies the characteristics of the operational system. The rationale for the c02 and the c03 experiments is to check whether the confinement of the SV areas of growth can give a larger spread among the ensemble members. These experiments have been run with a 36 h OTI, a time interval during which the time evolution of small perturbations with an amplitude similar to the analysis error (see subsection 4(a)) can be approximated by the linear Eq. (2). Although it is known that the linear approximation can be applied for time intervals not longer than 2–3 days for perturbations with such amplitudes, it was decided to run the c04 experiment with the LPO applied to the European area, but with a 7-day OTI. We want to verify if the c04 SVs can give us an indication of the structures and of the areas where the perturbations should have been located to generate the maximum growth over Europe after 7 days.

Before analysing the characteristics of the SVs, the reader is reminded of the definition of the 'similarity index' (Buizza et al. 1993) that can be used to compare unstable subspaces generated by the SVs of two different experiments. We define the
TABLE 2. EXPERIMENT CHARACTERISTICS

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Optimization time interval</th>
<th>Area of local projection operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPE</td>
<td>36 hours</td>
<td>Global</td>
</tr>
<tr>
<td>c02</td>
<td>36 hours</td>
<td>Northern hemisphere</td>
</tr>
<tr>
<td>c03</td>
<td>36 hours</td>
<td>Northern hemisphere with lat. &gt;30°</td>
</tr>
<tr>
<td>c04</td>
<td>7 days</td>
<td>Europe (-45° &lt; long. &lt; 45°; 30° &lt; lat. &lt; 80°)</td>
</tr>
</tbody>
</table>

_N-dim_ unstable sub-space relative to an experiment as the sub-space of the phase space of the system defined by the first _N_ most unstable perturbations. We can compare the unstable sub-spaces generated by the first _N_ SVs _v_i_ of two experiments A and B using a projection matrix _M(A, B, N)_ defined using the scalar product definition (1), as:

\[ m_{i,j}(A, B) = (\langle v_i(A); E v_j(B) \rangle)^2. \]  \hspace{1cm} (7)

Each element of this matrix is the squared scalar product between the _i_ th SV of the A experiment, and the _j_ th SV of the B experiment. In other words, it represents the amount of energy of the _i_ th SV of the A experiment that is explained by the _j_ th SV of the B experiment. The sum of the matrix elements with a fixed first index represents how well the _i_ th SV of the A experiment can be reconstructed from a linear combination of the first _N_ SVs of the B experiment.

The similarity index of two experiments A, B, which measures the similarity of the unstable sub-spaces generated by the first _N_ SVs of each experiment, is defined as:

\[ s(A, B; N) = \frac{1}{N} \sum_{i,j=1}^{N} m_{i,j}(A, B). \]  \hspace{1cm} (8)

Some reference values for the similarity index are listed below:

(a) parallel sub-spaces have \( s(A, B; N) = 100\% \);
(b) orthogonal sub-spaces have \( s(A, B; N) = 0\% \);
(c) statistics analysis performed using the first 3 months of the EPS results showed that the similarity index between the unstable sub-spaces generated by the first 20 36 h OTI SVs, computed for two consecutive days, has a mean value of \( s(\text{day}, \text{day} + 1;20) = 50\% \); and
(d) statistics analysis performed as in point (c), but for unstable sub-spaces computed 2 days apart, showed that the similarity index has a mean value of \( s(\text{day}, \text{day} + 2;20) = 30\% \).

More reference values are reported in Buizza (1994).

An indication of the regions where the selected perturbations of one experiment have maximum amplitude is given by the ‘overlap factor’ field, which gives, for each experiment, the number of selected SVs that cover a specified region, with the area covered by each SV defined to be the area where the local total energy of the SV is larger than 1% of its maximum value.

The first four sub-sections of section 4 refer to the case study for 14 February 1993. In sub-section 4(a) the structure of the OPE SVs is analysed. In sub-section 4(b) the c04 SVs are analysed (see Table 2), i.e. the optimal perturbations with maximum growth over Europe after an OTI of 7 days, and the unstable sub-space generated by these are compared with the OPE SVs. In sub-section 4(c) the c02 and the c03 SVs are analysed, optimal perturbations constrained to grow over the NH, and over the region of the NH with latitude \( \lambda \geq 30° \), during a 36 h time period. In sub-section 4(d) is a description of the result of an ensemble run using the c03 perturbations. Finally, two more case studies
are analysed in sub-section 4(e) to confirm the results relative to the 14 February 1993 case.

(a) The OPE optimal perturbations for 14 February 1993

Figure 4 shows the amplification-factor and the overlap-factor fields at model level 11 of the OPE SVs. It is not a case of a particularly large amplification rate, with the first four SVs characterized by a slightly larger amplification factor than the others. Looking at Fig. 4(b), we can identify three regions of maximum concentration of the SVs in the NH, one located over the Sahara and the Arabic peninsula, one over the western Pacific, and one over the sub-tropical eastern Pacific. Another local maximum is present over central USA. Note that some SVs have structures also in the SH, and that very few perturbations are located in the Atlantic storm track.

Figure 5 shows the root-mean-square amplitude of the perturbations (Figs. 5(a), (b) and (c)) and of the OI estimated analysis error (Figs. 5(d), (e) and (f)). Wind (Figs. 5(a), (b) and (d) and (e)) and temperature (Figs. 5(c) and (f)) components are shown at model level 11, which is close to where the optimal perturbation total energy peaks (Buizza and Palmer (1994), personal communication). Comparing Fig. 5(c) with Fig. 5(f) we can clearly see that the OPE perturbations have temperature components with magnitudes

![Figure 4](image-url)  
Figure 4. (a) Amplification factors of the OPE (solid line), c02 (dashed line) and c03 (dotted line) singular vectors (SVs). (b) Overlap factor of the OPE selected perturbations at model level 11 (i.e. almost 500hPa). Contour isolines are drawn every 2, starting from 1.
similar to the estimated error field, in the regions where the perturbations are located. On the contrary, the perturbation wind components are smaller than the OI estimates. Figure 5 clearly shows that the OPE perturbations have, at model level 11, local maxima of temperature not exceeding 2 K, and of wind components not larger than 2.1 m s\(^{-1}\) (in terms of 500 hPa geopotential height, these values correspond to not more than 10 m perturbations, not shown). A detailed comparison between the perturbation structure and the OI analysis-error estimate is beyond the scope of this work.

It is worth stressing again here that every time in this paper the validity of the linear approximation (2) is discussed, it always refers to perturbations with this typical amplitude. So whenever it is stated that the tangent linear model describes accurately enough the perturbation time evolution, it is not meant in an absolute sense, but it refers to these types of perturbations.

Let us consider the SVs numbers 1, 2 and 8, located in the three areas of maximum overlap. Figures 6(a), (b) and (c) show their stream function at model level 11. Six nonlinear T36L19 integrations are run, adding and subtracting these very localized and un-rotated perturbations to the control IC, to study whether they have any impact on the flows over Europe at fc-day 7. Figure 7 shows the difference between the perturbed and the control 500 hPa geopotential height fields, at fc-day 1 (left panels) and fc-day 7 (right panels). It can be clearly seen that after 1 day of nonlinear integration, the nonlinear effects are still small. Figure 7 shows that at fc-day 7 only the perturbations localized in the eastern Pacific have an impact on the flow over Europe.

It is worth mentioning that the SVs were linearly integrated for 7 days and it was found that the differences between the linear and the nonlinear integrations were small up to fc-day 2, but quite large after fc-day 4. If we consider the 2nd SV, for example, if linearly integrated it is growing over the European area after fc-day 3 (not shown), while its nonlinear evolution is not characterized by any amplification over this region (see Figs. 7(e)–(h)).

It was decided to run four more experiments, adding and subtracting the un-rotated SVs numbers 13 and 15 (see Figs. 6(d) and 6(e)). These two SVs were chosen among the others because, when linearly integrated for 7 days, they were giving a very large growth over the European region. The rationale for these time integrations is to check, once more, the accuracy and the usefulness of the 7-day linear integrations. Figures 8 and 9 show, respectively, the 500 hPa geopotential-height-field differences between the perturbed and the control runs, at fc-day 1 and 7, with the perturbed IC generated from these two SVs. It can be clearly seen that these two perturbations have an impact on the flow over Europe. In section 2 it was shown that the EPS members numbers 29 and 31 were the two forecasts with the highest scores at fc-day 7. Their initial conditions were generated adding rotated perturbations defined as linear combinations of the un-rotated perturbations. Looking at the matrix that characterizes the linear rotation (not shown) it can be seen that they have, respectively, the largest projection onto the un-rotated SVs numbers 13 and 15. It must be mentioned that forecasts started with the un-rotated initial conditions are characterized by a very large spread, with AC between them and the control of 60% at fc-day 2, but that their skills are very low.

(b) 7-day European optimal perturbations

Optimal perturbations can be used to identify ‘source regions’ where the SVs should be located to have an impact on the target region considered, i.e. Europe in our case. Clearly, the correctness of their indications is strictly related to the validity of the linear approximation up to optimization time. The comparison between the linear and the nonlinear time integrations of the SVs reported in sub-section 4(a) proved that the two
Figure 5. Root-mean-square amplitude of OPE perturbations and optimal interpolation analysis-error estimate at model level 11. Perturbation $u$, $v$ and $T$ components are plotted, respectively, in (a), (b) and (c), and analysis error $u$, $v$ and $T$ components in (d), (e) and (f). Contour isolines: (a) and (b) every 0.1 m s$^{-1}$, starting from 0.05 m s$^{-1}$; (c) and (f) every 0.25 K, starting from 0.125 K; and (d) and (e) every 1 m s$^{-1}$, starting from 0.5 m s$^{-1}$.
Figure 5. Continued.
Figure 6. Stream function at model level 11 of five OPE singular vectors (SVs). The 1st (a), 2nd (b), 8th (c), 13th (d) and 15th (e) SVs are plotted. The SVs are normalized to have unitary total energy norm. Contour isolines are drawn every $0.5 \times 10^8$, with solid (dashed) isolines referring to positive (negative) values.
evolutions can be very different when time intervals longer than 2–3 days are considered. In this sub-section we want to verify whether any useful indication can be given by SVs computed over a 7-day time interval.

Figure 10 shows the first six c04 SVs (stream function at model level 11) at initial (left panels) and after 7 days of linear integration (right panels). Generally speaking, at initial time they are more spread out in the physical space than the SVs computed with a shorter time interval. The structure of the fields at the end of the 7-day OTI clearly shows the impact of the LPO in constraining the area of growth of the perturbations (the contour interval at the final time is 20 times larger). Note that the 5th and 6th SV do not show, at model level 11, the same amplitude as the other four SVs because their amplification factors are three times smaller than the amplification factors of the first four SVs. Moreover, note that all the SVs have structures outside the LPO area because the LPO was applied only during their computation by the Lanczos algorithm, but not during their linear time integration.

These and the results reported in sub-section 4(a) indicate that the (linearly deduced) identification of the Sahara region as one of the source regions for SVs with maximum growth over Europe after 7 days is not correct. On the contrary, the importance of the USA area as a source region is supported by the nonlinear evolution of the 15th SV (see Figs. 8 and 9).

We can compare the OPE with the c04 unstable sub-spaces generated by the first 20 most unstable SVs, using the projection matrix. Table 3 reports the similarity indices computed between different pairs of experiments. The similarity index s(OPE,c04;20) = 11% indicates that there is a small but not negligible degree of similarity between the two unstable sub-spaces. The first four selected OPE SVs that have the largest projection on the c04 unstable sub-space can be identified using the projection matrix M(OPE,c04;20), and are listed in Table 4.

The difference between the linear and the nonlinear time evolution shows that if time intervals longer than the time of validity of the linear approximation are considered.
Figure 7. Spread between the perturbed forecasts and the control, of the perturbed forecasts with initial conditions generated adding and subtracting unrotated OPE singular vectors (SVs), at fc-day 1 (left panels) and fc-day 7 (right panels), computed using the 500 hPa geopotential height fields. Contour intervals are drawn every 1 m for left panels, and every 5 m for right panels. Figures (a)–(b) refer to perturbed forecasts generated adding the 1st SV, (c)–(d) refers to perturbed forecasts generated subtracting the 1st SV, and (e)–(f) refer to perturbed forecasts generated adding the 2nd SV.
Figure 7. Continued. (g)–(h) refers to perturbed forecasts generated subtracting the 2nd SV, (i)–(l) refers to perturbed forecasts generated adding the 8th SV, and (m)–(n) refers to perturbed forecasts generated subtracting the 8th SV.
then wrong results can be obtained. Nevertheless, and being conscious of this limit, the comparison of the c04 SVs with the SVs computed in the other configurations gives useful indications. For example, the comparison of the OPE and c04 unstable sub-spaces confirms that the SVs located in the eastern Pacific, USA and western Atlantic regions can have the largest impact on the forecast spread over Europe at fc-day 7.

(c) 36 h localized optimal perturbations

Figure 11 shows the overlap factors of the 16 perturbations selected by the EPS from the SVs computed in configurations c02 (Fig. 11(a)) and c03 (Fig. 11(b)). If we compare the c02 with the OPE overlap factors (see Fig. 4(b)), we can see that the restriction of the area of growth to the NH has cancelled the perturbations from the SH, while conserving almost the same pattern in the NH. The high value of the similarity index \( s(\text{OPE},c02;20) = 82\% \) indicates that the two unstable sub-spaces are very similar. The restriction of the area of growth to grid points with latitude \( \lambda \geq 30^\circ\text{N} \) have instead a large impact over the eastern sub-tropical Pacific and over the Sahara regions, since these regions of maximum OPE SVs concentration extend south of 30\(^\circ\text{N}\). Figure 4(a) shows
the OPE, c02 and c03 amplification-factor curves. The fact that the restriction of the area of growth to the NH has a small impact on the SVs is reflected by the very similar OPE and c02 amplification factors. Instead of this, the further restriction of the area of growth of configuration c03 reduces the SV amplification factors.

The analysis of the projection matrix between the OPE and the c02 experiment confirms the similarity between the two unstable sub-spaces (see Table 5). Table 3 shows that $s(OPE,c03;20) < s(OPE,c02;20)$, and this confirms the earlier comments made when comparing Fig. 4 with Fig. 11. The fact that $s(OPE,c03;20) = s(c02,c03;20)$ indicates that the restriction of the LPO area to the grid points with $\lambda > 30^\circ$N has a larger impact than the restriction to the NH only, as is expected considering the location of the OPE SVs.

We can compare the c02 and the c04 unstable sub-spaces. Although the similarity index has a small value, we can identify the c02 SVs characterized by the largest projection onto the c04 unstable sub-space (Table 4 gives the four SVs with the largest projection). It is interesting to analyse the area that they cover: the 15th, the 18th and the 20th SVs have maximum amplitude over the eastern Pacific and USA, and the 8th SV is localized in the eastern Pacific region. As a cross-verification test, Table 5 shows that the 8th, 15th
Figure 10. Stream function of the first six C04 singular vectors (SVs) at initial time (left-hand panels) and final time (right-hand panels). The SVs are sorted in decreasing order from top to bottom. They are normalized to have unitary total energy norm at initial time. At initial time, contour isolines are drawn as in Fig. 6, while the contour interval is 20 times larger at final time.
Figure 10. Continued.
TABLE 5. PROJECTION MATRIX COMPUTED BETWEEN THE FIRST 20 MOST UNSTABLE SINGULAR VECTORS (SVs) OF THE c02 AND THE OPE EXPERIMENTS (EACH MATRIX ELEMENT (%) IS THE NEAREST INTEGER TO THE SQUARED SCALAR PRODUCT)

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The first row identifies the OPE SV number, and the first column the c02 SV number. The last row shows the percentage of the norm of each OPE SV explained by the c02 unstable sub-space, and the last element of each column is the percentage of the norm of each c02 SV explained by the OPE unstable sub-space. A star characterizes the 16 SVs selected by the selection algorithm, when the algorithm is applied to each SV set to construct the Ensemble Prediction System initial conditions.

and 18th c02 SVs have maximum projection, respectively, onto the 8th, 13th and 15th OPE SVs. Instead of this, the 20th c02 SV has maximum projection onto the 19th OPE SV that was not chosen by the selection algorithm.

(d) Skill of an ensemble generated from localized SVs

Figure 12 shows that the spread between the perturbed forecasts and the control, when the perturbed ICs are generated from the c03 SVs, is larger than the spread between the OPE perturbed forecasts and the control (Fig. 2).

Comparing these two figures, we can see that enlarging the spread induces less skillful perturbed forecasts during the first five fc-days, but it also increases the chances to have better forecasts at the end of the forecast interval. Some statistics on the c03 scores are reported in Table 1. Table 1 confirms that the EPS forecasts generated from the c03 SVs have a better chance to have higher scores between fc-day 7 and 10, but this causes less forecasts to have 60% < AC < 70%.

The comparison between Fig. 12(c) and Fig. 2(c) confirms that the mean skill of the c03 ensemble is slightly worse than the mean OPE skill during the first 5 days, while it is slightly better between days 7 and 10. The c03 mean spread curve has larger values than the OPE curve up to fc-day 7, and it is closer to the mean error curve, confirming that the c03 spread is larger.

Figure 13 is the analogous of Fig. 3, and it shows the 500 hPa geopotential height forecasts over Europe predicted, for fc-day 7, by the ensemble with ICs generated from the c03 SVs. It clearly shows that the C03 ensemble gives more different weather types of circulation for the European area than the OPE ensemble (see, for example, the weather flows given by members numbers 17, 22, 29 and 32), confirming the impression
Figure 11. Overlap factors of the 16 perturbations selected by the Ensemble Prediction System from (a) c02 and (b) the c03 singular vectors. Contour isofines are drawn every 2, starting from 1.

given by the comparison between the spreads. Comparing the forecasts with the analysis, we can say that the verifying analysis is somewhere in between the ensemble forecasts.

Let us analyse in detail the EPS member number 30, which between fc-day 7 and 10 has the best AC scores among the forecasts generated using the c03 perturbations. Figure 14 shows the 500 hPa geopotential height field at fc-day 7. Apart from the trough over Europe that is still represented with a slightly wrong tilt, the general circulation over the western Atlantic and the European sector is well captured. The ICs of the 30th member are generated mainly from the 13th c03 SV, as it can be identified looking at the elements of the rotation matrix (not shown). The comparison between the c03 and the c04 unstable sub-spaces identifies this SV as one of the three c03 SVs with the largest projection onto the c04 unstable sub-space (31%, while the c03 SV with the maximum projection is the 6th with 37%). Figure 15 shows the 13th c03 SV at initial time, and after 36 h linear integration. Initially, it has maximum amplitude over the northern Atlantic and north-western Europe, and it evolves strongly over central and north-eastern Europe. It is worth mentioning that it is almost orthogonal to the OPE unstable sub-space, with only 2% of its total energy norm explained by the OPE unstable sub-space.
Figure 12. As Fig. 2 but for the Ensemble Prediction System 500 hPa geopotential height field run with the initial conditions generated from the localized c03 singular vectors.
(e) Two more case studies

The analysis of the 14 February 1993 case indicates that the confinement of the SV growth to the NH region with latitude $\lambda \geq 30^\circ$ can increase the spread among the ensemble members, and that this can have a positive impact on the EPS performances. In fact a larger divergence among the ensemble members increases the chances of having forecasts closer to the analysis. To verify this conclusion, ensemble experiments have been run in configurations OPE and c03 with two more starting dates, 21 March 1993 and 28 March 1993. Comparing the deterministic skill scores over the European area at fc-day 7, they can be classified as poor (28 March) and very poor (21 March) cases. Table 1 shows the AC skill score over Europe at fc-day 7 and 10.

At 12 GMT 21 March 1993 the flow over northern Europe was zonal, with an anticyclonic circulation over south-eastern Europe and a trough over the Arabian peninsula. Considering the whole NH, a particularly strong jet stream could be detected over the Pacific Ocean (not shown). Optimal perturbations computed in the OPE configuration are, in fact, mainly concentrated over north-eastern Africa and the western Pacific, with only a few of them localized in the Atlantic region.

Seven days later, a deep trough characterized the circulation over the eastern European countries. Considering the NH flow, a trough could be detected over Japan, and the Pacific jet stream was still very strong (not shown). With respect to the 21 March case, this further reduces the identification of SVs in the Atlantic sector, while it increases the number of the SVs localized in the Pacific region.

Contrary to the 14 February 1993 case, both the two OPE ensembles are characterized by large spread, with the 21 March case having the largest among the two, confirming the poor reliability of the deterministic forecasts. Table 1 summarizes the EPS performances: at fc-day 7, as it can be clearly seen, the unperturbed forecasts for 28 March 1993 have very low, but still positive, skill scores, while the unperturbed forecasts for 21 March 1993 are characterized by extremely low skill scores. The very poor performance of the unperturbed forecasts is reflected in the ensemble skill scores: for both the cases, none of the ensemble members gives a useful 7-day prediction for the European region, although some of them perform better than the unperturbed integrations.

If we consider the 28 March 1993 case, the ensemble run with SVs computed in configuration c03 shows better skill scores, with one member having a 70% AC skill score at fc-day 7. Instead of this, the small, but positive, impact (see below) of the LPO on the ensemble performances for the 21 March 1993 case is not evident from Table 1.

Figure 16 shows the root-mean-square spread and error of the OPE (left panels) and the c03 (right panels) for the two cases. First, consider the 28 March 1993 case. The two mean spread curves have very similar values, with the c03 curve characterized by slightly larger values for the whole 10-day period. The OPE and the c03 error curves are very similar up to fc-day 6, while the c03 is characterized by a slightly smaller error curve afterwards. Instead of this, a more detectable difference between the mean spread curves can be detected for the 21 March 1993 case. Similarly to the 14 February case, the mean c03 spread curve has much larger values between fc-day 4 and 7, and this induces slightly larger errors during the first 7 days of the forecast period. Contrary to the 14 February case, this does not reduce the mean error curve at the end of the forecast period.

These results are confirmed by the comparison of the NH mean spread and error curves: the differences between the c03 and the OPE curves are smaller, but clearly indicate that the action of the LPO increases the mean spread of the ensemble (not shown).
5. IMPACT OF THE LPO ON THE SPRINGTIME SVS

Figure 17 shows the root-mean-square amplitude of the perturbations (temperature at model level 11) that would have been added to the control IC by the operational EPS for three starting dates, 2, 9 and 16 May 1992. The seasonal change from winter to spring and summer induces a variation in the relative instability of the two hemispheres. During (NH) winter, the unstable areas are almost completely localized in the NH, and so the
perturbations added to the control IC generate a perturbed trajectory with a different flow pattern in the NH. On the contrary, during spring, SVs start being localized in the SH. As the season progresses it gets more and more difficult for the automatic algorithm to select 16 from among the 30–35 SVs that the Lanczos algorithm computes, which are not localized in the SH. The comparison of Fig. 5(c) with Fig. 17 clearly confirms that this is the case. Two more experiments were run with starting dates 23 and 30 May 1992:
for these two dates the selection algorithm was unable to select the 16 necessary SVs to generate the EPS ICs.

Figure 18 shows the root-mean-square amplitude of the perturbations generated from SVs computed with the LPO applied to the physical space with latitude $\lambda \geq 30^\circ$. These results show that the impact of the LPO is very large during the spring season. Moreover, experiments run with starting dates of 23 and 30 May demonstrated that the inclusion of the LPO is a necessary condition to be able to run the EPS system during this period of the year.

Table 6 gives the similarity indices for the three spring cases computed between the unstable sub-space generated in the two configurations. Taking into account also the similarity index between the 14 February 1993 OPE and c03 unstable sub-space, we can clearly see that, due to the seasonal variation of the position of the most unstable regions,
Figure 16. Mean root-mean-square spread (solid lines) and error (dotted lines), computed for the European area. (a) OPE 21 March 1993 ensemble, (b) c03 21 March 1993 ensemble, (c) OPE 28 March 1993 ensemble, and (d) c03 28 March ensemble.

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<td>$x_{\text{global}, \lambda \geq 30^\circ, 20}$</td>
<td>39%</td>
<td>22%</td>
<td>17%</td>
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the impact of the LPO on the definition of the unstable sub-space is larger during the warm seasons.

6. CONCLUSIONS

The analysis of the performances of the new Ensemble Prediction System run at the European Centre for Medium-range Weather Forecasts during the winter of 1992/93 indicated that, on some occasions, the spread between the perturbed and the control forecasts was too small, in the sense that the true state of the atmosphere did not appear as a plausible member of the ensemble. A small spread can indicate that the atmospheric flow is not characterized by instabilities that can be responsible for very fast amplification of the initial errors that reduces the skill of the forecasts, or it may be related to a poor efficiency of the added perturbations in giving trajectory divergence in the phase space of the system.
Figure 17. Root-mean-square amplitude of the selected perturbations (temperature at model level 11) for the singular vectors computed without the local projection operator for starting dates: (a) 2 May 1993, (b) 9 May 1993, and (c) 16 May 1993. Contour isolines are drawn every 0.25 K starting from 0.125 K.

The mean spread, computed by averaging the root-mean-square distance of the perturbed members from the control, can be used as indicative of the size of the ensemble cloud. Similarly, the mean error, computed by averaging the distance between the ensemble members and reality, can be used as a reference value for the distance between an ensemble forecast and the true state of the atmosphere.
Figure 18. As Fig. 17, but for the singular vectors constrained to be confined by the local projection operator to the grid points with latitude $\lambda \geq 30^\circ$.

A new operator, the local projection operator, has been introduced in the IFS (the new ECMWF model used to compute the optimal perturbations) to constrain the area of possible growth of the perturbations to localized regions. In the first part of this paper, the LPO has been applied to a case study to see whether its actions could improve the EPS performances, increasing the spread among the perturbed trajectories.
The analysed case study, based on forecasts with starting time 12 GMT 14 February 1993, is an extreme case of very small spread. This is partly due to the very predictable atmospheric flow that characterized the weekend of the 14th, as confirmed by the quite accurate predictions made by the unperturbed forecasts. A second reason that can explain the small spread among the EPS members could be that the optimal perturbations used to construct the ICs of the perturbed forecasts were not effective enough.

To test if this was the case, optimal perturbations were computed in four different configurations, and it was shown that only a few of the selected OPE SVs used to construct the perturbed ICs were not located in the ‘source regions’, i.e. regions where the perturbations should be located to have the largest impact over the target area, which is the European region in our case, after 7 days.

The comparison between linear and nonlinear integrations of optimal perturbations with local amplitude of the order of 2 m s\(^{-1}\) for the wind components, and of 2 K for the temperature component, showed that their time evolution can be described in the linear approximation up to 2–3 days. A study to find if SVs computed with a 7-day OTI can correctly identify these source regions was undertaken; only partial agreement between this and nonlinear results was found.

It has been demonstrated that the action of the LPO to confine the area of possible growth to the physical space with latitude \(\lambda \approx 30^\circ\) can guarantee a larger spread between the EPS members and the control. Considering the case study for 14 February 1993, it was proved that this confinement of the area of possible growth of the perturbations can improve the EPS performance by enlarging the possible predicted weather flows. Results from two other cases confirm that the localization of the optimal perturbations can induce a small but detectable increase of the mean ensemble spread, and that this can improve the probability of having ensemble members closer to reality between fc-day 7 and 10.

One reason why the spread among the perturbed trajectories increased only slightly, or let us say less than expected, can be related to model errors, as already mentioned in section 2. Adding small perturbations, although optimal, cannot compensate for deficiencies in the model parametrizations. This has been confirmed by the EPS performances during the winter of 1992/93: when situations difficult to be predicted occur, the control AC skill-score curve drowns with all the perturbed AC score curves, without any of them surviving above the 60% AC line after fc-day 7. Underestimation of analysis error (e.g. poor quality-control decisions) could be another reason. Moreover, the use of a simple model in the trajectory computation and the absence of physics (e.g. moist processes) when computing the optimal perturbations can explain the less than expected spread.

In section 5 another problem was focused upon, which is related to the different relative instability of the two hemispheres during the hot or cold seasons. Since the ECMWF is mainly interested in estimating the probability distribution of forecast states in the NH, the ICs of the EPS members have to be constructed from the SVs which guarantee diverging trajectories in this hemisphere. Springtime case studies showed that the LPO is necessary to identify optimal perturbations which amplify over the NH, when the relative instability of this hemisphere decreases. It was shown that this problem starts occurring in spring, and can be fatal during the NH hottest months, and proof was given that the action of the LPO can cure this problem completely.

The LPO was introduced into the EPS system at the ECMWF on 19 March 1993. The major result of its implementation is the possibility of identifying 16 optimal perturbations to generate the initial conditions of the EPS perturbed members during the hot seasons.
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