Gravity wave and equatorial wave morphology of the stratosphere derived from long-term rocket soundings

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SUMMARY

Fluctuations in vertical profiles of atmospheric temperature and horizontal wind in the 20–60 km altitude range have been isolated from meteorological rocket measurements during 1977–87 at 15 widely separated sites. The seasonal, geographical, and vertical variability of the variance of horizontal velocities, $u^2 + v^2$, and relative-temperature perturbations, $T^2$, were studied. The bulk of the variance of both quantities in the 2–10 km and 2–20 km vertical-wavelength bands was associated with gravity-wave motions, although in-depth study of the wave polarization shows that planetary-scale equatorial wave modes contribute to the variance at equatorial stations. Annual mean variances varied widely among the 15 stations, suggesting appreciable geographical variability in stratospheric wave activity. Whereas $u^2 + v^2$ values generally increased significantly with altitude throughout the stratosphere, $T^2$ values grew less substantially and often decreased with altitude at upper heights. Rotations of wave-velocity phasors with height were almost always more frequently clockwise than anticlockwise in the northern hemisphere, consistent with upward-propagating wave energy, yet these percentages (>50%) showed a marked semi-annual variation, with equinoctial maxima and minima at the solstices. At high latitudes ($\sim$50°N–80°N) variances exhibited a strong annual variation, with the minimum in summer and a strong peak during winter at both lower (20–40 km) and upper (40–60 km) heights. The annual variance cycle attenuated somewhat at mid-latitudes ($\sim$25°N–40°N), and a strong peak in August dominated the $u^2 + v^2$ variations at 40–60 km. The peak was also evident in $T^2$, but was smaller relative to the winter peak. At low latitudes ($\sim$15°N–25°N) the wave morphology was broadly similar to that at mid-latitudes, apart from an additional upper-level peak in the variance in May. This peak in May occurred in some years but not in others at mid-latitude stations. At the equatorial stations ($\sim$10°N–10°S) the low-level variance showed little systematic seasonal variability, but exhibited clear modulation over a quasi-two-year period. Much of this variance was consistent with the Kelvin modes thought to drive the eastward phase of the stratospheric quasi-biennial oscillation (QBO). However, the uniform east–west alignment of waves was inconsistent with the expected polarization of the mixed Rossby-gravity wave mode which is believed to drive the westward phase of the QBO. At 40–60 km, the variance was strongly attenuated around April–May and November, when both $u^2 + v^2$ and $T^2$ decreased with height around the 40–45 km range, indicating that wave dissipation occurs here. This produced a semi-annual variation at upper heights, with maxima around January and July, which may contribute significantly to the semi-annual wave driving of the equatorial upper stratosphere. Polarization studies showed that this variance in the 2–10 km band was mostly due to gravity waves, although equatorial modes contributed during December–February.

1. INTRODUCTION

The atmosphere between about 20 and 60 km in altitude, hereafter referred to as the stratosphere, has proved to be a notoriously difficult region to probe in fine detail. For many years in situ measurements from rocket-borne payloads provided the majority of information on the basic dynamics and thermal structure of the atmosphere at these heights. While remote-sensing experiments aboard satellites now provide excellent global data on the background structure and on planetary-scale wave disturbances throughout the stratosphere, smaller-scale motions, such as gravity waves, are poorly resolved by these instruments at present. While ground-based radar systems now regularly measure winds in the height intervals 0–30 km and 60–100 km with high time–height resolution,

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insufficient radiowave backscatter is received from the 30–60 km height range to provide similar mesoscale wind data here. Other experimental methods which can also measure small-scale atmospheric structure at upper heights (e.g. balloons, aircraft, acoustic sounders, meteor tracking, airglow observations, etc.) cannot be exploited in the upper stratosphere either.

Consequently, information on small-scale dynamics throughout the stratosphere remains elusive. Until quite recently, rocket soundings were still the only source of data with enough resolution to resolve the small-scale structure of this region. These measurements provide vertical profiles of horizontal winds and temperatures with good height resolution (1 km or better), and have been performed for many years at a number of sites around the world. The fluctuations evident in these profiles were studied by Hirota (1984) and Hirota and Niki (1985) to provide the first data on the seasonal and geographical variability of gravity-wave amplitudes in the upper stratosphere. Further gravity-wave studies using rocket data have been performed by Dewan et al. (1984), Hass and Meyer (1987), Eckermann and Vincent (1989), Hamilton (1991), and Tsuda et al. (1992).

While the vertical wave structure can be resolved from rocket profiles, the long and irregular time intervals between successive launches prevent the study of temporal gravity-wave fluctuations. However, ground-based Rayleigh lidars can now sound the upper stratosphere remotely, and give temperature and wind data with high resolution in both height and time. While it seems clear that these instruments will supersede rockets as the primary source of information on gravity-wave characteristics at stratospheric heights, their development is comparatively recent and observations are being performed at selected sites only, so that the data are currently limited in their climatological and geographical extent. Nevertheless, these instruments have already provided valuable information on the variability of stratospheric gravity waves with altitude and season (Shibata et al. 1986, 1988; Gardner et al. 1989; Wilson et al. 1990, 1991a, 1991b; Marsh et al. 1991; Mitchell et al. 1991; Beatty et al. 1992).

Since the initial studies of Hirota (1984) and Hirota and Niki (1985), the theory of atmospheric gravity waves has advanced considerably and, in light of these theoretical developments, there is now some uncertainty as to the precise interpretation of some wave variances which they calculated. The purpose of this study is both to resolve any uncertainties in these earlier results, while at the same time to extend the analysis to investigate features of the wave variances and covariances not investigated in previous studies.

The data, and the manner in which the fluctuations were isolated, are described in section 2. In section 3 we show how high-pass filtering of the profiles, while isolating a pure gravity-wave signal, also removes certain types of gravity waves, which can lead to biased mean results within narrow pass bands. On this basis we choose two pass bands within which to study the wave fluctuations. We present the azimuthal alignment of the velocity fluctuations in the broad band in section 4, and go on in section 5 to study variations in wave-induced horizontal-wind and relative-temperature variances within both wavelength bands. Geographical variations of the wave variance are discussed briefly in section 6, and the vertical variation of the variance is studied in section 7. The sense of rotation with height of horizontal-velocity phasors is presented in section 8. Section 9 investigates possible interannual variability in the climatological characteristics set forth in earlier sections, while section 10 collates the polarization results and uses them to infer the precise wave content of the data, particularly in equatorial regions. The major results are then brought together in section 11 to provide an overall picture of the observed characteristics of the wave oscillations evident in these measurements.
2. Data

The data used hereafter were obtained from meteorological rocketsonde experiments (see, for example, Kays and Olsen (1967)) and comprise a slightly extended version of the database used in the earlier studies of Hirota (1984) and Hirota and Niki (1985).

These data provide in situ measurements of atmospheric zonal wind, $U$, meridional wind, $V$, and temperature, $T$, at 1 km intervals between 20 km and 60 km in altitude. A decade of data from 1977 to 1987 was available from 15 different sites, with most of the stations concentrated in and around North America (see Fig. 1). The number of launches in each month of every year is plotted in Fig. 2. The characteristics vary a lot among the sites, but generally the greatest launch activity occurred between 1977 and 1981, with limited launchings after 1985. Nevertheless, apart from several of the high-latitude sites, the mean seasonal coverage of these data is fairly uniform, so that climatological statistics can be compiled. However, launches were usually timed to occur close to local noon at each observing site, so that the data have a very strong bias towards daytime conditions.

Wavelike oscillations superimposed on the background flow are frequently observed in the rocket data at these heights (e.g. Newell et al. 1966; Miller et al. 1968; Hirota 1984; Hirota and Niki 1985; Hass and Meyer 1987; Eckermann and Vincent 1989; Tsuda et al. 1992). Typical profiles are shown in Fig. 3. To isolate these fluctuations the following scheme was adopted.

Firstly, if more than 20% of the data in any given profile were missing, the profile was rejected. The percentage of the original data which remained after such discrimination is given in Table 1. After any limited missing data in an accepted profile were interpolated using a cubic spline fit, a least-squares cubic polynomial was fitted to the profile, which accurately defined the instantaneous background structure (see Fig. 3). The prevailing winds and temperatures were fitted in this manner, rather than calculating and subtracting climatological means (e.g. Eckermann and Vincent 1989;

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Figure 1. Geographical distribution of the rocket launching sites from where the data used in this study were obtained.
Figure 2. Histograms of the number of rocket launches in each month from 1977 to 1987 at the 15 sites shown in Fig. 1.
stratospheric wave morphology

<table>
<thead>
<tr>
<th>Site</th>
<th>Horizontal velocity</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thule</td>
<td>87.0% (430 out of 494)</td>
<td>82.2% (406 out of 494)</td>
</tr>
<tr>
<td>Chatanika</td>
<td>90.3% (265 out of 227)</td>
<td>77.5% (176 out of 227)</td>
</tr>
<tr>
<td>Fort Churchill</td>
<td>88.6% (279 out of 315)</td>
<td>74.6% (235 out of 315)</td>
</tr>
<tr>
<td>Primrose Lake</td>
<td>90.2% (604 out of 670)</td>
<td>80.7% (541 out of 670)</td>
</tr>
<tr>
<td>Shemya</td>
<td>35.9% (115 out of 320)</td>
<td>27.8% (89 out of 320)</td>
</tr>
<tr>
<td>Ryar</td>
<td>92.3% (223 out of 241)</td>
<td>91.7% (221 out of 241)</td>
</tr>
<tr>
<td>Wallops Island</td>
<td>71.4% (564 out of 790)</td>
<td>61.3% (484 out of 790)</td>
</tr>
<tr>
<td>Point Magu</td>
<td>73.4% (665 out of 906)</td>
<td>63.5% (575 out of 906)</td>
</tr>
<tr>
<td>White Sands</td>
<td>70.4% (626 out of 889)</td>
<td>55.2% (491 out of 889)</td>
</tr>
<tr>
<td>Cape Kennedy</td>
<td>80.1% (622 out of 777)</td>
<td>70.7% (549 out of 777)</td>
</tr>
<tr>
<td>Barking Sands</td>
<td>68.8% (474 out of 689)</td>
<td>57.8% (398 out of 689)</td>
</tr>
<tr>
<td>Coolidge Field</td>
<td>87.1% (392 out of 450)</td>
<td>79.3% (357 out of 450)</td>
</tr>
<tr>
<td>Fort Sherman</td>
<td>97.5% (348 out of 357)</td>
<td>79.0% (282 out of 357)</td>
</tr>
<tr>
<td>Kwajalein</td>
<td>99.2% (783 out of 789)</td>
<td>93.0% (734 out of 789)</td>
</tr>
<tr>
<td>Ascension Island</td>
<td>90.4% (622 out of 688)</td>
<td>88.4% (608 out of 688)</td>
</tr>
</tbody>
</table>

Hamilton (1991), because there can be considerable week-to-week and interannual variability in this vertical background structure, due to varying planetary-wave amplitudes, sudden stratospheric warmings (e.g., Labitzke 1981) and volcanic aerosol effects (e.g., Dunkerton and Delisi 1991). The fitted prevailing structure was then subtracted from the original profile, leaving only the fluctuations. After removing any small residual means that might exist, the data were numerically Fourier transformed and stored for later analysis.

### 3. Choosing an Appropriate Pass Band

In addition to the above reduction, it is often necessary to high-pass filter a rocket profile in order to isolate the fluctuating component (e.g., Hirota 1984). Yet the precise range of vertical wavelengths one should pass in order to retain only gravity-wave fluctuations is not generally clear. For example, Hirota (1984) argued that larger vertical wavelengths contained contaminating contributions due to tides and planetary waves, and he adopted a high-pass filter with a 10 km cut-off wavelength, so as to isolate a pure gravity-wave signal. A later study by Hirota and Niki (1985) used a 15 km cut-off. Hamilton (1991), on the other hand, argued that the contamination due to tides and planetary waves was limited, and so imposed no filtering of vertical wavelengths. Storing the Fourier form of the profile enabled us to adopt a more flexible approach and to test various types of high-pass filter.

In analysing fluctuating wave data, we shall make use of the Stokes parameters, introduced and discussed in relation to gravity waves by Vincent and Fritts (1987) and Eckermann and Vincent (1989). Briefly, for a gravity wave with a peak velocity amplitude of \( u_0^2 \) zonally and \( v_0^2 \) meridionally, the four Stokes parameters are the total wave-phasor length \( I = u_0^2 + v_0^2 \), the axial anisotropy \( D = u_0^2 - v_0^2 \), the linear-polarization parameter \( P = 2u_0v_0 \cos \delta \), and the circular-polarization parameter \( Q = 2u_0v_0 \sin \delta \), where \( \delta \) is the phase difference between the velocity components (\( 0^\circ \) for a linearly polarized wave, \( 90^\circ \) for a circularly polarized wave) and overbars indicate averaging over a number of profiles. These parameters uniquely characterize any partially polarized wave motion. For analysis of single profiles, as is done when analysing rocket data, then \( I \) equals twice the variance, \( D \) equals twice the difference between the zonal and meridional variance, \( P \) is four times
the covariance $u'v'$, and $Q$, which is harder to calculate directly\(^*\), can be calculated indirectly from the other parameters since $I^2 = D^2 + P^2 + Q^2$ for a single profile. The parameters are more easily and directly calculable in the Fourier domain (Eckermann and Vincent 1989).

A number of useful quantities can be calculated from the Stokes parameters. Of particular interest here is the mean azimuthal alignment of the horizontal-velocity fluctuations, $\bar{\phi}$, which is given by (e.g. Eckermann and Vincent 1989)

$$\bar{\phi} = \frac{1}{2} \arctan \left( \frac{P}{D} \right).$$  \hspace{1cm} (1)

* It can be evaluated either by integrating beneath the quadrature spectrum of $u'$ and $v'$, or by computing the covariance of the fluctuations with either $u'$ or $v'$ appropriately Hilbert transformed.
At high frequencies, gravity waves oscillate longitudinally in the horizontal plane, and so the alignment $\phi$ also gives the mean propagation orientation of the waves. This also holds true at low frequencies when the motion becomes elliptically polarized ($Q \neq 0$), as in this case $\overline{\phi}$ gives the orientation of the major axis of the motion ellipse, which again is parallel to the horizontal-propagation direction of the wave. However, (1) provides an azimuth that is $\pm 180^\circ$ uncertain, and so does not give the absolute direction of wave propagation, hence the nomenclature ‘alignment’. Correlation with simultaneous measurements of temperature oscillations can resolve this uncertainty (Kitamura and Hirota 1989; Hamilton 1991).

Wave frequencies as measured by ground-based systems are Doppler shifted by the background wind, and this leads to Doppler-shifted wave frequency spectra as measured by such devices (e.g. Scheffer and Liu 1986; Fritts and VanZandt 1987). Vincent and Eckermann (1990) and Lefrère and Sidi (1990) showed that this can lead to essentially ‘Doppler shifted’ $\phi$ values calculated from time-fluctuating ground-based data, which are not indicative of the actual wave-propagation directions.

While the vertical wavelength, $\lambda_z$, of a gravity wave does not suffer Doppler-shifting effects, it does vary with background wind speed $\overline{U}$ as

$$\lambda_z = \frac{2\pi |c - \overline{U} \cos \varphi| [1 - (f/\omega)^2]^{1/2}}{N}$$

where $c$ is the ground-based horizontal phase speed of the wave, $\varphi$ is the azimuth angle between the wind and wave-propagation vectors, $f$ is the inertial wave frequency, $\omega$ is the intrinsic wave frequency, and $N$ is the Brunt–Väisälä frequency. In the mid-latitude upper stratosphere, mean wind speeds increase with height substantially in summer and winter, and thus can produce large $\lambda_z$ values due to large values of the intrinsic phase speed $|c - \overline{U} \cos \varphi|$, according to (2). However, we note that this effect depends crucially on the direction $\varphi$ of wave propagation relative to the mean wind direction.

Figure 4(a) sets out the geometry of the problem, where the direction of the ground-based phase velocity $c$ varies around the lightly shaded circle, and the wind-velocity vector stays fixed. The intrinsic phase speed is given by the vector $\overrightarrow{PQ}$. Note that if $|c| \ll |\overline{U}|$, then small $|\overrightarrow{PQ}|$ values, and hence small $\lambda_z$ values, occur for $\varphi \sim \pm 90^\circ$, whereas larger wavelengths occur for $\varphi$ around $0^\circ$ or $180^\circ$.

Figure 4(b) shows the variation of $\lambda_z$ with $\varphi$, using representative values of $c$ (0, 10, and 20 m s$^{-1}$), $\overline{U} = 50$ m s$^{-1}$ eastward, and $N = 0.017$ rad s$^{-1}$. A 2–10 km vertical-wavelength range is shaded, the band of wavelengths used in a number of early studies of gravity waves using rocket data (e.g. Hirota 1984; Eckermann and Vincent 1989). The modelling indicates that, in high-wind conditions, the vertical wavelength of most zonally aligned waves will exceed 10 km, and so will be removed from the rocket data during bandpass filtering, so that waves within a 2–10 km wavelength band are mostly propagating at azimuths $\varphi \sim \pm (45^\circ–135^\circ)$ to the mean-wind direction.

We test these inferences by explicitly computing alignments $\overline{\phi}$ of rocket data from several sites, and high-pass filtering a 2–10 km range of vertical wavelengths. The results are displayed in Fig. 4(c), where $\overline{\phi}$ data are divided into conventional quarterly seasonal groupings across the page, and the number of counts within six alignment bins between $0^\circ$ and $180^\circ$ are plotted as histograms. Results for the various seasons are shaded differently to highlight the seasonal divisions better, and results from two height intervals (20–40 km and 40–60 km) are presented at each site. Indeed, the alignments become strongly clustered meridionally at upper heights during winter at Primrose Lake and White Sands, and similar characteristics are found at other mid-latitude stations (e.g. Point Mugu, Cape Kennedy, Wallops Island) where strong zonal wind jets arise during
Figure 4. (a) Phasor diagram showing the geometry of the variation of intrinsic phase speed $|c - \bar{U}\cos\varphi|$ with azimuth angle between the wind and wave-propagation vectors, $\varphi$. (b) Modeled variation of vertical wavelength, $\lambda_v$, with $\varphi$ for three different ground-based phase speeds $c$. Values of $\bar{U} = 50$ m s$^{-1}$ (eastward) and Brunt-Väisälä frequency $N = 0.017$ rad s$^{-1}$ were used in the calculation, and are typical of the mid-latitude winter stratosphere. (c) Histograms of the number of mean azimuthal alignments of the horizontal-velocity fluctuations, $\varphi$, within six azimuth bins, as calculated from rocket data in the 2–10 km vertical-wavelength band at three sites where earlier data were reinterpreted by Eckermann and Hocking (1989). The data are split into summer, spring, winter and autumn groupings. The lower and upper plot sequences at each site are results from a lower (20–40 km) and upper (40–60 km) height range. Error bars are 90% confidence intervals of the count as given by the Poisson distribution.
winter (see Fig. 15 later). At Thule the bias is more zonal, but mean winds are weaker here and so these results are more indicative of the total wave field.

Consequently, in the analysis to follow, we analyse fluctuations in the 20–60 km height range within two high-passed vertical-wavelength bands:

(i) a narrow band, in which the first three wavelength harmonics (40 km, 40/2 = 20 km, 40/3 = 13 km) were removed, leaving a vertical-wavelength range ~2–10 km. This ensures tidal, planetary-wave, and residual background signals are almost completely removed, leaving a pure gravity-wave signal;

(ii) a broad band, in which only the fundamental harmonic was removed, leaving a range of wavelengths ~2–20 km. We use this broader band to study more effectively the direction of horizontal wave propagation.

For comparison, results were also computed using the data-reduction method of Hamilton (1991), which involved subtraction of the climatological monthly-mean vertical profile of the background structure (as computed from the rocket data), followed by removal of any residual mean and linear trend in the profile. As in his analysis, these data were analysed over the 28–57 km height range.

4. AZIMUTHAL ALIGNMENT

Hamilton (1991) compiled climatological statistics of horizontal wave-propagation directions using a similar base of rocket data. Since his reduction technique passes a broad band of vertical scales, it should not be greatly affected by the narrow-band biasing effects discussed above. Consequently, we shall not aim to repeat his extended analysis here using \( \phi \) determinations. Instead, we shall briefly present selected \( \phi \) data in such a way that they highlight some additional aspects not investigated by Hamilton (1991). For example, we shall present results in two height ranges for the four seasons, to compare with Hamilton’s results in two six-monthly intervals computed over a single height range.

Azimuthal alignments were computed using the broad 2–20 km band of vertical wavelengths, and collated results are displayed in Fig. 5 at selected high-latitude, mid-latitude, low-latitude, and equatorial sites. At Primrose Lake the strong north-westward summertime alignment noted by Hamilton (1991) is evident at upper heights in June–August and in both height ranges during September–November. In other seasons the alignments are more evenly distributed, as also found by Hamilton (1991). At Point Mugu low-level alignments are preferentially zonal, but are more evenly distributed at upper heights. Hamilton (1991) noted NW–SE clustering of inferred propagation directions here during winter, which we do not observe, although this does arise at upper heights during winter at Wallops Island, where Hamilton’s data show similar attributes. It may be that these waves are still filtered from our 2–20 km band at Point Mugu. At Coolidge Field (referred to as Antigua in Hamilton’s analysis) during winter, strong zonal preference with some clustering in the NW–SE quadrant is observed, together with a zonal preference during summer, consistent with Hamilton’s findings. At the selected equatorial site of Ascension Island, alignments are strongly zonal at all times in the lower height range, whereas at upper heights the degree of zonal clustering is greatest in December–February, and weaker at other times. Strong eastward propagation directions were inferred by Hamilton (1991) at the equatorial sites in both summer and winter, as first inferred by Hirotta (1978) for waves with vertical wavelengths > 10 km.

The smaller-scale waves (\( \lambda_v < 10 \) km) in the equatorial upper stratosphere have generally been ascribed to gravity waves (Hirotta 1984; Hirotta and Niki 1985), yet reanalysis of earlier results by Eckermann and Hocking (1989) led them to question this
assumption. To investigate this issue, Fig. 6 displays alignments in the 2–10 km band at the three equatorial sites. The alignments at all sites in the lower height range 20–40 km are strongly zonal in all seasons. At the uppermost heights the zonal clustering persists strongly only during December–January. In other months the zonal preference is generally much weaker.

5. MEAN SEASONAL VARIATIONS IN WAVE ACTIVITY

Hirota (1984) investigated the seasonal variations of the gravity-wave ‘activity’ in stratospheric rocket data by calculating the intensity of the fluctuations in various atmospheric variables, $X$, over the height range $z_1 \leq z \leq z_2$, using the following height-integrated formula
Figure 6. As for Fig. 4(c), but using data in the 2–10 km vertical-wavelength band from the three equatorial sites.

$$\overline{X}^2 = \frac{1}{z_2 - z_1} \int_{z_1}^{z_2} \left( \frac{\text{d}^2X}{\text{d}z^2} \right)^2 \text{d}z. \quad (3)$$

Root-mean-square (r.m.s.) results of such calculations using the horizontal-velocity fluctuations, denoted $(u_z)_{\text{rms}}$, revealed a pronounced annual cycle at high latitudes, with a winter maximum, confirming earlier tentative findings (e.g. Theon et al. 1967). On progressing equatorward, the annual cycle attenuated and a weak semi-annual cycle became apparent.

Since this study, however, the theory of atmospheric gravity waves has advanced significantly. In particular, it is now believed that gravity-wave amplitudes are limited by saturation processes in the middle atmosphere (e.g. Fritts 1984, 1989; Hines 1991a), and this has led to the development of model vertical wave-number spectra produced by a wave ensemble (Dewan and Good 1986; Smith et al. 1987; Hines 1991b).

Analytical expressions have been presented for the vertical wave-number power spectra of horizontal winds and temperature, and are, respectively (Smith et al. 1987; Fritts et al. 1988),

$$F_{u_k}(m, z) = \frac{N^2}{6m_k^2 \{1 + (m/m_*)^3\}} \quad (4)$$

$$F_{T_k}(m, z) = \frac{N^4}{10g^2m_k^3 \{1 + (m/m_*)^3\}} \quad (5)$$

where $g$ is gravity, $m$ is the vertical wave number, and $m_*$ is the so-called 'characteristic
wave number', which varies with height as $e^{-z/H_E}$, where $H_E$ is some wave-energy scale height. This accounts for the height-dependence of the spectra. $T'_r = T'/T_0$ is the relative temperature fluctuation, $T_0$ is the background temperature, $F_{uv}(m)$ is the spectrum of horizontal velocities, and $F_{T'}(m)$ is the spectrum of relative-temperature fluctuations. While theories differ on the origin of this shape, the model fits above are generally well accepted and agree with observational data quite closely (e.g. Fritts and Chou 1987; Fritts et al. 1988; Shibata et al. 1988).

In appendix A these spectral models are used to study the variation of $(u_{zz})_{\text{rms}}$ with height. There it is demonstrated that this measure of wave activity is controlled by the large-$m$ portion of the spectrum, where variances have somewhat constant values due to the dynamical processes which shape the spectrum. The spectrum at small $m$ is more representative of the amplitude variations expected for a simple superposition of individual gravity waves, since it is less affected by amplitude-limiting effects within the interacting spectrum. Consequently, here we calculate seasonal variations in $\overline{u'^2} + \overline{v'^2}$ and $\overline{T'^2}$ as these variances should be controlled more by the larger wavelength (small $m$) motions in the spectrum, according to (4) and (5).

(a) Horizontal velocities

The monthly variations in $\overline{u'^2} + \overline{v'^2}$ at all 15 sites are plotted in Fig. 7 for fluctuations in the 2–10 km vertical-wavelength band. Successive decreases in station latitude from left to right and down the page.

It is immediately clear that $\overline{u'^2} + \overline{v'^2}$ increases with altitude, in line with theoretical expectations that the total wave variance should increase approximately as $e^{z/H_E}$, where $H_E > H_\rho$ (the density scale height). The general form of these height variations in the wave variances have been investigated by Hirota and Niki (1985). Selected cases will be investigated further in section 7.

For convenience, we now define four arbitrary latitude bands into which we shall classify the results: ‘high latitudes’ (50°N–80°N), ‘mid-latitudes’ (25°N–50°N); ‘low latitudes’ (10°N–25°N), and ‘equatorial latitudes’ (10°N–10°S).

At the five high-altitude sites a strong annual variation in $\overline{u'^2} + \overline{v'^2}$ is evident in both altitude ranges and at all sites, peaking in winter and minimizing during summer. While the normalized variations at all five sites are similar, the variances at Primrose Lake are almost twice those at either Thule or Fort Churchill. Variations at Chatanika and Shemya are more variable but less reliable, owing to low amounts of data from these sites.

Mean seasonal cycles at the four North American stations at mid-latitudes all show similar variations. In the 20–40 km range all sites show a wintertime peak, and much of the variability is quasi-annual. However, a weaker secondary maximum occurs at each station around August; in fact the two peaks are of similar magnitude at Cape Kennedy, producing a more semi-annual variation at low levels. Another small peak arises around April at three of the four stations (Cape Kennedy is the exception).

At the upper heights (40–60 km), however, the variations differ from those at lower altitudes. The wintertime peak is suppressed somewhat, and a large burst in variance, which occurs for about three months and peaks during August, dominates the variations. The small peak in April, observed at lower levels, also occurs at 40–60 km at White Sands and Point Mugu, whereas at Cape Kennedy and Wallops Island it is more of a plateau. Additionally, at all four sites, there is a deep minimum in wave activity around October; a minimum also occurs in June, most noticeably at Wallops Island. As was noted at high latitudes, while the normalized variations among the sites at Cape Kennedy, White Sands, and Point Mugu are all very similar, the absolute variances differ markedly from site to site. At Wallops Island, however, the normalized variations also differ
somewhat from those at the other three sites, in that the August maximum is not as large, and the winter maximum is less suppressed. These results contrast with the \( (u_{zz})_{\text{rms}} \) variations from White Sands calculated by Hirota (1984), who found small maxima in both April and September. The \( u^2 + v^2 \) data at White Sands reveal that the former maximum (in April) is very weak, whereas the latter maximum is very prominent and peaks earlier (around August).
At Ryori, the only non-American mid-latitude station, the variations from 20–40 km at this displaced longitude are similar to those at the North American stations, but in the 40–60 km range the cycle is distinctly annual, peaking in winter, and no maxima occur at the equinoxes. Similar annual variability was noted by Wilson et al. (1991b) in the wave potential energy per unit mass measured by lidar at two French sites (44°N) in the 30–45 km and 45–60 km height ranges. Curiously, similar data from a nearby lidar in Wales (52°N) are more like the mid-latitude North American variations (Mitchell et al. 1991).

The two stations within the low-latitude interval (Barking Sands and Coolidge Field) are separated by ~ 100° longitude, yet the mean seasonal variations of the gravity-wave variances over these two sites are very similar at 40–60 km. Here the variations are terannual, with peaks occurring around July–August, May, and January. Minima in activity in October (a deep minimum) and June are also observed at both sites, and are similar to the mid-latitude results. The high-altitude peak in May is the main difference between the low- and mid-latitude variations from 40–60 km. In the 20–40 km range the two stations exhibit different characteristics. The variations at Barking Sands are semiannual and similar to those at Cape Kennedy and White Sands, whereas the Coolidge Field data are somewhat acyclic, except for a small variance in April and a small peak in January. Finally, we note again the large differences between the mean variances at each site.

The three stations within the equatorial latitude belt are widely spaced in longitude, yet much of the upper-level structure among the three is similar. Firstly, in the 40–60 km height interval, there is a strong peak during January, and a broader secondary peak is centred around August (this is distorted somewhat at Fort Sherman). There is also strong diminution of wave variance at upper levels during April–May and October–November, especially at Ascension Island where the variances barely exceed the values at 20–40 km. The variations over each site differ within the lower height range. At Fort Sherman the lower-level variations are similar to those at upper heights, whereas at Kwajalein the variation is weakly annual, with a January maximum, and at Ascension Island the amplitudes from 20–40 km are seasonally acyclic.

These calculations were repeated using vertical wavelengths in the ~2–20 km range. Results from a selected high-latitude, mid-latitude, and tropical site are displayed in Fig. 8. The normalized cycles of these 'broad-band' variances in both height ranges were very similar to those in Fig. 7 among the various sites. The only general difference was that the annual variation of the broad-band variance was stronger, particularly at middle and low latitudes. The only site where the seasonal behaviour of the broad-band variance was significantly different to that in Fig. 7 was Fort Sherman, where a strong peak around October–November occurred. The broad-band variances were roughly three times larger at the tropics, and 1.5–2 times larger in the extratropics. The larger equatorial increases

Figure 8. As Fig. 7, but for the 2-20 km vertical wavelength band at three selected sites.
are probably due to the inclusion of large-amplitude Kelvin-wave disturbances in this broader band (e.g. Hirota 1978; Devarajan et al. 1985; see also Fig. 5).

The analysis was duplicated using the data-reduction technique of Hamilton (1991), with results computed over the 28–57 km range. As for the broad-band results, these data showed a more prominent annual cycle, giving large wintertime variances. Some of this variance may be due to interannual variability in the wintertime background vertical structure due to stratospheric warming events (e.g. Labitzke 1981). Nevertheless, the qualitative seasonal variations were generally the same.

(b) Temperature fluctuations

The seasonal variations in the variance of the relative-temperature fluctuations $\overline{T_r^2} = (T^r/T_0)^2$ are plotted in Fig. 9, again using data in the 2–10 km vertical-wavelength band. The four rows of plots correspond roughly to each of the four latitude bands defined earlier.

It is immediately apparent that $\overline{T_r^2}$ does not increase substantially with altitude, in contrast to $u^2 + v^2$ in Fig. 7. Additionally, monthly variations of $\overline{T_r^2}$ often differ from

![Figure 9](image-url)
the same site variations in the horizontal-velocity variance in Fig. 7. These results are not anticipated theoretically, as the model vertical wave-number spectra $F_{w}(m)$ and $F_{z}(m)$ have the same shape and height dependence, and thus their variances should be strongly coupled. Therefore, these findings require some evaluation.

An immediate consideration is whether such small measured temperature perturbations are reliable. Note from Fig. 9 that typical $T_{\text{rms}}$ values are $\sim 1\%$. Assuming a typical background temperature of 250 K, this translates into an r.m.s. absolute-temperature perturbation of $\sim 2.5$ K. The raw temperature data were digitized to the nearest degree Kelvin, yet this introduces a noise variance $T_{\text{dis}}^{2}$ of only $1/12$ K$^{2}$ (e.g. Kristensen and Kirkegaard 1987), which is too small to affect the $T_{\text{rms}}^{2}$ variations evident in Fig. 9 significantly.

If, however, the intrinsic accuracy of the original temperature measurements was only $\sim 1\%$, the variations in Fig. 9 may be due to random measurement errors. Schmidlin (1981) assessed the accuracy of temperatures measured by rocketsondes at Wallops Island by observing the repeatability of temperatures measured from successive launches separated by less than an hour. He found that the discrepancies increased as the time between successive launches increased, owing to natural atmospheric variability. He then used regression techniques to extrapolate the measured deviations for various inter-launch times back to an error value for a zero time difference, $\sigma_{T}$, which he took to be a measure of the intrinsic accuracy of rocketsonde temperature measurements. Using Schmidlin's results and mean temperatures computed from the Wallops Island data, measurement-error noise variances at various heights are estimated in Table 2. These values are typically around 10–30% of the values in Fig. 9, and so, while not an insignificant contribution, they appear unable to account for these fluctuations. Spectral analysis of these fluctuations revealed 'red' power spectra of the form $m^{-p}$, where $p \sim 2$, which also shows that neither measurement errors nor digitization noise are consistent with the bulk of this variance.

**TABLE 2. TABLE OF THE MEASUREMENT ERRORS, $\sigma_{T}^{2}$, IN ROCKETSONDE TEMPERATURES, AS INFERRED BY SCHMIDLIN (1981) FROM DATA AT WALLOPS ISLAND**

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>$\sigma_{T}^{2}$ (K$^{2}$)</th>
<th>$T_{0}$ (K)</th>
<th>$\sigma_{T}^{2}/(T_{0})^{2}$ (%)$^{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>0.70</td>
<td>240</td>
<td>0.04</td>
</tr>
<tr>
<td>40</td>
<td>1.00</td>
<td>250</td>
<td>0.16</td>
</tr>
<tr>
<td>45</td>
<td>1.34</td>
<td>260</td>
<td>0.20</td>
</tr>
<tr>
<td>50</td>
<td>1.73</td>
<td>270</td>
<td>0.24</td>
</tr>
<tr>
<td>55</td>
<td>1.65</td>
<td>260</td>
<td>0.24</td>
</tr>
<tr>
<td>60</td>
<td>5.55</td>
<td>250</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Background temperatures $T_{0}$ were computed from the Wallops Island rocket measurements analysed here, and are annual means (rounded to the nearest 10 K).

Another possibility is that this variance may result from processes other than gravity waves. For example, applying the analysis to the same winter temperature profiles at high-latitude sites occasionally produced an anomalously large perturbation variance. Inspection of the raw data revealed that these values were produced by isolated profiles where large impulsive temperature bursts occurred, and which were of small enough vertical scale to survive high-pass filtering. Such sudden stratospheric warmings are known to occur commonly during winter, and may enhance the observed winter variability. However, such impulses should approximate a delta-function, and so produce flat
spectra, yet, as mentioned above, spectral analysis does not reveal a white, noiselike spectrum.

Rejection of these ideas leads us to the conclusion that these temperature fluctuations are produced mostly by gravity waves. For a wave of intrinsic frequency \( \omega \) (such that \( \omega \ll N \)), the following polarization relation holds approximately for a zonally propagating wave:

\[
|T_*^r| \approx \frac{N}{g} \left( 1 - \left( f/\omega \right)^2 \right)^{1/2} |u'|
\]

(6)

For \( N \approx 0.02 \text{ rad s}^{-1} \), \( f/\omega \sim 0 \), and \( g \approx 10 \text{ m s}^{-2} \), then a value of \( u' \approx 5 \text{ m s}^{-1} \) (quite commonly observed in Fig. 7) produces a \( T_*^r \) amplitude of 1%. Therefore, the values in Fig. 9 are certainly of the order of those anticipated for gravity waves based on the earlier measurements of the horizontal-velocity amplitudes.

The height variations in \( T_*^r \) are studied in depth in section 7, but one can see from Fig. 9 that there is a clear lack of growth with altitude in the gravity-wave temperature perturbations in the mid-latitude, low-latitude, and equatorial belts, which is a surprising finding. Only at high latitudes is some increase in \( T_*^r \) with altitude evident. Here there are strong annual variations which peak in winter, as previously observed for \( \bar{u}^2 + \bar{v}^2 \) in Fig. 7. Summer values are very small at high latitudes and appear to approach the predicted values of the noise variance due to measurement errors as listed in Table 2.

At the mid-latitude sites the annual cycle in \( T_*^r \) persists more strongly than it did in the \( \bar{u}^2 + \bar{v}^2 \) results, where a prominent peak also occurred in August. Such a peak is weakly evident at Point Mugu and White Sands, but is quite apparent at Cape Kennedy.

The two low-latitude sites of Barking Sands and Coolidge Field both exhibit annual variations, but at Coolidge Field the August peak occurs much more strongly. There is only a suggestion of the subsidiary peak in May observed in \( \bar{u}^2 + \bar{v}^2 \) at both sites.

At the equatorial sites the variations are qualitatively similar to those detected in the horizontal velocities. There is a peak in January and a broad peak in August, with sharp minima occurring around April and October. At Ascension Island the mean \( T_*^r \) values in the 20–40 km height interval actually exceed those between 40 and 60 km.

As for \( \bar{u}^2 + \bar{v}^2 \), \( T_*^r \) values were recalculated using vertical wavelengths \~ 2–20 km, and again the normalized seasonal variations of these broad-band variances were much the same as those in Fig. 9. The stronger annual cycle at all sites and different seasonal structure at Fort Sherman, noted earlier in the broad-band \( \bar{u}^2 + \bar{v}^2 \) data, do not occur in \( T_*^r \). Broad-band \( T_*^r \) values are typically about three times larger than variances in the 2–10 km band at high latitudes, and roughly twice as large elsewhere.

6. Geographical variations

Strong site-to-site variations in ambient wave activity were noted in Figs. 7–9. Similar site-to-site variability was evident in the \( u_{zz} \) results of Hirota (1984).

Although there was some regularity in the normalized seasonal variations of the variance among sites within various latitude belts, little such latitudinal regularity is observed in these geographical differences in mean wave-variance levels. Some regularity is evident in the variances in the 20–40 km height range, where the variances at the equatorial and lower-latitude sites are generally larger than those at higher latitudes, although considerable site-to-site variability is also apparent. In the 40–60 km range no clear latitudinal trend is evident, and the site-to-site variations are generally larger than at lower heights. Similar variations were noted for mean variances in the 2–20 km wavelength band, except that mean equatorial variances were even larger again relative
to those at extratropical sites, due to the influence of equatorial wave disturbances at these longer vertical scales (e.g. Hirota 1978).

7. VARIATIONS WITH HEIGHT

Hirota and Niki (1985) studied the vertical variation of horizontal-velocity variances in summer, winter, autumn, and spring at several sites. They found that the variances increased with height quasi-exponentially, but with scale heights greater than the density scale height, suggesting dissipation of wave energy. Similar behaviour of the horizontal-velocity variances was noted in subsequent studies at single sites by Hass and Meyer (1987), Eckermann and Vincent (1989) and Tsuda et al. (1992), and in the wave potential energy per unit mass derived from stratospheric lidar measurements by Mitchell et al. (1991) and Wilson et al. (1991b).

However, vertical variations in $T_n^{ij}$ have not been investigated using rocket data. Figure 10 shows a collection of plot pairs of $\overline{u^2} + \overline{v^2}$ and $T_n^{ij}$ evaluated at 1 km height intervals at a specific location and during a given month, and plotted as a function of height. The first row of plots display data from the high-latitude site of Primrose Lake, where a strong annual variation in variances was observed. The first pair of plots are results for November, when variances are large, whereas the final pair are from July, when variances are much weaker. Despite the large variance contrast the normalized vertical variations are quite similar in both months. Horizontal-velocity variances increase with height quasi-exponentially, consistent with the findings of Hirota and Niki (1985). However, the vertical variations in $T_n^{ij}$ are different. From 20 to 40 km, amplitudes increase with height, as for $\overline{u^2} + \overline{v^2}$, but above ~40 km this growth stops fairly abruptly, and there is little increase or even some decrease in variance with height. Such variations are quite typical among the various sites.

The second row plots data from the mid-latitude site of Cape Kennedy; the first pair in September, when the variance is large, the second pair in the following month, when the upper-level variance is reduced substantially (see Fig. 7). Again, the normalized variations are rather similar. For $\overline{u^2} + \overline{v^2}$ we note similar increases with height in the 20–40 km interval, but above 40 km the variances grow with height far more in September than in October. In the $T_n^{ij}$ data, again the variances increase with height up to ~40 km, but then decrease somewhat with height thereafter. The decrease is more rapid in October than in September.

At the equatorial sites a strong semi-annual cycle in the variances was noted at upper levels, with weak wave variances around April–May and November (see Figs. 7–9). The bottom row of plots profile the variances at these times, the first pair for April data at Ascension Island, the second for November data at Kwajalein. At these times the $\overline{u^2} + \overline{v^2}$ data show a markedly different vertical variation. Quasi-exponential increases occur up to ~40 km, but above this height the variance decreases rapidly with altitude until, at around ~45–50 km, growth with height resumes. Similar behaviour is evident in the relative-temperature variances. Thus the region ~40–45 km of the equatorial stratosphere produces strong attenuation of wave amplitudes at these times. At other times when upper-level variances are larger, variations with height that are more typical are observed in this region.

8. SENSE OF ROTATION OF HORIZONTAL-VELOCITY PHASORS

In the northern hemisphere, clockwise (anticlockwise) rotation of the gravity-wave horizontal-velocity phasor with increasing height indicates upward (downward) wave-
Figure 10. Plot pairs showing the values of the variance of horizontal velocities, $u'^2 + v'^2$, and the relative temperature perturbations, $T'/T_s$, (2-10 km vertical-wavelength band) as a function of height at a selected site in a given month. The error bars are standard errors of the mean.

energy propagation, whereas the converse holds in the southern hemisphere (e.g. Andrews et al. (1987) p. 199). Therefore the sense of rotation with height of wave-induced horizontal-velocity vectors provides information on the vertical sense of propagation of the waves. Such calculations have verified that most upper-stratospheric gravity-wave energy originates from lower heights (Hirota and Niki 1985; Hass and Meyer 1987; Eckermann and Vincent 1989).

To study these features in greater depth, mean seasonal variations in the ratio of the clockwise-rotating variance to the total variance $u'^2 + v'^2$ (equal to $(I + Q)/2I$) are plotted as percentages in Fig. 11. Complete separation of upgoing and downgoing wave energy into the clockwise and anticlockwise spectra is only possible for a circularly polarized wave, whereas the gravity waves are elliptically polarized (see, for example, Fig. 1 of Hirota and Niki (1985)), and so the partitioning is 'blurred' somewhat (Eckermann and Vincent 1989). Therefore similar ratios were also computed, using another independent technique developed by Hirota and Niki (1985), in which the change in the wind phasor angle at adjacent heights is computed, and the wind profile rotation is classified according to whichever angular change (i.e. clockwise or anticlockwise) is the more common over the 20–60 km range. The results presented are for fluctuations in the
2–10 km band. The results were very similar when data in the 2–20 km band were used, differing in form only at selected tropical stations. Furthermore, calculations using Hirota and Niki's scheme within two height intervals 20–40 km and 40–60 km produced similar seasonal variations within each height regime at a given site.

The two calculation schemes produced different absolute values of the clockwise-rotation percentage, but gave very similar seasonal variations. As can be seen from
Fig. 11, nearly all the values lie above the 50% line, which indicates predominately upward-propagating wave energy. Furthermore, at virtually every site, the values peaked at the equinoxes (i.e. around April and September) and attained their minimum values at the solstices, of which the winter minimum was almost always the deeper. Note that anticlockwise percentages were plotted at Ascension Island, because the negative Coriolis parameter in the southern hemisphere reverses the sense of phasor rotation that any given wave there would have were it in the northern hemisphere.

9. INTERANNUAL VARIABILITY

So far the analysis has produced climatological results using a number of years of data. Now we investigate how reproducible mean seasonal variations in $u'^2 + v'^2$ and $T'^2$ are from year to year, using data from selected sites where good month-to-month data rates are maintained for several years.

The plots on the left of Fig. 12 show the variations in $u'^2 + v'^2$ and $T'^2$ in the 2–10 km wavelength band at a representative high-latitude site of Thule from 20 km to 60 km over four years from 1977 through 1980. We note that the mean annual cycles in $u'^2 + v'^2$ and $T'^2$, observed in Figs. 7 and 9, reproduce with similar characteristics from year to year in both height ranges. Similar reproducibility was found at Primrose Lake (55°N).

Mid-latitude results from Cape Kennedy from 1979 to 1982 are shown on the right-hand side of Fig. 12. The large peaks in August and smaller maxima around January, seen in Fig. 7, repeat from year to year, but a small subsidiary equinoctial peak is observed during 1980 and 1982, but not during 1981, perhaps explaining the 'plateau'

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Figure 12. Monthly-mean horizontal-velocity and relative-temperature fluctuations from 1977 to 1980 over Thule and from 1979 to 1982 over Cape Kennedy. Bars indicate the standard errors of these means. Results are for the 2–10 km vertical-wavelength band.
structure around April alluded to earlier when analysing the mean results at this site. Similarly, the peak around April is variable in strength in data from 1977 to 1980 at Point Mugu, perhaps explaining the weakness of this peak in the climatological mid-latitude results in Fig. 7. Peaks in the relative-temperature perturbations in August and January occur each year, but with somewhat variable intensity.

One location where one might anticipate some interannual variability is at the equatorial latitudes, where the lower stratosphere exhibits a quasi-biennial oscillation (QBO) in background wind and temperature. This structure is seen clearly at the lowest heights in the monthly-mean zonal-wind structure from 1977 to 1980 over Kwajalein and from 1979 to 1982 over Ascension Island, as plotted in the top panels of Fig. 13. Above

![Mean Zonal Wind at Kwajalein (9°N, 168°E)](image1)

![Mean Zonal Wind at Ascension Island (8°S, 14°W)](image2)

![Horizontal Velocity Variances at Kwajalein (9°N, 168°E)](image3)

![Horizontal Velocity Variances at Ascension Island (8°S, 14°W)](image4)

![Relative Temperature Variances at Kwajalein (9°N, 168°E)](image5)

![Relative Temperature Variances at Ascension Island (8°S, 14°W)](image6)

**Figure 13.** The top two plots show monthly-mean zonal winds from the rocket observations during 1977 through 1980 at Kwajalein and 1979 through 1982 at Ascension Island. Unshaded areas represent missing data, and the contour labels are in m s⁻¹. Positive zonal-wind values are eastward and have light shading. The corresponding plots below show narrow-band values of the variance of horizontal velocities, $u^2 + v^2$, and the relative temperature perturbations, $T^2$, in the 20–40 km and 40–60 km height intervals during these years. Error bars are standard errors of the mean.
~40 km one can also observe the well-known semi-annual oscillation (SAO) of the equatorial upper stratosphere (see, for example, Hirota 1978, 1980). The horizontal-wind and relative-temperature variances at these two sites in the 20–40 km and 40–60 km range over these years are shown in the remaining panels of Fig. 13.

In the lower height range (20–40 km) the variance shows evidence of a QBO-coupled effect. To highlight this better, the variances are replotted in Fig. 14. The time interval 1979–81, when the data from both sites overlap, is shaded. A seventh-degree polynomial was fitted to each data series by a least-squares method, and then overplotted with a thick curve on each plot. While this curve can have up to six turning points, in most cases only three or four turning points arise. Furthermore, in each case just two local maxima arise, separated by roughly two years, and occur in a similar position in each plot. The peak in the overlapping shaded region occurs around January 1980 for both $\overline{u^2} + \overline{v^2}$

![Figure 14](image)

Figure 14. Replotting of the variance data in Fig. 13. The top plots show the variance of horizontal velocities, $\overline{u^2} + \overline{v^2}$, at 20–40 km, the middle plots show the relative temperature perturbations, $\overline{T^2}$, at 20–40 km, and the bottom plots show $\overline{T^2}$ at 40–60 km. The period when there are data from both sites is shaded. The thick solid curves are least-squares fits to the data of a seventh-degree polynomial, highlighting variation over a quasi-two-year period.
and $T^{12}$ at both sites. A local minimum in the fitted curves is reproduced at around January 1979 at Kwajalein, and near January 1981 at Ascension Island, once again a separation of around two years. Thus there appears to be strong evidence of QBO-modulated wave variances in the 20–40 km range from 1977 to 1982.

In the upper height range (40–60 km) the data are more scattered and have a stronger semi-annual character. Nevertheless, in Fig. 14 the relative-temperature variances at this upper height range are plotted, and show evidence (albeit weaker) of similarly phased QBO modulation of wave variances. However, no clear evidence of a QBO signal could be discerned in the horizontal-velocity variances at these heights. Since we have only 4–5 years of semi-continuous data, this may be insufficient to pick out any weaker QBO signal from the strong semi-annual cycle in the wave variances here.

10. DISCUSSION

These results provide a great deal of new data on the morphology of gravity waves in the middle atmosphere, but attempts to explain all these features of the wave field are beyond the scope of this work. Here we shall focus upon the polarization data, so that we can cast light on the actual wave content of the data. We shall leave detailed appraisal of the variance characteristics of these waves for later studies. A start in this regard has been provided by Eckermann (1992, 1994), who employed simple modelling to propose explanations for aspects of the seasonal, latitudinal, and vertical variations in $u^{12} + v^{12}$ and $T^{12}$ observed here and in other studies.

(a) Seasonal variability of phasor-rotation percentages

Figure 11 showed that clockwise rotation of wave horizontal-velocity phasors was preferred at all but the equatorial sites in every month. This indicates that most gravity-wave energy propagates upwards from below, in agreement with earlier studies (e.g. Thompson 1978; Vincent 1984; Hirota and Niki 1985; Hass and Meyer 1987; Eckermann and Vincent 1989). Newly discovered seasonal features were: a deep minimum in the percentage of clockwise rotations during winter, and a smaller subsidiary minimum in summer; and maxima around March–May and August–October. While the seasonal features are clear, their cause is harder to identify.

The simplest interpretation is that the features arise owing to seasonal variations in the ratio of upward-propagating to downward-propagating wave energy. However, these rotary-component energy percentages can also be modified by seasonal variations in the intrinsic-frequency distribution and/or degree of superposition of the wave field (Eckermann and Hocking 1989), wave-packet localization (Dong and Yeh 1989), ducting effects (e.g. Fritts and Yuan 1989), and vertical vorticity or vertical shear in the mean wind (e.g. Hines 1989; Danielson et al. 1991).

The observed seasonal cycles correlate well with the seasonal variations in the magnitude of the mean zonal wind $U$. To illustrate this, climatological mean zonal winds, as computed from the rocket data taken over Point Mugu, are shown in Fig. 15. When the flow is weak the rotational ratios are high, whereas when the stratospheric jets intensify at the solstices the ratios are small, with the smallest ratios generally occurring during winter when the zonal flow is strongest. Furthermore, at Thule, where the flow is quite weak for most of the year, the variations in Fig. 11 are similarly small with season.

A possible explanation consistent with this correlation is as follows. We note that in order to observe this phasor rotation, we first require that the wave is elliptically polarized, which in turn implies that the wave's intrinsic frequency $\omega$ is of the order of
the inertial frequency $f$ (i.e. $f/\omega \neq 0$), where $\omega = k_h c - \bar{U}(z) \cos \phi$ and $k_h$ is the horizontal wave number. Consequently, the ellipticity depends on the background wind profile $\bar{U}(z)$. From Fig. 15 we observe that, at the equinoxes, mean winds are not only light, but the vertical shear in the mean wind is also small, whereas at the solstices mean winds increase in magnitude substantially with altitude, producing strong mean shear. Consequently, during April--May and September--October at Point Mugu, an elliptically polarized zonally propagating wave at 20 km will maintain roughly the same ellipticity as it propagates upwards, since $\bar{U}(z)$ varies so little with height. In summer or winter, however, an elliptically polarized zonally propagating wave at 20 km will increase in frequency on propagating upwards, owing to the strengthening zonal winds, so that $f/\omega \to 0$, the wave becomes linearly polarized, and phasor rotation cannot be observed.

While waves propagating orthogonal to the mean flow do not suffer from this background wind effect (e.g. Fig. 4(a)), on averaging over all azimuths, phasor rotations generally will be less coherent during the solstices, owing to those waves which propagate nearly zonally maintaining ellipticity over only a narrow range of heights. Such a lack of coherence with height due to mean wind shear can explain the lower rotation percentages at these times, as well as the peak percentages at the equinoxes when the wind-shear effect is least severe. Additionally, wind shear transverse to the wave propagation direction can directly induce horizontal ellipticity changes in both low- and high-frequency waves (Hines 1989).

This hypothesis, however, is neither quantitative nor definitive, and we cannot discount other explanations. For example, these variations may reflect real seasonal variations in the amount of upward-propagating wave energy. Indeed, the variation in the mean wind with height during summer and winter can lead to vertical reflection of wave energy in certain circumstances (e.g. Lindzen and Tung 1976; Chimonas and Hines 1986), and these results may be consistent with increased reflection of wave energy as the zonal wind jets intensify during the solstices. Indeed, ducting might also arise, producing trapped wave modes which also affect the polarization characteristics (e.g. Chimonas and Hines 1986; Fritts and Yuan 1989).
After reinterpretating some previous measurements made by Hirota and Niki (1985) of inferred $f/\omega$ distributions from four rocket stations (Thule, Primrose Lake, White Sands, and Kwajalein), Eckermann and Hocking (1989) concluded that the wave field at most sites was more directional during the winter months, and that annual-mean directionalities appeared to decrease as one moved progressively more poleward. We argued in section 3 that biasing of wave azimuth determinations occurs in wintertime rocket data within narrow pass bands, owing to stronger mean winds at such times, leading to tightly clustered azimuth determinations, bearing out Eckermann and Hocking’s reinterpretation of earlier data.

The modelling in section 3 showed that the propagation azimuths of small $\lambda_z$ waves were mostly orthogonal to the mean wind when the flow was strong. Eckermann and Vincent (1989) noted that the wave alignments calculated from rocket data from central Australia were clustered meridionally during winter. However, mean wind speeds at these times can near 100 m s$^{-1}$ eastward, so that any zonally propagating waves are likely to be missing from their band-passed (2–10 km) data. Thus their wintertime results are probably due to this effect. Similarly, Hamilton (1991) presented results within a range of different pass-bands, finding different annual-mean propagation directions at the smaller and larger pass-bands at Fort Churchill and White Sands. When propagation azimuths were computed using both unfiltered fluctuations and a pass-band of 7.5–15 km at White Sands and Cape Kennedy, the unfiltered results during winter showed more zonal propagation directions than the band-passed results, again consistent with our simple ideas of ‘azimuthal filtering’ when the mean winds are strong.

Eckermann and Hocking (1989) also argued that the wave field at equatorial locations appeared to possess ‘overdirectionality’; that is, the wave fluctuations appeared to exhibit more directionality in the horizontal than theoretically anticipated for a representative spectrum of gravity waves. They speculated that other types of waves may contribute to the fluctuations here, of which the Kelvin wave was identified as the most likely candidate.

Much lower mean clockwise-rotation percentages arose at the equatorial sites than at the extratropical locations, as previously noted by Hirota and Niki (1985). This was particularly evident during December–February. Yet the mean-wind shears in the equatorial upper stratosphere are not significantly stronger here; indeed, they are generally weaker than those around 30°N. Consequently, the aforementioned wind-shear explanation of the seasonal distributions cannot account for this equatorial decrease. Eckermann and Hocking (1989) addressed the similar findings of Hirota and Niki (1985), and suggested that an equatorial preponderance of Kelvin-wave motions, which are linearly polarized east–west in the horizontal (e.g. Andrews et al. 1987) even in the presence of realistic vertical and latitudinal shear in the background zonal wind (Boyd 1978), might explain these smaller clockwise-rotating percentages.

The alignment data presented in Fig. 6 bear out most of these indirect conclusions. The strong east–west alignments of the equatorial fluctuations are entirely consistent with the equatorial ‘overdirectionality’ of wave azimuths inferred by Eckermann and Hocking (1989), and are superficially consistent with a Kelvin-wave interpretation. The observation of rotation percentages $\sim$50% in December–February, when east–west alignments persist both at lower and upper heights, is evidence of linear polarization of these fluctuations ($Q = 0$), and this constitutes strong support for the Kelvin-wave interpretation of these motions at these times. However, in the upper stratosphere, alignments were often less zonally constrained (e.g. March–May). While this was not so in the lower stratosphere, the clockwise-rotation percentages in Fig. 11 exhibited semiannual variability in both height ranges, and only approached the 50% level around
December–February. Thus, Kelvin-wave dominance of the equatorial wave field does not appear to occur all year round, notwithstanding the strong zonal clustering of the fluctuations which was perpetually observed in the lower stratosphere. Consequently, we now study the equatorial wave field in more depth, so as to cast light on its intrinsic nature.

(c) Fluctuations in the equatorial lower stratosphere

We have noted that the velocity fluctuations in the equatorial lower stratosphere were strongly zonal throughout the year, whereas clockwise-rotation percentages evaluated over the full 20–60 km height range showed semi-annual variability. These values bottomed out ~ 50% in December–February and, when coupled with the strong east–west fluctuation alignments, indicate that Kelvin waves dominate the variance at these times.

Time-series analyses of radiosonde data from the equatorial lower stratosphere identified a large-amplitude (u' ~ 8 m s⁻¹) Kelvin wave of zonal wave number 1 and a 10–15 day ground-based period, which was calculated to have a vertical wavelength ~ 5–10 km (e.g. Wallace and Kousky 1968; Kousky and Wallace 1971; Angell et al. 1973). Thus much of the wave variance identified here in this spatial analysis may be due to this wave mode. Indeed, a range of numerical and laboratory experiments (e.g. Andrews et al. 1987) have concluded that this wave drives the eastward mean-flow accelerations of the QBO, a theory originally put forth by Holton and Lindzen (1972). Analysis has already confirmed that equatorial variance is QBO modulated in the 20–40 km height range.

In order to investigate this in greater depth we confine attention to the height range 20–30 km rather than 20–40 km, as above 30 km the stratospheric SAO starts to develop, whereas the QBO is distinct in the 20–30 km range, as can be seen in the monthly-mean zonal winds plots on the top row of Fig. 16. The second row of graphs in this figure show u² + v² in the 2–10 km wavelength band, over the restricted height range 25–28 km, which is centred within the QBO. Quasi-biennial modulation of the variances is clearly evident. The bottom row of plots show the alignment of the fluctuations and the clockwise-rotation percentages. Both quantities were evaluated over the 20–30 km height interval. While the alignment data exhibit strong east–west polarization with little or no interannual variability, the phasor rotations show a distinct QBO-related signal, although an annual signal is also superimposed which complicates the variations.

The increases in the phasor-rotation percentages occur during changeover times from westward to eastward flow, and decrease during changes from eastward to westward flow. The latter correlation is consistent with the penetration of the zonally aligned, linearly polarized Kelvin mode into the stratosphere, because this wave has an eastward ground-based phase speed, and so it propagates unhindered in the stratosphere when the flow becomes progressively more westward. The former correlation is consistent with an increasing absence of Kelvin-wave variance, since such an eastward-propagating wave experiences increasing attenuation as the flow becomes more eastward, until it is dissipated due to breaking, radiative damping, or critical-level absorption. While the increases are very gradual, the decreases are very rapid (e.g. September 1978, April 1981). This suggests that the Kelvin modes penetrate to these heights rather suddenly, dominating the wave field, and then dissipate in driving the eastward accelerations of the QBO and become less and less dominant in the fluctuations.

The question now arises as to the wave content of the data at the times of most-reduced Kelvin-wave effect. QBO studies suggest that a mixed Rossby-gravity wave mode of zonal wave number 4 and a ground-based phase speed around ~23 m s⁻¹ drives
the westward accelerations of the QBO, the so-called Yanai wave (Yanai and Maruyama 1966; Andrews et al. 1987; Dunkerton 1993). The wave is elliptically polarized in the horizontal, and the theoretical elliptical velocity polarization of this mode is shown as a function of latitude in Fig. 17, using the unsheared formulae presented by Andrews et al. (1987). At latitudes ~ 8–9°, corresponding to our equatorial station data, the formulae predict elliptical polarization with a meridional semi-major axis, so that analysis of this mode would give a meridional alignment value.

However, Boyd (1978) has demonstrated that the mixed Rossby-gravity wave is very susceptible to latitudinal wind shear, yet, while shear alters the latitudinal structure of the mode appreciably and can shorten the vertical wavelength by up to 60%, his numerical results still generally indicated an elliptically polarized motion ellipse with a meridional semi-major axis at these latitudes. While more complex modelling using a variety of background wind profiles has occasionally revealed zonal alignment of the Yanai mode in one hemisphere at these latitudes (Holton 1979; Dunkerton 1983), uniform zonal alignment of the wave ellipse at all times in both hemispheres never arises. Either the
semi-major axis of the motion ellipse of the Yanai wave is modified to become east-west through other effects, or else other types of wave motions with more zonally aligned velocity polarizations also exist with appreciable variance in the data. Possibilities include higher-order solutions to the equatorial beta-plane wave-eigenvalue problem, such as westward-propagating planetary-scale gravity-wave and/or Rossby-wave modes (e.g., Andrews et al. 1987). Small-scale gravity waves may also contribute, but there is no obvious reason why these waves should all be strongly polarized east-west. It is worth noting in this context that recent QBO modelling has cast doubt on whether the Yanai wave and Kelvin mode can provide sufficient mean-flow accelerations to drive the QBO (Boville and Randel 1992; Takahashi and Boville 1992), and the possibility was raised that integration over all wave modes might be necessary to resolve the requisite wave momenta.

The QBO modulation of wave variance in Fig. 16 is not related in a simple fashion to the phasor rotations. The peak variance occurs at the rapid changeover from westward to eastward flow, yet the onset of small rotation percentages occurs a number of months before this. It suggests that Kelvin waves progressively build up in variance until they become unstable, begin to dissipate, and then reduce in variance as they drive the eastward phase of the QBO, causing the gradual increase in the phasor-rotation percentages thereafter. Previous studies of the Kelvin mode found that it only arose around the times of onset of descending eastward winds of the QBO (e.g., Kousky and Wallace 1971; Angell et al. 1973). While the variances peak at this time, we find Kelvin-wave-like polarization some months before these times as well.
(d) Fluctuations in the equatorial upper stratosphere

In the upper stratosphere, fluctuations with vertical wavelengths $< 10$ km have usually been ascribed to gravity waves (Hirota 1984; Hirota and Niki 1985). However, in Fig. 6 we noted that fluctuations at Kwajalein and Fort Sherman (both 9$^\circ$N) in the 2–10 km wavelength range exhibited strong east–west alignment in the 40–60 km height interval during December–February. This fact, coupled with the observation of phasor-rotation percentages $\sim 50\%$ at these times (see Fig. 11), indicates that Kelvin-wave oscillations dominate the upper-stratospheric variance at these times, even at these short vertical scales.

We have no information on phase speeds or horizontal scales, so it is impossible to characterize the Kelvin mode(s) responsible for this variance using these data. Nevertheless, it is well established that Kelvin waves with vertical wavelengths $\sim 10$–40 km exist with appreciable variance in the upper stratosphere (Hirota 1978, 1979; Salby et al. 1984; Devarajan et al. 1985; Hitchman and Leovy 1988; Randel 1990; Randel and Gille 1991; Hamilton 1991; Gao and Stanford 1993). Salby et al. (1984) found evidence of a number of short vertical-wavelength Kelvin modes in their analysis of satellite data taken during October 1978 and January–February 1979. In particular, a strong horizontal wave-number 2 mode with a vertical wavelength of 7–13 km was identified. Devarajan et al. (1985) studied oscillations of the zonal wind from two years of equatorial rocket data at two sites (13.7$^\circ$N and 21.5$^\circ$N). They found that most of the waves had rather small vertical scales ($\sim 6$–12 km), which they also interpreted as being wave-number 2 Kelvin waves. Thus, the Kelvin-wave variance identified here may be due to this Kelvin mode.

At the southern equatorial site of Ascension Island, zonal alignments were also observed in Fig. 6 during December–February at 40–60 km. Indeed, reinspection of the seasonal variation of the vertical wave-number power spectral densities presented by Hirota (1978) reveals evidence of an oscillation in the zonal wind around February with a wavelength $\sim 9$ km. However, at this site, rotation percentages nearing 50% were not observed at the same time (see Fig. 11). To further the uncertainty at this site, during June–August, when zonal alignments were not strongly observed at these heights, phasor-rotation percentages became smaller. Thus it seems that the situation at Ascension Island is somewhat different.

Some possible clues come from comparing the mean zonal winds at Kwajalein and Ascension Island at 40–60 km in Fig. 18. At Kwajalein peak westward flow of $\sim$20–30 m s$^{-1}$ occurs in both January and July, whereas at Ascension Island the westward flow peaks nearer 60 m s$^{-1}$ in January, but only reaches $\sim 10$ m s$^{-1}$ during July. Since, from (2), $\lambda_z$ is proportional to $|c - U \cos \varphi|$, and since at Ascension Island $U$ is strong and westward whereas Kelvin-wave phase speeds $c$ are always eastward, this implies $\varphi \sim 180^\circ$ and thus vertical wavelengths significantly larger than 10 km for any Kelvin waves that might be present. Indeed, computation of phasor-rotation percentages at Ascension Island in the 2–20 km wavelength band revealed different seasonal behaviour, with four rather than two minima arising in December–January, April, June–July and October, suggesting Kelvin-wave effects become more obvious at the larger scales, as does the increased zonal clustering of the 2–20 km band alignment data at this site in Fig. 5. Thus it appears more likely that the zonally aligned activity in the 2–10 km band is due to waves with westward phase speeds, such as gravity waves or mixed Rossby-gravity waves. Around July, Kelvin waves impact upon the data more, but other nonzonal fluctuations also coexist.

Gravity waves seem to be the obvious explanation for the nonzonal fluctuating variance which arose at all three equatorial sites during March–May and September–
November when rotation percentages also became higher in Fig. 11. Another possibility is that these motions could be contributed to by mixed Rossby-gravity waves. Such waves are not expected to be prevalent at 40–60 km owing to their strong dissipation and thermal damping lower down (e.g. Hirota 1980). Nevertheless, Randel et al. (1990) detected mixed Rossby-gravity waves in upper-stratospheric satellite temperature data, but the measured amplitudes (~0.1–0.3 K) were far smaller than the temperature amplitudes encountered in these data, and their vertical wavelengths exceeded 10 km. Furthermore, we expect these motions to give alignment values either near zonal or near meridional.

Thus we interpret the quasi-randomly aligned motions encountered in the equatorial upper stratosphere as gravity waves, which is an important observation. Long-wavelength ($\lambda_z \sim 10–40$ km) Kelvin waves, first observed at these heights by Hirota (1978), were initially believed to provide the majority of the eastward forcing of the SAO at these heights (Holton 1975; Dunkerton 1979; Mahlman and Umscheid 1984). However, simulations with the Geophysical Fluid Dynamics Laboratory general circulation model (GCM) by Hamilton and Mahlman (1988) indicated that small-scale gravity waves provided most of the eastward forcing, and large-scale equatorial Kelvin waves contributed comparatively little. Indirect experimental support for this was inferred by analysis of satellite data from this region by Hitchman and Leovy (1988), who concluded that the relative importance of gravity-wave compared with Kelvin-wave forcing increased with altitude. Modelling of the antiphased mesospheric SAO certainly suggests that gravity waves are important at these higher altitudes (Dunkerton 1982; Hamilton and Mahlman 1988). Yet the most recent GCM simulations still differ on the relative contributions of Kelvin waves and gravity waves to the forcing of the SAO. Boville and Randel (1992) found that Kelvin waves produced the bulk of the eastward forcing of their well-simulated stratospheric SAO. Sassi et al. (1993), on the other hand, found that realistic Kelvin-wave driving produced a weaker eastward SAO phase in their model than is observed, and argued that gravity-wave driving was needed to improve their simulations.

We have studied the polarization and variance characteristics of the waves here, neither of which are easily related to the wave momentum flux variations which determine the effect of these waves on the SAO. Nevertheless, the seasonal behaviour of the long-
wavelength Kelvin-wave variance (e.g. Hirota 1978; Devarajan et al. 1985; Randel and Gille 1991) is strikingly similar to that of the small-scale equatorial variance in Fig. 7, with strong diminution of wave variance during April–May and October–November in the upper stratosphere clearly evident in Fig. 10. Thus it appears that both small-scale and large-scale waves dissipate during the eastward-acceleration phases of the stratospheric SAO.

11. SUMMARY AND CONCLUSIONS

It seems clear that the reduction and analysis of vertical variations of rocket-derived atmospheric winds and temperatures between 20 and 60 km in altitude has provided reliable and important new information on wave variance and polarization within this region of the atmosphere. We conclude with a point-by-point summary of the major findings of our analysis.

Systematic differences among different latitude belts were detected in the seasonal variations of the wave fluctuations within an upper and lower height interval. The clearest of these features were:

(1) The equatorial variance exhibited quasi-biennial variability at 20–40 km, and a semi-annual variation at 40–60 km, although there was some evidence of quasi-biennial modulation of $T_2^2$ at 40–60 km. Around April and November, when upper-level variances were weakest, variances decreased with height in the 40–45 km height range. Thus the variances appear to be correlated with the semi-annual and quasi-biennial oscillations of the background equatorial stratosphere. At 20–40 km, the small-scale fluctuations were aligned east–west, consistent with the expected QBO-related Kelvin modes, but inconsistent with the mixed Rossby-gravity mode (the Yanai wave) anticipated at other times. At 40–60 km the small-scale fluctuations were mostly gravity waves, although east–west polarization typical of Kelvin modes occurred during December–February.

(2) Large increases in mid-latitude wave variance occurred around August at upper levels, yet by October the variance was small and produced a seasonal minimum. A secondary upper-level peak during May occurred at the low-latitude sites.

(3) At high latitudes the wave variance had a strong annual variation, which peaked in winter and minimized during summer.

(4) The variance differed greatly among the 15 sites, even among sites quite close to one another, indicating appreciable geographical variability in wave activity.

Other features of the observations were common to all the sites. The relative-temperature variances grew insubstantially with altitude, and often decreased in value with altitude at upper heights, in contrast to the horizontal-velocity variances which increased quasi-exponentially with height throughout the stratosphere. The ratio of clockwise to anticlockwise rotations with height of horizontal-velocity phasors exhibited a semi-annual oscillation, with minima at the equinoxes and maxima during summer and winter. The variances at all but the equatorial sites had an annual component which peaked during winter. This annual cycle gradually attenuated as one moved equatorward from the polar sites. Finally, the azimuthal alignments of the velocity fluctuations in the 2–10 km band were clustered north–south when stratospheric mean winds became strongly zonal at mid-latitudes during the solstices, in qualitative accord with our modeling of this process.
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APPENDIX

Determining the 'height-averaged' gravity-wave spectrum

Formulae (4) and (5) are spectral models derived using simplified theoretical ideas (Smith et al. 1987; Fritts et al. 1988), yet these shapes agree quite well with spectra computed from both high vertical-resolution measurements of atmospheric winds and temperatures (e.g. Fritts and Chou 1987; Fritts et al. 1988; Shibata et al. 1988) and other theoretical approaches which have argued that different processes and dynamics produce the observed spectral shapes (e.g. Hines 1991b). Both the relative-temperature and horizontal-velocity spectra have the same normalized shape. For the horizontal velocities, waves which have wave numbers greater than \( m_\ast \) produce a spectrum asymptoting to the form \( N^2/6m^3 \), whereas waves with wave numbers less than \( m_\ast \) produce a flat spectrum of intensity \( N^2/6m^3 \), and so the spectral density here grows with height approximately as \( e^{3z/2H_E} \), because \( m_\ast \) varies with height as \( e^{-z/2H_E} \). Later studies have adopted a model in which this flat portion of the spectrum is altered to increase in intensity linearly with \( m \) and exponentially with height as \( e^{2z/H_E} \) (VanZandt and Fritts 1989; Hines 1991b). Furthermore, in the model of Hines (1991b) \( H_E \) equals \( 2H_p \). The integrated r.m.s. amplitude of the fluctuations over the whole wave-number range grows with height as \( e^{3z/2H_E} \).

For the horizontal-velocity fluctuations \( u' \) and \( v' \), we see that Hirota's measure of gravity-wave activity (3) gives

\[
\frac{u^2_{zz}}{2z - z_1} = \frac{1}{z_2 - z_1} \int_{z_1 = 20 \text{ km}}^{z_2 = 85 \text{ km}} \left( \frac{d^2 u'}{dz^2} \right)^2 + \left( \frac{d^2 v'}{dz^2} \right)^2 \, dz
\]

\[
= \frac{1}{z_2 - z_1} \int_{z_1}^{z_2} m^4 (u'^2(z) + v'^2(z)) \, dz
\]

\[
= \frac{1}{z_2 - z_1} \int_{z_1}^{z_2} \left( \int_{m_1}^{m_2} m^4 F_{uv}(m, z) \, dm \right) \, dz. \tag{A.1}
\]

Expression (A.1) results from the fact that the gravity-wave horizontal-velocity variance is given by integrating the model power spectrum \( F_{uv}(m, z) \). The ranges \( m_1 \) and \( m_2 \) correspond to the limits of the observational wave-number window, which in the data of Hirota (1984) were \( 2\pi(10 \text{ km})^{-1} \) and \( 2\pi(2 \text{ km})^{-1} \) respectively, although larger wave-number fluctuations may also be aliased in.

To proceed we assume that the vertical wave number \( m \) is independent of \( z \). For a given wave this is incorrect, as its vertical wave number depends on the height variation of the background wind \( \bar{U}(z) \), which can be considerable at these altitudes. Indeed, Hines (1991b) has argued that these spectral shapes arise owing to the stochastic ‘Doppler-spreading’ of the wave numbers of the various constituent waves. However, the \( m \) value
in the spectral formula (4) is just a parameter of the Fourier decomposition of the data series, and bears no direct relationship to the varying vertical wave number of any one wave. Thus the \( m \) parameter as defined in (4) is independent of \( z \), and so the integration order in \((A.1)\) can be interchanged, so that

\[
u z = \int_{m_1}^{m_2} m^4 \left( \frac{1}{z_2 - z_1} \int_{z_1}^{z_2} F_{u_0}(m, z) \, dz \right) \, dm
\]

\[
= \int_{m_1}^{m_2} m^4 \mathcal{F}_{u_0}(m) \, dm \tag{A.2}
\]

where the 'height-averaged' spectrum \( \mathcal{F}_{u_0}(m) \) can be derived analytically, and is given by

\[
\mathcal{F}_{u_0}(m) = \frac{N^2 H_E}{9(z_2 - z_1)m^3} \ln \left( \frac{1 + G(z_1)\mu^3}{1 + G(z_1)\mu^3} \right) \tag{A.3}
\]

where \( G(z) = e^{3(z-z_1)/2H_E} \), and \( \mu = m/m_0(z) \).

Therefore we obtain the useful result that \( \nu z \), the r.m.s. value of which Hirota (1984) used as a measure of wave activity, is given by the area beneath the height-averaged spectrum \( \mathcal{F}_{u_2z}(m) = m^4 \mathcal{F}_{u_0}(m) \). A similar result is easily derived for the relative-temperature fluctuations as well.

To plot this height-averaged spectrum \( (A.3) \), the parameters \( m_0(z) \) and \( H_E \) must be determined, and can be evaluated self-consistently from observational data as follows. It is a straightforward task to evaluate and plot the mean horizontal-velocity variances produced by these gravity-wave motions as a function of height (see, for example, Hirota and Niki 1985; Hass and Meyer 1987; Eckermann and Vincent 1989; see also Fig. 10). A theoretical estimate of the variance at any given height can be made by integrating the gravity-wave horizontal-velocity spectrum over the resolved wave-number range, say between wave numbers \( m_1 \) and \( m_2 \). A general analytical expression for this integral was derived by Eckermann (1990) (his Eq. (4)), and by evaluating \( N, u^2(z) + v^2(z), m_2 \) and \( m_1 \) from the data, an \( m_0(z) \) value can then be calculated self-consistently from this analytical formula. By doing this at an upper and a lower height, the parameter \( H_E \) can be estimated, from the change in \( m_0 \), using the fact that \( m_0 \) varies with height as \( e^{-z/2H_E} \). All the parameters are now specified, and so the height-averaged spectrum appropriate for the data in this height range is now defined and can be plotted using \((A.3)\).

Earlier calculations using these rocket data by Hirota and Niki (1985) showed that horizontal-velocity variances produced by upper-stratospheric gravity waves are typically around 10 m\(^2\)s\(^{-2}\) at a height of 30 km, and around 60 m\(^2\)s\(^{-2}\) at 60 km (see also Fig. 10). Using a representative value of \( N \sim 0.02 \) rad s\(^{-1}\) and values of \( m_1 = 2\pi (15 \text{ km})^{-1} \) and \( m_1 = 2\pi (2 \text{ km})^{-1} \) appropriate to the wave-number band analysed by Hirota and Niki (1985), then putting these numbers into Eq. (4) of Eckermann (1990) and solving for the only unknown parameter \( m_0 \), gives \( m_0(z = 30 \text{ km}) \approx 2\pi (2.7 \text{ km})^{-1} \) and \( m_0(z = 60 \text{ km}) \approx 2\pi (7.1 \text{ km})^{-1} \). This then gives a value of \( H_E \approx 17 \) km. The resulting spectrum is plotted in its energy-content form \( m^3 \mathcal{F}_{u_2z}(m) \) in Fig. A.1. Since \( \nu z \) is given by the area beneath \( \mathcal{F}_{u_2z}(m) \), one can see from Fig. A.1 that the majority of this variance is produced by the largest wave numbers (smallest vertical wavelengths).

According to the model of Smith et al. (1987), individual waves at high wave numbers \( m > m_0 \) are saturated. Therefore, according to their theory, the variations of \( \nu z \), noted by Hirota (1984) should be more indicative of seasonal changes in the local wave-
saturation environment rather than changes in unsaturated gravity-wave activity. According to the model of Hines (1991b) all individual waves are unsaturated, but it is the total variance of the vertical gradient of horizontal-velocity fluctuations $(du'/dz)^2 + (dv'/dz)^2$, divided by $N^2$, which saturates, and this variance is controlled principally by these smaller-scale motions in the spectrum too (Hines 1991a). Hines (1991b) argues, however, that the limited and quasi-invariant spectral form at large wave numbers is not due to saturation, but is instead due to the characteristic form that the wave–wave Doppler-spreading interactions take in the Fourier domain. We refer to this interpretation as 'spectral limiting' of variances at these high wave numbers.

Thus, according to these theories, the measure $u_{xz}^2$ is controlled more by wave saturation or spectral-limiting effects than by variations in unsaturated wave variances.

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