Entrainment and mixing in buoyancy-reversing convection with applications to cloud-top entrainment instability

By WOJCIECH W. GRABOWSKI*

National Center for Atmospheric Research, USA

(Received 28 March 1994; revised 28 June 1994)

SUMMARY

A consensus seems to exist throughout the cloud-physics community that buoyancy reversal associated with evaporative cooling affects not only the global (cloud-scale) dynamics of a convective cloud, but also the rate of mixing between the cloud and its environment. The latter effect is associated with the concept of the so-called 'cloud-top entrainment instability' (CTEI), which assumes a positive feedback between buoyancy reversal and the rate of entrainment. In this paper, effects of buoyancy reversal on cloud dynamics are discussed in the context of an unstratified anelastic two-fluid system. Convection in this system mimics some essential features of cumulus convection. Two-dimensional numerical experiments, with and without the effects of buoyancy reversal, have been performed. It was found that buoyancy reversal has a dramatic impact on the overall flow evolution, but that its effect on the rate of mixing between the two fluids is small, i.e. flows which differed dramatically depending on whether there was buoyancy reversal or not still resulted in similar amounts of mass being mixed. This result casts doubt on the concept of CTEI in cumulus dynamics. The distinction between strong global effects of buoyancy reversal and minor effects on the rate of mixing in buoyancy-reversing convection is supported by laboratory experiments with classical and buoyancy-reversing thermals.

1. INTRODUCTION

Atmospheric moist convection is an example of a buoyancy-reversing system. This is because mixing of positively buoyant cloudy air and dry environmental air usually leads to the formation of negative (reversed) buoyancy. This effect is a direct consequence of cooling associated with the evaporation of cloud droplets. Since the work of Squires (1958), numerous studies have considered effects of evaporative cooling and buoyancy reversal on cumulus entrainment. (See Reuter (1986) and Blyth (1993) for recent reviews of the subject.) Concepts that are usually invoked are 'cloud-top entrainment instability' (CTEI) and penetrative downdraughts. CTEI is based on the hypothesis that a positive feedback exists between buoyancy reversal due to evaporative cooling and the rate of entrainment. This enhanced entrainment is thought to be an effect of penetrative downdraughts that presumably bring environmental air deep into the convective cloud (Squires 1958). This paper attempts to clarify the role that buoyancy reversal plays in the problem of cumulus entrainment and, more generally, in the problem of entrainment in buoyancy-reversing convection.

Buoyancy reversal and CTEI have been considered important factors in controlling stability and entrainment into cloud-topped boundary layers (i.e. stratocumulus entrainment) and into isolated cumuli. For stratocumulus entrainment, the relevant references include Lilly (1968), Deardorff (1980), Randall (1980), Mahrt and Paumier

* Corresponding address: National Center for Atmospheric Research, PO Box 3000, Boulder, Colorado 80307-3000, USA. (NCAR is sponsored by the National Science Foundation.)

This paper considers effects of buoyancy reversal on cumulus dynamics in the context of the two-fluid system introduced by Grabowski (1993). This earlier paper is denoted hereafter as G93. The two-fluid system has several essential features of atmospheric moist convection (e.g. it mimics effects of latent-heat release and environmental stratification on the buoyancy of an adiabatic parcel rising from the cloud base). At the same time, it offers the simplicity of a system with only one scalar affecting the buoyancy field. Effects of buoyancy reversal on convection which develops in the two-fluid system are illustrated by considering two extreme situations, namely with the effects of buoyancy reversal (BR) and without them (NBR). As discussed in section 4(b) of G93, the effects of buoyancy reversal on the overall dynamics (as represented by time evolution of cloud-top height, total kinetic energy, convective mass-flux, etc.) were dramatic.

In both cases considered in G93 (i.e. NBR and BR), entrainment and subsequent mixing of the two fluids were associated with development of interfacial instabilities and entraining eddies, as suggested by Klaassen and Clark (1985) and Clark et al. (1988), and studied by Grabowski and Clark (1991, 1993a, b). These entraining eddies allow mixing of the two fluids and result in formation of negatively buoyant volumes when there is BR. The question this paper attempts to answer is to what extent does the presence of buoyancy reversal affect the rate at which the two fluids mix. If buoyancy reversal affects the rate of mixing significantly, then the concept of CTEI in cumulus dynamics is supported. However, if buoyancy reversal has little effect on the rate of mixing between the two fluids, then the positive feedback between buoyancy reversal and the rate of entrainment does not exist or is very weak. Numerical experiments discussed in this paper suggest that the latter seems to be the case.

Grabowski and Clark (1993a, section 6) argued that for cumulus entrainment, buoyancy reversal should have negligible effect on the dynamics of entraining eddies, considering their size (typically not much smaller than the size of a cloud) and the time required to reverse buoyancy through microscale homogenization (typically a few eddy-turnover time-scales). This argument applies, however, only to the initial stages of cloud development, when the mass of negatively buoyant fluid is still small. At later times (i.e. when the negatively buoyant mass is large and global effects of buoyancy reversal become significant), the rates of entrainment and mixing are likely to change. In classical fluid-mechanics the entrainment velocity (on which entrainment rate directly depends) is usually assumed to be proportional to the velocity of a convective element (cf. Turner 1986). Considering the simulations in G93 with BR (cf. G93 Fig. 10(h)), the presence in the later stages of strong downdraughts and corresponding increase of the total kinetic energy is likely to result in some enhancement of entrainment and mixing. The magnitude of the enhancement is the main topic of this paper.

The paper is organized as follows. Section 2 briefly discusses the two-fluid system and the numerical experiments. Detailed discussion of the set-up of the numerical experiments is presented in an appendix. Section 3 provides details of techniques that were applied to analyse entrainment and mixing in the numerical experiments. Section 4 focuses on the analysis of experiments. A discussion of results and their relevance to atmospheric moist convection is presented in section 5. Conclusions are summarized in section 6.
2. CONVECTION IN THE TWO-FLUID LABORATORY EXPERIMENT AND NUMERICAL SIMULATION OF THE CONVECTION

Numerical experiments discussed in this paper simulate two-dimensional convection and mixing in the unstratified, anelastic, two-fluid system introduced in G93. Essential features of this two-fluid system and details of the numerical experiments are presented in the appendix. The variable used to describe mixing of the two fluids is the mixing proportion \( \chi \), which is defined as \( \chi = (f - f_0)/(f_1 - f_0) \), where \( f \) is the extensive scalar-variable on which buoyancy depends (e.g., temperature, salinity), and subscripts 0 and 1 refer to upper and lower fluid respectively. (See discussion which follows (1) and (2) in section 2 of G93.) The conservation law for \( \chi \) is the advection–diffusion equation. The buoyancy field \( B \) depends only on the mixing proportion \( \chi \). The buoyancy of the undiluted lower fluid (\( \chi = 0 \)) below the level of free convection (LFC) and the buoyancy of the undiluted upper fluid (\( \chi = 1 \)) throughout the domain are both zero. The buoyancy of the undiluted lower fluid above the LFC is a prescribed function of height, and mimics the buoyancy of a moist-adiabatic parcel inside a cumulus cloud.

The lower fluid is brought to the LFC by superimposed low-level convection. The two fluids mix in entraining eddies that develop at the separating interface as the buoyant lower fluid rises. The buoyancy of a mixture of both fluids (i.e., \( 0 < \chi < 1 \)) is given either as a fraction of the buoyancy of undiluted lower fluid (the no-buoyancy-reversal, or NBR, regime), or as a bilinear function representing effects of buoyancy reversal (the BR regime). This is illustrated in Fig. 1. The magnitude of negative buoyancy in diluted volumes with the BR regime (\( B_r \) in Fig. 1) is typical for homogenized mixtures of cloud-base air and dry environmental air at a given level of a small cumulus.

The BR and NBR experiments have been performed in two different environmental situations. The first one is similar to that in experiments considered in G93, and the fluids have been assumed to be initially at rest (except for the low-level convergence). In the second situation a moderate environmental shear has been included above the LFC. The experiments with sheared environment do not consider the effects of gravity waves (Clark et al. 1986) because of the lack of environmental stratification. Thus, they include only the direct effect of the shear, as did the experiments discussed by Grabowski and Clark (1993b).

![Figure 1. Buoyancy as a function of the mixing proportion \( \chi \) for numerical simulations with effects of buoyancy reversal (BR) and without them (NBR). Note that the buoyancy of the pure lower fluid \( (B) \), the minimum buoyancy \( (B_s) \) and the corresponding mixing proportion \( \chi_s \) all depend on height. See appendix for details.](image-url)
Contrary to the discussion in G93, in the present paper the problem is considered in non-dimensional form. The horizontal scale, $L$, of the initially superimposed convergence and the maximum vertical velocity, $U$, associated with the convergence are used as length and velocity scales. For the entrainment problem, perhaps the scales of a convective element and its vertical velocity might be more appropriate (cf. Grabowski and Clark 1993a, section 6). However, scales selected should also be relevant: the scale of a convective element should be related to that of the low-level convergence, and the convective velocity scale is about $3U$ (see the appendix). The time scale is $L/U$ and maximum buoyancy of the lower fluid, $B_n$, serves as a buoyancy scale. All model results will be presented using these scales.

An additional difference between the experiments discussed in G93 and current numerical simulations is an inclusion of explicit diffusion terms in the model equations. This allows the overall flow regime to be defined in terms of the Reynolds and Schmidt numbers. The same viscosity, $\nu$, and scalar diffusivity, $D$, constant throughout the domain, were applied, i.e. Schmidt number $Sc = \nu/D = 1$. The values of $\nu$ have been chosen using the results of numerical experiments with moist thermals reported by Grabowski and Clark (1991). Twice as large values for $\nu$ and $D$ were used for calculations without environmental shear: the value of the Reynolds number, $Re = UL/\nu$, was used as $Re = 9600$ when there was environmental shear, and $Re = 4800$ when there was none.

As shown in the appendix, there are two further non-dimensional parameters which are essential in this problem. The first is the ratio $R_e = W/U$ of the convective velocity scale $W$ to the velocity scale associated with the external forcing $U$. The second parameter is a measure of the relative magnitude of the negative buoyancy in diluted parcels with respect to positive buoyancy of undiluted (adiabatic) parcels. In the numerical experiments discussed in this paper these two parameters have values typical of atmospheric moist convection. It should also be apparent that, because of the complex way the buoyancies of undiluted and diluted parcels depend on height, there are a few other relevant length-scales. Thus, a few other non-dimensional parameters are necessary to define the problem in a unique way. However, these parameters play a secondary role.

3. ENTRAINTMENT AND MIXING IN NUMERICAL EXPERIMENTS

In numerical experiments of the type considered in this paper, the definition of entrainment in general and, especially, entrainment rate are critical issues. In high-Reynolds-number natural flows, turbulent entrainment is usually envisaged as a sequence of events which begins when a large-scale structure engulfs the ambient fluid, and proceeds with the development of smaller and smaller structures down to the scale at which molecular homogenization occurs. In classical fluid-mechanics, the definition of entrainment rate is based on the increase of mass or mass flux of a fluid element (e.g. parcel). (See, for example, Turner (1986).) Typically, such a definition involves some elements of ensemble averaging in order to remove transient effects of entraining structures (eddies) whose dynamics is of limited interest so long as only the average properties of the flow are considered. Changes of those average properties may be deduced, for example, from the increase of volume of a thermal, from the angle of spread of a plume or jet, or from the change of the average height of an interface. For deducing the average properties, similarity theory has proved to be very successful (Turner 1986).

Although theoretically sound, application of the classical approach to the problem addressed in this paper is not straightforward. A rate of molecular mixing between the two fluids (i.e. the rate at which volumes with $0 < \chi < 1$ are generated) is used herein in discussing the effects of buoyancy reversal on entrainment and mixing. It should be
apparent that such an approach provides a very inaccurate estimate of the rate of turbulent entrainment because the mass of the ambient fluid that is already engaged in the turbulent mixing, but has not yet mixed down to the molecular level, is neglected. However, so far as the feedback problem is concerned, such an approach should be sufficient. If there is positive feedback between buoyancy reversal and the rate of entrainment, enhanced entrainment will also result in enhanced molecular mixing. In fact, molecular mixing is crucial because only through molecular mixing is more negative buoyancy generated as a result of enhanced entrainment.

Several characteristics will be used in section 4 in search of differences in entrainment and mixing between BR and NBR situations. They are briefly discussed in sub-sections 3(a) to 3(d).

(a) Mass of lower and upper fluid in grid-boxes containing diluted fluids

As a first test, the amount of either the lower fluid or the upper fluid in grid-boxes containing mixtures of both fluids will be considered. In this analysis, mixing both inside the whole domain as well as in a box covering the middle and upper part of the domain (i.e. above \( z = 1 \)) will be investigated. The reason for performing the analysis in this part of the domain is twofold. Firstly, an analysis using the box excludes undiluted fluid near the bottom of the domain. (Such fluid is continuously advected into the domain as a result of low-level convergence.) Secondly, such an analysis shows the relation of the mass of diluted lower (or upper) fluid to the mass of the undiluted lower fluid that passes through the LFC, i.e. the fluid that plays an active role in the convection and mixing.

(b) Mean mixing-proportion and its variance

Using the conservation equation for the mixing proportion \( \chi \) and the anelastic continuity-equation (Eqs. (A.2) and (A.3) in the appendix), the following conservation equations for the average over the entire domain \( \rho \chi \) and \( \chi \) may be derived:

\[
\frac{d\langle \rho \chi \rangle}{dt} = \frac{1}{V} \oint_S \left( -\rho \mathbf{u} \cdot \nabla \chi + \frac{1}{ReSc} \rho \nabla \chi \right) dS
\]

\[
\frac{d\langle \chi \rangle}{dt} = \frac{1}{V} \oint_S \left( -u \chi + \frac{1}{ReSc} \nabla \chi \right) dS - \frac{1}{\rho} \frac{d\rho}{dz} w \chi
\]

where angled brackets represent domain averages. Thus for a variable, say \( \psi \), \( \langle \psi \rangle = 1/V \int_V \psi dV \), \( V \) is the volume of the domain, \( S \) is the surface enclosing \( V \), \( w \) is the non-dimensional vertical-velocity component, and \( \rho \) is the non-dimensional anelastic density-profile. Terms not defined here are defined in the appendix. Note that the primes that mark a non-dimensional approach in the appendix are not used in the above equations; primes depict perturbations of \( \chi \), \( \chi' = \chi - \langle \chi \rangle \). Using (1) and (2), the equation for the average variance \( \rho \chi'^2 \) may be derived as:

\[
\frac{d\langle \rho \chi'^2 \rangle}{dt} = \frac{1}{V} \oint_S \left( -u \rho \chi'^2 + \frac{1}{ReSc} \rho \nabla (\chi'^2) \right) dS - \frac{2}{ReSc} \langle \rho (\nabla \chi')^2 \rangle.
\]

The first terms on the right-hand sides of Eqs. (1), (2) and (3) describe the rate of change of the average value which results from the advective and diffusive fluxes into the volume \( V \). These are the only terms that influence the domain average \( \rho \chi \). The domain-average variance \( \langle \rho \chi'^2 \rangle \) changes because of transport through the boundaries, and decreases as a result of the molecular mixing that reduces gradients of \( \chi \).
The analysis uses the above formulas in the following manner. When there are no fluxes through the boundaries, the domain average $\rho \chi$ does not change, i.e. $\langle \rho \chi \rangle$ is constant, no matter how differently BR and NBR calculations may evolve. The domain-average variance $\langle \rho \chi'^2 \rangle$, on the other hand, is influenced by the diffusion term. Because of this, the average variance decreases as the flow inside the domain evolves. Thus, significant differences in the rate of mixing in the BR regime and in the NBR regime result in different evolutions of the average variances with time.

When fluxes through domain boundaries do not vanish (as in the numerical experiments considered in this paper), time evolution of $\langle \rho \chi \rangle$ may be used to define the period in which both BR and NBR calculations may be meaningfully compared. The domain average $\rho \chi$ evolves in time as a result of the assumed low-level convergence as well as of fluxes of both fluids out of the domain associated with open boundary conditions. These fluxes influence both $\langle \rho \chi \rangle$ and $\langle \rho \chi'^2 \rangle$ inside the domain. However, as long as the fluxes are similar for BR and NBR calculations, $\langle \rho \chi \rangle$ should evolve similarly in both cases. During this period, the domain-average variance $\langle \rho \chi'^2 \rangle$ should change primarily as a result of the diffusion term, i.e. the last term on the right-hand-side of Eq. (3). Thus, any differences in the time evolution of the variance between the BR calculations and the NBR calculations during the period with the same evolution of the mean value are primarily associated with the differences in the rate of mixing between the two fluids.

(c) Probability-density function

Probability-density function (PDF), $p(\chi)$, is defined here in such a way that $p(\chi) \, d\chi$ is a probability of finding a grid-box with a mixing proportion between $\chi$ and $\chi + d\chi$ anywhere in the domain. (See, for example, Koochesfahani and Dimotakis (1986), section 4). Typically, the PDF is characterized by strong peaks at $\chi = 0$ and $\chi = 1$ that correspond to undiluted lower and upper fluids, and a broad peak for $0 < \chi < 1$ corresponding to already diluted grid-boxes. If there are significant differences in the mixing in the BR and NBR regimes, they should result in different PDFs. Analysis of PDFs should not extend beyond the point where global effects modify fluxes through domain boundaries, i.e. beyond times defined by the analysis described in section 3(b).

(d) Mixing rate

For the purpose of this study, the non-dimensional mixing rate has been defined as

$$\mu = \frac{1}{m} \frac{dm}{dt}$$

where $m$ is the mass with at least 1% of the lower fluid (i.e. $\chi = 0.99$) and $t$ is non-dimensional time as in Eqs. (1), (2) and (3). The values of the entrainment rate which result from calculations using $m$ defined in this way depend to some extent on the value of the threshold; however, the general features to be discussed in section 4 are independent of the value used. Such a definition of $m$ is likely to provide a reasonable estimate of the change of mass due to mixing. However, it certainly overestimates the mass of the fluid playing an active role in the mixing process, because the fluid near the bottom of the domain is also included in $m$.

4. Global dynamics versus dynamics of mixing

Figures 2 and 3 show the flow evolution at selected time levels for NBR and BR regimes in fluids without environmental shear, whereas Figs. 4 and 5 show results for
regimes with environmental shear. In Figs. 4 and 5, both the environmental wind vector above \( z \approx 0.9 \) and the shear vector in the layer between \( z \approx 0.9 \) and \( z \approx 3.4 \) are from left to right. The initial location of the low-level convergence has been moved upstream from the centre of the domain in sheared cases, shown in Figs. 4 and 5. Unfortunately, a few snapshots of the flow are not able to depict the complex evolution of the mixing process in real fluids; this process can, however, be appreciated by means of video animation. Nevertheless, some crucial differences between the cases considered are apparent. The non-sheared regimes (Figs. 2 and 3) evolve in a manner similar to the cases discussed in G93. The NBR regime (Fig. 2) produces a steady updraught in the middle of the domain and outflows around the level of neutral buoyancy aloft. Convection in the BR regime (Fig. 3) is much shallower as a result of the interaction between positively buoyant \( (\chi = 0) \) volumes and negatively buoyant diluted volumes. While descending, diluted volumes in the BR regime trigger secondary convection near the sides of the domain. These aspects of the flow evolution are discussed in G93.

The laboratory experiments with fluids with environmental shear, presented in Figs. 4 and 5, evolve in a quite a different way. In Fig. 4, NBR with shear leads to intermittent convection: the low-level convergence produces a narrow turret which gives a short-
lasting injection from the lower fluid into the upper part of the domain (Figs. 4(a) and (b)). Apparently, the interaction between the turret and the environmental shear detaches the turret from the supply of lower fluid, and convection ceases. As the mixing aloft proceeds, mixtures of the two fluids are advected out of the domain by the upper-level flow (Figs. 4(c) and (d)). At $t = 5.8$ (i.e. about 0.6 later than Fig. 4(d)), the next turret develops above the region of superimposed low-level convergence. The new turret evolves similarly to the one shown in Fig. 4. On the other hand, in the experiment with BR and shear (Fig. 5), the fluids develop a quasi-steady circulation in which the undiluted lower fluid rises on the upshear side, mixes with the upper fluid aloft while advected downwind, then sinks and produces a return flow toward the left (i.e. in the upwind direction) at lower levels.

As Figs. 2 to 5 demonstrate, the effects of buoyancy reversal on the global flow evolution are small before $t \sim 3$, i.e. in the early stages of convection and mixing. After $t \sim 3$, however, differences between NBR and BR experiments became significant. This demonstrates that buoyancy reversal plays an essential role in the overall flow evolution in both sheared and un sheared flows. This observation is consistent with the discussion in G93, and with laboratory experiments with buoyancy-reversing plumes (Turner 1966) and thermals (Johari 1992).
Figure 4. As Fig. 2, but with NBR and with environmental shear of 1.333 between $z = 0.9$ and $z = 3.4$. Both the flow field above $z = 0.9$ and the shear vector in the layer between $z = 0.9$ and $z = 3.4$ are from left to right.

Figures 6 to 14 show the results of an analysis whose purpose is to investigate differences of the mixing rate among the cases considered. Measures discussed in section 3 are used to show differences in entrainment and mixing between BR and NBR regimes, with and without environmental shear.

Figures 6 and 7 present time evolutions of the total mass of the lower ($\chi = 0$) fluid inside the box covering the middle and upper part of the domain. The curves marked as ‘no mixing’ in Figs. 6(a), 6(b), 7(a) and 7(b) show the mass of the lower fluid inside the box calculated from the time integral of mass flux through boundaries of the box. (In this analysis, the fluid was considered an undiluted lower fluid if its mixing proportion was $\chi \leq 0.03$.) Without molecular mixing inside the box, those curves would show the total mass of the undiluted lower fluid inside. However, because of the mixing, the mass of the undiluted lower fluid inside the box is smaller, as is shown by the curves marked as ‘mixing’ in Figs. 6(a), 6(b), 7(a) and 7(b). Thus, the mass of the lower fluid which is already diluted by the upper fluid is given by the difference between the two curves (i.e. ‘no mixing’ and ‘mixing’). Figures 6(c) and 7(c) show the difference between the two curves for NBR and BR regimes, whereas Figs. 6(d) and 7(d) show these differences expressed as a percentage of the mass of undiluted lower fluid that has entered the box (i.e. as given by the ‘no mixing’ curve). It is important to note that the same (though
arbitrary) units are used in panels (a), (b), and (c) of Figs. 6 and 7, and also in panels (a) and (b) of Figs. 8 and 9 (to be discussed later).

As Figs. 6(a), 6(b), 7(a) and 7(b) show, the majority of the lower fluid that rises above the LFC becomes diluted with the upper fluid. This is consistent with cumulus observations which typically show strong cloud-dilution by environmental dry air (e.g. Warner 1955). Note also that the amount of the lower fluid that rises in updraughts (i.e. as given by the ’no mixing’ curves in Figs. 6(a), (b) and 7(a), (b)) varies considerably when \( t \) exceeds about 3, i.e. when buoyancy reversal significantly influences the global flow-pattern. However, the rate at which molecular mixing proceeds in all calculations is affected by buoyancy reversal to a much smaller degree.

Figures 8 and 9 consider the total mass of the upper fluid contained in grid-boxes with the mixture of both fluids. For the purpose of this analysis, grid-boxes were considered diluted if \( 0 < \chi \leq 0.99 \). In Figs. 8 and 9, panel (a) represents the analysis for the whole computational domain, whereas panels (b) and (c) show results for the same box as used before. Calculations simulating shear and the absence of shear both give very similar results when either the whole domain or the upper part of it (i.e. the box) is used in the analysis. The large differences between results in calculations with NBR and with BR in the presence of environmental shear (panels (a) and (b) of Fig. 9) are associated with differences in global dynamics: a return flow at lower levels developed...
Figure 6. Numerical analysis of the mixing between the two fluids in the part of the domain above $z = 1$ for NBR and BR without environmental shear. Panels (a) and (b) show the time evolution of the mass of undiluted lower fluid in the box for NBR and BR, respectively. Dashed lines (marked 'no mixing') represent mass calculated as the time integral of flux of undiluted lower fluid throughout the box boundaries, whereas solid lines (marked 'mixing') represent the mass of undiluted lower fluid existing in the box at a given time. Panel (c) shows the time evolution of the difference between 'no mixing' and 'mixing' curves from panels (a) and (b) with BR (solid line) and without it (dashed line). Panel (d) shows the data from panel (c) expressed as the percentage of mass of the undiluted lower fluid shown as 'no mixing' curves in panels (a) and (b). Mass is expressed in arbitrary units in (a), (b) and (c).

when there was BR, whereas advection of diluted parcels out of the domain occurred when there was not. It is interesting to note, however, that when the mass of upper fluid in diluted parcels is scaled by the mass of the lower fluid that entered the box (Figs. 8(c) and 9(c)), calculations with both NBR and BR show similar time-evolution.

The analysis of the mean mixing-proportion ($\rho \chi$) and its variance ($\rho \chi^2$) are shown in Figs. 10 and 11. As discussed in section 3, the mean mixing-proportion ($\rho \chi$) decreases with time because of the presence of the low-level convergence and advection of diluted
Figure 7. As Fig. 6, but with environmental shear of 1.333 between \( z \approx 0.9 \) and \( z \approx 3.4 \). Mass expressed in the same arbitrary units as in Fig. 6.

Parcels out of the domain at later times. The time evolution of the BR and NBR regimes suggests that \( \langle \rho' \chi' \rangle \) is about the same for both up to time \( t = 3.8 \) when there is no environmental shear, and up to \( t \approx 3.4 \) when there is shear. These times are shown in the Figures as vertical lines and will be used throughout the paper to set limits for a meaningful comparison between the results of calculations with BR and with NBR. The evolution of the mean variance \( \langle \rho' \chi'^2 \rangle \) is characterized by the slow decrease of variances up to time \( t \approx 2 \), i.e., before flows shown in Figs. 2 to 5. After \( t \approx 2 \), the rate of change of variances increases significantly in all the cases considered. Again as discussed in section 3, time evolution of variances is affected by the transport throughout lateral boundaries for times beyond those marked in Figs. 10 and 11. In general, although BR and NBR lead to some differences in the flows in both sheared and shear-free environments, the differences are small if one considers the dramatic effects of buoyancy reversal on the global flow evolution.
Figure 8. Analysis of the mixing between the two fluids for NBR and BR without environmental shear: (a) in the whole domain and (b) and (c) in the part of the domain above \( z = 1 \). Panels (a) and (b) show the time evolution of the mass of the upper fluid contained in diluted grid-boxes with NBR (dashed line) and with BR (solid line). Panel (c) shows the mass as in (b) but divided by the mass of the undiluted lower fluid shown as 'no mixing' curves in panels (a) and (b) of Fig. 6. Mass is expressed in the same arbitrary units as in Figs. 6 and 7.

Figure 12 compares whole-domain probability-density functions for BR and NBR with environmental shear for \( t \approx 3.4 \) and without environmental shear for \( t \approx 3.8 \). All PDFs have been calculated using a bin size of \( \Delta \chi = 0.05 \). The vertical scale in both panels has been chosen to show only the central part of the PDFs, and the peak values for \( \chi < 0.05 \) and \( \chi > 0.95 \) are about 3 and 15, respectively. As before, the presence or absence of BR leads to only minor differences.

Finally, Fig. 13 shows the time evolution of the mixing rate calculated using Eq. (4), with and without environmental shear and with the change of mass \( m \) based on a time increment of 0.225 with BR and with NBR. Only results for times smaller than \( t \approx 3.8 \) (without shear) and \( t \approx 3.4 \) (with shear), as shown in Figs. 10 and 11, are displayed. For
Figure 9. As Fig. 8, but with environmental shear of 1.333 between \( z = 0.9 \) and \( z = 3.4 \).

later times, mixing rate is affected by advection of diluted parcels out of the domain and even becomes negative in shear and NBR. Until the development of interfacial instabilities (at approximately \( t = 1.5 \)), mixing rates for both cases are almost the same. As instabilities develop, and form entraining eddies, mixing rates increase sharply. With an increasing amount of diluted mass, the two calculations begin to diverge as do the mixing rates. However, the differences with and without BR are not large considering the dramatic differences in the overall flow evolution. This applies especially where there is no environmental shear, where the average mixing-rate for the period between \( t = 2.475 \) (Figs. 2(a) and 3(a)) and \( t = 3.8 \) (i.e. the end of the period when those two calculations give similar results) is about 0.21 (i.e. \( 3.9 \times 10^{-4}\text{s}^{-1} \) in dimensional units) with BR, and about 0.19 (i.e. \( 3.5 \times 10^{-4}\text{s}^{-1} \)) without BR. When there is environmental shear, the effect of BR on the rate of mixing seems to be stronger than this 10% difference. Note
that the values of mixing coefficients defined according to Eq. (4) are smaller than expected from similarity theory (e.g. Blyth 1993); this is consistent with the discussion in section 3(d).

5. DISCUSSION

The purpose of the analysis presented in section 4 is to contrast the dramatic effects of BR on the overall flow pattern inside the domain with the rather limited effects on the rate of mixing between the two fluids. The set-up of experiments with the low-level convergence supplying lower fluid into the main updraught was designed to mimic the atmospheric situation where larger-scale processes provide forcing for a single cloud. Unfortunately, such an approach resulted in some difficulties in defining precisely the
rate of mixing between the two fluids because of the advection of the lower fluid into the domain and the transport of diluted parcels out of the domain at later times. These processes were influenced differently by changes in the overall dynamics resulting from the presence or absence of BR.

Buoyancy reversal has been thought to influence the entrainment process in two different ways (Albrecht et al. 1985; Siems et al. 1990; Krueger 1993). The first is associated with the direct influence on small-scale entraining eddies. In this process the enhanced entrainment is thought to occur because kinetic energy of a small-scale entraining eddy is directly enhanced by BR. This 'interfacial instability' is thought to operate in laboratory studies of runaway entrainment (Shy and Breidenthal 1990) and was studied by Siems and Bretherton (1992). The second mechanism is associated with the indirect influence of BR on entrainment: enhancement of entrainment occurs through the enhancement of global flow structures (e.g. large-scale eddies entraining air into a cloud-capped boundary layer), that in turn leads to more entrainment and mixing.
Figure 12. Probability-density functions: (a) when there is no environmental shear and \( t = 3.825 \) and (b) when there is environmental shear and \( t = 3.375 \).

Apparently this is the indirect influence that seems to dominate in the numerical experiments discussed in this paper.

The interpretation in which a clear distinction is made between the dramatic effects of BR on the overall dynamics on the one hand and small effects on the dynamics of entraining eddies on the other hand is strongly supported by laboratory experiments with classical and buoyancy-reversing thermals in liquids (Johari 1992). Figure 14, redrawn using data taken from Figs. 9 and 14 of Johari (1992), shows the distance (normalized by the cube root of released volume \( V_0 \)) a thermal has to rise before all the released volume is mixed with the environmental fluid to at least the volumetric ratio, \( \phi \), as a function of \( (1 + \phi)^{1/3} \). This distance is called reaction (or flame) length in combustion literature; \( \phi \) is a ratio between the entrained-fluid volume and the released volume (or \( 1 + \phi \) is the ratio between the volume of the thermal to the released volume). Experiments with several buoyancy-reversing parameters \( D_s \)† were performed; see Johari (1992) for

† \( D_s \) is the ratio between the minimum negative buoyancy attainable in dilated parcels and the positive buoyancy of the undiluted fluid. \( D_s \) and \( x_s \), the mixing proportion corresponding to the minimum buoyancy (\( x \) in the notation used in this paper), are two of the parameters used to define effects of buoyancy reversal in buoyancy-reversing systems (e.g. Siems et al. 1990).
more details. The linear relation evident in Fig. 14(a) comes from similarity considerations for classical thermals (Johari 1992). What is more important, however, is that the best fit for the data for classical (or no-buoyancy-reversing) thermals, shown as a solid line in Fig. 14(a), also provides a very good estimate for the data obtained for buoyancy-reversing thermals. Also, the minimum reaction-length (i.e. the reaction length for very small values of the volumetric ratio, \( \phi \), see Johari (1992)) was very similar for both classical and buoyancy-reversing thermals. These results suggest that the same entrainment mechanism operates in both classical and buoyancy-reversing thermals, despite dramatic differences in the overall dynamics discussed by Johari (1992). A similar result is found in the present numerical experiments.

Reaction-length measurements described above (Johari 1992) do not, however, provide any information regarding differences in the mean (throughout the domain) mixing rate with and without BR. The numerical results discussed in this paper suggest that the mean mixing-rate is not substantially enhanced, even in the late stage of convection and mixing. The magnitude of this enhancement may still be a matter of debate and fully three-dimensional computations at higher Reynolds numbers, \( Re \), or with a proper formulation of the subgrid-scale mixing, are required to clarify this issue. A sensitivity of current results to variations of model parameters such as \( Re \) should also be considered. In addition, a simpler experimental set-up could be used to avoid complications arising from transport through lateral boundaries.

Atmospheric moist convection is three-dimensional and is characterized by extremely large values of \( Re \). Thus, care should be taken in using the current results to represent processes in the atmosphere. As discussed in G93, the use of the two-dimensional framework may be justified by the results of the numerical experiments with moist thermals considered by Grabowski and Clark (1991, 1993a, b). Those experiments clearly showed that, as long as the development of entraining eddies is of interest, both two- and three-dimensional frameworks yield very similar results. It is the later evolution of
Figure 14. Reaction length $L_R$ (normalized by the cube root of initial volume $V_0$) as a function of $(1 + \phi)^{1/3}$, where $\phi$ is the volumetric ratio between entrained and released volume: (a) for classical thermals and (b) for buoyancy-reversing thermals. Redrawn using data from Figs. 9 and 14 of Johari (1992). Solid line in (a) represents best fit to the experimental data for classical thermals. This line is also shown in (b).

entraining eddies that is affected by dimensionality; two-dimensional dynamics leads to vortex pairing and kinetic energy cascades up-scale (Grabowski and Clark 1991), whereas in three dimensions fully three-dimensional turbulence develops (Grabowski and Clark 1993a). Beyond a certain threshold the value of $Re$ is generally supposed to have only a small effect on the entrainment rate. (See a discussion by Johari (1992).) There is no evidence, however, that Reynolds numbers in the current experiments exceed that threshold.

It is very likely that the differences between the two-dimensional, relatively low $Re$, numerical experiments considered in this paper and natural three-dimensional, very high $Re$, atmospheric convection will lead to somewhat modified details of the entrainment process (such as entrainment rates or the times required to achieve small-scale homogenization). However, the issue is whether similarities between entrainment rates found with and without BR apply in the atmosphere. Although the laboratory experiments discussed above suggest that this is likely to be so, this issue should be a subject of further investigation.

6. Conclusions

This paper has discussed two-dimensional numerical experiments simulating convection, and mixing in the anelastic two-fluid system introduced in G93. Experiments were performed with and without effects of BR in environments with no shear, and with moderate shear located in the layer where convection developed. The results of modelling the flows suggest that a clear distinction should be made between significant effects of BR on the global flow evolution and minor effects on the rate of mixing between the two fluids. Several characteristics were used in G93 to document the influence of BR on the overall dynamics (e.g. cloud-top height, total kinetic energy and convective mass-flux). In this paper, the rate at which molecular mixing between the two fluids proceeded in
numerical experiments was used to highlight the dynamics of entrainment and mixing. In the early stage of the mixing process (i.e. before BR influenced the flow pattern in the whole domain), the effect of BR on the mixing rate was very small. At later times, the rate of mixing between the two fluids was affected by the dramatic differences in flows evolving with and without BR. As a result of these differences the enhancement of the mixing process was difficult to assess: there were, however, some indications that the enhancement was minor.

Numerical experiments discussed in this paper and laboratory experiments with classical and buoyancy-reversing thermals reported in Johari (1992), both suggest that a clear distinction should be made between cause and effect in the problem of entrainment in buoyancy-reversing convection. The entrainment is caused by interfacial baroclinic instabilities (Klaassen and Clark 1985; Clark et al. 1988; Grabowski and Clark 1991, 1993a, b). Buoyancy reversal, on the other hand, is an effect of the entrainment and, as this paper suggests, does not significantly influence the rate of mixing between convecting fluid and its environment.

In relation to atmospheric moist convection, the results discussed in this paper provide evidence against the concept of cumulus entrainment driven by cloud-top entrainment instability: CTEI assumes a strong positive feedback between BR and the rate of entrainment, but in the numerical experiments discussed in this paper only a very weak feedback was observed.

ACKNOWLEDGEMENTS

The numerical model used in this work is based on a two-dimensional computer model provided by Piotr Smolarkiewicz. Comments on the manuscript by Brad Baker, Terry Clark and Steve Siems, as well as the editorial support of Hope Hamilton and Mary Ann O'Meara, are gratefully acknowledged. Critical reviews by S. Krueger, D. Lilly and two anonymous reviewers led to the final version of this paper.

APPENDIX

Details of numerical experiments with the two-fluid system

Using the velocity scale, $U$, and the length scale, $L$, of the low-level convergence, and the maximum parcel-buoyancy, $B_m$, as a buoyancy scale*, anelastic equations are written in the following non-dimensional form:

$$\frac{du'}{dt'} = -\nabla' \cdot x' + \frac{k}{U^2} B' + \frac{1}{\rho' Re} \nabla' \cdot (\rho' \nabla' u')$$  \hspace{1cm} (A.1)

$$\nabla' \cdot (\rho' u') = 0$$  \hspace{1cm} (A.2)

$$\frac{dz'}{dt'} = \frac{1}{\rho' ScRe} \nabla' \cdot (\rho' \nabla' \chi).$$  \hspace{1cm} (A.3)

In Eqs. (A.1), (A.2) and (A.3), primed variables and operators are used to depict a non-dimensional approach. Symbols used have the following meaning: $x' = x/L$, and $z' = z/L$ are non-dimensional distances in the horizontal and vertical directions, $t' = tU/L$ is

* Note that, as in G93, buoyancy is defined as the ratio $\Delta p/\rho$, i.e. not multiplied by $g$. Also, $L$ depicted half of the actual width of the low-level convergence in G93.
non-dimensional time, \( u' = u/U \) is the non-dimensional velocity, \( \nabla' = L \nabla \) is the non-dimensional gradient-operator, \( \pi' = p/(\rho_0 U^2) \) (\( p \) is the perturbation pressure and \( \rho_0 \) is the base-state density-profile), \( B' = B/B_m \) is the non-dimensional buoyancy (\( B \) is the buoyancy defined as described below), \( Re = UL/v \) is the Reynolds number, \( Sc = v/D = 1 \) is the Schmidt number (\( \nu \) is fluid viscosity and \( D \) is the scalar diffusivity), \( W = (gLB_m)^{1/2} \) is the convective-velocity scale, and the non-dimensional density is defined as \( \rho' = \rho_0(z)/\rho_0(0) = (1 - gLz'/(c_p \theta))(z/R_e)^{-1} \), i.e. as in the neutrally stratified (\( \theta = \text{constant} \)) environment. The actual values of the environmental profiles were assumed using \( \theta = 300 \) K and \( \rho_0(0) = 1.16 \) kg m\(^{-3}\). The vector \( \mathbf{k} \) in Eq. (A.1) is the unit vector in the vertical direction.

There are four non-dimensional parameters that primarily affect the flow regime. These are: (i) \( Re \), (ii) \( Sc \), (iii) \( R_1 = W/U \) which is the ratio of the convective updraught to the updraught associated with external forcing, and (iv) the ratio \( R_2 \) of the negative buoyancy that can be generated in a diluted parcel to the positive buoyancy of an undiluted parcel. In atmospheric moist convection, the ratio \( R_2 \) depends on height above the cloud base (cf. section 4(a) in G93) and it cannot be defined globally. The definition chosen here is \( R_2 = -B_2(z_2)/B_m \), where \( B_2 \) is the minimum buoyancy that can be generated in the mixing between the two fluids and \( z_2 \) is the height between the level of maximum buoyancy \( B_m \) and the theoretical cloud-top which is used to define the reversed-buoyancy profile (see below).

The Reynolds number has been chosen as \( Re = 4800 \) and \( Re = 9600 \) for calculations without and with environmental shear, respectively. The Schmidt number \( Sc = 1 \) has been assumed. The assumed vertical-profiles of positive buoyancy of the undiluted lower fluid and negative buoyancy of diluted volumes when there is BR has resulted in an \( R_1 \) ratio of about three, and a ratio \( R_2 \) of two. These values should be considered as typical of atmospheric moist convection (see G93).

As in G93, random velocity-perturbations were applied to provide excitation for the interfacial instabilities (see Grabowski and Clark 1991, 1993a). Further details of the experimental set-up are as follows:

- length scale, \( L \): 1.6 km
- velocity scale, \( U \): 3.0 m s\(^{-1}\)
- time scale, \( T \): \( \sim 9 \) min
- initial height of the interface: 1.2 km
- domain size in the horizontal direction: 8.533 km
- domain size in the vertical direction: 6.4 km
- spatial resolution (\( \Delta x = \Delta z \)): 16.667 m
- viscosity \( \nu \) and Reynolds number \( Re = UL/\nu \):
  - with shear: 0.5 m\(^2\)s\(^{-1}\)
  - without shear: 1 m\(^2\)s\(^{-1}\)
- diffusivity \( D \) and Schmidt number \( Sc = \nu/D \): 0.5, 1 m\(^2\)s\(^{-1}\)
- amplitude of random excitation: 0.1 m s\(^{-1}\)
- random excitation applied every: 0.5 min
- shear-layer specification (shear cases only):
  - bottom height: 1.5 km
  - top height: 5.5 km
  - shear: \( 2.5 \times 10^{-3} \) s\(^{-1}\)

Buoyancy of undiluted lower fluid (Fig. 6 in G93) is zero below the LFC. Theoretical cloud-top is defined as the level at which buoyancy changes from positive to negative. Linear functions are used to describe the change of buoyancy with height.
Buoyancy reversal (Fig. 7 in G93) is considered above height $z_0$. Between $z_0$ and $z_1$, decrease of the buoyancy occurs but not buoyancy reversal. Buoyancy reversal occurs above height $z_1$, and data at height $z_2$ are used to determine the rate of change of minimum reversed-buoyancy, $B_2$, and corresponding mixing-proportion, $\chi_2$, with height. (See Fig. 7 in G93.)

| Height $z_0$ | 1.3 km | 0.8125$L$ |
| Height $z_1$ | 1.6 km | $L$ |
| Mixing proportion, $\chi_0$, at $z_1$ | 0.3 | 0.3 |
| Height $z_2$ | 3.0 km | 1.875$L$ |
| Mixing proportion, $\chi_0$, at $z_2$ | 0.7 | 0.7 |
| Negative buoyancy, $B_2$, at $z_2$ | $-1 \times 10^{-2}$ | $-2B_m$ |

The numerical model, the same as that described in G93, uses the semi-Lagrangian technique described by Smolarkiewicz and Pudykiewicz (1992). The computational domain was $513 \times 385$ grid points. An aspect of these experiments not explained in G93 was the set of boundary conditions. Free-slip, rigid-lid boundary conditions were applied at the bottom and top of the computational domain. Davies’s (1983) relaxation scheme was incorporated near lateral boundaries.

**REFERENCES**


Turner, J. S. 1966 Jets and plumes with negative or reversing buoyancy. *J. Fluid Mech.*, 26, 779–792
