Evaluation of physical processes in an idealized extratropical cyclone using adjoint sensitivity

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SUMMARY

An adjoint model is used to examine the sensitivity of an idealized dry extratropical cyclogenesis simulation to perturbations of predictive variables and parameters during the cyclone life cycle. The adjoint sensitivity indicates how small perturbations of model variables or parameters anywhere in the model domain can influence cyclone central pressure. Largest sensitivity for both temperature and wind perturbations is located between 600 and 900 hPa in the baroclinic zone above the developing cyclone. Perturbations of a given size have more influence on cyclone intensity when located in high-sensitivity regions (the middle and lower troposphere in this simulation). The effects of physical processes can be interpreted with adjoint sensitivity by considering perturbations that are proportional to temperature and wind tendencies in the basic state (nonlinear forecast). In the early phase of the cyclone life cycle, temperature advection near the steering level in the lower troposphere (about 800 hPa) is strongly cyclogenetic and resembles a Charney mode of baroclinic instability. During the phase of most rapid deepening, temperature advection in the lower troposphere remains important, while interpretation of sensitivity to wind perturbations suggests that increased vorticity in the middle and upper troposphere above the surface low-pressure centre may also be significant for cyclone intensification.

Adjoint techniques can provide insight into spatial and temporal sensitivity not easily obtained from other methods. Higher sea surface temperature (SST) has a cyclogenetic effect mainly in a localized region corresponding to the cyclone warm sector. Outside the areas of high sensitivity, small perturbations of SST have very little effect on central pressure of the forecast cyclone. When strong upward sensible-heat flux, \( F_s \), exists, it can have a cyclogenetic (preconditioning) influence early in the cyclone life cycle, although downward \( F_s \) in the cyclone warm sector is anticyclogenetic during the phase of most rapid deepening. The sensitivity indicates that \( F_s \) can be cyclogenetic in one location and anticyclogenetic at the same time in another location, so that \( F_s \) effects on cyclone intensity are partially self-cancelling. Surface momentum stress is anticyclogenetic, with sensitivity highly localized in the cyclone warm sector.

KEYWORDS: Adjoint methods Air–sea interaction Baroclinic instability Cyclogenesis Sea surface temperature Surface stress

1. INTRODUCTION

The study of physical processes that occur during extratropical cyclogenesis has been the subject of many numerical model-based investigations. Often these have taken the form of sensitivity studies in which certain variables or parametrizations in a model are altered to determine their effect on a forecast of a cyclone event. For example, the effects of sea surface temperature (SST) in a particular region might be evaluated through a series of simulations with a numerical model.

Adjoint methods provide a new and powerful approach to numerical sensitivity analysis in meteorology and oceanography. An adjoint model can be used to identify regions where changes to variables or parameters have the largest impact on a selected forecast measure. In these regions of high ‘sensitivity’, small perturbations can grow rapidly, and strongly influence the growth of forecast error. An adjoint model is the only practical method to estimate this sensitivity in a comprehensive manner.

The earliest uses of adjoint methods in meteorology can be attributed to Lorenz (1965), who investigated predictability using tangent-linear and adjoint operators, and Marchuk (1974). Hall et al. (1982) and Hall (1986) demonstrated that the adjoint approach could be used efficiently to evaluate parameter sensitivity in atmospheric models. Thomson and Sykes (1990) used the adjoint method to examine sensitivity in a sea-ice model. More
recently, the adjoints of primitive-equation meteorological models have been developed and used in sensitivity studies by Errico and Vukičević (1992), Rabier et al. (1992), Errico et al. (1993b) and Rabier et al. (1994). These studies have identified preferred regions for high sensitivity in middle-latitude situations and examined several forecast sensitivity measures. They have also demonstrated that adjoint sensitivity provides acceptable accuracy for describing the effects of perturbations in nonlinear models.

The possible uses of adjoint models include a wide range of research and operational applications. Adjoint methods provide a basis for next-generation four-dimensional variational data assimilation (Rabier et al. 1993; Thépaut et al. 1993; Zupanski 1993; Courtier et al. 1994). Ensemble forecasting in the operational environment can use dynamically conditioned perturbations derived using adjoint models (Murca et al. 1993; Molteni et al. 1994; Buizza 1994). Predictability can be studied using adjoint-derived singular vectors (Molteni and Palmer 1993), and optimal perturbations (Farrell and Moore 1992; Ehrendorfer and Errico 1995).

For sensitivity studies, the adjoint method has several advantages compared with conventional sensitivity tests performed in the forward sense. A single adjoint run can provide the sensitivity to all model fields and to model parameters, such as albedo, roughness length, or drag coefficient. Similar guidance can be obtained from forward sensitivity only by re-running and successively perturbing each variable in the initial conditions at every grid point. In a forward sensitivity experiment, it is not usually known in advance which variables or parameters actually have the most influence on the forecast, so a given experiment may not relate to the most significant variable or process. For example, a perturbed SST field may appear to have a beneficial effect on cyclone prediction in a forward sensitivity experiment, but the specific areas of positive contributions may be difficult to determine, and more significant improvements may be controlled by other factors that an adjoint model can identify. In an adjoint experiment, a forecast aspect is selected as a starting condition, and the adjoint model determines, in a quantitative sense, the sensitivity gradients of the forecast aspect with respect to perturbations of variables and parameters at earlier times during the forecast. This allows identification of the most important sensitivity effects.

In this study, we use the adjoint of a primitive-equation atmospheric model to evaluate sensitivity for an idealized cyclogenesis in a dry channel model. The objectives are to demonstrate that the adjoint sensitivity information is consistent with previous understanding of physical processes in extratropical cyclogenesis, and to use adjoint sensitivity to provide new insights into cyclogenesis.

Section 2 describes the nonlinear, tangent-linear and adjoint models, and the sensitivity definition. The initial conditions and forecast of the idealized cyclogenesis are discussed in section 3. Selection of the adjoint forecast aspect, \( J \), and tangent-linear error estimates are described in section 4. Section 5 describes the adjoint sensitivity interpretation for the initial conditions and at several times during the cyclone life cycle. Section 6 is a discussion of sensitivity to sea surface temperature and surface sensible-heat flux. Section 7 describes the sensitivity to surface momentum stress, and section 8 is a summary and discussion of the study.

2. THE NUMERICAL MODEL AND ADJOINT SENSITIVITY DEFINITION

The model is version 1 of the National Center for Atmospheric Research (NCAR) Mesoscale Adjoint Modeling System (MAMS1, Errico et al. 1994). MAMS1 includes a nonlinear hydrostatic primitive-equation model based on the Pennsylvania State University (PSU)/NCAR mesoscale model (MM4), a tangent-linear model, and an adjoint model. The
nonlinear model is similar to the MM4 as described by Anthes et al. (1987), Anthes (1990), and Warner and Seaman (1990); differences between the nonlinear model used here and MM4 are described by Errico et al. (1994).

The model dynamics include second-order horizontal advection with flux form variables on an Arakawa B-grid, and a time-splitting procedure (Madala 1981) added to allow a larger time step. Horizontal diffusion is fourth-order in the interior, and second-order near the north and south boundaries. Vertical turbulent mixing is performed with a stability dependent first-order closure (K-theory) parametrization. Surface transfers of heat and momentum involve a drag coefficient and bulk differences of temperature and wind. The vertical coordinate is sigma-pressure, \( \sigma_p = (p - p_T)/(p_s - p_T) \), where \( p_s \) is surface pressure and \( p_T \) a fixed pressure representing the top of the model. Model grid specifications and certain parameter values are summarized in Table 1.

**TABLE 1. NONLINEAR-MODEL SPECIFICATIONS**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sigma levels (temperature, wind)</td>
<td>0.0125, 0.0375, 0.0625, 0.1275, 0.23, 0.33, 0.44, 0.55, 0.65, 0.75, 0.85, 0.925, 0.965, 0.99</td>
</tr>
<tr>
<td>Model top, ( p_T )</td>
<td>25 hPa</td>
</tr>
<tr>
<td>Predictive variables, ( p^*u, p^*v, p^<em>T, p^</em> )</td>
<td>( (p_s - p_T) )</td>
</tr>
<tr>
<td>Time step, ( \Delta t )</td>
<td>180 s (split-explicit)</td>
</tr>
<tr>
<td>Coriolis parameter, ( f )</td>
<td>( 10^4 ) s(^{-1}) (( f )-plane)</td>
</tr>
<tr>
<td>Horizontal grid spacing, ( \Delta x )</td>
<td>60 km</td>
</tr>
<tr>
<td>Horizontal diffusion coefficient, ( K_M )</td>
<td>( 3.24 \times 10^{14} ) m(^4) s(^{-1}) (( K_H ) V(^4))</td>
</tr>
<tr>
<td>Surface heat-transfer coefficient, ( C_H )</td>
<td>( 1.0 \times 10^{-3} )</td>
</tr>
<tr>
<td>Surface momentum-transfer coefficient, ( C_M )</td>
<td>( 1.0 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

See text for explanation of symbols.

The nonlinear model may be expressed as

\[
\frac{dx}{dt} = M_N(x, t) \tag{1}
\]

where \( M_N \) is a nonlinear function of the model components, \( x \), defined in orthogonal Cartesian coordinates. The discretized nonlinear model is

\[
x_t = R_N(x_0) \tag{2}
\]

where \( R_N \) denotes the resolvent nonlinear operator. The resolvent represents a complete integration of the model from the initial conditions \( x_0 \) to the final state \( x_t \).

The tangent-linear model (TLM) is a linearized version of the discretized nonlinear model, with the linearization performed about the evolving nonlinear forecast. The TLM can be used to forecast the evolution of perturbations on the nonlinear (basic state) forecast trajectory, as

\[
x'_t = R_L(x'_0) \tag{3}
\]

where \( x' \) is a first-order approximation of the difference between an unperturbed and a perturbed nonlinear solution, and \( R_L \) is the resolvent tangent-linear operator.

We define a forecast aspect, \( J \), as a differentiable (scalar) function of the components of \( x \), \( J(x) \). The nonlinear response of \( J \) to a change in \( x_0 \) is

\[
\Delta J = J(\tilde{x}_t) - J(\tilde{x}_t) = J[R_N(\tilde{x}_0 + x'_0)] - J[R_N(\tilde{x}_0)] \tag{4}
\]

where \( \tilde{x}_t \) is a perturbed and \( \tilde{x}_t \) an unperturbed nonlinear forecast. Thus \( \Delta J \) must be obtained from two integrations of the nonlinear model. For sufficiently small \( x' \), \( \Delta J \) can be
approximated using (3)

\[ J' = \frac{\partial J}{\partial x_t} \cdot x'_t. \]  

(5)

The adjoint of (3) can be written as

\[ \hat{x}_0 = R^T_L(\hat{x}_f) = R^T_L \left( \frac{\partial J}{\partial x_f} \right) \] 

(6)

where \( R^T_L \) is the adjoint operator corresponding to the transpose of \( R_L \). The vector of adjoint variables \( \hat{x} \) denotes the gradient (sensitivity) of \( J \) with respect to the components of \( x \) (the basic-state vector)

\[ \hat{x} = \frac{\partial J}{\partial x} \] 

(7)

and the operator \( R^T_L \) represents a complete integration of the adjoint model from the starting condition \( \partial J/\partial x_f \) to \( \partial J/\partial x_0 \). The tangent-linear and adjoint operators satisfy the scalar dot product relationship

\[ J' = \langle \hat{x}_f, R_t x'_0 \rangle = \langle R^T_L \hat{x}_f, x'_0 \rangle \] 

(8)

and \( J' \) may be expressed as a sum of the product of the adjoint sensitivity and perturbation values over all model grid points

\[ J' = \sum_{ijk} \hat{x}_{0_i} \cdot x_{0_j}' \] 

(9)

Equation (9) is a first-order (Taylor series) estimate of \( J' \) resulting from a change to the initial conditions. Note that (4) provides the response \( \Delta J \) for a particular \( x'_0 \), while (6) provides the sensitivity \( \partial J/\partial x_0 \) for a particular \( J \).

To perform a tangent-linear or adjoint-model run, the nonlinear model is first integrated forward, and the forecast (basic state) fields are saved at some prescribed interval. These saved basic-state variables become time-dependent coefficients in the tangent-linear and adjoint equations. The TLM coefficients are defined by slopes tangent to the nonlinear trajectory, hence the term 'tangent linear'. If the basic state is not updated at every time step, the TLM is not truly tangent linear, but can still provide a good approximation to the nonlinear trajectory (Errico et al. 1993a).

The nonlinear model may be considered as a sequence of operators. The tangent-linear operators are then the Jacobians of the nonlinear operators applied to the perturbation variables. The TLM is a complete linearization of the nonlinear model except for parametrization of dry convection and an effect of surface-pressure variation acting on potential temperature that may influence wind speed near the surface (considered a negligible effect (Errico et al. 1994)). The TLM also includes moist convective and grid-scale precipitation parametrizations; these are not used in the present study.

The adjoint code is developed by writing the transpose ( interchange of rows and columns) of the tangent-linear operators and reversing the sequence of code execution. The starting condition of the adjoint variables is determined by the choice of \( J \). The adjoint model is integrated backward in time from the ending time of the nonlinear forecast to the initial time. An adjoint run requires slightly more computational time than a nonlinear run, because a file containing the saved basic state is read in, and additional terms related to linearization of the model must be evaluated. A single integration of the adjoint model provides the sensitivity gradients for all model prognostic variables. Complete mathematical development of MAMS1 is described by Errico et al. (1994). Considerations of tangent-linear and adjoint accuracy are discussed in section 4 and by Errico et al. (1993a).
3. **Description of the Idealized Cyclogenesis**

Adjoint models provide sensitivity with respect to perturbations on a particular nonlinear (basic state) forecast. In this case, the basic state describes an idealized cyclogenesis in an $f$-plane channel domain with periodic east and west boundaries. Zero-gradient conditions for temperature, pressure, and zonal wind, with meridional wind equal to zero, are specified on the north and south boundaries. The horizontal grid spacing is 60 km, with 121 grid points in the east–west direction, 62 grid points in the north–south direction, and 14 vertical levels between the surface and 25 hPa (sigma levels specified in Table 1).

The initial conditions include a background westerly jet in the centre of the channel, with a corresponding north–south temperature gradient in hydrostatic and geostrophic balance (Fig. 1). As this jet is uniform in the east–west direction, a cyclone development will not occur if the nonlinear model is integrated forward without a perturbation. To initiate a cyclogenesis, an anomaly of temperature and wind is placed in the upper troposphere. The temperature anomaly (Fig. 1, dotted contour) has a maximum of approximately +1.7 K at 225 hPa, a minimum of −4.3 K at 425 hPa, and a radius of 700 km, with perturbation magnitude decreasing horizontally according to a cosine-squared relation. Assuming geostrophic balance, the maximum perturbation of zonal and meridional wind components is 8 m s$^{-1}$, which establishes a jet streak of approximately 45 m s$^{-1}$ near 250 hPa. The position of the dynamic tropopause can be approximated by the 2 PVU contour (1 Potential Vorticity Unit (PVU) = $10^{-6}$ m$^2$ K s$^{-1}$ kg$^{-1}$) and is located near 350 hPa beneath the specified initial anomaly.

An initial temperature lapse rate of approximately 8 K km$^{-1}$ exists throughout the troposphere. Above the tropopause, initial temperature is constant to 30 hPa, and increases at 1 K km$^{-1}$ above 30 hPa. The model surface is assumed to be water at all points, with

![Figure 1. North-south vertical cross-section of the initial background zonal-wind component (m s$^{-1}$, solid contour) and temperature, (°C, dashed contour). Initial temperature anomaly (deg C, dotted contour) and 2 PVU contour (heavy solid line, 1 Potential Vorticity Unit (PVU) = $10^{-6}$ m$^2$ K s$^{-1}$ kg$^{-1}$). S indicates location of the 45 m s$^{-1}$ jet streak associated with temperature anomaly.](image-url)
SST constant in time, and an initial air–sea potential-temperature difference of about 0.2 K (stable lapse). The initial surface pressure is 1000 hPa at all grid points. No special initialization is performed to remove gravity waves from the initial conditions.

The cyclogenesis that results from this initial condition deepens over 120 h to a minimum surface pressure of 982 hPa. The maximum deepening rate of approximately 0.5 hPa h\(^{-1}\) is attained near 80 h (Fig. 2). The e-folding time of the disturbance is approximately 30 h, which is typical of synoptic-scale extratropical cyclones. Between the initial time and 60 h, the incipient cyclone moves eastward below the jet axis and gradually intensifies. After 60 h, surface pressure decreases more rapidly and the storm moves northeastward into the colder air. At 90 h, the central pressure is 985.7 hPa and the ridge–trough wavelength of the system is about 1500 km. A cold front extends south from the cyclone. After 100 h, the cyclone enters a barotropic decay phase with no further intensification. Certain features of marine cyclogenesis, such as a warm air seclusion or bent-back warm front (Shapiro and Keyser 1990), do not appear in this simulation, and may depend on finer grid resolution, moist processes, or more sophisticated parametrizations of surface heat and momentum interchange. More description of features during the idealized cyclogenesis will be provided in later sections related to the adjoint-sensitivity results.

![Figure 2](image_url) Time series of minimum surface pressure in nonlinear simulations with initial anomalies in upper troposphere (solid line) and lower troposphere (dotted line). Time series for simulation with upper-tropospheric initial anomaly and warm sea-surface-temperature anomaly (dashed line). Forecast aspect \(J\) is pressure at centre of 90 h cyclone.

The cyclone scale here is somewhat smaller than the idealized simulations of Rabier et al. (1992, RCT92 hereafter), or Simmons and Hoskins (1978, SH78 hereafter) using global models. This difference in scale is probably due to the greater north–south extent of the initial baroclinic zone and jet in SH78 and RCT92. The initial jet speeds in SH78 and
RCT92 are similar to that used here. SH78 specified small-amplitude initial disturbances of normal-mode form corresponding to wave number 6, including surface-pressure perturbations. As noted by Thornicroft et al. (1993), the basic features of these idealized cyclone life cycles (scale and intensity) depend more on specification of the mean zonal flow than on initial perturbation amplitude. Over periods of several days or more, small-amplitude perturbations of various forms will evolve asymptotically to a ‘preferred’ normal-mode structure, as noted, for example, by Reinhold (1986).

Of course, initial perturbation structure can have significant effects on the timing of cyclone development. A larger scale or more intense initial temperature anomaly in this simulation will produce surface development more quickly, but the final cyclone pressure and scale are similar to Fig. 3. A cyclone can also be initiated by placing an anomaly in the lower troposphere instead of in the upper troposphere as described above. In this case, the surface disturbance begins to intensify more quickly (dotted line in Fig. 2), but the final cyclone scale and intensity are again similar to the original simulation. The dashed line in Fig. 2 corresponds to a simulation including a large surface-temperature anomaly, and is discussed in section 6.

![Diagram](image)

Figure 3. Surface pressure (hPa, solid contour) and near-surface air temperature (°C, dashed contour) at 90 h in the nonlinear run. Solid dots indicate cyclone positions at 30, 70, and 90 h (985.7 hPa). Forecast aspect $J$ is pressure at centre of 90 h cyclone. Heavy solid line surrounding cross-hatched area corresponds to the 40 m s$^{-1}$ contour of the jet streak at 250 hPa in the initial conditions.

The deepening rate of 12 hPa day$^{-1}$ in this simulation is somewhat greater than the rate of 8 hPa day$^{-1}$ in RCT92. This difference could be related to the smaller scale of this cyclone, and differences in tropospheric temperature lapse rate. In this simulation and those of SH78 and RCT92, the basic mechanism for development is baroclinic instability with some modification by barotropic processes. The features of the simulated cyclones are similar, with well-defined life cycles, including westward-tilting troughs, eastward-tilting temperature perturbations and frontal structures typical of middle-latitude disturbances.
4. FORECAST ASPECT AND ACCURACY CONSIDERATIONS

Adjoint sensitivity provides quantitative information concerning how changes to model variables and parameters will influence the feature represented by the forecast aspect \( J \). For the forecast change to occur, physical processes and instability mechanisms must provide a link between the initial perturbation and the forecast aspect. To determine what processes are involved, it is necessary to select an appropriate forecast aspect, and examine in detail the adjoint-sensitivity fields at various times during the cyclone life cycle.

In data-assimilation applications, \( J \) is a costfunctional measuring the fit between the forecast and observations, e.g. Courtier et al. (1994) or Li et al. (1994). In singular-vector (Buizza and Palmer 1995) or optimal-perturbation applications (Ehrendorfer and Errico 1995), \( J \) might be a function involving a norm used to measure total perturbation energy in a specified region.

In this paper, all results correspond to a forecast aspect \( J \) that represents surface pressure in the centre of the cyclone at 90 h (Fig. 3). That is, \( \partial J/\partial p_s = 1.0 \) at a single grid point is the starting condition for the adjoint model. For the adjoint sensitivity presented in this paper, the term 'cyclogenetic' will imply a decrease of 90 h pressure at the location where \( J \) has been defined. Sensitivity for several other choices of \( J \) (including pressure over a region) has been investigated, but does not lead to any substantial differences in the interpretation of the cyclogenesis described here. An example of sensitivity using \( J \) as kinetic energy is shown in section 5(a).

The sensitivity fields shown here do not take into account any variations in grid volume size, since the adjoint operator in this study is defined with respect to the ordinary scalar dot product (Eq. 8). The adjoint sensitivity will thus appear smaller in high-resolution regions, since the total contribution for a given volume is coming from more grid points. The model grid has higher vertical resolution near the surface, below about 900 hPa, with more or less evenly spaced layers above 900 hPa. The effect of grid spacing on the sensitivity has minimal implications for this simulation, since perturbations near the surface are damped by boundary-layer processes, and the sensitivity is diminished primarily for that reason.

Before discussing the adjoint-sensitivity results, it is necessary to evaluate the accuracy that can be expected for this simulation. This is best done by comparing forecasts of the TLM with difference fields taken from nonlinear-model forecasts, to inspect the error pattern over the entire domain for a particular initial perturbation. The adjoint and tangent-linear accuracy are identical, in the sense that (8) is an identity.

Tangent-linear error arises from two factors, the neglect of terms involving second-(and higher) order perturbations, and the frequency of basic-state update. Here, the basic-state fields for the TLM and adjoint are updated at 30 min (10 time-step intervals). For reasons of file size, it is impractical to increase the update frequency beyond 30 min for this 90 h integration. Error growth in tangent-linear systems is discussed by Lacarra and Talagrand (1988), Farrell (1990), Rabier and Courtier (1992), and Errico et al. (1993a). In general, tangent-linear forecasts longer than 48–60 h are subject to more significant error. In this simulation with idealized initial conditions, there is a spin-up period of about 30 h during which perturbation growth is relatively slow, which allows the total forecast length to be extended.

An accuracy test is made by perturbing the initial temperature field by 2 K in a square of nine grid points on model level 10 (near 750 hPa) in the region where \( \partial J/\partial T \) is most strongly negative (location shown as a small square in Fig. 8(a)). That is, the adjoint results have been used to select a location at which to perturb the initial conditions. At this location, positive temperature perturbations will produce a relatively large pressure reduction in the 90 h surface cyclone compared with the same perturbation in regions of
weaker sensitivity. Comparison of the TLM 90 h perturbation pressure (Fig. 4(a)) and the corresponding nonlinear difference field (Fig. 4(b)) indicates that, for this perturbation, the TLM accurately depicts the region where 90 h pressure is reduced, but overestimates slightly the magnitude of the pressure fall. Similarly, comparison of the 90 h TLM forecast of 250 hPa zonal wind (Fig. 4(c)) with the nonlinear difference field (Fig. 4(d)) shows good correspondence in terms of positive and negative response, with relatively small differences in perturbation wind magnitude.

The values of the largest negative forecast perturbations at 90 h resulting from temperature perturbations at 0 h, 30 h, and 70 h are provided in Table 2. In each case the initial perturbation is placed in a region of strongly negative $\partial J/\partial T$. We examine the 90 h forecast perturbations of the zonal and meridional wind components, $u$ and $v$, near 250 hPa, temperature near 750 hPa, and surface pressure. The decision to examine negative forecast perturbations is arbitrary; similar errors occur for positive perturbations.

<table>
<thead>
<tr>
<th>TABLE 2. COMPARISON OF TANGENT-LINEAR AND NONLINEAR PERTURBATION FORECASTS VALID AT 90 H (LARGEST NEGATIVE PERTURBATION AND GRID-POINT LOCATION)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast length</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>20 h Tangent</td>
</tr>
<tr>
<td>Nonlinear</td>
</tr>
<tr>
<td>60 h Tangent</td>
</tr>
<tr>
<td>Nonlinear</td>
</tr>
<tr>
<td>90 h Tangent</td>
</tr>
<tr>
<td>Nonlinear</td>
</tr>
</tbody>
</table>

See text for explanation of symbols. The perturbations consist of +2 K at nine grid points on sigma level 10 (near 750 hPa) in region of minimum $\partial J/\partial T$. Perturbations are made at 0 h (90 h forecast), 30 h (60 h forecast), and 70 h (20 h forecast). 0 h perturbation location shown in Fig. 8(a).

As shown in Table 2, the differences between the tangent-linear perturbations and perturbations from the nonlinear model are generally smallest for the shortest forecast interval (20 h). For the perturbation inserted at 70 h, the difference between the TLM and nonlinear forecasts of 90 h pressure is 0.018 hPa (5%), with the TLM perturbation shifted one grid point to the south. Over the full 90 h, the TLM has a 17% error in surface pressure and a 6% error in 750 hPa temperature.

The forecast perturbations are largest over 90 h, which indicates that the selected perturbation projects onto a growing mode. As shown in Figs. 4(c) and 4(d), and in Table 2, perturbations of temperature in high-sensitivity regions of the lower troposphere can change the wind field in the upper troposphere (250 hPa) by several m s$^{-1}$. By way of comparison, the same temperature perturbation (2 K, nine grid points) applied in a high-sensitivity region at 250 hPa results in relatively weak 90 h perturbations of surface pressure and upper-tropospheric wind (Table 3).

Other perturbations of the initial conditions will produce different error patterns. In general, the TLM forecast will be less accurate for larger perturbations, and errors will also depend to some extent on the forecast situation. Errors due to nonlinearity will be largest for perturbations in high-sensitivity regions, that is, for the fastest-growing perturbations. As perturbation size approaches zero, the tangent-linear error will asymptotically approach zero. The effects of perturbation size and basic-state update interval on adjoint accuracy are discussed by Errico et al. (1993a).
Figure 4. (a) 90 h forecast of tangent-linear-model (TLM) surface-pressure perturbations (contour = 0.5 hPa) resulting from initial temperature perturbations near 750 hPa (location shown in Fig. 8(a)). (b) As in (a) except difference between two nonlinear runs. (c) 90 h forecast of TLM zonal wind perturbations (contour = 0.5 m/s) near 250 hPa resulting from initial perturbation as in (a). (d) As in (c) except difference between two nonlinear runs. Hatching denotes negative values.
In summary, the TLM does a satisfactory job of forecasting the location and magnitude of wind, temperature, and pressure perturbations for this simulation, even over 90 h. We therefore have considerable confidence in the validity of the adjoint-sensitivity patterns. Section 5 will describe sensitivity results at the initial time (0 h), in which the adjoint model is integrated backwards in time for 90 hours, at 30 h (60 h adjoint integration), and at 70 h (20 h adjoint integration).
TABLE 3. COMPARISON OF TANGENT-LINEAR AND NONLINEAR PERTURBATION FORECASTS
(LARGEST NEGATIVE PERTURBATION AND GRID-POINT LOCATION)

<table>
<thead>
<tr>
<th>Forecast length</th>
<th>$u(\sigma = 0.23)$</th>
<th>$v(\sigma = 0.23)$</th>
<th>$T(\sigma = 0.75)$</th>
<th>Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m s$^{-1}$</td>
<td>m s$^{-1}$</td>
<td>K</td>
<td>hPa</td>
</tr>
<tr>
<td>90 h Tangent</td>
<td>-0.391 (108.35)</td>
<td>-0.224 (104.31)</td>
<td>-0.057 (104.30)</td>
<td>-0.201 (73.38)</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>-0.353 (110.34)</td>
<td>-0.210 (104.31)</td>
<td>-0.055 (106.29)</td>
<td>-0.237 (73.39)</td>
</tr>
</tbody>
</table>

See text for explanation of symbols. The initial perturbation consists of +2 K at nine grid points on sigma level 5 near 250 hPa in region of minimum $\partial J/\partial T$. Forecast length is 90 h.

5. TROPOSPHERIC SENSITIVITY

(a) Initial conditions

Adjoint-sensitivity patterns are first examined for the initial conditions, as determined by a backward integration of the adjoint model over 90 h. The sensitivity to initial temperature ($\partial J/\partial T$) and meridional wind ($\partial J/\partial v$) is shown in east–west vertical cross-sections through the centre of the zonal jet (Figs. 5(a) and 5(b)). In areas of negative (positive) sensitivity, a positive perturbation will decrease (increase) 90 h pressure ($J$). Note that the units of the adjoint variables depend here on the choice of $J$.

The sensitivity to both temperature and meridional wind is largest in the lower troposphere between 600 and 900 hPa, and tilts westward with height. Closer to the surface, the sensitivity is damped by boundary-layer processes. The decrease in model-layer thickness near the surface also causes the sensitivity to appear smaller below 900 hPa, as discussed in section 4. Damping of near-surface perturbation growth is consistent with other adjoint-sensitivity studies, e.g., Buizza et al. (1993), and with modeling studies, including Valdes and Hoskins (1988), that show surface friction reduces baroclinic growth rates. The horizontal scale of the sensitivity structures in Fig. 5 is similar to that of the 90 h cyclone (Fig. 3) and the strongest sensitivity is located below the initial upper-tropospheric anomaly (jet streak).

The adjoint sensitivity to initial surface pressure ($\partial J/\partial p_s$) indicates that the 90 h cyclone will be intensified if the initial pressure under the jet streak is reduced (Fig. 6, location 'L'). The surface-pressure sensitivity is geostrophically consistent with lower-tropospheric $\partial J/\partial v$ (Fig. 5(b)) and $\partial J/\partial u$ (not shown), which imply a cyclonic circulation around L at the initial time. In addition, a reduced pressure at L is hydrostatically consistent with the temperature sensitivity, in that $\partial J/\partial T$ implies a net warming of the column (lower average density) above L intensifies the 90 h cyclone.

The vertical variations in sensitivity imply that temperature perturbations at 250 hPa must be on the order of 10 times those at 750 hPa to have comparable effects on the forecast surface pressure. The second column of Table 4 summarizes the response of $J$ to one-unit perturbations placed at single grid points having the largest sensitivity on model levels near 250 hPa and 750 hPa. Perturbations of wind and temperature in the lower troposphere can produce a greater response than equal perturbations in the upper troposphere. The same conclusion applies for perturbations scaled to five per cent of the initial basic-state field value (third column of Table 4). Although zonal wind speed is greater at 250 hPa than at 750 hPa, the effect on $J$ is still larger from lower-tropospheric perturbations. The example in Table 4 does not rule out possible effects of strong temperature or wind perturbations over large areas of the upper troposphere. However, the impact of very strong perturbations cannot be evaluated using tangent-linear or adjoint models. In such cases, the complete sensitivity must be resolved with a nonlinear simulation.
Figure 5. East–west cross-sections (A–B indicated in Fig. 6) of adjoint sensitivity to (a) initial temperature, \( \partial J/\partial T \) (contour = 0.01 hPa K\(^{-1} \)) and (b) meridional wind, \( \partial J/\partial v \) (contour = 0.001 hPa m\(^{-1} \) s\(^{-1} \)) at 0 h. Negative values are hatched. Heavy solid line surrounding cross-hatched area corresponds to the 40 m s\(^{-1} \) contour of jet streak in the initial conditions.
Figure 6. Field of adjoint sensitivity to surface pressure, $\partial J/\partial p_s$, at 0 h (contour interval = 0.001). Negative values are hatched. Location 'L' indicates a region where a negative pressure perturbation in the initial conditions will deepen the 90 h cyclone. A–B and C–D denote cross-sections used for other figures. Forecast aspect $\mathcal{F}$ is pressure at centre of 90 h cyclone.

TABLE 4. The 90 h pressure change ($\mathcal{F}$) resulting from perturbations of basic-state wind and temperature near 250 and 750 hPa, implied by adjoint sensitivity

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>One unit</th>
<th>Five per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 hPa $u$</td>
<td>-0.0055 hPa</td>
<td>-0.0042 hPa</td>
</tr>
<tr>
<td>250 hPa $v$</td>
<td>-0.0030</td>
<td>-0.0012</td>
</tr>
<tr>
<td>250 hPa $T$</td>
<td>-0.0048</td>
<td>-0.00495</td>
</tr>
<tr>
<td>750 hPa $u$</td>
<td>-0.0122 hPa</td>
<td>-0.0067 hPa</td>
</tr>
<tr>
<td>750 hPa $v$</td>
<td>-0.0080</td>
<td></td>
</tr>
<tr>
<td>750 hPa $T$</td>
<td>-0.1186</td>
<td>-1.573</td>
</tr>
</tbody>
</table>

Perturbations are located in region of largest negative sensitivity on each level at 0 h. Column 2 uses single-point positive signed one unit perturbations (1 m s$^{-1}$, 1 K). Column 3 uses positive-signed perturbations scaled to five per cent of basic-state value at same grid point. There is no initial basic-state meridional wind at 750 hPa.

Above 500 hPa, the sensitivity structure (Fig. 5) is nearly vertical, which suggests a barotropic pattern. The $u$–component wind sensitivity near 250 hPa (Fig. 7) is zonally elongated, with maximum amplitude on either side of the jet core. This pattern seems generally consistent with the discussion of optimal perturbations for barotropic flows by Farrell and Moore (1992) and Molteni and Palmer (1993). As noted by Simmons and Hoskins (1980) and Thornicroft et al. (1993), barotropic mechanisms can have significant influence on baroclinic flows. The sensitivity in Fig. 7 depicts how barotropically unstable initial perturbations may be configured to destabilize the shear of the mean zonal flow, and
lead to intensification of the 90 h cyclone. However, zonal wind sensitivity also includes baroclinic influences that are dynamically consistent with changes to temperature at lower levels.

The temperature sensitivity near 750 hPa (Fig. 8(a)) is largest in the centre of the baroclinic zone and is suggestive of normal-mode perturbations, with the crescent shape on the sigma level resulting from a combination of upshear tilt of the sensitivity structure and zonal elongation to the side of the jet core. The sensitivity resembles an $x$-periodic Rossby wave with an alternative $+/−$ isentropic potential vorticity (IPV) pattern as shown in Fig. 17 of Hoskins et al. (1985, HMR85 hereafter).

A north-south vertical cross-section of $\partial J/\partial T$ through the centre of the initial jet streak (Fig. 8(b)) shows that a positive temperature perturbation at 700 hPa (in the region of negative $\partial J/\partial T$ below the jet core) or negative temperature perturbations to the north of the jet axis would be favourable for cyclone intensification. That is, an increase of the north-south temperature gradient and corresponding vertical wind shear in this location is cyclogenetic. However, the effect of warming (or cooling) the entire column under the jet core would be small because $\partial J/\partial T$ has areas of both positive and negative sensitivity, and the effect of the perturbations would partially cancel.

The westward tilt of sensitivity to wind perturbations in the lower troposphere (Fig. 5(b)) corresponds to a stream-function axis of a developing baroclinic wave. A trough axis is implied by the zero contour of $\partial J/\partial v$, with negative sensitivity to the east and positive sensitivity to the west (see conceptual diagram of Fig. 9). A cyclogenetic perturbation involving the wind components will slope westward with height according to Fig. 5(b). The sensitivity structure for temperature perturbations ($\partial J/\partial T$) also slopes westward with height in the lower troposphere (Fig. 5(a)). Near 800 hPa, $\partial J/\partial T$ is offset by about one half wavelength from the trough axis implied by $\partial J/\partial v$, so that initial perturbations configured...
Figure 8. (a) Field of 750 hPa adjoint sensitivity to initial temperature, $\partial J/\partial T$, at 0 h (contour = 0.01 hPa K$^{-1}$). Solid square is the location of the test perturbation used in Table 2 and Fig. 4. (b) $\partial J/\partial T$ at 0 h as in Fig. 5(a), except for north–south cross-section (C–D indicated in Fig. 6). Negative values are hatched. Heavy solid line surrounding cross-hatched area in (b) corresponds to 40 m s$^{-1}$ contour of the jet streak in the initial conditions. Forecast aspect $J$ is pressure at centre of 90 h cyclone.
to enhance thermal advection are strongly cyclogenetic. That is, warm advection ahead of the trough and cold advection behind the trough will decrease $J$ (90 h surface pressure). Above 800 hPa, the maximum temperature sensitivity is closer to the implied trough axis, so that thermal advection will contribute less. This corresponds qualitatively to effects of differential temperature advection, which is the primary mechanism for amplification of mid-latitude cyclones in the quasi-geostrophic system (Holton 1992, Chapter 6).

It is interesting that the westward tilt of the temperature sensitivity differs from the typical eastward tilt of temperature in a baroclinic wave (e.g. Hoskins and Heckley 1981). A westward tilt of temperature sensitivity is also noted by RCT92, who used response functions different from that used here, so this sensitivity feature is probably not anomalous. The axis of warm air in the basic state extends upward from the surface low toward the upper-tropospheric ridge, as must be true for hydrostatic reasons. The adjoint sensitivity therefore implies that cyclone development is not enhanced by temperature perturbations along the basic-state thermal axis in the middle and upper troposphere. Instead, middle- and upper-tropospheric temperature perturbations are most influential in the neighbourhood of the stream-function axis, where the strongest sensitivity to wind perturbations is also found. The sensitivity field does not necessarily have to resemble the basic-state temperature pattern, since $\partial J/\partial T$ is not a temperature field, but rather a gradient field that describes how temperature perturbations can change the forecast aspect ($J$).

It is possible to construct a sensitivity to a linearized form of quasi-geostrophic potential vorticity using $\partial J/\partial u$, $\partial J/\partial v$ and $\partial J/\partial T$. The sensitivity to this quantity is also
Figure 10. East-west cross-sections of $\partial J/\partial T$ as in Fig. 5(a), for (a) initial anomaly in lower troposphere (dotted contour = 1 deg C), (b) $J = \text{kinetic energy } [u^2 + v^2]$, $\partial J/\partial T$ contour = 0.05 m$^2$ s$^{-2}$ K$^{-1}$, and (c) purely zonal basic state (no initial upper-tropospheric jet streak). Negative values are shown hatched. Heavy solid line surrounding cross-hatched area in (b) corresponds to the 40 m s$^{-1}$ contour of the jet streak in the initial conditions.
largest in the lower troposphere (indicated by PV' in Fig. 9). When an IPV anomaly is said to 'induce' a cyclonic circulation, this corresponds to effects of wind, temperature and pressure perturbations that represent the anomaly. Conceptually, the potential-vorticity viewpoint is another way of describing the integrated effects of forcing that can be considered separately in a quasi-geostrophic context as (differential) thickness and vorticity advection. Adjoint sensitivity includes all physical processes of the nonlinear model, and does not depend on assumptions such as conservation of potential vorticity to transmit the effects of selected perturbations.

According to Fig. 5, a temperature or wind perturbation at, say, 500 hPa must be larger and farther upstream than one at 750 hPa to have equivalent effects on the 90 h surface pressure. If these perturbations represent a positive IPV anomaly near 500 hPa, they will induce a cyclonic circulation (according to 'PV thinking') with warm advection below, and slightly ahead of, the anomaly. If the warm advection is to reinforce the 500 hPa IPV anomaly, the sensitivity \( \partial J / \partial T \) should be negative in the region of warming, which is the pattern in Fig. 5. The warm advection ahead of the trough axis has a cyclogenetic effect by inducing cyclonic flow above it that advects higher potential vorticity southward into the upper anomaly, which creates a 'phase-locking' situation (HMR85; McIntyre 1988), so long as the upper anomaly is slightly west. The upper IPV feature might be a short-wave trough combining wind and temperature anomalies, as in a type-B development (Petterssen and Smelby 1971).

When the initial anomaly used to initiate a cyclone in the nonlinear model is placed in the lower troposphere (centred at 750 hPa, directly below position in Fig. 1), the resulting adjoint-sensitivity pattern (Fig. 10(a)) is not that different from the sensitivity for the original basic state (Fig. 5(a)). Maximum sensitivity is still in the lower troposphere and has about the same magnitude. In a nonlinear forecast with the lower-tropospheric anomaly, the surface pressure begins to decrease much more quickly (dotted line, Fig. 2) than in
the forecast with the initial upper-tropospheric anomaly (solid line, Fig. 2). Apparently, placing the anomaly in the lower troposphere causes baroclinic instability to develop more rapidly. This is a good example of how changing the initial perturbation structure can influence transient growth rates according to the ideas of Farrell (1989).

The sensitivity is also not highly dependent on the specification of the initial lapse rate or drag coefficients (not shown). Changes to these factors can alter the intensity or location of the 90 h cyclone to some extent, but the sensitivity to perturbations of initial wind and temperature remains localized in the lower troposphere.

As an example of sensitivity using a different forecast aspect, $J$ is defined as a simple approximation to near-surface kinetic energy ($u^2 + v^2$) in the centre of the 90 h cyclone (Fig. 10(b)). The sensitivity remains concentrated in the lower troposphere, although $\partial J/\partial T$ has changed sign, which indicates that a decrease in 90 h pressure is accompanied by an increase in lower-tropospheric kinetic energy. Domain (or global) kinetic energy may not be an appropriate forecast measure for surface development, as the sensitivity will propagate upwards approaching validation time, since kinetic energy is concentrated in the upper-tropospheric jet, and the jet can be strengthened without intensification of the surface-pressure disturbance.

In this section, we have examined how possible perturbations of the initial conditions might influence the forecast central pressure of the idealized cyclone. There are any number of ways the initial conditions could be modified, and the adjoint sensitivity can be used to determine the response of $J$ within a certain neighbourhood of the basic-state trajectory. However, the adjoint sensitivity does not, by itself, confirm that processes in various regions were important to the development of the cyclone in a particular nonlinear forecast. To illustrate this principle, a vertical cross-section of $\partial J/\partial T$ for a basic state in which all fields are purely zonal over a 90 h period is shown in Fig. 10(c). No cyclone development occurs, yet the sensitivity pattern is generally similar to one in which the basic state does contain a cyclone (Fig. 5(a)).

Therefore, the adjoint sensitivity pertains only to the effect of possible changes and is not an indicator of physical processes such as temperature advection or surface-heat fluxes that actually occurred in the nonlinear basic-state forecast. An alternative approach is to consider the adjoint sensitivity in association with tendencies of temperature, wind, or pressure obtained from the nonlinear forecast. This is discussed in sections 5(b) and 5(c) for conditions at 30 h and 70 h, respectively.

(b) Pre-deepening phase

Conditions at 30 h are representative of the environment before the period of most rapid surface-pressure falls. The central pressure of the incipient cyclone is 998 hPa, located below the right-rear quadrant of the main 250 hPa jet streak, which has propagated ahead of the surface disturbance. The basic-state trough axis that extends from the surface to 250 hPa tilts westward with height and the axis of basic-state temperature tilts eastward with height. The IPV anomaly associated with the jet streak is not extremely strong in this simulation compared with many observed cases of extratropical cyclogenesis.

In addition to using adjoint sensitivity to estimate the effect of possible changes to basic-state variables, sensitivity effects that correspond to physical processes, such as temperature advection, may also be considered. Sensitivity to a physical process involving more than one predictive variable cannot be obtained directly from the adjoint model. For example, temperature advection depends on zonal and meridional wind components and temperature. Various combinations of $u^r$, $v^r$, and $T^r$ can produce different forecast responses ($J^r$), so the sensitivity with respect to the process is not uniquely defined. However, a physical process may be considered in terms of an effect on a prognostic
variable. For example, temperature advection produces a tendency of temperature. If the process is intensified at a particular location, the effect may be considered as a temperature perturbation, and the response of $J$ can be estimated using the adjoint-sensitivity field $\partial J/\partial T$. The response ($J'$) implied by adjoint sensitivity does not depend on the source of a perturbation; $T'$ could be an arbitrary modification, the result of an analysis change, or represent a change to a physical process.

The ‘total’ tendencies (sum of all processes) are obtained from differences of zonal and meridional wind and temperature between 29 and 31 h. These tendencies (Fig. 11(a)) represent the effect of physical processes in an average sense at this stage of the cyclone evolution. The tendencies of wind and temperature are largest near jet level in the upper troposphere, decrease towards the middle troposphere, and have a secondary maximum closer to the surface.

Perturbations of wind and temperature are then proportional to the size of these nonlinear tendencies, with larger perturbations in locations where tendencies are largest. Thus, the perturbations represent an increase in the nonlinear tendency, while keeping the relative strength equal at all locations. At each grid point, the perturbation magnitude is equal to an adjustment of wind or temperature over a one-hour interval, as determined from the nonlinear tendencies. The product of these perturbations and the adjoint sensitivity at each grid point (e.g. Eq. (9)) are summed on each model level (Fig. 11(b)). Although temperature and wind tendencies are smaller on average in the lower troposphere (Fig. 11(a)), the sensitivity there is considerably larger (as at 0 h), and perturbations of lower-tropospheric tendencies can be more significant to cyclone development. In particular, strong cyclogenetnic effects appear related to the meridional wind and temperature below 600 hPa, which implies effects of temperature advection. The spike in Fig. 11(b) related to temperature near 850 hPa suggests a critical layer, although the largest sensitivity is closer to 750 hPa. When this method is applied to the example of the purely zonal basic state (Fig. 10(c)), there will be no basic-state tendencies and therefore no attribution of cyclogenetnic effects to any physical processes, which is the correct interpretation.

The temperature sensitivity ($\partial J/\partial T$) near 850 hPa (Fig. 12) depicts localized areas of strong sensitivity near the developing surface-pressure centre at 30 h. The effect of temperature perturbations will be cyclogenetnic for warming ahead of the low, and cooling behind the low. In these locations, horizontal temperature advection is the most significant process producing temperature tendencies. An increase in warm advection ahead of the low would be strongly cyclogenetnic, since the position of maximum warm advection in the basic state is close to the minimum of $\partial J/\partial T$. Strengthening cold advection behind the low does not appear to have as strong an effect. Adiabatic cooling in the region of upward motion ahead of the cyclone is anticyclogenetnic.

Physical processes may produce tendencies of temperature or wind in regions of weak sensitivity. For example, temperature advection in certain regions of the upper troposphere produces relatively large temperature tendencies, but an increase or decrease in strength of this process will have a relatively small effect on the forecast measure ($J$) in this cyclogenesis. A rule of adjoint-sensitivity interpretation is suggested:

Physical processes that produce large tendencies in regions of strong adjoint sensitivity are significant to the feature or statistic represented by the forecast aspect ($J$).

The importance of lower-tropospheric processes in this early stage of the cyclone life cycle is supported by conceptual ideas of baroclinic development presented by Hoskins et al. (1983) and HMR85. According to this view, eddy activity in a storm track originates in the lower troposphere (steering level in the Charney model) through linear baroclinic instability, with subsequent propagation of the instability into the upper troposphere. The
Figure 11. (a) Root-mean-square (on sigma levels) of nonlinear-model tendencies of zonal and meridional wind (m s$^{-1}$ h$^{-1}$) and temperature (K h$^{-1}$) obtained as a two-hour difference centred at 30 h. (b) Dot product of perturbations (one-hour wind or temperature adjustment) and 30 h adjoint sensitivity, summed on sigma levels. Units of $J'$ are hPa, negative values indicate an increase in nonlinear tendency produces a negative pressure perturbation ($J'$) at centre of 90 h cyclone. Crosses correspond to model levels.
steering level in this simulation is between 750 and 800 hPa, where the disturbance phase speed (12 m s⁻¹) and basic-state zonal wind are approximately equal, and this is also the region where the adjoint indicates the maximum sensitivity. As shown by Edmon et al. (1980), large upward Eliassen–Palm fluxes (implying poleward heat flux) exist near the steering level in the lower troposphere for the Charney mode. Studies by Davis and Emmanuel (1991), Whitaker and Barcilon (1992) and Black and Dole (1993) using potential-vorticity interpretations also demonstrate the importance of lower-tropospheric processes within a storm track. The adjoint sensitivity for this case confirms that eddy meridional heat flux in the lower troposphere ahead of the developing cyclone is very important in the early stage of the cyclone development. The steering-level instability is initiated by a relatively weak upper-tropospheric anomaly, and significant pressure falls at the surface begin when the lower-tropospheric instability is established.

(c) Deepening phase

At 70 h, the central pressure is 992.8 hPa, and the cyclone is deepening at approximately 0.5 hPa h⁻¹. The surface low-pressure centre is below the right-rear quadrant of a 42 m s⁻¹ jet streak at 250 hPa and the left-front quadrant of an upstream 37 m s⁻¹ jet streak at 250 hPa. Strong temperature advection is present in the lower troposphere and also in association with an upper-tropospheric trough and sloping tropopause. The maximum sensitivity to wind and temperature determined by the adjoint model is still in the lower troposphere, and localized above the surface cyclone as at 30 h.

Tendencies of wind and temperature from the nonlinear forecast (obtained from differences of wind and temperature between 69 and 71 h) are depicted in Fig. 13(a). Compared with 30 h, the average tendencies are larger on nearly all levels, with most significant increases in the middle and lower troposphere. Following the method of the previous sec-
tion, perturbations that are proportional to the nonlinear tendencies are projected onto the 70 h adjoint sensitivity at each grid point and summed for each model level. As shown in Fig. 13(b), increases in lower-tropospheric temperature tendencies (primarily due to temperature advection) are strongly cyclogenetic, as are perturbations related to meridional wind tendencies in a layer from about 350 hPa to 750 hPa.

The product \((v' \cdot \partial J/\partial v)\) on a level near 450 hPa is represented in Fig. 14. In locations
where $v'$ is negative (southward) and $\partial J/\partial v$ is positive (+ in Fig. 14), an increase in the nonlinear tendency creates a negative $J'$ (decreases the 90 h cyclone pressure). A stronger southward wind at + will increase the gradient of cyclonic vorticity ($\partial v/\partial x$) above the surface low, and divergence ($\partial v/\partial y$) in the jet-exit region upstream of the surface low, conditions which are known to be cyclogenetic. Temperature sensitivity (not shown) indicates that cooling below + (implying increased trough amplitude) is cyclogenetic. Amplifying the upper-tropospheric ridge ahead of the surface disturbance also has a cyclogenetic effect. In this phase of the cyclone life cycle, the forecast central pressure is strongly influenced by physical processes in both the upper and lower troposphere.

![Figure 14](image-url)

Figure 14. Product of 450 hPa meridional wind sensitivity, $\partial J/\partial v$, (70 h) and $v'$, with $v'$ proportional to one-hour basic-state tendency (contour = 0.0001 hPa). Negative values are stippled and represent regions in which increasing the nonlinear tendency of meridional wind decreases 90 h central pressure. Basic-state surface pressure at 70 h (contour = 2 hPa, dashed). Jet axis indicated by heavy solid line. S = positions of jet streaks at 250 hPa. (+) and (−) indicate centres of positive and negative sensitivity ($\partial J/\partial v$). Arrows pointing upward (downward) indicate nonlinear tendency of meridional wind is northward (southward). Forecast aspect $J$ is pressure at centre of 90 h cyclone.

It is noted that the regions of significant sensitivity at 30 h and 70 h are much more localized than the sensitivity for the initial conditions, and the sensitivity magnitude is larger at the initial time than at 30 h or 70 h. The increase in sensitivity magnitude at longer times indicates that perturbations in high-sensitivity regions result in forecast perturbations that grow with time (demonstrated in Table 2). Whereas the initial-time sensitivity appears related to normal-mode-type structures, the sensitivity at 30 h and 70 h probably includes non-modal (continuous spectrum) features. That is, the cyclone scale and intensity over the full 90 h are determined by normal-mode considerations, so the most effective perturbations are those that enhance the normal-mode structure (e.g. Figs. 5, 7, and 8). Perturbations of non-modal form may affect short-term growth, but will decay before 90 h. Non-modal perturbations are much more likely to be significant for shorter forecast intervals (Farrell 1984). The highly localized sensitivity at 30 h and 70 h represents the type of non-modal perturbation structures that can influence cyclone development on shorter time-scales.
The studies of Borges and Hartman (1992) and Molteni and Palmer (1993) also note increasingly localized structures as optimization time decreases. For intervals of less than about six hours, a gravity-wave signal may appear relatively large in the sensitivity. For longer times, the gravity-wave contribution to sensitivity remains, but is masked by the effects of other processes.

6. **Sea Surface Temperature and Sensible-Heat Flux**

The adjoint method can be used to describe sensitivity to external forcing conditions, such as perturbations of SST. For example, the hypothesis that a higher SST in a certain region results in a more intense cyclone may be tested. When examining the SST sensitivity it is appropriate to consider the accumulated sensitivity over the entire forecast, since SST (in this model) is a lower-boundary condition that remains constant in time.

The adjoint field $\partial J/\partial T_s$ in Fig. 15 depicts the sensitivity to SST perturbations accumulated over 90 h. The most significant sensitivity is negative and located to the east and south of the 70 h cyclone position. This suggests that higher (lower) SST in the cyclone warm sector after 70 h will cause the 90 h central pressure to decrease (increase). There are regions where $\partial J/\partial T_s$ is positive, so that higher SST will increase 90 h central pressure, but this sensitivity is relatively weak.

![Figure 15](image)

**Figure 15.** Field of surface-temperature sensitivity, $\partial J/\partial T_s$, accumulated from 0–90 h (contour = 0.0004 hPa K$^{-1}$) for basic state without sea-surface-temperature anomaly. Negative values are stippled and are regions where a higher surface temperature will deepen the 90 h cyclone. Solid dots are 30, 70, 90 h cyclone positions. Solid square is location of test perturbation used in Table 5. Forecast aspect $J$ is pressure at centre of 90 h cyclone.

An example of the effect of perturbing SST in the nonlinear and tangent-linear models is provided in Table 5. Here, the SST perturbation consists of 2 K at nine grid points in a region of negative $\partial J/\partial T_s$ (see Fig. 15). The TLM provides fairly accurate magnitude of the surface pressure and 750 hPa temperature perturbations over 90 h although, in general, accuracy is better for perturbations of atmospheric temperature (e.g. Tables 2 and 3). The
maximum accumulated sensitivity to SST perturbations is only about 10% of the largest sensitivity to atmospheric temperature perturbations at 0 h, which is found near 750 hPa.

The effects of very large perturbations to SST cannot be evaluated with high accuracy over long forecast intervals using tangent-linear or adjoint models. In such cases, the perturbation forecast can be highly nonlinear, and the complete sensitivity must be resolved with the nonlinear model. Sensitivity in the neighbourhood of the new basic-state trajectory can then be evaluated with the tangent-linear or adjoint models. For instance, tangent-linear systems cannot be expected to handle situations accurately in which SST perturbations change the surface-layer stability over a large area. The surface-layer parametrization in the nonlinear component of MAM51 includes a ‘convective velocity’ (Anthes et al. 1987) that can greatly increase upward heat transfer under conditions of low wind speed in an unstable surface layer (upward heat flux). If surface-temperature perturbations change the stability, the convective velocity acts as a switch that can cause the trajectory of the tangent-linear forecast to enter a different regime.

The sensible-heat flux may be expressed as

\[ F_s = C_H \rho C_p V_K (\theta_s - \theta_K) \]  \hspace{1cm} (10)

where \( F_s \) has units of \((W \text{ m}^{-2})\), \( \rho \) is surface-layer air density, \( C_p \) is specific heat at constant pressure, \( \theta_s \) is surface potential temperature, and \( \theta_K \) potential temperature on the lowest model level. In this simulation, the surface heat-transfer coefficient \( C_H \) is equal to \( 1.0 \times 10^{-3} \). The velocity \( V_K \) in (10) includes the convective velocity.

The original nonlinear simulation contained a zonally invariant SST and relatively small contributions from surface-heat transfer. This was useful for isolating the sensitivity to air temperature and winds with a simplified basic state. To explore surface-temperature sensitivity under more interesting conditions, an alternative basic state is considered, with a circular warm SST anomaly placed somewhat south of the main baroclinic zone (Fig. 16). The anomaly has a magnitude of 12 K in the centre, and decreases outward to zero over a radius of 960 km. The initial air–sea temperature difference of 12 K is similar to conditions near the Gulf Stream north wall during cold-air outbreaks (Bane and Osgood 1989). Since the adjoint sensitivity \( \partial J / \partial T_s \) cannot be used to approximate accurately the effects of this large a temperature perturbation over 90 h, some trial and error was necessary to select the anomaly location. This location was selected because the presence of higher SST in the nonlinear model intensifies the cyclone development, and the nonlinear forecast made with the anomaly provides a basic state in which surface-heat fluxes have more influence on the cyclone development.

The time series of surface pressure for this nonlinear forecast with the SST anomaly appears as a dashed line in Fig. 2. The central pressure at 90 h in the simulation with the SST anomaly is reduced by 2.2 hPa and the adjoint approximation to this effect is
Figure 16. Sea-surface temperature (SST) (solid contour = 2 K) and 90 h surface pressure (dashed contour = 2 hPa) in the nonlinear simulation with the SST anomaly. 90 h central pressure = 983.4 hPa. Forecast aspect J is pressure at centre of 90 h cyclone.

a pressure reduction of 1.0 hPa. Although the adjoint identifies the correct sign of the pressure perturbation, the difference is indicative of the nonlinear effects that may occur over 90 h with a large SST perturbation.

The effect of the warm SST anomaly is communicated through the surface sensible-heat flux \( (F_s) \), which warms the lower troposphere and creates a strong 'preconditioning' effect. The heated air is drawn into the cyclone warm sector, where it contributes to a more intense cyclogenesis. The remaining discussion in this section pertains to sensitivity on the basic-state trajectory with the warm SST anomaly present. The forecast aspect \( (J) \) is defined at the centre of the 90 h cyclone shown in Fig. 16.

The effects of surface sensible-heat flux may be investigated by studying the sensitivity to the parameter \( C_H \), which appears only in the equation for surface-air heat transfer. The tangent-linear temperature equation has one term involving \( C_H \)

\[
\frac{\partial (pT')_k}{\partial t} = \ldots + \frac{1}{\Delta \sigma_k} (C_H \bar{\rho} g \bar{V}_k(\bar{\theta}_s - \bar{\theta}_K))
\]

(11)

where \( (') \) indicates a perturbation variable, and \( (\cdot) \) indicates a basic-state variable. Therefore, the adjoint equation for \( C_H \) is

\[
\hat{C}_H = \frac{\partial J}{\partial C_H} = \int \frac{1}{\Delta \sigma_k} ((pT')_k \bar{\rho} g \bar{V}_k(\bar{\theta}_s - \bar{\theta}_K)) \, dt
\]

(12)

where \( \partial J/\partial C_H \) can be accumulated over the entire adjoint integration or examined at any selected time. The interpretation of positive \( \partial J/\partial C_H \) is that \( F_s \) is anticyclogenetic, since a positive perturbation of \( C_H \) will increase \( F_s \) (Eq. (10)) and increase 90 h central pressure \( (J) \). A negative \( \partial J/\partial C_H \) implies that \( F_s \) is cyclogenetic, since a positive perturbation of \( C_H \) will increase \( F_s \), but \( J' \) will be negative.
In considering the effects of sensible-heat flux, it is useful to divide the cyclogenesis with the SST anomaly into two phases, before and after 60 h. The period before 60 h can be considered a 'preconditioning' phase, and from 60 to 90 h the 'deepening' phase. During the deepening phase, a strong sensitivity to sensible-heat flux exists in the cyclone warm sector, where downward heat fluxes exist (shown at 70 h in Fig. 17(a)), but $\partial J / \partial C_H$ from 60 h to 90 h is strongly positive (Fig. 17(b)). Thus, the surface heat flux in the warm sector during the deepening phase opposes cyclogenesis. That is, if the downward heat flux in the warm sector were increased (by a positive perturbation of $C_H$), the 90 h cyclone central pressure would be higher. The upward heat flux in the cold sector is also anticyclogenic, but the sensitivity is much smaller. The spatial correspondence between $\partial J / \partial C_H$ and $F_s$ confirms the relation of $C_H$ to the surface sensible-heat flux.

Downward (or weak upward heat fluxes) are typical for the area immediately east of strong cold fronts in mid-latitude cyclones (Petterssen et al. 1962; Fleagle and Nuss 1985; Neimann and Shapiro 1993). The anticyclogenic effect of sensible-heat fluxes during the later stages of cyclogenesis has been noted by Danard and Ellenton (1980), as related to the Laplacian of surface heating, and general weakening of horizontal temperature gradients across the storm centre. Other studies that have shown anticyclogenic (or minimal) effects of surface fluxes during the deepening phase include Kuo and Reed (1988) and Kuo et al. (1991). However, Nuss and Anthes (1987) show there can be some cyclogenic effect if surface heat fluxes are upward in parts of the warm sector.

In this cyclogenesis, there must also be a period during which $F_s$ is cyclogenic (and $\partial J / \partial C_H$ is negative) since surface heat flux associated with the warm SST anomaly produces a deeper cyclone by adding heat and potential energy. A time series of the sensitivity for $C_H$ (Fig. 18) at location 'P' (position shown in Fig. 17) indicates that the heat flux in that region contributes to cyclone deepening from 0 h to about 50 h, as the cyclone approaches and the surface heat flux is upward over the SST anomaly. The interpretation of negative $\partial J / \partial C_H$ here is that larger $C_H$ and increased upward heat flux will deepen the low (decrease $J$). The cyclogenic effects of surface heat fluxes during the early stage of cyclogenesis are noted by Mailhot and Chouinard (1989), Grotjahn and Wang (1989), Fantini (1990), Kuo et al. (1991) and others.

The time series of $\partial J / \partial C_H$ at location 'W' in Fig. 18 indicates that the anticyclogenic effects of sensible-heat flux occur between 60 h and 80 h, when $W$ is in the cyclone warm sector and downward heat fluxes exist. Inspection of $\partial J / \partial C_H$ at 70 h (Fig. 17) and other times (not shown) reveals the simultaneous existence of both positive (anticyclogenic) and negative (cyclogenic) sensitivity. Thus, the effects of sensible-heat flux at various times during the cyclone life cycle may be partially self-cancelling. This type of effect may explain why previous numerical sensitivity experiments in which sensible-heat flux was entirely removed during various phases of cyclone development have produced ambiguous results (Kuo et al. 1991) or concluded that the net effect of sensible-heat flux is small (Reed and Simmons 1991). However, these results concerning surface-heat-flux sensitivity should not be generalized to other basic states in which the surface heat flux is much stronger or weaker, or has a different distribution of upward and downward heat transfer.

The sensitivity to instantaneous perturbations of SST ($\partial J / \partial T_S$) at 70 h (Fig. 19) is consistent with the surface heat-transfer-coefficient sensitivity just discussed. Higher SST in the warm sector where $\partial J / \partial T_S$ is negative will reduce the atmospheric heat loss associated with downward sensible-heat flux, increase the baroclinicity between the warm and cold sectors of the cyclone, and result in lower central pressure. A lower SST can be cyclogenic in some areas (positive $\partial J / \partial T_S$), for example, in a small region west of the surface cold front, but this sensitivity is much weaker.

Observational and modelling studies demonstrate the relation of intense cyclogenesis
Figure 17. (a) Sensible-heat flux ($F_s$, contour = 10 W m$^{-2}$, positive values are upward) at 70 h in the nonlinear forecast with sea-surface-temperature anomaly. (b) Field of sensitivity to surface heat-transfer coefficient, $\partial J/\partial C_H$ (contour = 10 hPa) accumulated between 60 and 90 h. Positive $\partial J/\partial C_H$ indicates $F_s$ is anticyclonic for 90 h central pressure. Locations 'P' and 'W' refer to Fig. 18. Negative values are hatched.
Figure 18. Time series of sensitivity to surface heat-transfer coefficient, $\partial J/\partial C_H$, (0.01 hPa) at locations 'P' (solid) and 'W' (dashed) for basic state with sea-surface-temperature anomaly. Negative values (heavy stippling) indicate period when upward sensible-heat flux, $F_s$, near P acts to deepen the 90 h cyclone. Positive values (light stippling) indicate period when downward $F_s$ near W is anticyclogenetic. Locations of P and W shown in Fig. 17.

Figure 19. Sensitivity to instantaneous perturbations of surface temperature, $\partial J/\partial T_s$, at 70 h (contour = $10^{-5}$ hPa K$^{-1}$) for basic state with sea-surface-temperature anomaly. Negative values, stippled, are regions where warmer surface temperature will decrease 90 h central pressure. 70 h basic-state surface pressure (contour = 2 hPa, dashed). Forecast aspect J is pressure at centre of 90 h cyclone.
to certain ‘antecedent’ features in the lower troposphere, including increased vorticity (Gyakum et al. 1992), and coastal fronts (Bosart and Lin 1984). These features of the marine boundary layer are very sensitive to the SST distribution (Doyle and Warner 1993). Climatologies of rapidly deepening cyclones show a preference for development near the western boundary currents of the Atlantic and Pacific Oceans (Sanders and Gyakum 1980; Roebber 1984; Sanders 1986; Chen et al. 1992). Many explosive cyclones follow tracks just poleward of the north walls of the Gulf Stream and Kuroshio currents, with the largest SST gradients to the right of the direction of motion. The adjoint-sensitivity results (e.g. Figs. 15 and 19) are consistent with these climatologies, as they imply that stronger SST gradients in the warm sector of the idealized cyclone will result in lower central pressure.

7. Surface Momentum Stress

The surface momentum stress may be written as

$$\tau = \rho C_M V_K^2$$  \hspace{2cm} (13)

where $\tau$ is stress (Pa) and $\rho$ and $V_K$ are defined as in (10). In this simulation, the surface momentum-transfer coefficient $C_M$ is specified as a constant $1.0 \times 10^{-5}$ over the entire model domain. An adjoint equation for $\partial J/\partial C_M$ can be derived in an analogous manner to that for $\partial J/\partial C_H$ (Eqs. (11) and (12)) to study the sensitivity of $J$ with respect to variations in surface stress. We consider sensitivity with respect to the basic state without the SST anomaly, and show the accumulated sensitivity between 0 and 90 h in Fig. 20. The interpretation of $\partial J/\partial C_M$ in Fig. 20 is relatively simple. The surface momentum stress

![Figure 20](image-url)  

Figure 20. Field of sensitivity to surface momentum-transfer coefficient, $\partial J/\partial C_M$, (contour = 2 hPa) accumulated between 0 and 90 h. Positive values indicate surface momentum stress is anticyclogenetic. Cyclone positions at 30 and 70 h indicated by solid dots. Negative values are stippled. Forecast aspect $J$ is pressure at centre of 90 h cyclone.
is anticyclogenetic during the entire life cycle, with sensitivity strongly localized in the
cyclone warm sector after 70 h.

Frictional damping can have significant effects on baroclinic development, as noted by
Danard and Ellenton (1980), Branscombe et al. (1989), and Hines and Mechoso (1993). An
increase in stress will damp thermal advection in the lower troposphere, which may explain
the large sensitivity in the cyclone warm sector, where sensitivity to temperature is also
greatest. In regions outside the cyclone warm sector, the parameter $C_M$ could be changed
with little or no effect on the forecast pressure of this cyclone. Boundary-layer stress is a
damping process, as shown by the decrease in temperature and wind sensitivity close to
the surface (Fig. 5), and lower surface stress over sea surfaces is a possible explanation for
more frequent occurrence of rapidly deepening extratropical cyclones over oceans than
over land.

The effect of increasing $C_H$ and $C_M$ from $1.0 \times 10^{-3}$ to $1.5 \times 10^{-3}$ in the nonlinear
model (basic state with no SST anomaly) is compared in Table 6 with the response estimated
using the adjoint sensitivity. Increases in both parameters are anticyclogenetic, with a larger
effect (about 1 hPa) from increasing $C_M$. The adjoint accuracy is better for the perturbation
related to surface stress ($C_M$), since $C_H$ is related to more significant nonlinear effects
involving surface-layer stability and convective velocity.

<p>| TABLE 6. COMPARISON OF SURFACE-PRESSURE PERTURBATIONS AT CENTRE OF 90 H CYCLONE RESULTING FROM AN INCREASE OF 0.0005 (50%) IN COEFFICIENTS FOR SURFACE FLUXES OF SENSIBLE HEAT ($C_H$) AND MOMENTUM FLUXES ($C_M$). |</p>
<table>
<thead>
<tr>
<th>Forecast</th>
<th>$C_H + 0.0005$</th>
<th>$C_M + 0.0005$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 h Adjoint</td>
<td>+0.2449 hPa</td>
<td>+1.0174 hPa</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>+0.1713</td>
<td>+1.0028</td>
</tr>
</tbody>
</table>

Nonlinear-model result ($\Delta J$), and $J'$ (see text) obtained as dot product of perturbation and adjoint sensivity. The surface flux coefficients are perturbed over the entire domain, from 0–90 h. Basic state without sea-surface-temperature anomaly.

8. DISCUSSION

Adjoint sensitivity has been examined in this study for an idealized extratropical cyclone using dry dynamics, with surface stress and sensible-heat flux. Baroclinic instability is initiated by a relatively weak upper-tropospheric wind and temperature anomaly in the initial conditions. The early phase of the cyclone life cycle is characterized by increasingly strong thermal advection near the lower-tropospheric steering level (about 800 hPa), with the most rapid surface intensification occurring later in the life cycle when the disturbance has also propagated into the upper troposphere.

The adjoint method efficiently determines sensitivity of a forecast measure to perturbations of model variables at earlier times in a forecast. In this case, the sensitivity describes how perturbations can intensify central pressure of an idealized cyclone. A single adjoint run determines sensitivity for all variables and selected parameters. An adjoint-sensitivity pattern, by itself, does not provide information about the relative importance of anomalies or physical processes in the original nonlinear forecast. Rather, the sensitivity shows how the forecast feature could be modified in various alternative nonlinear forecasts. It is possible to consider the effects of hypothetical perturbations using adjoint sensitivity. In this
study, we also consider the effects of perturbations for which the relative size and location is based on tendencies of temperature and winds in the original basic-state (nonlinear) forecast. The interpretation of these perturbations provides insight into forecast sensitivity that is closely related to physical processes and model dynamics, rather than simply considering how hypothetical perturbations might change the forecast. Physical processes that produce large wind, temperature or pressure tendencies in regions of strong adjoint sensitivity are significant to the feature represented by the forecast aspect ($J$). Processes that occur in regions of weak sensitivity may be related to the general evolution of the basic state, but changes in their intensity do not have as much effect on $J$.

Adjoint sensitivity is examined at three times during the 90 h forecast: the initial conditions, at 30 h (just before the deepening phase), and at 70 h (during the period of most rapid deepening). At the initial time the sensitivity resembles normal-mode-type structures, with maximum sensitivity to temperature and wind in the lower troposphere below the jet core. At 30 h the sensitivity shows a strong cyclogenetic effect for heating the lower troposphere in the warm sector of the incipient cyclone. A diagnosis of the nonlinear (basic state) forecast indicates that positive temperature tendencies associated with horizontal temperature advection exist in the region of maximum sensitivity. Thus, lower-tropospheric temperature advection is an important physical process in the development of this cyclone, and an increase in its intensity would be cyclogenetic. Thermal advection in the lower troposphere is still an important process at 70 h, but localized increases of meridional wind in the middle and upper troposphere are also strongly cyclogenetic. In terms of physical processes, this appears related to stronger vorticity advection above the surface cyclone. The intensification of this cyclone is primarily a baroclinic development, and the sensitivity indicates where perturbations will grow by tapping the potential energy of the basic-state thermal gradient. The sensitivity structure in the upper troposphere also relates to barotropic effects, with perturbations drawing kinetic energy from the upper-tropospheric jet. The areas of strong sensitivity become more localized for shorter forecast intervals.

Explosive cyclogenesis is more sensitive than other types of flows to errors in initial conditions (Mullen and Baumhefner 1989; Kuo and Low-Nam 1990), and observations in regions of high baroclinicity are more likely to impact forecast outcome (Hollingsworth et al. 1985; Källén and Huang 1988). Adjoint results show that sensitivity can be highly localized even within a baroclinic zone, so that perturbations a short distance apart may have radically different effects on forecast error. The interpretation of adjoint sensitivity is thus an example of how a numerical model can provide physical insight that would be very difficult to obtain from observations alone. Cyclone predictability (and forecast skill) will be strongly influenced by conditions in high-sensitivity regions (primarily the middle and lower troposphere in this simulation), since small changes to model variables or parameters in those areas can cause forecast trajectories to diverge more quickly. In addition, model parametrizations that produce large tendencies in high-sensitivity regions are likely to have relatively large impact on forecast skill.

The Eady baroclinic instability model considers the effects of potential vorticity on only the upper and lower boundaries (tropopause and surface Rossby waves). Since the adjoint sensitivity describes also how perturbations in the interior can affect baroclinic development, the Eady model is probably too simplistic to provide a complete framework to explain the adjoint sensitivity. The Charney baroclinic instability model does consider interior potential-vorticity anomalies, and the strong sensitivity to temperature and wind in the lower troposphere implies the possible importance of an unstable Charney mode, with strong instability near the steering level in the lower troposphere. As discussed by HMR85, the ‘upper’ potential-vorticity anomaly is found near the steering level for the
fastest-growing Charney mode. Lindzen et al. (1980) and Robinson (1989) show that Charney and Green modes are essentially 'critical layer' instabilities that exist between the steering level and the 'surface' (thermal anomalies just above the boundary layer), even if unstable eigenmodes exist above the steering level. The adjoint sensitivity represents (in terms of $\partial J/\partial u$, $\partial J/\partial v$, $\partial J/\partial T$) how this type of instability can intensify the cyclone.

The adjoint sensitivity described in this study clearly identifies the primary elements that can intensify extratropical cyclones and provides insight into spatial and temporal variations of sensitivity related to physical processes of cyclogenesis. The most significant factors that can intensify the cyclone in this simulation are increased baroclinicity and thermal advection in the lower troposphere, higher SST in the cyclone warm sector, and lower surface stress. The surface field sensitivity suggests that air–sea heat transfer in the cyclone warm sector could be a primary physical mechanism explaining the preference for marine cyclogenesis near strong SST gradients. This includes preconditioning effects of upward sensible-heat flux early in the cyclone life cycle, and suppression of downward heat flux during the rapid deepening phase by higher SST in the cyclone warm sector. The effects of surface moisture fluxes are a contributing process in marine cyclogenesis, but are not included in this study. Adjoint sensitivity also indicates possible effects of upper tropospheric wind and temperature anomalies, consistent with the role of a short-wave trough or IPV anomaly in a type-B development as described, for instance, by HMR85 and Uccellini (1990).

It remains for further study to verify the generality of these adjoint-sensitivity results for other extratropical cyclones. The great variety in extratropical cyclone developments will almost certainly include situations in which the adjoint-sensitivity pattern will indicate a stronger response to upper-tropospheric thermal or wind anomalies. This may be likely when an upper-level anomaly is more intense or originates outside the baroclinic zone in which the surface cyclone ultimately forms. Adjoint models will be useful for interpreting and understanding cyclogenetic processes under many types of conditions.

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