Dependence of surface exchange coefficients on averaging scale and grid size

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SUMMARY

The value of the effective exchange coefficient for area-averaged fluxes can depend significantly on the averaging scale. This dependence implies that the exchange coefficient in numerical models should depend on grid size. The main goal of this study is the assessment of the importance of such scale-dependence. When the large-scale flow is weak, the mesoscale motions on scales smaller than the averaging width (grid size) may generate significant turbulence which is not represented by the area-averaged wind vector (resolved flow). In this case, the effective exchange coefficient must be larger than that predicted by similarity theory in order to predict the grid-averaged turbulent flux.

This study reviews different approaches for representation of spatially-averaged surface fluxes over heterogeneous surfaces. The scale-dependence of the effective exchange coefficient is posed in terms of spatially averaging the bulk aerodynamic relationship. This scale-dependence is evaluated in terms of observed fluxes from three different field programs. The effective drag coefficient is found to be more scale-dependent than the effective exchange coefficients for heat and momentum. The effective conductance is found to be less scale-dependent than the effective exchange coefficient and effective resistance. Except for the case of weak flow over strong surface heterogeneity, the exchange coefficient is nearly independent of grid size.

KEYWORDS: Boundary layer Exchange coefficients Grid-square averaging Heterogeneous terrain Scale dependence Surface fluxes

1. INTRODUCTION

In a typical grid area or domain of observations, surface properties simultaneously vary spatially on a variety of scales. The relationship between the spatially-averaged fluxes, mean variables and surface properties depend on the scale of grid-averaging. For convenience, the term ‘subgrid area’ is defined to be either the surface area of a grid box in a numerical model or the observational area of a field program. Unfortunately, specific parametrizations of grid-averaged fluxes over heterogeneous terrain are not available. As a result, grid-averaged fluxes are normally computed from the bulk aerodynamic formulation using local similarity theory for the exchange coefficients and grid-averages for the other variables.

However, similarity theory has been constructed from observations of the time-averaged flux at a fixed location. The bulk aerodynamic formulation for the time-averaged flux can be written in the form

$$\bar{w} \bar{\phi} = C_\phi \bar{V}(z)(\bar{\phi}_0 - \bar{\phi}(z))$$

where $\bar{V}$ is the time-averaged wind speed and $\bar{\phi}$ is the time-averaged transported variable at the observational level or the first model level. With Monin–Obukhov similarity theory, $\bar{\phi}_0$ is the locally averaged variable at the surface roughness height. The exchange coefficient, $C_\phi$, depends on stability, surface roughness height, and the observational level $z$.

In numerical models, $\bar{V}$ is replaced by the speed computed from the spatially-averaged wind components $\langle |\mathbf{V}| \rangle$ (Table 3, section 2) since models compute the grid-averaged wind components from the equations of motion and then compute the speed from these components. The wind vector $\langle \mathbf{V} \rangle$ will be referred to as the ‘resolved flow, although only motions on scales equal to or greater than four times the grid size are fully resolved (Avissar et al. 1990). The ‘effective exchange coefficient’, corresponding to use of $\langle |\mathbf{V}| \rangle$ as the velocity

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scale in the bulk aerodynamic relationship, behaves quite differently from that in Eq. (1) (Mahrt and Sun 1995). In order to be relevant to numerical models, this study will examine the behaviour of the effective exchange coefficient for spatially-averaged fluxes based on the speed of the spatially-averaged wind components |\( \mathbf{V} \)|.

The predictability of this effective exchange coefficient is not known. It may be possible to avoid explicit parametrization of surface heterogeneity by appealing to 'statistical heterogeneity' (Mason 1988) and assume that the surface consists of a limited number of surface types randomly distributed within the grid box. While the flow may not achieve equilibrium within a given small patch of uniform surface, the flow above the 'blending height' achieves collective equilibrium with the various surface types. At these levels, the fluxes are horizontally invariant and can be related to effective surface properties such as surface temperature and roughness lengths for the spatially-averaged flow. The datasets analyzed in this study seem inadequate for defining the blending height because the heterogeneity is poorly defined and occurs simultaneously on a variety of scales, or, occurs on scales which are too large. In the later case, the blending height is too high in the boundary layer to assist in the estimation of surface fluxes. In spite of these complications, it may be possible to employ alternative methods to estimate spatially-averaged fluxes over heterogeneous surfaces which improve upon existing parametrizations based on local similarity theory.

(a) Approach 1: Homogeneous subareas

In some ideal cases, the grid area may consist of only a few types of surface conditions, say grassland and forest. Then it may be necessary only to measure or parametrize each of these surface types and then to calculate pro-rata the corresponding fluxes according to the percentage of grid area occupied by each surface type as in the modelling philosophy of Wetzel and Chang (1988), Avissar and Pielke (1989), Claussen (1990), Ducoudré et al. (1993), and Huang and Lyons (1995). Fitzjarrald and Moore (1993) found that this approach can estimate regional fluxes within 50% accuracy based on aircraft observations over tundra containing significant coverage by lakes. This method performs much better in the analysis of fluxes over well-defined surface heterogeneity (section 3).

While each surface type may occur in several independent subgrid areas, this approach implicitly assumes that the surface flux transition regions between different surface types occupy only a small percentage of the total area. In other terms, this approach assumes that the scale of the heterogeneity is sufficiently large that the boundary-layer flux is in equilibrium with local surface conditions over most of the subgrid area. In this case, the theoretical blending height would be above the top of the boundary layer.

(b) Approach 2: Evaluation of spatial Reynolds terms

Section 4 evaluates extra spatial Reynolds terms for the grid-averaged flux due to sub-grid variation of the mean variables (Esbensen and Reynolds 1981; Mahrt 1987; Claussen 1991; Pinty 1991; Garratt 1992; Mahrt et al. 1994a,b). These corrections to the grid-averaged flux cannot be parametrized in a practical way because their behaviour does not appear to be systematic between different ambient situations. However section 4 derives a somewhat different form of these terms which assist the interpretation of the observations in section 5.

(c) Approach 3: Direct evaluation of the effective exchange coefficient

A more direct approach would be to compute the effective exchange coefficient or effective resistance directly from the observed fluxes and variables, grid-averaged over an
area of width $\Delta x$. In other words, the effective exchange coefficient $C_{\text{eff}}(\Delta x)$ is the parameter which predicts the grid-averaged fluxes in terms of the grid-averaged variables used in the bulk aerodynamic formula. This effective exchange coefficient may be a function of grid-averaging scale ($\Delta x$), stability, and the degree of surface heterogeneity. Existing similarity theory cannot predict $C_{\text{eff}}(\Delta x)$ since it is valid only for time-averaged fluxes over homogeneous surfaces. The effective exchange coefficients are evaluated from data as a function of grid size ($\Delta x$) in section 5.

(d) Approach 4: Effective roughness heights for similarity theory

Similar to Approach 3, the effective roughness height is computed by matching the exchange coefficient for grid-averaged fluxes to the existing Monin–Obukhov similarity theory (Wood and Mason 1991; Mahrt and Ek 1993; Wieringa 1986; Beljaars and Holtslag 1991, section 4b; see Claussen 1990, and Wieringa 1993 for surveys). The definition of the effective roughness length varies between studies and also depends on horizontal scale. As with any roughness length, the effective roughness length depends on the particular version of the stability-dependent similarity theory. This approach is not considered in this study.

(e) Approach 5: Regression model using remotely sensed variables

Recently, regression models have been constructed to relate surface fluxes to surface variables remotely sensed by satellites (Desjardins et al., 1992; Nemani et al. 1993; Sun and Mahrt 1994). However, this approach needs re-examination over a variety of surface conditions including forests, wet bare soil, vegetation with strong stomatal control, a variety of sun angles and different cloudiness regimes. This approach is not considered in this study.

(f) Approach 6: Spatial averaging coefficients

It may be possible to approximate the effective exchange coefficient for the grid-averaged flux in terms of an appropriate spatial average of the local exchange coefficient. Dolman (1992) shows that, at least for sufficiently moist surfaces, the spatial averaging problem for the moisture flux can be simplified by expressing fluxes in terms of the conductance. Using a numerical model, Blyth et al. (1993) found that different averaging methods must be used for momentum, heat and moisture and that the required averaging technique for heat must be quite complex. These techniques require that the atmospheric information is obtained at, or above, the blending height which cannot be defined in the present data. Nonetheless, these results suggest the utility of expressing the averaging algorithm in terms of the effective resistance or inverse of the effective resistance (effective conductance), which will be included in section 5.

2. Data

This study analyzes aircraft data from the California Ozone Deposition Experiment (CODE), the Hydrological and Atmospheric Pilot Experiment (HAPEX) in south-west France (Les Landes) and the bunny Fence experiment (buFex) (Table 1). The wind speed was roughly 5 m/s except for CODE flight 19 where the wind was only 1–2 m/s. The CODE data consist of 30 km runs taken by the Canadian Twin Otter over 5–10 km patches of well organized cool irrigated surfaces and warm dry unirrigated surfaces in the San Joaquin Valley of California. The HAPEX data are taken by the NCAR King Air over a 125 km wide flat pine forest in south-west France (Les Landes). The buFex data are
taken over a region of mixed agriculture sharply divided by a semi-arid region of sparse vegetation in the Lakes District of Great Southern Region of Western Australia.

The aircraft runs are divided into 5 km segments. Turbulent fluctuations are mathematically defined as deviations from 5 km averages. These averages are a substitute for a local time-average and are symbolized by \( \bar{\phi} \) in Table 2. Choosing an averaging scale smaller than 5 km increases sampling errors (Sun and Mahrt 1994). To compute fluxes, the products of the perturbations are averaged over scales \( \Delta x = 5, 10, 15 \) and 30 km, symbolized as \( \langle w^p \phi^q \rangle \). Regardless of the averaging scale \( \Delta x \), turbulent fluctuations are always computed as deviations from the 5 km averages.

A special alternative calculation is made for the CODE data by dividing the flight leg into seven sub-legs according to natural boundaries based on changes of land use such as boundaries between irrigated and non-irrigated areas (Mahrt et al. 1994b). To compute fluxes on larger scales, these natural sub-legs are combined into larger sub-legs. The results for these natural sub-legs will be discussed only when they differ significantly with the results for the 5 km sub-legs.

The effective exchange coefficients, resistances and conductances for the grid-averaged fluxes of heat, moisture and momentum are defined, respectively as:

\[
\begin{align*}
C_{\text{Heff}}(\Delta x) & \equiv \frac{\langle w^p \theta^q \rangle}{|\langle V \rangle| |\langle \Delta \theta \rangle|}, \\
C_{\text{qeff}}(\Delta x) & \equiv \frac{\langle w^p q^q \rangle}{|\langle V \rangle| |\langle \Delta q \rangle|}, \\
C_{\text{Deff}}(\Delta x) & \equiv \frac{\langle u^p \rangle^2}{|\langle V \rangle|^2}, \\
r_{\text{Heff}}(\Delta x) & \equiv \frac{\langle \Delta \theta \rangle}{\langle w^p \theta^q \rangle}, \\
r_{\text{qeff}}(\Delta x) & \equiv \frac{\langle \Delta q \rangle}{\langle w^p q^q \rangle}, \\
r_{\text{Deff}}(\Delta x) & \equiv \frac{\langle u^p \rangle^2}{|\langle V \rangle|}, \\
g_{\text{Heff}}(\Delta x) & \equiv \frac{\langle w^p \theta^q \rangle}{\langle \Delta \theta \rangle}, \\
g_{\text{qeff}}(\Delta x) & \equiv \frac{\langle w^p q^q \rangle}{\langle \Delta q \rangle}, \\
g_{\text{Deff}}(\Delta x) & \equiv \frac{\langle u^p \rangle^2}{|\langle V \rangle|}.
\end{align*}
\]

where:

\[
\Delta \theta \equiv \langle \theta_{\text{sfc}} \rangle - \langle \theta \rangle \\
\Delta q \equiv q_{\text{sat}}(\langle T_{\text{sfc}} \rangle) - \langle q \rangle.
\]

Note that all effective quantities are defined in terms of the speed based on spatially-averaged wind components. For the data analysis in this study, \( \theta_{\text{sfc}} \) is the potential temperature corresponding to surface radiative temperature in which case \( C_{\text{Heff}} \) is the effective exchange coefficient for spatially-averaged fluxes, defined in terms of the spatially-averaged
3. Homogeneous Subgrid Areas (Approach 1)

(a) Flux formulation in homogeneous sub-areas

This subsection uses a simple example to illustrate one mechanism for the enhancement of the effective exchange coefficient corresponding to grid-averaged fluxes. This development assumes that the grid area can be partitioned into homogeneous sub-areas corresponding to Approach 1 in the Introduction.

The grid-averaged heat flux can then be written as

$$\langle w' \theta' \rangle = \sum_{n=1}^{N} F(n)C_h(n)V(n)\{\theta_o(n) - \theta(n)\}$$  \hspace{1cm} (5)

where $n$ is an index for the different subgrid regions, $N$ is the total number of subgrid regions, $F(n)$ is the fractional coverage of the $n$th sub-area characterized by aerodynamic temperature at the roughness height, $\theta_o(n)$, air temperature at the observational height, $\theta(n)$, and wind speed, $V(n)$. Since each sub-area is homogeneous, the sub-area averaged wind vector is equal to the subgrid averaged speed $V(n)$. Relationship (5) is nonlinear in $\theta_o(n) - \theta(n)$ since the exchange coefficient depends on the stability.

To illustrate the role of subgrid variability on the heat flux, we divide the total area into a dry heated part ($n = 1$) and a cool wet part ($n = 2$), in which case (5) becomes

$$\langle w' \theta' \rangle = F(1)C_h(1)V(1)\{\theta_o(1) - \theta(1)\} + F(2)C_h(2)V(2)\{\theta_o(2) - \theta(2)\}.$$  \hspace{1cm} (6)

For the CODE data, the heat flux over the wet area is an order of magnitude smaller than over the dry area and even locally vanishes over the wet area. Then the area-averaged heat flux is approximately

$$\langle w' \theta' \rangle \approx F(1)C_h(1)V(1)\{\theta_o(1) - \theta(1)\}. \hspace{1cm} (7)$$
Although temperatures at the roughness height are not available, we observe that in
the CODE data, the air-surface temperature differences are on the order of 1 degC, or less,
over the wet areas and 10–30 degC over the dry area. Therefore, we assume that
\[
\langle \theta_0 \rangle - \langle \theta \rangle \simeq F(1) \{ \theta_0(1) - \theta(1) \}.
\]
(8)
The effective exchange coefficient for the spatially-averaged flow, \(C_{\text{Heff}}\) defined in Eq. (2),
can now be estimated by substituting the spatially-averaged temperature difference (8) and
the estimate of the spatially-averaged heat flux (7) into (2), in which case
\[
C_{\text{Heff}}(\Delta x) = \frac{C_n(1)}{\{\langle V \rangle \}}.
\]
(9)
Neglecting spatial variation of wind speed, the grid-averaged heat flux should then be
estimated using the exchange coefficient for the heated part of the grid area. Existing simi-
larity theory predicts the exchange coefficient to increase monotonically with increasing
instability. Then the required effective exchange coefficient for grid-averaged fluxes will
be larger than the exchange coefficient computed from the grid-averaged temperature dif-
ference. Therefore, the usual modelling approach would underestimate the grid-averaged
heat flux. The model estimate fails because the heat flux relationship is nonlinear in that
the exchange coefficient depends on the temperature difference. This nonlinearity is ex-
pressed in terms of spatial Reynolds averaging in section 4. In addition, the surface wind
speed is expected to be greater over the heated area due to stronger downward mixing of
momentum. Then \(V(1)/\|\langle V \rangle \|\) acts to further increase the value of the effective exchange
coefficient for grid-averaged fluxes.

However the above arguments are only valid for application of similarity theory using
the surface aerodynamic temperature at the surface roughness height. In most modell-
ing situations, this temperature is replaced by the temperature computed from the surface
energy budget which is equivalent to the surface radiation temperature. When the heat
flux is related to the surface radiation temperature, the exchange coefficient is no longer
observed to increase systematically with increasing instability (Sun and Mahrt 1995). Then
the dependence of the effective exchange coefficient on averaging scale is also reduced.
For example, if the exchange coefficients for the two sub-areas in the above example are of
similar value, then the effective exchange coefficient for predicting the area averaged flux
is similar to the value for the two sub-areas. In this case, the effective exchange coefficient
would not depend significantly on scale.

(b) A simplified flux calculation in homogeneous sub-areas

Without further simplification, implementation of Eq. (5) in modelling studies is
numerically expensive since it is analogous to increasing the spatial resolution in the lower
part of the model. A major simplification can be imposed by noting that the spatial variation
of the surface air temperature, \(\theta(n)\), is an order of magnitude smaller than the spatial
variation of \(\theta_{\text{sfc}}(n)\) for conditions of significant differential heating over land, where \(\theta_{\text{sfc}}(n)\)
is based on the surface radiation temperature. This suggests that the spatial variability of
the surface–air temperature difference might be estimated in terms of the spatial variability
of the surface temperature. Then (5) is approximated as
\[
\langle w'\theta' \rangle = \sum_{n=1}^{N} F(n) C_n(n) V(n) (\theta_{\text{sfc}}(n) - \langle \theta \rangle)
\]
(10)
where \(\langle \theta \rangle\) is the grid-averaged air temperature near the surface. Application of (10) in
numerical models would require specification of spatial coverage of the surface types and
would require solution of the surface energy balance for each sub-area in order to estimate \( \theta_{\text{dc}}(n) \).

This approach seems to be the simplest possible method for explicitly including some subgrid variability at a cost which is less than simply increasing the horizontal resolution. The bulk aerodynamic formula estimates surface fluxes separately for each sub-area of the grid box but uses atmospheric variables which are spatially constant for the entire grid box. Relationship (10) is now evaluated from CODE flights 13 and 19 by defining each sub-area as a ‘natural record segment’ constructed in terms of sharp changes of surface conditions (section 2). The exchange coefficient \( C_\theta(n) \) is that value which predicts the correct flux for the \( n \)th natural segment. The natural segments for CODE flights 13 and 19 are approximately of equal width so that \( F(n) \) will be assumed constant.

From (10), the turbulent flux averaged over the grid area is then approximated as

\[
\langle w' \theta' \rangle = \frac{1}{N} \sum_{n=1}^{N} C_\theta(n) |\langle V \rangle| (\theta_{\text{dc}}(n) - \langle \theta \rangle)
\]

(11)

where

\[
\langle \theta \rangle = \frac{1}{N} \sum_{n=1}^{N} \theta(n).
\]

(12)

Again, the angle brackets indicate averaging over the grid area and \( |\langle V \rangle| \) represents the resolved flow. The same procedure is applied to calculation of the moisture flux.

Equation (11) slightly overestimates the heat flux composited over the 30 km flight, by about 7% for CODE flight 19 and 5% for CODE flight 13. For the daytime heated case, the use of the grid-averaged air temperature over-predicts air-surface temperature difference and the heat flux in regions of warmer surfaces. This is largely compensated by under-prediction of the air-surface temperature difference and the heat flux in regions of cooler surfaces. Use of the grid-averaged specific humidity, predicts the composited moisture flux for the entire flight leg with less than 1% error for both flights 13 and 19. If we divide the record into arbitrary 5 km segments instead of natural segments, (11) again commits only small errors. For the buFex and HAPEX data sets, (11) overestimates the grid-averaged heat flux by about 10% and predicts the grid-averaged moisture flux within a few percent.

We conclude that for the present data, assuming spatially constant atmospheric variables and spatially varying surface conditions closely approximates the area composited heat flux. This supports the modelling philosophy of Approach 1 in the Introduction. The analysis in this section used the local exchange coefficient computed from the data rather than one predicted by similarity theory. Part of the success of the method of homogeneous sub-areas is due to the fact that the exchange coefficient, based on surface radiation temperature, does not depend systematically on stability, at least for the neutral and unstable cases (Sun and Mahrt 1994) and that the surface radiative temperature varies spatially much more than the air temperature. This approach may be less useful over the ocean where the spatial variation of air temperature is often comparable to, or greater than, the spatial variation of air temperature.

4. Spatial Reynolds Averaging (Approach 2)

This section estimates the flux over a grid-area by spatially averaging the bulk aerodynamic relationship. Similarity formulations of the exchange coefficients can not be applied directly to grid-averaged variables. The relationship between the grid-averaged
flux and the grid-averaged variables is quite different than the 'known' bulk relationship between the local time-averaged flux and the local time-averaged variables.

For physical interpretation, the following is posed in terms of heat flux although the development can be applied to the flux of any variable. The averaging width, $\Delta x$, will range from 5 km up to the record length in order to simulate various grid sizes. The dependence of $C_{\text{Hef}}(\Delta x)$ on averaging scale, $\Delta x$, serves as a measure of the scale-dependence of the effective exchange coefficient. To facilitate this analysis, the local average, $\phi$, is decomposed into the grid-averaged value, $\langle \phi \rangle$, and the deviation of the local average from the grid-averaged value, $\phi^*$, so that:

$$
\bar{u} = \langle u \rangle + u^* \\
\bar{v} = \langle v \rangle + v^* \\
\bar{\Delta \theta} = \langle \Delta \theta \rangle + (\Delta \theta)^* \\
\bar{w'\theta'} = \langle w'\theta' \rangle + (w'\theta')^* 
$$

where the term $(w'\theta')^*$ is the mesoscale subgrid variation of the turbulence flux. The direct transport by the mesoscale vertical motion $w^*$ is not included in the bulk aerodynamic formulation.

A simplified expression for the local exchange coefficient can be derived by rotating the coordinate system such that $\langle v \rangle = 0$ and $\langle u \rangle$ is positive, and by assuming $(u^*, v^*)$ are small compared to the grid-averaged flow $\langle u \rangle$. With these assumptions, the speed of the locally averaged wind can be written as

$$
(u^2 + v^2)^{1/2} = (\langle u \rangle^2 + 2u^*\langle u \rangle + u^* + v^* + v^2)^{1/2} \approx \langle u \rangle + V^* 
$$

where

$$
V^* \equiv u^* + \frac{u^* + v^*}{2\langle u \rangle}. 
$$

The local exchange coefficient for the turbulent heat flux can then be written in terms of decomposed variables as

$$
C_h = \frac{\langle w'\theta' \rangle + (w'\theta')^*}{\langle u \rangle + V^*}(\langle \Delta \theta \rangle + (\Delta \theta)^*) \\
= \frac{\langle w'\theta' \rangle + (w'\theta')^*}{\langle u \rangle \langle \Delta \theta \rangle (1 + \epsilon)} 
$$

where

$$
\epsilon \equiv \frac{\langle u \rangle (\Delta \theta)^* + V^* (\Delta \theta) + V^* (\Delta \theta)^*}{\langle u \rangle \langle \Delta \theta \rangle}. 
$$

Since subgrid variations are assumed to be small when compared to grid-averaged quantities, $\epsilon \ll 1$, so that

$$
\frac{1}{1 + \epsilon} \approx 1 - \epsilon + O(\epsilon^2). 
$$

Then neglecting higher order terms, (16) becomes approximately

$$
C_h \approx \frac{\langle w'\theta' \rangle + (w'\theta')^*}{\langle u \rangle \langle \Delta \theta \rangle} \left(1 - \frac{\langle u \rangle (\Delta \theta)^* + V^* (\Delta \theta) + V^* (\Delta \theta)^*}{\langle u \rangle \langle \Delta \theta \rangle}\right). 
$$
Note that the series expansion is not unique to higher order since one could also expand separately in terms of individual perturbation quantities. However, these differences are not important here. Substituting (15) into (19), neglecting triple products of subgrid perturbations, and then grid-averaging (19), the grid-average of the local exchange coefficient is

$$
\langle C_h \rangle \approx C_H(\Delta x) \left( 1 - \frac{\langle u'^2 + v'^2 \rangle (\Delta \theta) / (2 \langle u \rangle) + \langle u^* (\Delta \theta)^* \rangle}{\langle u \rangle (\Delta \theta)} \right)
$$

$$
- \frac{1}{\langle u \rangle (\Delta \theta)} \left( \frac{\langle (w \theta')^* \Delta \theta^* \rangle}{\langle \Delta \theta \rangle} + \frac{\langle (w \theta')^* u^* \rangle}{\langle u \rangle} \right)
$$

(20)

where again $C_{H_{eff}}(\Delta x) \equiv \langle w \theta' \rangle / \langle \langle \theta \rangle \rangle (\Delta \theta)$ is the effective exchange coefficient for the grid-averaged flux and the speed of the vector averaged flow $\langle \langle \theta \rangle \rangle$ is, here, equal to $\langle u \rangle$. Solving for the effective exchange coefficient for the grid-averaged flow

$$
C_{H_{eff}}(\Delta x) \approx \left( \langle C_h \rangle + \frac{1}{\langle u \rangle (\Delta \theta)} \left( \frac{\langle (w \theta')^* \Delta \theta^* \rangle}{\langle \Delta \theta \rangle} + \frac{\langle (w \theta')^* u^* \rangle}{\langle u \rangle} \right) \right)^{-1} \left( 1 - \frac{\langle u'^2 + v'^2 \rangle (\Delta \theta) / (2 \langle u \rangle) + \langle u^* (\Delta \theta)^* \rangle}{\langle u \rangle (\Delta \theta)} \right)
$$

(21)

Equation (21) shows how the effective exchange coefficient for the area averaged flux, $C_{H_{eff}}(\Delta x)$, is different from the grid-averaged local exchange coefficient $\langle C_h \rangle$. This difference is due to ‘spatial Reynolds terms’ corresponding to covariances between the subgrid deviations $\langle \theta \rangle^*$. Numerical models require the effective exchange coefficient for the area-averaged flux, $C_{H_{eff}}(\Delta x)$. However, existing similarity theory predicts only the local exchange coefficient, $C_h$.

To apply the above decomposition to the aircraft data listed in section 2, local averaged values are computed as averages over 5 km segments of the record (see section 2), and then the exchange coefficient is computed from the locally averaged values. For these data, the most important spatial Reynolds term is the ‘mesoscale kinetic energy term’ occurring in the denominator of the right hand side of (21), equal to $\langle u'^2 + v'^2 \rangle / (2 \langle u \rangle^2)$. This term is significant when the large scale flow is weak and subgrid variation of the mesoscale flow is large. This ‘mesoscale flow’ Reynolds term is due to the smaller magnitude of the speed computed from the grid-averaged wind components when compared to the grid-average of the local wind speed. To compensate for this difference, the effective exchange coefficient for the grid-averaged flow, $C_{H_{eff}}$, is expected to be greater than the grid-average of the local exchange coefficient, $C_h$. In still other terms, the effective exchange coefficient for the grid-averaged flux must increase to compensate for turbulent flux driven by mesoscale subgrid variations $(u^*, v^*)$ which are not resolved by the grid-averaged flow.

For the data listed in section 2, the mesoscale-flow Reynolds term is large for CODE flight 19 which represents weak winds over strong surface heterogeneity. This term is sufficiently large that the linearization assumption leading to (21) breaks down. However, (21) qualitatively indicates that the large observed value of $C_{H_{eff}}(\Delta x = 30 \text{ km})$ for CODE flight 19 (section 5) is probably due mainly to that part of the turbulent flux driven by the subgrid mesoscale motion $(u^*, v^*)$. The positive spatial correlation between the heat flux and the vertical temperature difference in (21) is also important for CODE flight 19. Spatial variations of the vertical temperature difference are dominated by variations of the surface radiation temperature so that the heat flux is larger where the surface temperature is greater. The other two spatial Reynolds terms are much smaller.
For CODE flight 13, several of the spatial Reynolds terms appear to be significant although the net effect on the scale-dependence of the effective exchange coefficient is small. For the remaining data sets, the mesoscale-flow Reynolds term and the spatial correlation between the heat flux and vertical temperature difference (21) alter the effective exchange coefficient by only a few percent or less. Consequently, the spatial Reynolds terms do not lead to significant scale-dependence of the effective exchange coefficient, except for the case of weak flow over strong heterogeneity.

The relationships for conductance and resistance defined in (2) are considerably simpler because they do not depend explicitly on the spatial variation of the wind. The effective conductance for the grid-averaged heat flux is approximately

$$g_{\text{eff}}(\Delta x) \approx \bar{g}_H + \frac{\langle (\Delta \theta)^* (w^* \theta^*)^* \rangle}{\langle \Delta \theta \rangle^2}$$

and the effective resistance for heat is approximately

$$r_{\text{eff}}(\Delta x) \approx \bar{r}_H + \frac{\langle (\Delta \theta)^* (w^* \theta^*)^* \rangle}{\langle w^* \theta^* \rangle^2}$$

where $\bar{g}_H$ and $\bar{r}_H$ are the grid-averages of the local values of the conductance and resistance, respectively. The effective conductance and resistance are expected to be less scale-dependent because the extra Reynolds terms associated with subgrid variations of wind do not occur. The data analysis of section 5(b) indicates that the effective conductance, in particular, is relatively independent of averaging scale.

5. **Direct evaluation of the effective exchange coefficients, resistances and conductances (Approach 3)**

(a) **Formulation**

The meaning and numerical value of the effective exchange coefficient, conductance and resistance depend on the grid size over which the fluxes are averaged. To study the dependence of these quantities on averaging scale, the effective exchange coefficients, $C_{\text{eff}}(\Delta x)$, resistances, $r_{\text{eff}}(\Delta x)$, and conductances, $g_{\text{eff}}(\Delta x)$, defined in (2) are calculated from the fluxes and variables averaged over $\Delta x$ for different values of $\Delta x$. For each value of $\Delta x$, the values of the effective exchange coefficients, resistances and conductances are composited over all of the segments of width $\Delta x$, for a given flight leg, such that:

$$[C_{\text{eff}}(\Delta x)] = \frac{1}{K} \sum_{k=1}^{K} C_{\text{eff}}(\Delta x)$$

$$[r_{\text{eff}}(\Delta x)] = \frac{1}{K} \sum_{k=1}^{K} r_{\text{eff}}(\Delta x)$$

$$[g_{\text{eff}}(\Delta x)] = \frac{1}{K} \sum_{k=1}^{K} g_{\text{eff}}(\Delta x)$$

where $K$ is the number of segments of width $\Delta x$ for each flight leg and the operator $[ \ ]$ indicates compositing all of the values at scale $\Delta x$ over the flight leg. These composited values are then averaged over all of the flight legs for a given flight day. This procedure
is repeated for different values of $\Delta x$ to examine the scale-dependence of the effective exchange coefficients, resistances and conductances.

Special care must be exercised when interpreting composited values of a ratio since very large values of the ratio occur when the denominator becomes small. For example, the resistance becomes very large when the flux becomes small and the mean air-surface difference does not. These large values disproportionately affect the composited value as will be seen in section 5(b).

Compositing the conductance according to (26) is equivalent to compositing resistances in parallel, as in Blyth et al. (1993) for averaging at the blending height. Wood and Mason (1991) and Blyth et al. (1993) suggest more complex methods to grid-averaged resistances for the case of small scale heterogeneity where the first model level is at, or above, the blending height. Their approach cannot be applied to the present case of relatively large scale heterogeneity because the observational level is well below the blending height. Also, the similarity theory required for their approach does not describe the spatial variation of the exchange coefficients when applied to the surface radiation temperature. Nonetheless, the study of Blyth et al. (1993) indicates that the specific method of grid-averaging the resistances can strongly influence the value of the horizontal average.

(b) Weak heterogeneity

The effective exchange coefficients for heat and moisture for buFex, CODE flight 13 and the two HAPEX cases show little dependence on averaging scale (Figs. 1 and 2), as predicted by the small values of the spatial Reynolds terms (section 4). Significant scale-dependence of the composited effective exchange coefficients for heat and moisture $[C_{\text{eff}}(\Delta x)]$ occurs only for CODE flight 19 (section 5(c)), where the grid-averaged flow is weak and the surface heterogeneity is well defined on a scale which is larger than the scale of the largest turbulent eddies, but smaller than the grid size. For the HAPEX data, the surface heterogeneity is due mainly to clearings in the forest and blocks of different
tree heights resulting from different planting dates. These variations occur on the scale of order of 1 km and do not affect the scale dependence of the effective exchange coefficient examined here. For buFex, the surface spatial variability occurs on small scales in the agricultural region, and the larger scale variation between the agricultural region and the region of semi-arid native vegetation in buFex is weaker than that for the CODE flight track.

In contrast to the effective exchange coefficients for heat and moisture, the composited effective drag coefficient [$C_{deff}(\Delta x)$] (Fig. 3) tends to decrease with averaging scale due to decreasing stress with increasing averaging scale. This scale-dependence occurs because the stress is spatially more variant than the wind vector. For example, the stress components are more likely to reverse sign across the averaging area, leading to reduced magnitude of the grid-averaged stress components. The variability of the momentum flux appears to be associated with transient mesoscale motions which do not cause significant spatial variability of the heat and moisture fluxes. The cause of the mesoscale motions is not known.

The effective drag coefficient is approximately three times larger over the rough forest in HAPEX when compared to the other field programs where the surface roughness is less. The drag coefficient might be enhanced by the bluff roughness effects associated with the clearings and blocks of different tree heights. A net enhancement of spatially averaged momentum fluxes due to roughness transitions is predicted by Schmid and Bünzli (1995) and references therein. The composited effective exchange coefficient for moisture, [$C_{qeff}(\Delta x)$], is much smaller than that for heat; this is due to the soil moisture deficit and use of the surface saturation value of the specific humidity in the bulk aerodynamic relationship.
SURFACE EXCHANGE COEFFICIENTS—AVERAGING SCALE AND GRID SIZE

![Graph](image)

Figure 3. Variation of the spatially composited effective drag coefficient \(C_{\text{Def}}(\Delta x)\) for buFex (circles), CODE flight 13 (squares), HAPEX 19 May (triangles) and HAPEX 25 May (diamonds) as a function of averaging scale \(\Delta x\).

(c) Strong heterogeneity effect

For CODE flight 19 (Fig. 4), the ambient wind is weak and the wind field is strongly influenced by the surface heterogeneity. The cool air over irrigated cotton and the warmer air over the adjacent unirrigated lands produce horizontal variation of hydrostatic pressure which generates divergent outward flow from the irrigated area (Mahrt et al. 1994b) known as an ‘inland’ or ‘irrigation breeze’. This mesoscale circulation is on the scale of 10–20 km and is, therefore, part of the mean flow for \(\Delta x = 5\) km but becomes ‘subgrid’ flow for \(\Delta x = 30\) km. As a result of the reversal of wind direction along the flight track, \(|\langle V\rangle|\) decreases with increasing grid size \(\Delta x\). In this case, the speed of the wind vector averaged over 30 km is particularly small since averaging across the sign reversal of the wind components leads to significant cancellation. If the stress vector was everywhere aligned opposite to the wind vector, then similar cancellation would be expected when grid-averaging the stress vector. However, the stress vector is not aligned opposite to the wind vector probably due to the rapid change of wind direction with height. In particular, the momentum flux at the surface is partly associated with mixing between the thin inland breeze and the overlying ambient flow which is almost perpendicular to the surface flow. The direction of this stress is not determined by the wind direction in the surface layer but rather by the shear between the local circulation and overlying synoptic flow. The magnitude of the stress vector does not decrease as much with increased averaging scale, as compared to the decrease of the magnitude of the wind vector. Therefore, the effective drag coefficient for the 30 km grid-averaging is large, about an order of magnitude larger than the 5 km effective drag coefficients (Fig. 4).

The decrease of the speed of the grid-averaged wind vector with increasing grid size substantially increases the effective exchange coefficients for grid-averaged heat and
moisture fluxes. The effective exchange coefficient for heat for flight 19 increases by about 400% between the 15 and 30 km averaging scales (Fig. 4) partly to compensate for the decrease of the speed of the grid-averaged wind vector $\langle \mathbf{V} \rangle$. This effect is represented by the mesoscale flow Reynolds term in the expression for $[C_{\text{Reff}}(\Delta x)]$ in Eq. (21) of section 4. For flight 19, the effective exchange coefficient for moisture increases by almost 300% as the averaging extends from 15 km to 30 km.

From another point of view, the velocity vector, averaged over the largest averaging scale, does not resolve those mesoscale scale motions which occur on smaller scales and, therefore, does not include the turbulent flux generated by such mesoscale motions. As a result, the effective exchange coefficient for the larger averaging scales must be larger than that for the smaller averaging scales. When the above analysis is repeated using natural sublegs based on land-use boundaries, the results are similar except that the scale-dependence is captured partly at smaller averaging scales instead of concentration of the change between the 15 km and 30 km averaging scales.

The problem of the scale-dependent velocity in the bulk aerodynamic relationship for the heat and moisture flux can be reduced by using the spatial average of the wind speed. However, this information is not available in numerical models. Alternatively, the fluxes can be formulated in terms of the effective resistance or the effective conductance. These quantities for heat and moisture do not depend directly on the wind and, therefore, do not contain the spatial Reynolds terms for subgrid velocity variations (section 4). For momentum, the effect of the scale-dependence of the velocity is reduced by using the effective resistance or effective conductance since these quantities depend only linearly on the wind, whereas the drag coefficient depends quadratically on the velocity.

However, the problem of averaging ratios causes an additional scale-dependence for the effective resistance. At small averaging scales, the fluxes of moisture and momentum occasionally become quite small leading to large values of the local resistance. These cases disproportionally affect the composited value of the ratio and cause the composited effective resistance at small scales to be larger than that for large scales (Fig. 5). This averaging problem also leads to scale-dependence of the effective resistance for the other
Figure 5. The composited effective resistance for heat \( r_{\text{heat}}(\Delta x) \)(10^3 s^{-1}) (circles), moisture \( r_{\text{moist}}(\Delta x) \)(10^3 s^{-1}) (squares) and momentum \( r_{\text{momentum}}(\Delta x) \)(10^3 s^{-1}) (triangles) as a function of averaging scale \( \Delta x \) for CODE flight 19.

Figure 6. The composited effective conductance for heat \( g_{\text{heat}}(\Delta x) \)(10^{-3} m s^{-1}) (circles), moisture \( g_{\text{moist}}(\Delta x) \)(10^{-3} m s^{-1}) (squares) and momentum \( g_{\text{momentum}}(\Delta x) \)(10^{-3} m s^{-1}) (triangles) as a function of averaging scale \( \Delta x \) for CODE flight 19.

data sets, albeit significantly weaker than that for CODE flight 19. This problem is not a fundamental difficulty with the use of the resistance but only occurs when attempting to average resistances.

This averaging problem does not affect the conductance which is defined with the flux in the numerator (Fig. 6). Since the effective conductance also avoids the problem of small grid-averaged velocity for the largest averaging scale, the effective conductance is
less scale-dependent than both the effective exchange coefficient and effective resistance for the datasets in this study. The generality of these results are not known.

6. CONCLUSIONS

The above analysis of data from three different field programmes indicates that the effective exchange coefficients for spatially-averaged fluxes of heat and moisture are nearly independent of averaging scale or grid size (section 5(b)) except in the case of weak winds over heterogeneous terrain (section 5(c)). The scale-independence of the effective exchange coefficient is related to the insignificance of Reynolds terms associated with spatial correlation between the flux, wind speed and the air-surface moisture or temperature difference (section 4). These results apply to the bulk aerodynamic formulation, expressed in terms of the speed computed from the spatially-averaged (grid-averaged) wind components, in analogy to numerical models. The heat and moisture fluxes are related to the surface radiation temperature or, equivalently, the temperature computed from the surface energy budget.

The extra Reynolds terms are important only in the case of strong heterogeneity and weak grid-averaged flow in which case the effective exchange coefficient depends on the averaging scale. With stronger grid-averaged flow, the scale-dependence of the effective exchange coefficient is significantly reduced even over the same strong surface heterogeneity. For the general case, where the effective exchange coefficients for heat and moisture are approximately independent of averaging scale, local similarity might be formally justified for prediction of the grid-averaged surface fluxes. This justification is contingent upon adjustment for use with the surface radiation temperature, either measured or computed from the surface energy budget.

The effective drag coefficient is characterized by significant scale-dependence for more cases than that for the effective exchange coefficients for heat and moisture; this is due to the complex spatial variability in the relationship between the surface stress and mean wind vector. Since the grid-averaged stress vector tends to decrease with grid size at a greater rate than the speed of the grid-averaged wind vector, the effective drag coefficient generally decreases with grid size.

Effective conductances computed from the observations exhibited less scale-dependence than the effective exchange coefficients or resistances. The effective conductance does not contain the scale-dependence terms related to the grid-averaged wind vector (section 4), as occurs with the effective exchange coefficient. The scale-dependence of the effective resistance is partly related to locally small fluxes at small Δx. This occasionally leads to very large values of the small scale effective resistance (section 5(a)).

The bulk aerodynamic formulation, based on spatially constant atmospheric variables and spatially varying surface conditions (section 3), closely approximates the area averaged heat flux, at least for the present datasets. This supports the modelling philosophy of Wetzel and Chang (1988), Avissar and Pielke (1989), Claussen (1991), Ducoudré et al. (1993), and Huang and Lyons (1995).

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