The response of a variable resolution semi-Lagrangian NWP model to changes in horizontal interpolation

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SUMMARY

The effect of changing the horizontal interpolation for evaluating the quantities at departure points in the global spectral semi-Lagrangian variable resolution NWP model ARPEGE/IFS is described. The two interpolations involved are conventional cubic Lagrangian and Hermite with estimates of derivatives by fifth-order compact upwind differencing (CUD-5). Their properties are compared with examples of different one-dimensional and two-dimensional linear-advective tests. With each kind of horizontal interpolation, twelve 5-day forecasts are performed, beginning on the fifth day of each month from February 1994 to January 1995, using the same vertical non-interpolating scheme.

On average, the difference between forecasts with these interpolations in geopotential and mean sea-level pressure fields is negligible within the first three days, but noticeable on the fifth day, especially in the mean sea-level pressure field. However, these improvements are present beyond the range for which the variable resolution strategy is designed.

The root-mean-square errors for the relative humidity fields at 1000 hPa and 850 hPa demonstrate some improvement as a result of the change of interpolation, from the second day of the forecasts onwards. Hermite CUD-5 interpolation also results in a reduction of the amount of negative moisture.

KEYWORDS: Interpolation Numerical weather prediction Semi-Lagrangian model

1. INTRODUCTION

Semazzi and Dekker (1994) and Ritchie and Beaudoin (1994) have examined the accuracy of different components in a semi-Lagrangian model. One of their conclusions is that one can successfully use interpolation of an order lower than fourth to evaluate the velocities at the midpoints (which are necessary for the trajectory research algorithm) and that interpolation of the fourth order applied to estimate the quantities at departure points is quite enough. Nevertheless there are several kinds of fourth-order interpolation. This paper examines the impact of changing the horizontal interpolation for departure-point fields from conventional cubic Lagrangian to Hermite cubic interpolation with fifth-order compact, upwind differencing (CUD-5) estimates of derivatives (Tolstykh 1992) implemented with the approximate inversion of the tridiagonal operators involved.

The study is performed on the basis of the French global spectral variable-resolution NWP model ARPEGE/IFS (Courtier et al. 1992) using a vertical non-interpolating scheme (Ritchie 1986). In this model, the resolution changes gradually from a maximum at the northern pole of the working sphere (which is placed over France) to the minimum at the opposite pole (Courtier and Geleyn 1988).

The Hermite CUD-5 interpolation and its properties are presented in section 2, and section 3 describes the one-dimensional (1-D) and two-dimensional (2-D) linear-advective tests with cubic Lagrangian and Hermite CUD-5 interpolations. Some five-day forecasts computed by ARPEGE/IFS using cubic Lagrangian and Hermite CUD-5 interpolations are described in section 4. The results are discussed in section 5, and an appendix gives a brief description of CUD-5.

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2. Hermite interpolation with CUD-5 derivatives estimates

Tolstykh (1994b) described the application of the CUD-5 suggested by Tolstykh (1992), who presented the Eulerian formulation of problems of atmospheric modelling. It turned out that this method is appropriate when solving the problem of transport of substances which vary rapidly, such as moisture in the atmosphere. However, this method uses a lot of computer time, since it requires accurate time-stepping. The obvious idea was to apply CUD-5 as a part of some interpolation algorithm in the semi-Lagrangian scheme, where the limitation on the CFL number is much weaker and hence the cost of CUD-5 can be compensated by its accuracy.

Smolarkiewicz and Rasch (1991), Smolarkiewicz and Grell (1992) and Smolarkiewicz and Pudykiewicz (1992) proposed using Eulerian advection finite-difference algorithms as interpolation operators in semi-Lagrangian schemes. They show that the interpolation procedure can be reduced either to the solution of multidimensional Eulerian advection problems, or to the set of one-dimensional ones, with constant velocity and an absolute value of the Courant number of less than 1/2. The sample computations given in these papers show that the basic Eulerian finite-difference scheme should be of high order in space and time. In this approach, ‘time’ means the relative displacement from the nearest grid point, so the ‘time’ integration procedures in these schemes can use only two time-levels. (See Smolarkiewicz and Grell (1992) for detailed discussion.) Hence, for the fourth-order interpolation method we need fourth-order two-time-level time-stepping in the corresponding part of the Eulerian transport algorithm (iterative, based on the Lax–Wendroff method, implicit or a combination of them), which is expensive in computer time.

Another way to apply the CUD-5 algorithm is to use it for the first-derivative estimates in the well-known interpolation procedures, such as Hermite or rational cubic. It follows from the results of Williamson and Rasch (1989) that the quality of these interpolations improves with increasing accuracy of the estimate of the derivative.

The proposed interpolation procedure uses CUD-5 approximation to the first derivative estimate in Hermite cubic interpolation (see, for example, Delbourgo and Gregory 1985) which can be written as

\[
\begin{align*}
    f(x) &= \alpha f_{i+1} + \beta f'_{i+1} + \gamma f'_{i} + \epsilon f_{i}, \quad x \in [x_i, x_{i+1}] \\
    \theta &= (x - x_i)/h, \quad h = x_{i+1} - x_i, \\
    \alpha &= (3 - 2\theta)^2, \\
    \beta &= -(1 - \theta)^2, \\
    \gamma &= (1 - \theta)^2\theta, \\
    \epsilon &= (1 - \theta)^2(1 + 2\theta),
\end{align*}
\]  

(1)

where \( f \) is a function with known values at grid points, \( f_i = f(x_i) \) and \( f' \) is its derivative defined on the same grid. The implementation of CUD-5 based on the approximate inversion of the tridiagonal operators involved is described in an appendix.

In the 2-D case we use the tensor-product approach (Williamson and Rasch 1989), which means that first we perform the interpolations in the longitudinal direction and then in the latitudinal direction. We do not need to have the same grid-step for different longitudinal lines and so can use the reduced grid on the sphere (Hortal and Simmons 1991). The tensor-product form also allows us to add, at any moment, the shape-preserving constraints modifying the values of derivatives (Williamson and Rasch 1989). Note that
Figure 1. The ratio of the 'scheme' phase velocity to the exact one, for Hermite CUD-5 interpolation as a function of grid wave-number and CFL number.

Figure 2. The amplification factor of Hermite CUD-5 interpolation as a function of grid wave-number and CFL number.

the longitudinal derivatives can be precomputed whereas to calculate the latitudinal ones we need the values interpolated in longitude.

The amplitude and phase errors of Hermite CUD-5 interpolation as functions of non-dimensional grid wave-number and CFL number are depicted in Figs. 1 and 2 respectively. Only the worst case $0 \leq \text{CFL} \leq 1$ is considered. We can see that for long waves this method is neutral (i.e. the amplification factor $\equiv 1$) and the shortest wave is well damped.

When observing the distribution of the phase error, we may conclude that the phase error is absent except for waves with lengths $2h$ and $3h$. (Recall that the grid wave-number
is defined as $2\pi/kh$, $k = 1, 2, \ldots$.) The phase velocity of the wave $2\pi$ (wave number $\pi$) is equal to zero but here we have a very small amplification factor for most CFL numbers so this error does not affect the solution.

One can also notice that these errors are significantly less than for cubic Lagrangian interpolation (McDonald 1984).

3. Linear-advection tests

Now we consider 1-D linear-advection tests suggested by Carpenter et al. (1990) for Hermite CUD-5 interpolation with the approximate inversion of the tridiagonal operators involved using exact estimates of departure points. For comparison, we also present the results for cubic Lagrangian interpolation. These tests were performed in the periodical interval of 40 points for a triangle distribution and 80 grid points for a Gaussian initial distribution with constant velocity. One hundred time-steps were performed for the triangle distribution, while for the Gaussian distribution 800 time-steps were performed. The results of these two tests for CFL = 0.5 for cubic Lagrangian and for CUD-5 Hermite interpolation are shown in Fig. 3. The same tests were also carried out for CFL = 0.333. 960 steps were performed for the Gaussian distribution and 150 for the triangle one. The results are depicted in Fig. 4. The conservation of the $L_2$ norm of the transported substance in all the cases considered is presented in Table 1.

![Image of graphs showing linear-advection tests](image-url)

Figure 3. One-dimensional advection in the periodical interval for (a) triangular and (b) Gaussian initial distributions, CFL = 0.5: (i) cubic Lagrangian; (ii) Hermite CUD-5 interpolation. Numbers between square brackets show minimum and maximum function values.
Figure 4. One-dimensional advection in the periodical interval for (a) triangular and (b) Gaussian initial distributions. CFL = 0.333: (i) cubic Lagrangian; (ii) Hermite CUD-5 interpolation. Numbers between square brackets show minimum and maximum function values.

| TABLE 1. $L_2$ NORM OF THE TRANSPORTED SUBSTANCE FOR THE ONE-DIMENSIONAL ADVECTON TEST |
|-----------------------------------------------|-----------------------------------------------|
| Gaussian                                      | Triangle                                      |
| CFL = 0.33                                    | CFL = 0.5                                    |
| Initial values                                | CFL = 0.33                                    |
| 2.663                                         | 1.843                                         |
| Cubic Lagrangian                              | CFL = 0.5                                    |
| 2.559                                         | 1.731                                         |
| Hermite CUD-5                                 | 2.654                                         |
|                                               | 1.830                                         |

One can see that the results of these tests are better for Hermite CUD-5 interpolation. (Note that in the case of the Gaussian initial distribution, the only negative value has an amplitude of the order $10^{-5}$ for the CFL numbers tested.) The Hermite CUD-5 demonstrates better behaviour than the Eulerian version of CUD-5 with different third-order time-stepping (Tolstykh 1994a).

A comparison of many Eulerian schemes for the case CFL = 0.5 was given by Carpenter et al. (1990), the best results being for the Piecewise Parabolic Method (PPM). We may note that, with a Gaussian initial distribution, CUD-5 gives a better curve than the PPM, where the maximum decreases to 0.881. For the triangle distribution they are comparable; the PPM decreases the maximum to 0.768 when no steepening procedure is applied and to 0.828 with steepening (when the shape of the distribution is similar to
a rectangle). PPM generates no spurious negative values. In the case of Hermite CUD interpolation we have a maximum value of 0.907 and a most negative value of $-0.013$.

The CFL numbers used in these tests allow us to compare the properties of semi-Lagrangian transport algorithms with Eulerian ones. To compare the properties of both interpolations with CFL numbers greater than one, a test with the triangle initial distribution of unit height in the periodical domain of 60 points was carried out for CFL numbers in the range between 1 and 4. In this case, the initial distribution occupies 13 grid points. The graphs of maximum undershoot, maximum value and relative deviation of $L_2$ norm, after six revolutions around the computational domain, are presented in Figs. 5 to 7 respectively. Again we see that Hermite CUD-5 behaves better than cubic interpolation.

It is interesting to note that in this test the difference between exact implementation of CUD-5 and the symmetrized version based on the approximate inversion of the tridiagonal operators as described in the appendix was found to be limited to the third meaningful digit, thus verifying our approach.

To investigate the shape-preserving property of the resulting interpolation algorithm, the 2-D advection test on the sphere described in detail by Williamson et al. (1992) was also carried out. The velocity field is the solid rotation and the initial distribution is the cosine bell shown by dashed lines in Fig. 8.

The Gaussian grid contained 128 points in longitude and 64 points in latitude. These grid parameters are the same as those used by Williamson and Rasch (1989), Smolarkiewicz and Rasch (1991) and some others.
The results for ‘quasi-cubic’ Lagrangian interpolation implemented in the ARPEGE/IFS model, i.e. with linear interpolation at the southernmost and northernmost longitudinal rows of the stencil (Ritchie et al., 1995), and Hermite CUD-5 with flow along the equator (CFL = 0.5, 256 time-steps) are shown in Fig. 8.

Figure 9 shows the results for the same test, but with flow over the poles. The pictures for the flow over the poles and CFL number 0.333 (384 time-steps) are similar to Fig. 9 and not shown. In the case of Hermite CUD-5 we can see that the shape of the distribution is well preserved without any artificial tool such as quasi-monotonicity and the area covered with small negative values is smaller.

Now we present normalized errors as defined by Williamson et al. (1992) (p. 218) for the maximum value $h_{\text{max}}$, the minimum value $h_{\text{min}}$ and the mean value $M$. Time-averaged and maximum values of these errors are summarized in Table 2. This table shows that the results are free from overshoots and the error measures are better for Hermite CUD-5 interpolation. It is interesting to note that semi-Lagrangian implementation of CUD-5 gives smaller errors than Eulerian implementation of CUD-5 (Tolstykh 1994b).

4. Forecast experiments with different interpolations

To study the sensitivity of the global three-dimensional model to change of horizontal interpolation, a series of experiments was carried out using the French global spectral semi-Lagrangian variable-resolution NWP model ARPEGE/IFS (Courtier et al. 1992) with a vertical non-interpolating scheme (Ritchie 1986). The model employs a three-time-level
Figure 7. The relative deviation of the $L_2$ norm in a one-dimensional advection test for triangular initial distribution after six revolutions for cubic and Hermite CUD-5 interpolation as functions of CFL number, where int(CFL) is the integer part of CFL number, $1 \leq \text{CFL} \leq 4$.

Figure 8. Two-dimensional advection on the sphere with flow along the equator: (a) cubic Lagrangian interpolation; (b) Hermite CUD-5 interpolation. CFL = 0.5. The initial distribution is shown by dashed lines. The maximum of the initial distribution is 1000. Contour interval is 100 with zero contour.
scheme, hybrid vertical coordinate and reduced Gaussian grid. The adiabatic part of the model is as described by Ritchie et al. (1995) but adding a variable resolution feature. In this model, the resolution changes gradually from a maximum at the northern pole of the work sphere (which is placed over France) to the minimum at the opposite pole (Courtier and Geleyn 1988). The map of the stretched hemisphere is given in Fig. 10. The resolution of the model was T119c3.5 with 24 levels in the vertical, which corresponds to a horizontal grid-step of about 25 km at the area of maximum resolution. The time-step chosen was 10 min or slightly more than 3 times that in the Eulerian version of the model. The increase of CPU time per time-step resulting from Hermite CUD-5 interpolation was about 8%.

With each kind of horizontal interpolation, twelve 5-day forecasts were performed beginning on the fifth day of each month from February 1994 to January 1995. The root-mean-square (r.m.s.) difference between the forecast and analysis fields was computed in the region bounded by latitudes 45 and 90 degrees in the northern hemisphere of the work sphere (see Fig. 10). One can see that this region covers most of Europe, with its centre over France. The graph of this difference, averaged over the twelve cases, for the mean sea-level pressure field (MSLP) is depicted in Fig. 11. The graphs for 500 hPa and 850 hPa geopotential demonstrate similar behaviour with a smaller response to the change of interpolation, and are not shown. The scatter plot of MSLP r.m.s. error for the fifth day of integration is given in Fig. 12. For the forecast on 5 November 1994, the reduction of r.m.s. error of the MSLP field because of the change of interpolation is about 27%.
Figure 10. The map of the stretched hemisphere in the ARPEGE/IFS model with stretching factor $c = 3.5$.

Figure 11. The mean r.m.s. error, over 12 cases, of the MSLP field as a function of forecast period.
Figure 12. The r.m.s. error (hPa) of the MSLP field on the fifth day of integration. (The cross marks the mean value and the numbers denote the month of the initial data.)

We can see that, on average, the difference between the forecasts with cubic Lagrangian and Hermite CUD-5 interpolations is negligible within the first three days, but is noticeable on the fifth day, especially in the MSLP field. However, the variable-grid strategy is limited to the short-range forecasts, since, for medium-range forecasts, the high-resolution region will come under the influence of weather systems that at initial time are far away, and hence are poorly resolved in the analysis. One can consider our results as an indication of the probable range at which the change of interpolation can improve the forecasts.

The r.m.s. errors of relative humidity near the surface demonstrate some improvement as a result of change of interpolation, starting from the second day of forecasts; for temperature there is no improvement in the first three days (not shown). For the humidity field, we do not have an error due to linearization of non-linear terms (Semazzi and Dekker 1994) and the response to the change of interpolation is more visible. Hermite CUD-5 interpolation also results in a reduction of the accumulated flux of the negative moisture. The value of this flux on the fifth day of forecasts, averaged over 12 cases, is less by 5% when CUD-5 is used. In the ARPEGE/IFS model in the variable resolution mode, the humidity field is currently treated in spectral space; consequently, we cannot expect any significant reduction of this flux due to the Gibbs phenomenon intrinsic to the spectral method.

5. Discussion

In order to estimate quantities at departure points in the global semi-Lagrangian model, two different horizontal interpolations have been applied. These were cubic Lagrangian interpolation and Hermite interpolation with estimates of derivatives by fifth-order compact upwind differencing (CUD-5). The results of using these two types of interpolation have been compared. In tests with linear advection, the Hermite CUD-5
interpolation had the advantage; in the global spectral variable-resolution NWP model ARPEGE/IFS, its advantage was less straightforward, especially within the first three days of forecasts. Indeed, in a spectral model only waves of length greater than three grid steps are usually included and high frequency harmonics are well damped. Thus the scales on which Hermite CUD-5 interpolation has an advantage over cubic Lagrangian interpolation are partially masked. The introduction of the reduced-resolution grid by Côté and Staniforth (1988) can change the situation. One may expect that in a spectral model with a reduced-resolution grid or in a grid-point model where all grid scales are treated, the difference between two interpolations will be more pronounced. Besides, as was pointed out by Semazzi and Dekker (1994), the error due to interpolation in a semi-Lagrangian model is only one component of the model error; the error due to the semi-implicit scheme (i.e. the linearization of non-linear terms) is also important.

In sum:

- For a short-range weather forecast with a spectral semi-Lagrangian NWP model, the cubic Lagrangian horizontal interpolation applied to estimate the quantities at departure points seems to be sufficient in most cases, except for humidity fields, where the use of Hermite CUD-5 interpolation gives some advantage.
- The change of interpolation results in an improvement of MSLP and 850 hPa geopotential fields on the fifth day of forecasts. However, these improvements are present beyond the range for which the variable-resolution strategy is designed.
- In spectral models with a reduced-resolution grid, and in grid-point semi-Lagrangian models, one may expect the response to the change of interpolation to be more pronounced.

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Appendix

Fifth-order compact upwind differencing

Let us review compact differencing. Consider classical second-order central differencing for calculating the first derivative of a function

$$\left( \frac{\partial \varphi}{\partial x} \right)_i = \frac{\varphi_{i+1} - \varphi_{i-1}}{2h} + O(h^2)$$

and the first of the compact schemes—the fourth order Numerov (1927) scheme

$$\frac{1}{6} \left( \frac{\partial \varphi}{\partial x} \right)_{i-1} + \frac{2}{3} \left( \frac{\partial \varphi}{\partial x} \right)_i + \frac{1}{6} \left( \frac{\partial \varphi}{\partial x} \right)_{i+1} = \frac{\varphi_{i+1} - \varphi_{i-1}}{2h} + O(h^4).$$

This scheme can be also obtained from finite-element considerations on a uniform mesh (Cullen 1973). We can see that the Numerov scheme differs from the usual scheme in averaging derivative values. This averaging is absent in more usual schemes. In general, compact schemes possess the following advantages:
• high-order accuracy on the three-point stencil (Numerov’s scheme is six times more accurate than explicit fourth-order central differencing), and
• small phase and amplitude errors.

The disadvantage of such schemes is the necessity to solve linear systems with constant tridiagonal matrices. Part of this process can be performed in advance.

Fifth-order compact upwind differencing (CUD-5) was introduced by Tolstykh (1992). (See Tolstykh (1994a) for its complete description.) On a grid with constant step \( h \) we construct the fifth-order-accurate operator of the first differentiation using the first-order generalized upwind operator

\[
\Delta(s)f = \left( \Delta_0 + s \Delta_2 \right)f.
\]

\[
\Delta_0 f = f_{i+1} - f_{i-1}, \quad \Delta_2 f = f_{i+1} - 2f_i + f_{i-1}.
\]

One can see that with \( s = \pm 1 \), \((1/2h)\Delta(s)\) reduces to the usual upwind first-order discretization. The CUD-5 operator can be then written as

\[
L_s(u) = \frac{1}{2h} \left\{ \Delta(s) + s R^{-1} Q \left( 1 + \frac{\Delta_2}{12} \right)^{-1} \Delta_2 \right\} u,
\]

where \( |s| = 2/\sqrt{5} \), I is the identity operator,

\[
R = I + \left( \frac{b - s_1}{2} - \frac{s_1}{4} \right) \Delta_0 + \left( \frac{1}{6} - \frac{b}{2} \right) \Delta_2,
\]

\[
Q = I + \left( \frac{a - s_1}{2} - \frac{s_1}{4} \right) \Delta_0 + \left( \frac{1}{6} - \frac{a}{2} \right) \Delta_2,
\]

\[
a = -\frac{2}{15 s_1}, \quad b = \frac{1}{5 s_1}, \quad |s_1| > 5/24.
\]

Parameters \( s \) and \( s_1 \) should change their sign as usual in upwind schemes.

Note that the CUD-5 operator consists of a first-order upwind operator and Padé approximant (Baker and Graves-Morris 1981), annihilating corresponding terms in the Taylor expansion for the truncation error. One can recognize in \((1/h^2)(I + \Delta_2/12)^{-1} \Delta_2\) the fourth-order Padé approximation to the second derivative. CUD-5 possesses small phase-errors and dissipation concentrated on short waves (Fig. 13 of Tolstykh (1994b)).

In the interpolation algorithm set out as Eqs. (1), we can use either the corresponding upwind operators \( L_+ \) with negative \( s \), \( s_1 \) at points \( i \) and \( L_- \) with positive \( s \) and \( s_1 \) at \( i + 1 \) in Eqs. (1), or the symmetrized form of CUD-5 \((1/2)(L_+ + L_-)\) in both points. The second possibility is more attractive from the viewpoints of accuracy, simplicity and computer-memory requirements.

We can see that this approximation to the first spatial derivative requires inversion of the constant tridiagonal operators \((I + \Delta_2/12)\), \( R^+ \) and \( R^- \). The two last operators are used in \((1/2)(R^+)^{-1} Q^+ + (R^-)^{-1} Q^-\), where \( + \) and \( - \) signs correspond to the sign of \( s_1 \). One can apply the idea of the approximate inversion of the tridiagonal operators involved. With an appropriate choice of the parameter \( s_1 \), which also has an effect on the phase error, all matrices possess strong diagonal dominance. This means that their inverse matrices (which are, of course, dense) will also possess this property, which, in turn, enables replacement, without significant loss of accuracy, of the inversion of these matrices by the multiplication by the band matrices composed of the set of main diagonals of their inverse.
The approximate inversion of such matrices is widely used in the finite-element world and has also been used in meteorological applications (Cullen 1973).

While using the approximate inversion, the computations may be optimized by performing all matrix multiplications in advance. The calculation of longitudinal derivative thus involves multiplication by the 14-diagonal skewsymmetric matrix, while the latitudinal derivative estimate uses 10-diagonal matrix multiplication; (some simplifications are performed in this case).

REFERENCES

Baker Jr., G. A. and Graves-Morris, P.

Carpenter, R. L., Droegemeier, K. K., Woodward, P. R. and Hane, C. E.

Côté, J. and Staniforth, A.

Courtier, P. and Geleyn, J.-F.

Courtier, P., Freydidier, C., Geleyn, J.-F., Rabier, F. and Rochas, M.

Cullen, M. J. P.

Delbourgo, R. and Gregory, J. A.

Hortal, M. and Simmons, A.

McDonald, A.

Numerov, B. V.

Ritchie, H.

Ritchie, H., Temperton, C., Simmons, A., Hortal, M., Davies, T., Dent, D. and Hamrud, M.

Ritchie, H. and Beaudoin, C.

Semazzi, F. H. M and Dekker, P.

Smolarkiewicz, P. K. and Grell, G. A.

Smolarkiewicz, P. K. and Pudykiewicz, J.

Smolarkiewicz, P. K. and Rasch, P. J.

Tolstykh, A. I

Tolstykh, M. A.

Williamson, D. L. and Rasch, P. J.


1927 Astronom. Nachr., 230, 359

1986 Eliminating the interpolation associated with the semi-Lagrangian scheme. Mon. Weather Rev., 114, 135–146


1994a High accuracy non-centered compact difference schemes for fluid dynamics applications. World Scientific, Singapore

