Large-eddy simulation and parametrization of the baroclinic boundary-layer

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(Received 27 November 1995; revised 15 February 1996)

Summary

Large-eddy simulations of the non-entraining dry boundary-layer, in which the geostrophic wind varies with height, are described. Results are presented for various turbulence statistics, and the large-eddy model results are used to evaluate the performance of two simple closure-models. It is shown that the performance of these models is not significantly degraded, in either neutral or convective conditions, by the presence of shear in the geostrophic wind.

Keywords: Geostrophic wind-shear Large-eddy model Turbulence closure

1. Introduction

In recent years, the use of large-eddy simulation (LES), to produce turbulence datasets describing the atmospheric boundary-layer, has become increasingly popular. Studies have been made of the boundary layer under a wide range of conditions—free convective (e.g. Mason 1989; Schmidt and Schumann 1989), driven by both shear and buoyancy forcing (e.g. Moeng and Sullivan 1994), neutral (e.g. Mason and Thomson 1992; André 1994) and stable (e.g. Brown et al. 1994; André 1995). Typically, the model flows are driven by an imposed surface heat-flux and a large-scale horizontal pressure-gradient \( \partial P_0/\partial x, \partial P_0/\partial y \), which can be related to a geostrophic wind \((u_g, v_g)\) through

\[
\begin{align*}
    u_g &= -\frac{1}{f\rho} \frac{\partial P_0}{\partial y}, \\
    v_g &= +\frac{1}{f\rho} \frac{\partial P_0}{\partial x}.
\end{align*}
\]

(1)

(2)

Here \( f \) is the Coriolis parameter \((\sim 10^{-4} \text{s}^{-1} \text{ at } 45^\circ \text{N})\) and \( \rho \) is the atmospheric density. Almost invariably, earlier LES studies have used geostrophic winds that were constant with height. This paper presents results from large-eddy simulations of the boundary layer in which the geostrophic wind is allowed to vary with height \((z)\), both in speed and in direction.

Differentiating (1) and (2) with respect to \( z \) and using the hydrostatic approximation \( \partial P_0/\partial z = -\rho g \) where \( g \) is the acceleration due to gravity and the ideal gas law \( P_0 = \rho RT \) where \( R \) is the gas constant and \( T \) the absolute temperature), the thermal-wind equations can be obtained

\[
\begin{align*}
    \frac{\partial u_g}{\partial z} &= -\frac{g}{fT} \frac{\partial T}{\partial y} + \frac{u_g}{T} \frac{\partial T}{\partial z}, \\
    \frac{\partial v_g}{\partial z} &= +\frac{g}{fT} \frac{\partial T}{\partial x} + \frac{v_g}{T} \frac{\partial T}{\partial z}.
\end{align*}
\]

(3)

(4)

As discussed by Arya and Wyngaard (1975), the second terms on the right-hand side of (3) and (4) can usually be neglected; assuming that the temperature gradient is close to the dry adiabatic lapse rate, these terms lead to a fractional change in geostrophic wind speed of less than 4% per kilometre. The shear in the geostrophic wind in the atmosphere is therefore largely determined by the horizontal temperature-gradients. These can be large

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in the vicinity of mesoscale systems such as fronts and orographically induced flows (of order 10 K per 100 km), but are also likely to be significant in situations which appear to be more idealized. For example, a temperature gradient of only 1.5 K per 100 km at 45 N, is consistent with a shear in the geostrophic wind of 0.005 s^-1 (i.e. a change of 5 ms^-1 through a boundary layer 1000 m deep). Temperature gradients of this order of magnitude occur frequently: assuming a surface temperature difference of 50 K between equator and pole, even the mean north-south temperature-gradient must be around 0.5 K per 100 km. It is concluded that significant shear in the geostrophic wind in the atmospheric boundary-layer is commonplace, but its effects remain poorly understood.

Further motivation for an LES study of the baroclinic boundary-layer, in which the geostrophic wind varies with height, is provided by the suggestion that simple closure schemes may perform poorly in such conditions. Hollingsworth (1994) identified a systematic error in the performance of the European Centre for Medium-Range Weather Forecasts (ECMWF) weather-prediction model by comparing composited radiosonde observations and short-range forecasts of wind shear across the boundary layer (between the surface and 850 hPa). In regimes in which the observed wind veered with height (in the northern hemisphere), the model performance was reasonable, but it failed to reproduce backing of the wind with height when this was indicated by the observations. Hollingsworth suggested that this might be caused by the failure of the simple eddy-viscosity closure-scheme used by the boundary-layer parametrization. Hence, in this study, the predictions of two simple closure-models are compared with LES, both in cases in which the LES wind veers with height, and in cases in which the geostrophic shear is strong enough to cause (northern hemisphere) backing with height.

The structure of the remainder of this paper is as follows. First, in section 2, a description is given of the large-eddy model used in the present study. The one-dimensional closure-models tested are described in section 3. Although very simple, they have been chosen as typical of the types of closure commonly used in large-scale climate and numerical weather-prediction models. Section 4 details the simulations performed, and the results are presented in section 5. Finally, conclusions are given in section 6.

Cartesian (x_1 = x, x_2 = y, x_3 = z) coordinates are used, with the z direction normal to the surface, and the velocity vector having components (u_1 = u, u_2 = v, u_3 = w). Angled brackets indicate an average (over the horizontal domain and time, for the results of the large-eddy model), while primes denote perturbations from these averages.

2. THE LARGE-EDDY MODEL

This section details the large-eddy model used in the present study. A filter operation is applied to the Navier–Stokes equations and a solution is sought to the following continuous equations for the resolved velocity, (\bar{u}, \bar{v}, \bar{w}) = (\bar{u}, \bar{v}, \bar{w})$, and potential temperature \( \bar{\theta} \) in a Boussinesq fluid with reference temperature \( \theta_r (= 300 \text{ K}) \) and density \( \rho_r \). Einstein summation notation is used, and the overbars denote resolved quantities.

\[
\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_i} = -\frac{1}{\rho_r} \frac{\partial p}{\partial x_i} - \frac{1}{\rho_r} \frac{\partial P}{\partial x_i} + \delta_{ij} \left( \frac{\partial \bar{\theta}}{\partial x_i} \right) \bar{\theta} - \frac{\partial \tau_{ij}}{\partial x_i} + f \left( \delta_{1j} u_2 - \delta_{2j} u_1 \right)
\]

\[
\frac{\partial \bar{u}_i}{\partial x_i} = 0
\]

\[
\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial (\bar{u}_i \bar{\theta})}{\partial x_i} = - \frac{\partial H_i}{\partial x_i}.
\]
Note that effects of molecular viscosity on the resolved fields are neglected. This assumption that the Reynolds number is infinite is valid because of the scale separation which exists between the filter scale and the length scale for molecular dissipation. \( p \) is the pressure and \( \partial P_0/\partial x_i \) the imposed pressure-gradient (which can be related to an imposed geostrophic wind as described in section 1). Note that in this Boussinesq framework, mention of density will be suppressed (equivalent to choosing units of mass so that the density is unity), so that both energy and stress have units of \((\text{m}^2\text{s}^{-2})\). Similarly the vertical flux of potential temperature \( \langle u'\theta' \rangle \) is referred to as a heat flux (units of \( \text{K m s}^{-1} \)).

\( \tau_{ij} \), the subgrid stress-tensor and \( H_i \), the subgrid heat-flux have to be parametrized. This is usually done deterministically, often using the model of Smagorinsky (1963) which is essentially a three-dimensional mixing-length closure

\[
\tau_{ij} = -\nu \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]

\[
H_i = -\nu_h \frac{\partial \bar{\theta}}{\partial x_i}.
\]

In the present model, the subgrid eddy viscosity \( \nu \) and diffusivity \( \nu_h \) are given by

\[
\nu = \lambda^2 S f_m(\text{Re}_t)
\]

\[
\nu_h = \lambda^2 S f_b(\text{Re}_t),
\]

where

\[
S^2 = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)^2
\]

and \( \lambda \) is a length scale which, in the interior of the flow, has a constant value \( \lambda_0 \) which is related to the filter-scale. Stability dependence is introduced through \( f_m(\text{Re}_t) \) and \( f_b(\text{Re}_t) \) which are functions of local gradient Richardson number (calculated pointwise). For details see Brown et al. (1994).

Close to the surface the length scale of the subgrid motions must decrease so that \( \lambda \) is proportional to the distance from the wall \( z \). The matching relation is written

\[
\frac{1}{\lambda^2} = \frac{1}{\lambda_0^2} + \frac{1}{(\kappa z)^2}
\]

where \( \kappa \) is the von Kármán constant. This match is fairly arbitrary, but Mason and Thomson (1992) found it gave the best results. The backscatter model (see below) is also sensitive to this match, and Mason and Brown (1994) showed that approximately the correct amount of energy was backscattered when using this matching relation.

The boundary conditions used are periodic in the horizontal. The lower boundary is a no-slip wall, with imposed surface heat-flux (characterized by roughness lengths \( z_0 = 0.1 \text{ m} \) for momentum and \( z_{0h} = 0.01 \text{ m} \) for heat). The surface layer is considered to be in local equilibrium between the surface and the lowest grid-point of each column, and Monin–Obukhov similarity is applied pointwise. Provided that this is applied sufficiently close to the surface for the large-scale acceleration terms to have a negligible effect on the stress budget, this approach is justified and results will be independent of the exact height of the lowest grid-points. The top boundary is a rigid lid at height \( h \) where the stress and heat flux are set to zero.
Deterministic models such as the Smagorinsky model have been used extensively and with considerable success in modelling the atmospheric boundary-layer. However, Mason and Thomson (1992) pointed out that previous simulations of the neutral-static-stability boundary-layer had all shown excessive shear in the semi-resolved near-wall region. They argued, following Chasnov (1991), that the subgrid model should not be deterministic as the subgrid motions are influenced, but not fully determined, by resolved motions. The addition of stochastic subgrid stress fluctuations to the model has the effect of transferring some energy from small to large scales, against the turbulent-energy cascade (hence ‘backscatter’) and was shown to lead to a much more realistic simulated velocity-profile. Mason and Brown (1994) performed various tests concerning the implementation of the backscatter process in the model, studying for example the effects of changing the spatial and time scales of the stochastic variations. They concluded that the proposed model of Mason and Thomson (1992) was fairly optimum, and used it to perform a high-resolution simulation of the neutral boundary-layer. The backscatter model was extended to include buoyancy effects by Brown et al. (1994). They showed in simulations of the stable boundary-layer that the use of backscatter led to non-dimensional velocity- and temperature-gradients in much better agreement with observations. Only a very brief description of the model is given here as full details can be found in the papers cited.

The backscatter process is modelled by adding random fluctuations of stress to the standard Smagorinsky model. The space and time scales of these variations (which should be the implied filter-scale and subgrid turbulence time-scale) are dealt with approximately by using a 1:2:1 filter and changing the random numbers every two time-steps. The model ensures that the rates of backscatter of energy and scalar variance are given by

\[
\left( \frac{\partial (\overline{u^2} / 2)}{\partial t} \right)_{sct} = C_B \left( \frac{\lambda_t}{\lambda_0} \right)^5 \epsilon,
\]

\[
\left( \frac{\partial (\overline{\theta^2} / 2)}{\partial t} \right)_{sct} = C_{B\theta} \left( \frac{\lambda_t}{\lambda_0} \right)^5 \epsilon_{\theta},
\]

where \( \epsilon \) is the dissipation, \( \epsilon_{\theta} \) is the dissipation of scalar variance, and \( \lambda_t \) is a typical subgrid length-scale defined through

\[ v = \lambda_t^2 S (1 - Rf_p)^{1/2}, \]

where \( Rf_p \) is the flux Richardson number, calculated pointwise. \( C_B \) and \( C_{B\theta} \) are constants whose magnitude can be estimated using EDQNM theory (Chasnov 1991). However, their exact values depend on the (unknown) filter shape, and the amount of backscatter is sensitive to the precise form of the matching relation so there is a degree of empiricism. Mason and Brown (1994) found that good results were obtained using matching relation (5) along with the values \( C_B = 1.4, C_{B\theta} = 0.45 \) and this has been adopted here. Subgrid estimates of energy and scalar variance are made using Eqs. (13) and (14) of Brown et al. (1994).

The numerical methods used are similar to those discussed by Mason and Callen (1986). The variables are stored on a staggered mesh and the standard model uses Piasek and Williams’s (1970) form of the nonlinear terms which, with the leapfrog time-stepping scheme, ensures conservation of energy and scalar variance. \( S \) and \( v \) are calculated and stored on \( u \)-points to avoid averaging of vertical derivatives.

Unless otherwise stated in the text, all turbulence statistics presented are total quantities, i.e. the sum of the resolved and subgrid contributions. For example, \( \langle u' w' \rangle \) is the sum
of the resolved part \((\bar{u} \bar{w})\) and the subgrid part \((\tau_{13})\). Similarly, \((w'\theta')\) is the sum of \((\bar{w} \bar{\theta})\) and \((H_b)\), \((w'w')\) is the sum of \((\bar{w} \bar{w})\) and two-thirds of the estimate of subgrid energy, and so on.

3. One-dimensional closure models

The starting point for many low-order-closure models is to relate the turbulent shear stress to the mean velocity gradient using an eddy viscosity \(K_m\):

\[
\langle u'w' \rangle = -K_m \frac{\partial \langle u \rangle}{\partial z},
\]

\[
\langle v'w' \rangle = -K_m \frac{\partial \langle v \rangle}{\partial z}.
\]

Similarly the heat flux is related to the temperature gradient through

\[
\langle w'\theta' \rangle = -K_h \frac{\partial \langle \theta \rangle}{\partial z},
\]

where \(K_h\) is the diffusivity. The viscosity and diffusivity have to be expressed in terms of known or calculable quantities. Two different models are considered here—the first is a local mixing-length model, and the second is the non-local model of Holtslag and Boville (1993) in which profiles of the eddy coefficients are prescribed as functions of overall (rather than local) boundary-layer stability.

(a) Mixing-length model

The mixing-length approach writes the viscosity as the product of a turbulent length-scale \(l_m\), and a velocity scale equal to \(l_m\) times the magnitude of the velocity shear vector i.e.

\[
K_m = l_m^2 \left\{ \left( \frac{\partial \langle u \rangle}{\partial z} \right)^2 + \left( \frac{\partial \langle v \rangle}{\partial z} \right)^2 \right\}^{1/2}.
\]

The mixing length, \(l_m\), is prescribed algebraically and will typically be related to height, boundary layer depth and the local stability. Often (e.g. Louis 1979) \(l_m^2\) is written as the product of the square of a basic mixing-length \(\lambda_m\) and a function of stability \(F_m(Ri)\), i.e.

\[
l_m^2 = \lambda_m^2 F_m.
\]

The diffusivity is set equal to the viscosity divided by a Prandtl number \((Pr)\) which is itself a function of stability

\[
K_h = \frac{K_m}{Pr} = K_m \frac{F_h(Ri)}{F_m(Ri)}.
\]

The model tested in this paper uses a scheme of this type. In unstable conditions \((Ri \leq 0)\), the stability functions are set so as to be broadly consistent with observations in the atmospheric surface-layer

\[
F_m = 1 - \frac{g_m Ri}{1 + (g_m/e_m)(-Ri)^{-1/2}}
\]
\[ F_h = 1 - \frac{g_m \, Ri}{1 + (g_m / e_h) \, (-Ri)^{-1/2}}, \]

where \( g_m = 10 \), \( e_m = 4 \) and \( e_h = 25 \). In stable conditions the scheme uses

\[ F_h = F_m = \frac{1}{1 + g_m \, Ri}. \]

The basic length-scale \( \lambda_M \) is written

\[ \frac{1}{\lambda_M} = \frac{1}{\lambda_{M0}} + \frac{1}{\kappa \, z_i}, \]

where \( \lambda_{M0} \) is associated with the size of the turbulent eddies, and is related to the boundary-layer depth \( z_i \) through

\[ \lambda_{M0} = \beta \, z_i, \]

where \( \beta = 0.15 \).

\[(b) \quad \text{Non-local model}\]

The non-local model of Holtslag and Boville (1993) continues to use (6) and (7) to relate the stresses to the mean gradients, but the \( K_m \) profile is prescribed directly as follows

\[ K_m = \kappa \, w_m \, \frac{z - z_i}{z_i}; \quad (10) \]

Here \( w_m \) is a turbulent-velocity scale, which is set equal to \( u_*/\phi_m \) (where \( u_* \) is the friction velocity and \( \phi_m \) is the non-dimensional velocity-gradient) for \( z/z_i \leq 0.1 \), but is constant above this height. A \( K_h \) profile is specified in a similar manner (and a small counter-gradient correction-term is also introduced into the heat-flux parametrization). Thus, momentum fluxes in the boundary-layer interior are no longer affected by local temperature-gradients, but instead depend on bulk measures of boundary-layer stability. This is potentially advantageous, particularly in unstable conditions, but the most appropriate forms for the eddy-coefficient profiles are not well established and it is conceivable that a \( K_m \) profile which performed well with constant geostrophic wind might not produce satisfactory wind-profiles in cases with significant geostrophic shear. This possibility is tested in the present study.

The mean fields near the top of the boundary layer are highly sensitive to the form chosen for the eddy coefficients. For example, the cubic profile of (10) leads to large shears just below the inversion (Nieuwstadt 1983) which are not present in the LES. Accordingly, near the top of the boundary layer, the model of Holtslag and Boville (1993) was set up to use the values of \( K_m \) and \( K_h \) calculated based on local gradients, as in the mixing-length model, if those values were larger than those given by the prescribed profiles.

Note that the tests performed in the present study examine only one aspect of the model of Holtslag and Boville (1993), namely the use of (10) rather than (8) and (9) to provide the eddy viscosity. Their full model also incorporates a counter-gradient correction to the heat-flux parametrization (used here, but not important for the mean-wind and stress profiles), and a method of diagnosing a value of \( z_i \) (which was imposed directly in the present tests).
4. Procedure

The large-eddy simulations described in this section have been designed to permit examination of the effects of geostrophic shear as functions of its magnitude and direction, in both neutral stability and highly convective conditions.

In order to allow direct comparisons with the equilibrium predictions of the closure models, it was decided to perform relatively low-resolution simulations (40 x 40 x 32 points), and to run them for long time. The size of the inertial oscillation was reduced by initializing the large-eddy simulations with profiles obtained from a one-dimensional boundary-layer model, and runs of 100,000 s were then found to be sufficient for the mean winds to reach approximate equilibrium with the geostrophic forcing. Averages of turbulence statistics were taken over the last 10,000 s of each simulation. To prevent excessive inversion-rise during these long runs, it was decided to use stress-free rigid lids to form the boundary layer tops at \( z_t = 1000 \text{ m} \). This use of rigid lids also has the advantage of enabling the effects of varying shear in the geostrophic wind on the performance of the closure models to be assessed, without their being masked by differences in the modelled entrainment rates. However, the absence of entrainment in these simulations means that any interpretation of their results as being directly relevant to the atmospheric boundary-layer (and to the errors pointed out by Hollingsworth (1994)) must be made with care. Tests in neutral conditions, with a constant geostrophic wind, have shown very similar results for the velocity profiles below a stress-free rigid lid and below a more realistic temperature-inversion (Brown 1995). In convective conditions, entrainment effects tend to be more significant, and, consistent with the earlier LES results of Moeng and Sullivan (1994), in simulations of the entraining convective boundary-layer, Brown (1995) found significant shear above \( z/z_t = 0.8 \). Nevertheless, the velocity profiles below this height remained well-mixed and similar to those obtained when using a stress-free rigid lid to represent the inversion. Hence, it is believed that the present simulations also give results relevant to the real atmospheric boundary-layer, and represent a useful first step towards the modelling of more realistic baroclinic cases. Nevertheless, it must be borne in mind that baroclinicity may lead to enhanced shear across the inversion and thus potentially to enhanced entrainment effects, and so this is certainly an area for further study.

All simulations used imposed surface geostrophic winds \((u_0, v_0)\) of \((10, 0) \text{ m s}^{-1}\), and various geostrophic shears \((\partial u/g/\partial z, \partial v/g/\partial z)\) were applied (at angle \(\gamma\) to the surface geostrophic-wind direction). Parallel series of runs were performed in neutral conditions for which the surface heat-flux \((w')_o\) was set to zero, and in convective conditions for which \((w')_o = 0.305 \text{ K m s}^{-1}\). The neutral simulations used domains of horizontal extent \(3 \text{ km} \times 2 \text{ km}\), with \(\lambda_0 = 10 \text{ m}\), while the convective simulations used \(4 \text{ km} \times 4 \text{ km}\), with \(\lambda_0 = 23 \text{ m}\). The vertical grids used were non-uniform, with the lowest points at \(1.8 \text{ m}\) from the surface, and the spacing gradually increasing to \(\sim 20 \text{ m}\) at \(200 \text{ m}\). It was then constant at this value until about \(800 \text{ m}\), above which there was a gradual decrease to \(4 \text{ m}\) at the top of the domain. A summary of all the simulations performed can be found in Table 1.

Equations (3) and (4) indicate that, in thermal-wind balance, a shear in the geostrophic wind is associated with a horizontal temperature-gradient. It could be argued that an extra term should have been incorporated in the equation for \(\partial \theta/\partial t\), allowing for advection of this mean temperature gradient. Such a modification to the model is easily made, and would provide non-uniform heating or cooling as the mean wind speed is a function of height. However, a test simulation confirmed that the effect of such a modification was not significant in convective conditions as the change in heat flux required to balance the non-uniform advective heating was negligible compared to the total flux, i.e. the flux profile remained almost exactly linear. With zero surface-flux (the neutral runs) the effect was
TABLE 1. SUMMARY OF LARGE-EDDY SIMULATIONS, EXTERNAL PARAMETERS

<table>
<thead>
<tr>
<th>Run</th>
<th>$(u'/u'_0)$</th>
<th>$u_{gl}$</th>
<th>$v_{gl}$</th>
<th>$(\partial u_g/\partial z)$</th>
<th>$(\partial v_g/\partial z)$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNLR</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
</tr>
<tr>
<td>N0</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0.005</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>N90</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0.000</td>
<td>0.005</td>
<td>90.0</td>
</tr>
<tr>
<td>N180</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>-0.005</td>
<td>0.000</td>
<td>180.0</td>
</tr>
<tr>
<td>N270</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0.000</td>
<td>-0.005</td>
<td>270.0</td>
</tr>
<tr>
<td>N135</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>-0.010</td>
<td>0.010</td>
<td>135.0</td>
</tr>
<tr>
<td>M134</td>
<td>0.305</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>C0</td>
<td>0.305</td>
<td>10</td>
<td>0</td>
<td>0.005</td>
<td>0.000</td>
<td>0.0</td>
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<tr>
<td>C90</td>
<td>0.305</td>
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<td>0.000</td>
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<tr>
<td>C180</td>
<td>0.305</td>
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<td>270.0</td>
</tr>
<tr>
<td>C135</td>
<td>0.305</td>
<td>10</td>
<td>0</td>
<td>-0.010</td>
<td>0.010</td>
<td>135.0</td>
</tr>
</tbody>
</table>

Notation and abbreviations as in text.

not found to be completely negligible as heat fluxes of maximum magnitude of around $3 \times 10^{-3}$ K m s$^{-1}$ were set up to balance the non-uniform heating/cooling caused by the advection term. A surface flux of this magnitude would result in a non-dimensional stability ($-z_i/L$, where $L$ is the Monin–Obukhov length) of around 0.5. Nevertheless, it was decided to concentrate on runs without this advective term, in order to study the purely dynamical effects of a height dependent geostrophic wind, without complicating stability effects. The closure-model tests were set up with the same forcing and boundary conditions as the large-eddy simulations. They were run to equilibrium using 40 mesh points, although sensitivity studies showed that the results were almost independent of vertical resolution. The sensitivity of the LES results to resolution has not been tested in the present study, although past studies using the same model (in conditions without shear in the geostrophic wind) have shown an encouraging level of resolution independence (e.g. Mason 1994; Brown 1995).

TABLE 2. DEFINITIONS OF ANGLES

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Angle between directions of geostrophic wind shear and surface geostrophic wind imposed in the present simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>Angle between directions of surface wind and surface geostrophic wind</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angle between directions of geostrophic wind shear and surface wind (i.e., $\beta = \gamma - \alpha_0$)</td>
</tr>
<tr>
<td>$\phi_-$</td>
<td>Angle between directions of surface wind and wind just below $z_i$,</td>
</tr>
<tr>
<td>$\phi_+$</td>
<td>Angle between directions of surface wind and geostrophic wind at $z_i$,</td>
</tr>
</tbody>
</table>

Various parameters which will be used in presenting the results are introduced at this point. The baroclinicity of the boundary layer is usually characterized (e.g. Arya and Wyngaard 1975) using $M$ and $\beta$. For constant geostrophic shear, $M$ is simply equal to the magnitude of the geostrophic shear vector normalized by $u_s/z_i$ and $\beta$ is the angle between that vector and the surface wind (positive for shear which is backed relative to surface wind). In the present simulations, it is $\gamma$, the angle between the geostrophic shear vector and the surface geostrophic wind, which is imposed, but these two angles are simply related through $\beta = \gamma - \alpha_0$, where $\alpha_0$ is the surface cross-isobaric flow angle. $\phi_-$ is defined to be the angle between the surface wind and the wind just below $z_i$, and $\phi_+$ is that between the surface wind and the geostrophic wind at $z_i$. Note that it is assumed that the wind above $z_i$
TABLE 3. RESULTS OF LARGE-EDDY SIMULATIONS

<table>
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<tr>
<th>Run</th>
<th>( u_* ) (m s(^{-1}))</th>
<th>(-z_i/L)</th>
<th>( M )</th>
<th>( \gamma ) (deg)</th>
<th>( u_0 ) (deg)</th>
<th>( \beta ) (deg)</th>
<th>( \phi_- ) (deg)</th>
<th>( \phi_+ ) (deg)</th>
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<td>–0.8</td>
<td>–21.8</td>
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</tbody>
</table>

Notation and abbreviations as in text.

is geostrophic, so \( \phi_+ \) gives the total turning across the boundary layer, i.e. turning across the boundary-layer interior (\( = \phi_- \)) plus turning across the inversion. For ease of reference, definitions of all these angles are given in Table 2. All are positive when the first direction is backed relative to the second; for example, a positive value of \( \phi_- \) indicates that the surface wind is backed relative to the geostrophic wind at \( z_i \) or, alternatively, that the wind veers with height across the boundary layer.

5. RESULTS

Key characteristics of the large-eddy-simulation results are shown in Table 3. Values of \( -z_i/L \) are between 8.8 and 26.1 for the convective runs, indicating that buoyancy effects should be expected to dominate over shear effects throughout most of the boundary layer in all of these simulations. The values of non-dimensional baroclinicity \( M \) obtained in the simulations with geostrophic shears of 0.005 s\(^{-1}\), are of order 10. Grant and Whiteford (1987) found that values of \( M \) in flights from the KONTUR experiment in near-neutral conditions over the North Sea were between 2.9 and 13.7 with a mean of 7.5, so it can be concluded that the LES values are by no means excessive. Indeed, in none of these cases is the baroclinicity strong enough to cause a net backing of the wind with height, although \( \phi_+ \) does become close to zero in simulation N90. The two more strongly baroclinic runs, N135 and C135, were specifically designed to make the wind back with height, with the geostrophic wind backing by 90° across the boundary layer. The values of \( M \) in these simulations are around 30, which are comparable with those found in convective conditions over eastern parts of the China Sea during the AMTEX experiment (Lenschow et al. 1980).

Examination of time series of \( u_* \) (not shown) suggests that the mean wind profiles are close to achieving equilibrium after 100 000 s, with variations in \( u_* \) associated with the inertial oscillation estimated to be only around \( \pm 0.01 \) m s\(^{-1}\). Thus, it is believed that a meaningful comparison can be made with the equilibrium predictions of the two closure-models.

The effects of shear in the geostrophic wind on turbulence statistics and closure-model performance in neutral and convective conditions are now examined in turn.

(a) Neutral LES and closure-model results

Figure 1 shows normalized profiles of total stress \( \langle \tau \rangle = (\langle u'w' \rangle^2 + \langle w'w' \rangle^2)^{1/2} \) for
simulations BNLR, N0, N90, N180 and N270. For clarity, the subgrid contributions are not shown separately, but it is noted that the stresses, which must be wholly subgrid at the surface, are typically 70% resolved by $z/z_i = 0.1$ and 85% resolved by $z/z_i = 0.2$. In the limit of $(u_*/f z_i) \to \infty$, the shape of the total-stress profiles is expected to be linear, even with shear in the geostrophic wind. This is because the timescale of the Coriolis effect ($f^{-1}$) is large with respect to the timescale for momentum exchange $z_i/u_*$ (Nieuwstadt 1983). Approximately linear stress-profiles have been observed in shallow boundary-layers (e.g. Brost et al. 1982; Grant 1986), both for cases where $u_*/(f z_i) \approx 10$, and it can be seen that simulation BNLR ($u_*/(f z_i) \approx 4.5$, and a constant geostrophic wind) also has a profile which is roughly linear. However, the LES boundary-layers are too deep to be able to sustain linear stress-profiles when $M \approx 10$.

These changes in the stress profiles (and consistent changes in the mean wind profiles) make the shear-production term in the turbulence kinetic energy (TKE) budget sensitive to the presence of shear in the geostrophic wind. Nevertheless, it is found that this remains almost exactly balanced by the dissipation term in lower and mid boundary-layer in these five simulations, i.e. the transport terms remain small. An approximate balance between production and dissipation is maintained even in the upper boundary-layer (except in simulation N90 in which the shear production becomes very small in this region and upward transport of TKE is not negligible for $z/z_i \approx 0.6$). For example, Fig. 2(a) shows the TKE budget for simulation N0. In contrast, Fig. 2(b) shows that, in the more highly baroclinic simulation N135, the transport term becomes larger than the shear-production term over a limited vertical extent around 0.25$z_i$.

Assuming that the turbulent-transport terms are small in neutral conditions, various diagnostic relations can be deduced from the parametrized budget equation for the second order moment $\langle u'_i u'_j \rangle$ (Grant 1992). These include the result that the stress–energy ratio should be constant with height within the neutral boundary-layer (as long as the transport terms remain small). Figure 3(a) confirms that the diagnosed stress–energy ratio profiles from the moderately baroclinic runs do indeed all give approximately the same constant value (around 0.22) in mid boundary-layer where the variances are approximately 85% resolved. The lower values in the upper part of the boundary layer of simulation N90 are not particularly significant as both stress and turbulence energy are small, but probably occur because of the non-negligible TKE transport terms in this region. Figure 3(b,c,d)
Figure 2. Turbulence kinetic energy budgets. The shear-production (SHR), dissipation (DIS) and total-transport (TRANS) terms are shown, all normalized by \((u_0^3/z_0)\). (a) Simulation N0; (b) simulation N135.

Figure 3. Profiles from neutral simulations N0, N90, N180, N270 and BNLR. (a) Stress-energy ratio; (b) \((\langle u' u' \rangle)/u_*^2\); (c) \((\langle v' v' \rangle)/u_*^2\); (d) \((\langle w' w' \rangle)/u_*^2\).
shows the scaled variance profiles, with the $x$-axis aligned with the surface stress. It can be seen that the introduction of shear in the geostrophic wind causes changes of $O(u_x^2)$ in the magnitudes of the variances in mid boundary-layer. However, the diagnostic relationships (Grant 1992) also predict that the fraction of TKE in each of $\langle u'u' \rangle$, $\langle v'v' \rangle$ and $\langle w'w' \rangle$ should be constant and independent of height, provided that the $x$-axis is aligned with the local stress. Equivalently, as the stress–energy ratio has been shown to be constant, $\langle u'u' \rangle/\langle \tau \rangle$, $\langle v'v' \rangle/\langle \tau \rangle$ and $\langle w'w' \rangle/\langle \tau \rangle$ should all be constant. This is well supported by the LES results (not shown), with the constants equal to 4.2, 2.7 and 2.0 (with a scatter of roughly ±0.2) in mid boundary-layer. These values are broadly consistent with the experimental results of Grant (1986), although the LES result for $\langle u'u' \rangle/\langle \tau \rangle$ is on the lower edge of the data scatter. As discussed by Mason and Brown (1994), this may be the result of the limited size of the domain’s excluding some larger-scale turbulent motions.

It is concluded that the effects of geostrophic shear on the variance profiles in these moderately baroclinic neutral simulations can be explained with reference to the changes in the stress profiles. This is because the basic balance between shear production and dissipation of TKE is not altered. However, Fig. 2(b) shows that with greater non-dimensional shear in the geostrophic wind (N135), the shear production goes to zero at around 0.25$z_i$ and the transport terms are no longer negligible in the TKE budget at this height. Thus, strong baroclinicity may make the use of the diagnostic relationships discussed above inappropriate, even in the lower boundary-layer. For example, the diagnosed profile of stress–energy ratio from N135 (not shown) is similar to those from the moderately baroclinic simulations for $z/z_i \leq 0.1$ and for $z/z_i \geq 0.4$, but shows a minimum of around 0.1 at 0.25$z_i$.

Although the addition of geostrophic shear leads to significant changes in the stress and velocity profiles, it is found that the stress remains closely parallel to the local shear in all the neutral runs (not shown). Hence the mixing length $\lambda_M (= l_M)$ can be diagnosed from the LES results using (6) and (8). Figure 4 shows profiles of $\lambda_M/z_i$ from the five neutral simulations with geostrophic shear (results from BNLR are omitted for clarity). Below around 0.6$z_i$, the length scales diagnosed from the different runs are in surprisingly good agreement, with the differences between them being only slightly larger than the statistical uncertainties associated with any one of them (estimated by comparing results.
from successive averaging periods). The differences in the upper boundary-layer are more marked, but probably not particularly significant as both stress and shear become small. Therefore, although simulation N180 shows slightly low values of $\lambda_M/z_i$ in mid boundary-layer and the results from the highly baroclinic simulation N135 differ somewhat close to the surface (possibly due to the transport terms being non-negligible), it is concluded that the mixing length in the neutral boundary-layer is largely independent of $M$ and $\beta$.

Figure 4 also shows that this length scale is well parametrized using $1/\lambda_M = 1/(0.15z_i) + 1/(\kappa z_i)$.

In view of this finding, it is not surprising that runs of the mixing-length model using this parametrization are highly successful in reproducing the LES wind-profiles—the dashed lines in Fig. 5 show the mixing-length model predictions and in many cases they are indistinguishable from the LES results (solid lines). The dot-dash lines in this plot are the predictions of the non-local model of Holtslag and Boville (1993), and are also in excellent agreement with the large-eddy results. Initially this might seem surprising, since if $\lambda_M$ diagnosed from LES is not affected by geostrophic shear, then diagnosed $K_m$ profiles would be expected to be affected due to changes in wind shear. In fact, Fig. 6, shows that the diagnosed profiles of $[K_m/(u^+,z_i)]^{1/2}$ are still fairly insensitive to the geostrophic shear, although the scatter is rather larger than in the $\lambda_M/z_i$ profiles of Fig. 4. Thus the $K_m$ profile prescribed by Holtslag and Boville (1993) remains in reasonably good agreement with those diagnosed from simulations N0, N90, N180 and N270 in lower and mid boundary-layer and, while the discrepancy is more serious in N135, Fig. 5 shows that it does not lead to major errors in the wind profiles.

Table 4 confirms that $u^+, \alpha_0, \phi_-$ and $\phi_+$ are all well predicted, both by the mixing-length model and by the non-local model, even in the case where the geostrophic shear is strong enough to cause backing of the wind with height. The conclusion is that inclusion of shear in the geostrophic wind does not, on its own, lead to failure of these simple parametrizations.

<table>
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<tr>
<th>Run</th>
<th>Model</th>
<th>$u^+$ (m s$^{-1}$)</th>
<th>$\alpha_0$ (deg)</th>
<th>$\phi_-$ (deg)</th>
<th>$\phi_+$ (deg)</th>
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Notation and abbreviations as in text, except ML denotes results of mixing-length model and HB denotes results of non-local model.
Figure 5. Mean wind profiles from neutral runs. Solid lines, LES; dashed lines, mixing-length model; dot-dash lines, non-local model of Holtslag and Boville (1993). The geostrophic wind components $u_g$ and $v_g$ are shown as dotted lines. Note that in N135 $<u> > <v>$.

(b) Convective LES and closure-model results

The effect of geostrophic shear in convective conditions is rather different from that in neutral conditions. Figure 7 shows the mean wind profiles from the convective simulations. It is clear that these simulations are all convective enough for the profiles to remain well mixed even in the presence of considerable geostrophic shear. Table 3 confirms that the values of $\phi_-$, the amount of wind turning within the boundary layer, stay close to zero in
all cases, although the surface cross-isobaric flow-angles and geostrophic departures just below $z_i$ are affected. The dashed lines on Fig. 7 show the results obtained using the mixing-length model and the dot-dash lines show the predictions of the non-local model of Holtslag and Boville (1993). The performance of eddy-coefficient-based closure-models is expected to become less satisfactory with increasing instability, as the transport terms in the TKE budget become non-negligible and so the fluxes are not necessarily closely related to the local gradients. Thus the tendency of both models to predict too much shear in the boundary-layer interior (as they require a mean gradient in order to sustain a flux) is not surprising. The aim of the present paper is to assess whether these problems are exacerbated by the presence of shear in the geostrophic wind, and it is not clear that the results are systematically poorer in the cases with geostrophic shear (C0, C90, C180, C270 and C135) than in the case with constant geostrophic wind (M134).

Figure 8 shows the variation of $u_*$, $\alpha_0$, $\phi_-$ and $\phi_+$ with $\gamma$, predicted by various models when given the same forcing as the convective large-eddy simulations with shear in the geostrophic wind of 0.005 $\text{s}^{-1}$. The LES results can be seen to be in excellent agreement with those obtained iteratively using the mixed-layer model of Garratt et al. (1982) which assumes that Monin–Obukhov similarity applies up to 0.1$z_i$, with constant velocities above that height. The mixing-length model systematically underestimates $u_*$, with values between 7% and 12% below the LES results, while the non-local model gives values between 4% and 9% below the LES. Note however, that in convective conditions without geostrophic shear, the mixing-length model predicts a $u_*$ value 12% below that in simulation M134, while the non-local model gives $u_*$ 7% below the LES. The discrepancies of a few degrees between the closure model and LES predictions for $\alpha_0$, $\phi_-$ and $\phi_+$ (Fig. 8(b,c,d)) also do not appear to be systematically increased by the presence of shear in the geostrophic wind.

Hollingsworth (1994) found the largest errors in forecasts of wind shear across the boundary layer in cases where a net backing with height was observed, but all the convective cases considered in Fig. 8 show veering ($\phi_+$ positive). The velocity profiles in Fig. 7 suggest that the performance of the closure models in reproducing the LES results in the case where the wind backs with height (C135) is not especially poor, and this is confirmed by the LES
Figure 7. Mean wind profiles from convective runs. Solid lines, LES; dashed lines, mixing-length model; dot-dash lines, non-local model of Holtslag and Boville (1993). The geostrophic wind components \( u_g \) and \( v_g \) are shown as dotted lines. Note that in C135 \( (v) > (u) \).

and simple closure-model results in Table 5. All the predicted values of \( u_* \) are within 7% of the LES result, and the discrepancies in the angles are also still small. In fact, the closure models predict slightly more backing across the boundary layer than LES. Thus it is concluded that the performance of the closure models in convective conditions, although not as good as in neutral conditions, is not significantly degraded by the presence of shear in the geostrophic wind. Furthermore, these models are capable of sustaining backing of the wind with height (at least in these cases without entrainment) and so these results do not explain the findings of Hollingsworth (1994).
Figure 8. Results obtained using various models with \((u' \theta')_0 = 0.305 \text{ K m s}^{-1}\), \(z_i = 1000 \text{ m}\), \(z_0 = 0.1 \text{ m}\), with a surface geostrophic wind speed of 10 m s\(^{-1}\) and a shear in the geostrophic wind of 0.005 s\(^{-1}\) at angle \(\gamma\) to the direction of the surface geostrophic wind. Squares, LES; diamonds, mixing-length model; crosses, non-local model of Holtslag and Boville (1993); solid lines, mixed-layer model of Garratt et al. (1982). (a) \(u_*\) versus \(\gamma\); (b) \(\alpha_0\) versus \(\gamma\); (c) \(\phi_-\) versus \(\gamma\); (d) \(\phi_+\) versus \(\gamma\).

<table>
<thead>
<tr>
<th>Run</th>
<th>Model</th>
<th>(u_*) (m s(^{-1}))</th>
<th>(\alpha_0) (deg)</th>
<th>(\phi_-) (deg)</th>
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Notation and abbreviations as in text, except MIX denotes the mixed-layer model, ML denotes the mixing-length model and HB denotes the non-local model.
Finally, it is noted that, as the wind profiles remain well mixed, buoyancy production of energy continues to dominate over shear production throughout most of the boundary layer, and it is found that turbulence statistics continue to scale convectively. For example, Fig. 9 shows profiles from the convective simulations of the scaled vertical-velocity variance and skewness, and clearly demonstrates that these quantities are not sensitive to magnitude and direction of the shear in the geostrophic wind. Note that this result is often implicitly assumed when non-baroclinic LES results are compared with the data of Lenschow et al. (1980), which were obtained in convective, baroclinic conditions (M between 20 and 40). In such conditions, the effects of geostrophic shear on the turbulence are likely to be significant only close to the surface and close to $z_t$, both regions where shear production may be significant.

6. Conclusions

This paper has presented results from large-eddy simulations of the non-entraining boundary-layer in which the geostrophic wind is a function of height. Various turbulence statistics have been presented, but the most important results have concerned the performance of two simple closure-models. It has been shown that the local mixing-length and Holtslag and Boville (1993) schemes both perform well in neutral conditions, even with considerable shear in the geostrophic wind. In convective conditions, the performance is less good, but does not appear to be significantly worsened by the presence of a shear in the geostrophic wind. Both closures are capable of reproducing backing of the wind with height, both in neutral conditions, when there is significant turning within the boundary layer, and in convective conditions, when the boundary-layer flow is approximately unidirectional and most of the backing is assumed to occur across the capping inversion.

These tests are undoubtedly idealized as they have considered only the steady state and the dry boundary-layer without entrainment. In many ways, these simplifications can be advantageous, as they allow a clean comparison to be made between LES and closure-model results, and allow the effects of changing stability and geostrophic shear to be studied separately in a way which would be almost impossible using experimental data alone. Nevertheless, it is clear that the present results are not directly relevant to complex frontal
situations, and further work is required to assess the importance of entrainment in baroclinic conditions. However, it seems unlikely that the effects of non-stationarity, moisture and entrainment could cause sufficient degradation of the boundary-layer parametrization-scheme performance (relative to the present results) to account for the large systematic error identified by Hollingsworth (1994). One possibility is that the forecast model failed to provide the correct geostrophic forcing in the cases where the observed wind backed with height. It is also noted that when the wind backs with height the air will often be unstable to convective ascent (for example, when cold air moves briskly over relatively warm sea). Thus, another possibility is that the problem is related to the action of the model’s convection scheme and the way in which it interacts with the boundary-layer scheme.

ACKNOWLEDGEMENTS

The author wishes to acknowledge the contributions to this work of Paul Mason and Alan Grant of the Meteorological Office, and of Ian Castro of the University of Surrey.

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