Towards improved radar estimates of surface precipitation rate at long range

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SUMMARY

A method has been developed for estimating surface precipitation rates using reflectivities measured with a radar scanning at several different angles of elevation. In each pixel of a radar image, an idealized profile of the reflectivity factor is constructed. Each profile is defined in terms of three unknowns: the reflectivity in the rain beneath the melting layer, and the slope of the profile of the reflectivity in each of two layers above the melting layer. In these layers above the melting layer, it is necessary to invoke assumptions of horizontal homogeneity in the shape of the profile. Simple parametrizations are used for low-level orographic growth and for the bright band, and the profile is diagnosed with the help of non-radar information. The difference between the idealized and observed profiles is expressed as a single value for an entire radar-image; the difference is minimized by iteration and this avoids using complicated methods of inversion. A simulation experiment was carried out for 15 different cases; in most of them, the incorporation of data from several elevation angles reduced bias errors at long range.

The new method was then used on real radar-data. In typical frontal rainfall, it showed no consistent improvement. Because it avoids using corrections which can otherwise become detrimental at long ranges, however, the new method gave improved results when reflectivity profiles were atypical.

Unresolved spatial variability of the reflectivity profile continues to be a problem.

KEYWORDS: Bright band  Radar reflectivity  Rainfall rate  Remote sensing

1. INTRODUCTION

Correction of radar data from ranges where the lowest elevation scan is above the level of the 0°C isotherm (‘freezing level’) is extremely difficult because of the enormous variability in the shape of reflectivity profile within the ice. The variability arises on a wide range of scales down to that of individual radar pixels (Kitchen and Jackson 1993). In a study of weather-radar performance, Fabry et al. (1992) concluded that beyond the range where the radar horizon intercepts the freezing level, ‘any attempt to obtain quantitative rainfall estimates is futile. Furthermore, this range is an upper bound to the maximum useful range for all scanning strategies . . .’. Radar coverage of the United Kingdom is relatively good by international standards. Nevertheless, the spacing of the radars is such that over most of the land area, the lowest scan is centred more than 1 km above sea level and, for 5% of the area, the beam centre is at more than 3 km altitude. As the freezing level is commonly below 1 km in the winter, corrections at ranges where the radar beam is mainly above the freezing level (referred to here as ‘long range’ corrections) must be attempted if complete radar-coverage is to be maintained. Kitchen et al. (1994), hereafter referred to as KBD, described a correction scheme in which observational data, combined with simple parametrizations of the bright band and low-level orographic growth, were used to construct an idealized profile of reflectivity factor profile at each pixel. A weakness of their method was that, between the ‘freezing’ level and the observed cloud top, a climatological profile shape was assumed. This assumption gave acceptable results in instances of widespread frontal precipitation similar to those for which the climatology was derived, but gave rise to large bias errors at long range if the observed profile deviated significantly from that assumed. There was also the problem that if the top of the precipitation layer was much lower than the observed height of the cloud top, the radar beam started to overshoot the precipitation at shorter ranges than anticipated.

The overall objective of the work described here was to try to improve the performance of the KBD correction-scheme at long range by using radar data from several

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scan-elevations to assist in the diagnosis of the reflectivity profile. Almost all radars in the UK network scan at 4 different elevation-angles from 0.5° up to 2.5° or 4°, with the data output as averages over 5-km pixels. Use of these upper scans in a correction scheme was first proposed by Harrold and Kitchingman (1975). The shape of the vertical profile of reflectivity factor was to be derived directly from the ratio between measurements at just two elevations by inversion of the equations that provide the ratio. Their inversion method was criticized by Carpenter (1983), who proposed an alternative. The method of Smith (1986) was somewhat similar, in that data from two different elevations were used to correct for the bright band only. The theory was simpler than in the Carpenter scheme and it was implemented operationally for a time. More recently Koistinen (1991) and Gray (1991) have developed methods, for use in Finland and New Zealand respectively, in which an average reflectivity-factor profile is derived from all available scans close to the radar. The average profile is then used to calculate corrections which are applied to data from longer ranges. Andrieu and Creutin (1995) have described an improved method of retrieving the profile which can make use of radar data from all ranges in a consistent manner. Their method also has the potential to make use of other information, such as the observed height of the freezing level, through input of an a priori reflectivity-profile.

A potential problem with these methods is that assumptions of spatial homogeneity in profile shape are required in the derivation of the profile and/or in the application of the corrections to data over a wide area. In contrast, the height and intensity of the bright band, and the magnitude of any orographic growth, cause related variations in the profile shape on small scales. Smith (1990) suggested that the most effective correction methods would be those which could resolve as much as possible of this variability, and this is the reasoning behind the pixel-by-pixel parametrizations adopted by KBD. However, methods involving direct profile-retrieval offer advantages at long range because they avoid the assumption of a climatological-average ice-profile shape. Thus, it was considered advantageous to combine the best features of both approaches.

The single variable used to define the idealized reflectivity-profile in the KBD method was the reflectivity factor in the rain beneath the melting layer in each pixel. This was varied iteratively to reconcile the measurement of reflectivity in the lowest elevation-scan with a value calculated from the idealized profile. In the present development, the measurements from a number of scan elevations are utilized to add to the number of profile variables which describe the shape of the reflectivity-factor profile above the freezing level. The difference between the idealized profile and the radar measurements is expressed as a penalty function which is minimized by iteration. The new method is referred to subsequently as the multiple-scan method and is described in more detail in section 2. Initial testing of the method was in a simulation experiment described in section 3. In section 4, the results of a limited verification using operational radar-data are presented. The evaluation did not include cases dominated by orographic enhancement, so this parametrization remains to be tested. Finally, some conclusions and ideas for further development are noted in section 5.

2. The multiple-scan method

(a) The idealized reflectivity-profile

The idealized reflectivity-factor profile is illustrated in Fig. 1. Below the freezing level, the parametrizations for the bright band and orographic growth are similar to those used by KBD. The turning points in the profile are defined by the following expressions.

Let the reflectivity factor and the precipitation rate at the ground be denoted by \( Z_{\text{surf}} \) and \( R_{\text{surf}} \) respectively. \( Z_{\text{rain}} \) is the reflectivity factor in the rain just beneath the melting layer. The orographic enhancement, \( R_{\text{orog}} \), expressed as a rainfall rate, is taken to be an
additive correction and is estimated from a consideration of low-level winds and humidity; see Hill (1983). A standard $Z-R$ relationship of the form $Z = A R^\mu$ is assumed.

$$
R_{\text{surf}} = \left( \frac{Z_{\text{rain}}}{A} \right)^{1/\mu} + R_{\text{orog}} \tag{1}
$$

$$
Z_{\text{surf}} = A \left( \left( \frac{Z_{\text{rain}}}{A} \right)^{1/\mu} + R_{\text{orog}} \right)^\mu.
$$

Any orographic growth is assumed to be linear in $Z$ and to begin at a height of $\Delta h_{\text{orog}}$ (= 1.5 km) above the ground.

Figure 7 in a paper by Davies (1992) shows a scatter plot of values of the area of the bright-band peak, $B$, in units of $\text{mm}^6\text{m}^{-2}$, and $Z_{\text{rain}}$ in $\text{mm}^3\text{m}^{-3}$ on logarithmic scales. A straight line fitted by eye through these data provides the relationship

$$
\log B = 1.42 \log Z_{\text{rain}} + 2.1. \tag{2}
$$

In the idealized profile, the bright band is assumed to be an isosceles triangle of depth $\Delta h_{\text{bb}}$ and area given by Eq. (2). The peak reflectivity in the idealized profile, $Z_{\text{bb}}$, is then given by

$$
Z_{\text{bb}} = Z_{\text{rain}} + \frac{252 Z_{\text{rain}}^{1.42}}{\Delta h_{\text{bb}}}. 
$$

$\Delta h_{\text{bb}}$ was taken to be 500 m.
Estimates of height of the freezing level ($h_f$) in each pixel are provided by a mesoscale numerical forecast model that has a horizontal resolution of 17.5 km.

Examination of reflectivity profiles suggests that they commonly follow an approximately exponential decay of reflectivity factor with height (equivalent to a linear decrease in reflectivity expressed in dBZ) above $h_f$. Variability in the profile shape typically increases with height. In the present method, two layers are defined between $h_f$ and cloud top ($h_c$). The depth of the layer immediately above the freezing level (the lower layer) was chosen to be 2 km. This layer thickness is comparable to the radar beam width at all except extreme range so there is the realistic expectation that the radar data is capable of resolving gradients on this scale. In the lower layer

$$Z(h) = Z_f e^{a_{\text{lower}}(h-h_f)}$$  \hspace{1cm} (3)

and in the upper layer

$$Z(h) = Z_{f+2} e^{a_{\text{upper}}(h-h_{f+2})},$$

where the subscript ‘$f+2$’ denotes the level 2 km above the freezing level. It is assumed that $Z_f = Z_{\text{rain}}$.

Thus, the $Z$ profile at each pixel is specified in terms of three unknowns; $Z_{\text{rain}}$, which varies from pixel to pixel, and two decay constants, $a_{\text{lower}}$ and $a_{\text{upper}}$, which are uniform over the radar image.

The long-range corrections are sensitive to the value of $a_{\text{lower}}$. The value of $a_{\text{upper}}$ only becomes important at long range when the surface precipitation is snow. The upper layer is introduced to provide an extra degree of freedom and avoid the more variable part of the reflectivity profile dominating the determination of the long-range corrections. Also, Harris (1977) noted that a common observation in RHI (Range–Height Indicator) scans is of ice precipitation generated in moist layers which completely evaporates in dry layers beneath. In these conditions, the application of climatologically-based long-range corrections will produce a gross overestimate of the precipitation reaching the surface. By defining two layers above the freezing level, the correction scheme has the potential for diagnosing total or partial evaporation of ice, if the conditions are sufficiently widespread.

The potential of this idealized profile to resolve variations above the freezing level was first examined by analysing measured reflectivity-profiles. RHI scans recorded by the Rutherford Appleton Laboratory’s Chilbolton radar were divided into 5 km range bands and an average profile constructed for each band (see Kitchen and Jackson 1993). Those profiles where the maximum in reflectivity (assumed to be associated with the bright band) was at heights between 1.0 and 2.5 km were selected for further analysis. The lower height-limit was necessary to ensure the reflectivity in the rain beneath the melting layer could be estimated; the upper limit ensured that there were sufficient data from above the freezing level for the analysis. The profile slope in a layer approximately 2 km deep above the freezing level (i.e., $\sim a_{\text{lower}}$) was estimated, based upon the assumptions used to construct the idealized reflectivity-factor profile. The freezing level was taken to be 250 m above the level of maximum reflectivity and a straight line was fitted to the reflectivity factors (expressed in dBZ$_c$) in the layer 350–2350 m above the freezing level. The line was constrained to pass through the freezing level at a point where the reflectivity factor was the same as the minimum in the rain beneath the bright band. Figure 2 shows this construction schematically. The slopes of the fitted lines for those profiles in the period January to March 1988 are plotted as a series in Fig. 3. Note that this is not strictly a time series since the data are not continuous. However, to provide some sort of timescale for the $x$-axis, a continuous series would provide about 45 profiles per hour. This series is typical of the whole data-set (3 years in total) and shows that for much of the time, the point-to-
point scatter in $a_{lower}$ was almost as large as the total range of values. In these conditions, the assumption of horizontal homogeneity is flawed and the idealized profile will not be representative. However, the series also exhibits periods (e.g. profile numbers 230-270 and 350-380) when $a_{lower}$ deviated systematically from the mean value and a correct determination of $a_{lower}$ should result in reduced bias-errors in surface precipitation-rate at long range.

(b) The penalty function

The form of the penalty function was arrived at after some experimentation.

$$J = \sum_{i} \sum_{n} \frac{1}{(n+1)^2} \left[ \log \left( \frac{(Z_{\text{calc}} + \epsilon)}{(Z_{\text{raw}} + \epsilon)} \right) \right]^2,$$

where $J$ is the penalty for a given radar-image and is a sum over all 'wet' pixels in the image, $i$, and over all available radar scans in those pixels, $n$. The different scans are denoted 0 to 3; $n = 0$ being the lowest elevation scan. $Z_{\text{calc}}$ is the calculated reflectivity-factor that the radar scan $n$ would measure in pixel $i$ given the idealized reflectivity-factor.
profile in that pixel. $Z_{raw}$ is the corresponding measured reflectivity-factor. $\epsilon$ is a constant (in reflectivity units).

The reasons behind the particular formulation of $J$ are as follows:

- $Z_{calc}$ and $Z_{raw}$ range over 5 orders of magnitude so the use of log $Z$ in the expression reduces the range of $J$ and assists in the iterative solution. The log ratio is squared so that $J$ has a minimum when all $Z_{calc} = Z_{raw}$;
- most information on the surface precipitation rate is provided by scan 0, particularly where it is beneath the freezing level. The preferred solution is one in which the fit of the idealized reflectivity-profile to the scan 0 measurement is closer than for the other scans. The $1/(n + 1)^2$ term reduces the relative weight given to upper-scan data;
- the constant $\epsilon$ fulfils two purposes. It is necessary to avoid numerical problems when $Z_{calc} = 0$, which arises when the whole depth of a radar beam is entirely above $h_{ct}$. Secondly, precipitation detected by the lowest elevation scan may be undetected by one or more upper scans. In these pixels, the upper-scan data are providing some information, but this information is limited because whether the signal is identically zero or just below the minimum detectable is not known. If, in all such instances, the true signal is assumed to be zero, then in situations where the true signal is closer to the minimum detectable, the result could be an unrealistically large negative value of $a$. The parameter $\epsilon$ reduces the contribution to the total penalty of pixels and scans where the measurement is close to or less than $\epsilon$, compared to the contribution of those where the signal is larger. The choice of $\epsilon$ needs to reflect the detection limit for a particular radar and the probability that, given a zero measurement, the true signal is zero. A reasonable choice of $\epsilon$ was found to be 1.0 mm$^6$m$^{-3}$ $\equiv$ 0 dBZ. A
change in the value of $\epsilon$ of an order of magnitude results in a small, but significant, change in the slopes $a_{\text{lower}}$ and $a_{\text{upper}}$. This formulation is not ideal, and a more formal treatment of errors in both the radar data and the profile parametrizations would be preferable. (See also section 5.)

If there are $N$ wet pixels in a scan 0 radar-image, there are $N + 2$ unknowns and up to $4N$ measurements. However, a large fraction of the upper-scan data are typically zeros and so the number of measurements providing unambiguous information is usually much less than $4N$. The large dimensions of the penalty-minimization problem place restrictions on the choice of the functional form of both the idealized profile and the beam power-profile (see below). The following simple function was fitted to the two-way beam power-profile

$$P(\phi) = \frac{\cos(k\phi) + 1}{2},$$

where $k$ is a constant $= 225$ and $\phi$ is the off-axis beam-angle in radians. The function is valid for values of $\phi$ in the range $-0.014$ to $0.014$. It provides a good fit to the power profile measured by the radar manufacturers for $\phi < 0.006$ rad, whereas it underestimates the power transmitted at angles $> 0.010$ rad (see Fig. 4). The differences are considered to be insignificant in view of the uncertainty of about 0.002 rad in the true scan-elevation.

The solution also requires that the vertical coordinate in the idealized reflectivity-factor profiles is $\phi$ rather than height $h$.

$$h = h_{\text{radar}} + \frac{0.5r^2}{1.33R_0} + r \sin(\gamma + \phi),$$

where $h_{\text{radar}}$ is the height of the radar antenna above sea level, $r$ is the radar range, $R_0$ is the radius of the earth and $\gamma$ is the scan elevation-angle. The factor 1.33 is the usual
empirical correction for the effect of refraction. For the operational radars in the UK network, \( \gamma \approx 0.070 \) rad. Also, we are interested only in layers in the idealized reflectivity profile which are intersected by the radar beam. These are typically a few kilometres deep, so, for ranges greater than about 25 km, relevant values of \( \phi \) are less than 0.2 rad. The maximum error arising from the approximation \( \sin(\gamma + \phi) \approx (\gamma + \phi) \) is therefore approximately 0.003 rad and may be ignored as being similar to the uncertainties in scan elevation mentioned above. Thus we have

\[
\phi = \frac{1}{r}(h - h_{\text{radar}} - \frac{0.376r^2}{R_0} - \gamma).
\]

At very short radar-ranges (<25 km), the approximation may effectively distort the idealized profile where there are large gradients in reflectivity factor in the vertical. However, at these ranges, the radar scans are commonly within rain beneath the freezing level where gradients (and corrections) are relatively small. Above the freezing level, for small \( \gamma \) and \( \phi \) (see above), \( h - h_n \approx r(\phi - \phi_n) \) and we can rewrite Eq. (3) as

\[
Z(\phi) \approx Z_{\text{rain}} e^{a_{\text{lower}}(\phi - \phi_n)}.
\]

\[\text{(c) Minimization of the penalty}\]

As a first guess, \( Z_{\text{rain}} \) was set equal to \( Z_{\text{raw}} \) as measured by the lowest elevation scan; \( a_{\text{lower}} \) and \( a_{\text{upper}} \) were set equal to a typical average value of \(-1.4 \text{ km}^{-1}\).

\( Z_{\text{calc}} \) is calculated for an individual pixel and radar scan by weighting the idealized reflectivity-factor profile by the beam power-profile.

\[
Z_{\text{calc}} = \sum_j \int_a^\beta Z(\phi) \frac{P(\phi)}{\int_a^\beta P(\phi) \, d\phi} \, d\phi,
\]

where the sum is over all the layers in the profile, \( j \), which are intersected by the radar beam. The limits \( \alpha \) and \( \beta \) are the boundaries of the profile layer or of the radar beam, except where the beam is occluded (when the radar horizon is used as the lower integration limit on the top line of Eq. (4)). The integration is in one dimension only whereas, strictly, the integration should be over the two-dimensional beam power-profile. However, for a Gaussian power-profile, the results of the integration will be the same. The differences between the function fitted to the beam power and a Gaussian are small compared to the uncertainty in elevation angle (see Fig. 4) and the simplification is adequate for this purpose.

Minimization of the penalty function was accomplished using standard NAG* routine E04DGE which is suitable for large-scale problems. No bounds can be placed on the variables during the iteration but \( Z_{\text{rain}} \) was limited to a reflectivity factor equivalent to a maximum of \( 10 \times R_{\text{raw}} \) in the lowest elevation scan (\( Z_{\text{raw}} \) expressed as an equivalent rainfall-rate) for output. To improve the scaling of the problem, \( \log Z_{\text{rain}} \) rather than \( Z_{\text{rain}} \) is the variable used in the minimization routine. When applied to operational radar-data, with up to a few thousand wet pixels in an image, no more than 10 iterations are normally sufficient to locate the minimum to within a small tolerance. The NAG routine requires expressions to be provided not only for \( J \) but also its gradient with respect to the unknowns, \( \partial J/\partial \log Z_{\text{rain}} \) (\( N \) values), \( \partial J/\partial a_{\text{lower}} \) and \( \partial J/\partial a_{\text{upper}} \). As \( J \) is a simple function of \( Z_{\text{calc}} \), the task is essentially that of deriving expressions for \( \partial Z_{\text{calc}}/\partial Z_{\text{rain}} \), \( \partial Z_{\text{calc}}/\partial a_{\text{lower}} \) and \( \partial Z_{\text{calc}}/\partial a_{\text{upper}} \). Some key steps involved in their calculation are provided in an appendix.

Once solutions for $Z_{\text{rain}}$, $a_{\text{lower}}$ and $a_{\text{upper}}$ are found, $R_{\text{surf}}$ is obtained from Eq. (1) with $A = 200$, $\mu = 1.6$ ($R$ in units of mm h$^{-1}$ and $Z$ in mm$^3$m$^{-3}$).

3. Evaluation using synthetic data

A simulation method was adopted for initial testing so as to minimize the impact of measurement and sampling errors on the results.

Operational radar-measurements were synthesized from the Chilbolton radar reflectivity-factor profile data-set (see Kitchen and Jackson 1993). Twelve one-hour periods were selected, from occasions when the freezing level was below 2.5 km (to provide an adequate test at long range) and there was negligible orographic enhancement (for simplicity). The periods were also chosen to provide a range of profile shapes above the freezing level. The average profiles for each period are plotted in Fig. 5.

The operational radar was assumed to be at a height of 100 m and to have its horizon at 0.0° elevation. Data for three scans at 0.5°, 1.0° and 1.5° of elevation were synthesized. From each profile, radar-measured rainfall-rates ($R_{\text{raw}}$) at 8 different ranges (25, 50, 75 . . . 200 km) were calculated. At ranges up to 100 km, the minimum detectable signal was taken to be 7.2 mm$^3$m$^{-3}$, increasing with the square of the range beyond 100 km. This corresponds to a conservative estimate of the operational radar’s detection-capability (Kitchen and Jackson 1993). If $R_{\text{raw}}$ was calculated to be less than the minimum detectable, then $R_{\text{raw}}$ was set to zero. A series of hour-long Chilbolton profiles contained about 50 profiles and thus radar data for about 400 pixels were created. These pixels were assumed to form the ‘wet’ pixels in a single instantaneous operational radar-image. The Chilbolton profiles were examined to estimate a typical height of the freezing level which was then assumed to be constant over the image. In reality, variations in the freezing-level height were observed during the hour-long periods but the magnitude of the variations was no larger than the uncertainty in estimating the height on an operational basis ($\sim$ 200 m). $h_{\text{ef}}$ was taken to be the typical maximum height for which the Chilbolton radar detected precipitation during each period. The true surface precipitation-rate $R_{\text{truth}}$ was estimated from the Chilbolton reflectivity-measurements at a height of 500 m above sea level, assuming the same $Z$–$R$ relationship as in the correction scheme.

The multiple-scan method was used to produce estimates of $R_{\text{surf}}$ from the synthesized image.

The comparisons between $R_{\text{surf}}$ and $R_{\text{truth}}$ were grouped according to the radar range. To act as control and to measure the impact of the data from higher-elevation scans, a simpler correction-method was also applied to the synthetic data. This used the same bright-band parametrization, but only radar data from the lowest elevation scan were processed. Above the freezing level, a single layer was assumed in which $a$ was set to a typical climatological value of $-1.4$ km$^{-1}$ $\equiv -6$ dB km$^{-1}$.

In Fig. 6, ($R_{\text{surf}} - R_{\text{truth}}$) is plotted as a function of radar range for both the control and the multiple-scan methods. ($R_{\text{raw}} - R_{\text{truth}}$) is also plotted for comparison. The results for ranges where the radar beam is mainly below the freezing level are very similar for both methods. The bright-band parametrization was mainly successful in reducing bias errors at short and moderate range. The multiple-scan scheme reduced the mean error at the range of the bright-band peak in all but one case (10 April 1989). The mean absolute bias at 75 km range was reduced by 80% compared to the raw data and the mean root-mean-square (r.m.s.) error was reduced by 64%.

In some cases (e.g. 19 November 1987), the reflectivity factor decreased only slowly with height above the freezing level and only very small range-corrections were required, even at extreme range. Assumption of a fixed profile-slope caused the control method to
Figure 5. Hourly-mean profiles of reflectivity factor for the first 12 cases used in the simulation experiment. The values of the reflectivity factor in mm$^3$m$^{-3}$ from each profile were averaged together but converted to dBZ for plotting.
Figure 6. Bias errors as a function of radar range from the simulation experiment. \( (R_{\text{surf}} - R_{\text{truth}}) \) from the multiple-scan method is shown by the closed circles and from the control method by the open circles. \( (R_{\text{raw}} - R_{\text{truth}}) \) (which is also the error in the first guess) is shown by the crosses and solid line for comparison.
overestimate surface precipitation rates. By contrast, for the period 0000–0100 13 January 1988 UTC and 200 km range, the required correction was about a factor of 20, which the control method underestimated. In most cases the multiple-scan scheme produced a worthwhile reduction in both the bias and r.m.s. errors at long range compared to the control method.

The multiple-scan method produced poor results in only two cases, probably because of large variations in the shape of the Chilbolton profile during the hour. Some failures are to be expected, given the scatter in Fig. 3 and the method’s assumption of horizontal homogeneity. Problems may also be expected when the error in the height of the freezing level is large compared to the depth of the melting layer. The large gradients in the vertical profile of reflectivity factor associated with the bright band are then misinterpreted as being associated with the profile above the freezing level.

Overall, the mean absolute bias-error at 200 km range from the multiple-scan method was 64% lower than that for the raw radar-data and 35% lower than for the control method.

The final penalty per pixel \((J/N)\) was typically about an order of magnitude smaller than the initial penalty and is a measure of the fit of the idealized reflectivity-factor profiles to the radar data. Figure 7 is a plot of r.m.s. \(\log(R_{\text{surr}}/R_{\text{truth}})\) against \((J/N)\). The correlation suggests that the penalty per pixel may be a predictor of the error in surface precipitation-rate estimates and may have a useful quality-control function (see section 5).

To test the performance of the scheme on occasions of evaporation, three additional hour-long periods were selected. The average profiles of reflectivity factor are plotted in Fig. 8. In the first case (1700–1800 14 October 1987 UTC), the bright-band parametrization overestimated its intensity and the corrections were relatively ineffective as a result (Fig. 9). An outstanding result was achieved for the other occasions of evaporation, and, over all three, the mean absolute bias-error was reduced by 48% compared to the raw data, whereas the control method increased the error by over 200%.

The results of the simulation experiment suggest significant benefit from the multiple-scan method in cases where the reflectivity-factor profile differs markedly from the climatological norm. Some tests on real radar-data were made, so as to validate this conclusion.
4. Evaluation of the Method Applied to Operational Radar-data

Periods from four days were specially selected during which the climatologically-based long-range corrections in the KBD method produced poor results. Figure 10 shows time series of \( a_{\text{lower}} \) produced by the multiple-scan correction scheme applied to operational radar data for these periods. Point-to-point scatter was confined to order 0.1 which suggests that the solution was generally stable. For 15 October 1987, the multiple-scan method was applied to data from two adjacent radars in the network (Clee Hill and Chenies) and the values of \( a_{\text{lower}} \) compared with estimates derived directly from Chilbolton radar RHI scans. The transition from evaporation to rapid growth of precipitation just above the freezing level was successfully diagnosed by the correction scheme. On 13 November 1991 there was vigorous convection over Wales and central England. Profiles of reflectivity factor in convection commonly show a large gradient in reflectivity factor near the top of the precipitation layer and relatively smaller gradients close to the freezing level. (See, for example, Illingworth et al. (1987).) This is reflected in the low values of \( a_{\text{lower}} \) diagnosed from data from the Dyfed radar and shown in Fig. 10. On 8 and 9 January 1992, there was
Figure 10. Time series of $a_{\text{corr}}$ produced by the correction scheme applied to operational radar-data from selected case studies (solid lines). For 15 October 1987, the solid line is from analysis of Clee Hill radar data whereas the pecked line is for the Chenies radar. The circled crosses are average values of $a_{\text{corr}}$ estimated directly from the hourly-average profiles of reflectivity from the Chilbolton radar.
moderate frontal rainfall over Wales. The Dyfed-radar images suggested that the required range-correction generally decreased with time, with a rapid change around 0400 9 January 1992. However, this trend was only weakly reflected in the values of $a_{lower}$.

In Table 1, estimates of surface precipitation-rates $R_{surf}$ at ranges greater than 100 km are compared with ground truth $R_{truth}$. Results obtained using the multiple-scan method are shown alongside those obtained using the KBD method (which serves as control) and raw data received from the radar site. The 'raw' data include a correction for the effects of range, based on long-term comparisons with measurements by rain gauge. The multiple-scan method reduced the average bias-error by 38% and the r.m.s. errors by 10%, although benefit was not evident in every case. By contrast, the KBD scheme produced increases of 100 and 80% respectively (dominated by the 15 October case). Given the similarity between the bright-band and orographic-growth parametrizations in the multiple-scan and KBD methods, it is not surprising that the comparison results at ranges $<100$ km were comparable for these methods and are therefore not shown in detail here. The multiple-scan method reduced bias errors by 63% and r.m.s. errors by 65% on average, compared to 51% and 58% respectively for the KBD method.

Next, the corrections were applied to data from seven occasions of more typical, widespread, winter frontal rainfall recorded by the Warden Hill radar and previously used in the evaluation of the KBD method. Kitchen and Brown (1992) found that this radar has a lower effective minimum detectable signal than the other radars in the UK network. It was therefore considered that a value of $\epsilon = 0.1$ would be more appropriate for this radar.

Time series of the values of $a_{lower}$ (not shown) showed that values were mainly in the range $-1$ to $-2$ km$^{-1}$; i.e. close to the climatological average. Hourly gauge-comparisons were again used to assess the performance of the corrections at long range. In the 6 cases which yielded adequate comparison data, the multi-scan method reduced the average bias-error by 29% and average r.m.s. difference by 13% compared to the raw data, whereas the KBD method produced an increase of 1% in the bias and a reduction of 12% in r.m.s. differences. Verification in these cases was also provided by near-surface measurements of reflectivity from the Chilbolton radar. (See KBD for details of the comparison method.) At ranges between 40 and 125 km from Warden Hill, where the corrections were dominated by the bright band, the multi-scan and KBD methods produced almost identical results, both reducing the average r.m.s. difference by 53%.

It is interesting to note that values of $J/N$ from tests on operational radar-data were typically a few times larger than in the simulation experiments. This suggests that the level of spatial variability in profile shape incorporated in the simulation method may have underestimated the true level. The penalty may also have been increased by measurement uncertainties (e.g. in the radar horizon and scan elevation-angles).

5. Conclusions

To meet a requirement for quantitative estimates of surface precipitation-rate over the UK in wintertime, an attempt must be made to correct radar data from ranges where the lowest elevation scan is mainly above the freezing level. A method has been devised in which radar data from several scan-elevations are used in an attempt to improve long-range corrections. The method is similar to that of Andrieu and Creutin (1995) in two important respects. It can make use of other sources of meteorological data, and the difference between an idealized profile and the radar observations is minimized by iteration. Unlike other methods, the reflectivity-factor profile below the freezing level is constrained by simple pixel-scale parametrizations of the bright band and of orographic growth.
<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Synoptic conditions</th>
<th>Melting level (km)</th>
<th>No.</th>
<th>$R_{\text{truth}}$ (mm h$^{-1}$)</th>
<th>Raw $R_{\text{surf}}$ (mm h$^{-1}$)</th>
<th>KBD $R_{\text{surf}}$ (mm h$^{-1}$)</th>
<th>M-S $R_{\text{surf}}$ (mm h$^{-1}$)</th>
<th>r.m.s. $R_{\text{surf}} - R_{\text{truth}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 Oct 87</td>
<td>1000–1100</td>
<td>evaporation</td>
<td>1.5</td>
<td>56</td>
<td>0.01</td>
<td>-0.52</td>
<td>-2.88</td>
<td>-0.09</td>
<td>0.54</td>
</tr>
<tr>
<td>15 Oct 87</td>
<td>1100–1200</td>
<td>partial evap'n</td>
<td>2.0</td>
<td>81</td>
<td>0.20</td>
<td>-0.39</td>
<td>-3.12</td>
<td>-0.16</td>
<td>0.48</td>
</tr>
<tr>
<td>13 Nov 91</td>
<td>0000–1200</td>
<td>showers</td>
<td>0.8</td>
<td>53</td>
<td>0.64</td>
<td>-0.25</td>
<td>0.21</td>
<td>-0.33</td>
<td>0.67</td>
</tr>
<tr>
<td>08 Jan 92</td>
<td>1800–2400</td>
<td>warm front</td>
<td>1.5</td>
<td>37</td>
<td>2.28</td>
<td>-1.22</td>
<td>1.06</td>
<td>-1.26</td>
<td>1.94</td>
</tr>
<tr>
<td>09 Jan 92</td>
<td>0000–0400</td>
<td>occlusion</td>
<td>1.5</td>
<td>15</td>
<td>2.31</td>
<td>-0.81</td>
<td>-0.33</td>
<td>-0.29</td>
<td>1.17</td>
</tr>
<tr>
<td>09 Jan 92</td>
<td>0400–0900</td>
<td>occlusion</td>
<td>0.5</td>
<td>20</td>
<td>3.89</td>
<td>-0.56</td>
<td>1.08</td>
<td>-0.19</td>
<td>2.79</td>
</tr>
</tbody>
</table>

No. denotes the number of comparisons.

Three estimates of $R_{\text{surf}}$ were compared: Raw (as received from the radar site); KBD (from the KBD scheme); M-S (using the new multiple-scan method. Other symbols are defined in the text.

Verification for 15 October 1987 was against instantaneous near-surface rates derived from the Chilbolton radar. In all other instances, the comparison was between hourly gauge-accumulations and integrations of 5-minute radar-data.

A single difference of more than 10 mm h$^{-1}$ was eliminated from the statistics of the comparison for 9 January 1992.
Despite the large scale of the minimization problem, with up to a few thousand variables, there were no problems in arriving at a satisfactory solution.

A simulation experiment demonstrated that the method had considerable potential to reduce bias errors at long range. Tests using operational radar-data showed less benefit, but in some cases where the reflectivity-factor profile above the freezing level deviated markedly from the climatological average, the use of the higher elevation-angle data produced better results than a method using data from a single elevation-angle. This was achieved without sacrificing performance at shorter ranges where the simpler method performed adequately.

Future development should be focused on resolving the spatial variability between profiles. One approach could be to apply the method to all the data initially, and then examine the contribution to the penalty \( J \) from each pixel. If some pixels were found to be contributing disproportionately to the penalty, this could be indicative of systematic differences in the reflectivity-factor profile. These pixels could then be placed in a separate correction domain and the method reapplied to the two regions. It may be imagined that this approach would be advantageous where, for example, deep convective cells were embedded within more widespread frontal rainfall.

The magnitude of \( J \) in individual pixels may also be of use in quality control, and Eyre et al. (1993) describe such an application relating to satellite temperature-soundings. The value of the penalty could be given further quantitative significance by using a more standard formulation of the penalty function, in which the penalty is normalized by the error variance

\[
J = \sum \frac{X^2}{\sigma_X^2},
\]

where, in this case,

\[
X = \sum_i \sum_n \log \frac{Z_{\text{calc}}}{Z_{\text{raw}}}.
\]

The sums are over \( i \) wet pixels in the radar image and \( n \) radar scans. The error variances \( \sigma_X^2 \) include contributions from errors in the radar measurements, the reflectivity-profile specification and in the various parametrizations, all of which have to be estimated. This penalty formulation has not yet been adopted because, despite attempts to tune the error variances, in the simulation experiment it has so far failed to deliver better results than the empirical version.

When there is small-scale convection, there may be few wet pixels and insufficient information to derive stable, representative values of \( a_{\text{lower}} \). In such circumstances, it may be preferable to revert to the simpler KBD method.

**Acknowledgements**

The idea of using a penalty minimization method to estimate surface precipitation-rates from radar data from several elevation-angles was suggested by both Cesar Beneti (Reading University) and Bruce Macpherson (Meteorological Office) independently. Bruce Macpherson also gave advice on the solution method. Rod Brown of the Meteorological Office provided general advice and direction to the work. The Chilbolton radar data were supplied by the Radio Communications Research Unit of the Rutherford Appleton Laboratory. John Goddard and Kevin Morgan of the Unit gave advice on its processing. Some of the hourly gauge-data were provided by the Thames region of the National Rivers Authority.
APPENDIX

To compute $Z_{\text{calc}}$ from Eq. (4), expressions for $\int_\phi^\beta Z(\phi) P(\phi) \, d\phi$ for each layer in the idealized profile are required. Below the freezing level, where the reflectivity factor is assumed to change linearly with height (Fig. 1) and for small values of $(\gamma + \phi)$, the reflectivity factor will also be approximately linear in $\phi$. For these layers, the expressions are therefore relatively simple and are not shown here.

For the layer immediately above the freezing level

$$\int Z(\phi) P(\phi) \, d\phi = Z_{\text{rain}} \int e^{a_{\text{lower}}r(\phi - \phi_H)} P(\phi) \, d\phi$$

$$= \frac{Z_{\text{rain}} \ e^{-a_{\text{lower}}r\phi_H}}{2} \int e^{a_{\text{lower}}r\phi} \cos(k\phi) \, d\phi + \frac{Z_{\text{rain}} \ e^{-a_{\text{lower}}r\phi_H}}{2} \int e^{a_{\text{lower}}r\phi} \, d\phi.$$

$\int e^{C\phi} \cos(c\phi) \, d\phi$, where $c$ and $C$ are constants, is a standard integral with the solution

$$\frac{e^{C\phi}}{C^2 + c^2} (C \cos(c\phi) + c \sin(c\phi)).$$

Therefore,

$$Z_{\text{rain}} \int e^{a_{\text{lower}}r(\phi - \phi_H)} P(\phi) \, d\phi = \frac{Z_{\text{rain}} \ e^{-a_{\text{lower}}r\phi_H}}{2} \left[ \frac{e^{a_{\text{lower}}r\phi}}{(a_{\text{lower}}r)^2 + k^2} (a_{\text{lower}}r \cos(k\phi) + k \sin(k\phi)) \right]_a^\beta$$

$$+ \frac{Z_{\text{rain}} \ e^{-a_{\text{lower}}r\phi_H}}{2a_{\text{lower}}r} \left[ e^{a_{\text{lower}}r\phi} \right]_a^\beta.$$

(A.1)

A similar expression can be derived for the profile layer up to cloud top. The expression for the contribution of each layer to $Z_{\text{calc}}$ is then differentiated to find $\frac{\partial Z_{\text{calc}}}{\partial Z_{\text{rain}}}$.

$\frac{\partial Z_{\text{calc}}}{\partial a_{\text{lower}}}$ is found by differentiating the left-hand side of Eq. (A.1):

$$\frac{\partial}{\partial a_{\text{lower}}} \left( Z_{\text{rain}} \int e^{a_{\text{lower}}r(\phi - \phi_H)} P(\phi) \, d\phi \right) = \frac{\partial Z_{\text{rain}}}{\partial a_{\text{lower}}} \int e^{a_{\text{lower}}r(\phi - \phi_H)} P(\phi) \, d\phi$$

$$+ Z_{\text{rain}} \frac{\partial}{\partial a_{\text{lower}}} \left( \int e^{a_{\text{lower}}r(\phi - \phi_H)} P(\phi) \, d\phi \right).$$

(A.2)

An expression for the second term on the right-hand side of Eq. (A.2) is provided by differentiation of the terms on the right-hand side of Eq. (A.1):

$$Z_{\text{rain}} \frac{\partial}{\partial a_{\text{lower}}} \left( \int e^{a_{\text{lower}}r(\phi - \phi_H)} P(\phi) \, d\phi \right) = \frac{Z_{\text{rain}}}{2} \left[ e^{a_{\text{lower}}r(\phi - \phi_H)} \left( \frac{r}{(a_{\text{lower}}r)^2 + k^2} \right) \cos(k\phi) \right.$$

$$+ e^{a_{\text{lower}}r(\phi - \phi_H)} \left( -\frac{2a_{\text{lower}}r^2}{((a_{\text{lower}}r)^2 + k^2)^2} \right)$$

$$\times (a_{\text{lower}}r \cos(k\phi) + k \sin(k\phi))$$

$$\left. + r(\phi - \phi_H) e^{a_{\text{lower}}r(\phi - \phi_H)} \left( \frac{1}{(a_{\text{lower}}r)^2 + k^2} \right) \right].$$
\[
\begin{align*}
&\times (a_{\text{lower}} r \cos(k\phi) + k \sin(k\phi)) \\
&+ e^{\phi_{\text{lower}} r} \left(\frac{-1}{a_{\text{lower}}^2 r}\right) \\
&+ \frac{r (\phi - \phi_0)}{a_{\text{lower}} r} e^{\phi_{\text{lower}} r} \left(\phi - \phi_0\right)^\beta
\end{align*}
\]

An analogous equation to (A.3) is used to calculate \(\partial Z_{\text{calc}} / \partial a_{\text{upper}}\). Note that there is a contribution to \(\partial Z_{\text{calc}} / \partial a_{\text{lower}}\) from the uppermost layer, as the reflectivity factor at the bottom of the upper layer is a function of \(a_{\text{lower}}\).

\[
Z_{\beta+2} = Z_{\text{rain}} e^{\phi_{\text{lower}} r} \left(\phi_{\beta+2} - \phi_0\right).
\]

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