A theory for vortex Rossby-waves and its application to spiral bands and intensity changes in hurricanes

By MICHAEL T. MONTGOMERY* and RANDALL J. KALLENBACH
Colorado State University, USA

(Received 12 June 1995; revised 26 February 1996)

SUMMARY

In this paper we examine further the physics of vortex axisymmetrization, with the goal of elucidating the dynamics of outward-propagating spiral bands in hurricanes. The basic physics is illustrated most simply for stable vorticity monopoles on an $f$-plane. Unlike the dynamics of sheared disturbances in rectilinear shear flow, axisymmetrizing disturbances on a vortex are accompanied by outward-propagating vortex Rossby-waves whose restoring mechanism is associated with the radial gradient of storm vorticity. Expressions for both phase and group velocities are derived and verified; they confirm earlier speculations on the existence of vortex Rossby-waves in hurricanes. Effects of radially propagating vortex Rossby-waves on the mean vortex are also analysed. In conjunction with sustained injection of vorticity near the radius of maximum winds, these results reveal a new mechanism of vortex intensification. The basic theory is then applied to a hurricane-like vortex in a shallow-water asymmetric-balance model. The wave mechanics developed here shows promise in elucidating basic mechanisms of hurricane evolution and structure changes, such as the formation of secondary eye-walls. Radar observations possessing adequate temporal resolution are consistent with the predictions of this work, though more refined observations are needed to quantify further the impact of mesoscale banded disturbances on the evolution of the hurricane vortex.

KEYWORDS: Hurricane dynamics  Shallow-water model  Spiral bands  Vortex Rossby-waves

1. INTRODUCTION

Radar observations in hurricanes suggest the existence of outward-propagating spiral bands emanating near the eye-wall and traversing distances of the order of a hundred kilometres or more. The bands are believed to be caused by convective forcing or frictional convergence. Their radial scale and radial speed of propagation are observed to be of the order of 10 km and 4 m s$^{-1}$ respectively (Tuttle and Gall 1995). In addition to their radial propagation, spiral bands are often observed to move azimuthally at speeds less than the local mean tangential wind. These inner bands are distinct from the quasi-stationary convective bands associated with the eye-wall cloud and outer rain-bands (e.g., Shapiro 1983; Willoughby et al. 1984). Despite our knowledge of the thermodynamic and convective structure of both inner and outer bands (Barnes et al. 1983; Powell 1990a), understanding of their dynamical significance remains incomplete. In this paper we focus on the problem of outward-propagating bands in vortex flows. The bands are described as outward-propagating vortex Rossby-waves on an $f$-plane that are progressively sheared by the differential rotation of the vortex. Before presenting the theory, and to place the present work in broader context, previous work on outward-propagating bands is reviewed.

Two main theories have been proposed to explain the physics of outward-propagating hurricane bands. The first describes these features as gravity–inertia waves (Abdullah 1966; Kurihara and Tuleya 1974; Kurihara 1976), while the second describes them as sheared potential-vorticity (PV) disturbances (Guinn and Schubert 1993; henceforth GS). Given the fine-scale structure and outward propagation of the inner bands, one might naturally suspect them to be gravity–inertia waves. However, since both inner and outer bands have been observed to move more slowly than the local mean tangential flow (Senn and Hiser 1959; Powell 1990b; May 1995), their basic dynamics may instead lie rooted in the ‘slow-manifold’ associated with the advective (i.e. ‘low-frequency’) component of the flow. Spiral bands were qualitatively described as vortex Rossby-waves by MacDonald (1968).

* Corresponding author: Department of Atmospheric Science, Colorado State University, Fort Collins, CO 80523-1371, USA.
Unlike planetary Rossby-waves, which owe their existence to the meridional gradient of planetary vorticity, vortex Rossby-waves were postulated to exist on the radial gradient of storm vorticity. Following this theme, GS presented perhaps the most penetrating investigation of hurricane spiral-bands to date. In the context of PV dynamics, GS demonstrated that a shallow-water primitive-equation model evolves banded features with minimal projection on the gravitational linear-wave manifold. Although the GS study represents a milestone in our understanding of hurricane spiral-bands, a variety of questions concerning their dynamical characteristics remains unanswered. Are there fine-scale outward-propagating waves that accompany 'the symmetrization process'? If so, can one then predict their principal characteristics, such as phase and group velocity, eddy-momentum flux, and wave–mean-flow interaction?

In this work we extend previous studies of vortex axisymmetrization by developing a wave mechanics that unifies the physics of barotropic vortex axisymmetrization and vortex Rossby-wave propagation in rapidly rotating vortices. The theory developed herein extends previous work of Smith and Montgomery (1995—hereafter SM) and GS. The theory provides simple formulae for phase and group velocities that may be used in distinguishing vortex Rossby-waves from gravity–inertia waves in observational data. The theory also reveals a wave–mean-flow interaction mechanism for vortex intensification. In hurricanes, this mechanism may play an important role in changes in structure, such as formation of a secondary eye-wall. Further applications of these results are discussed in section 4.

2. VORTEX ROSSBY-WAVES: THEORETICAL BASIS

(a) Prototype model.

The prototype model is that of two-dimensional non-divergent inviscid flow on an $f$-plane. Applications to more realistic atmospheric vortices are considered in section 3.

For vanishing flow at infinity, the dynamics is described completely by material conservation of vorticity. Because observations show small deviations from circular symmetry in the near-vortex region of hurricanes (Shapiro and Montgomery 1993), a useful zeroth-order approximation is to consider linearized dynamics on a circular vortex in gradient balance. In a stationary cylindrical coordinate-system, the linearized vorticity-equation is

$$\left( \frac{\partial}{\partial t} + \frac{\mathbf{\tilde{v}} \cdot \mathbf{\nabla}}{r \partial \lambda} \right) \zeta' - \frac{\partial \psi'}{r \partial \lambda} \frac{d\tilde{\eta}}{dr} = 0. \quad (1)$$

Here $\psi'$ denotes the perturbation streamfunction, $\zeta' = \nabla^2 \psi'$ the perturbation vorticity, $\mathbf{\tilde{v}}$ the basic-state tangential velocity at radius $r$, $\lambda$ azimuthal angle, $t$ time, $\nabla^2$ the horizontal Laplacian, $\tilde{\eta} = f + (1/r) \frac{d(r\tilde{v})}{dr}$ the basic-state vertical vorticity, and $f$ the Coriolis parameter. Although a constant $f$ does not affect interpretation of results, it will be retained for later sections.

Upon solving (1), the perturbation radial and azimuthal winds are obtained from

$$u' = -\frac{\partial \psi'}{r \partial \lambda}, \quad v' = +\frac{\partial \psi'}{\partial r}.$$

(b) Analytical results reviewed

Despite extensive research on vortex dynamics employing nonlinear contour dynamical models (Dritschel and Legras 1993) and Cartesian spectral models (McWilliams et al. 1994), a thorough investigation of the linear initial-value problem (1) for all azimuthal
wave-numbers has not yet been carried out. In this section, known analytical solutions relating to the theme of the present paper are reviewed.

The solution to (1) is conveniently obtained in azimuthal-Fourier space. Letting \( \tilde{\psi}_n(r, t) \) denote the Fourier amplitude for azimuthal wave-number \( n \), the linearized vorticity equation in Fourier space becomes

\[
\left( \frac{\partial}{\partial t} + i\Omega \right) \left[ \frac{\partial}{\partial r} \left( r \frac{\partial \tilde{\psi}_n}{\partial r} \right) - \frac{n^2}{r^2} \tilde{\psi}_n \right] - \frac{in}{r} \frac{\partial \tilde{\psi}_n}{\partial r} = 0,
\]

where \( \Omega = \tilde{\nu}/r \) is the angular velocity of the basic-state vortex. Explicit solutions have been constructed for a few special cases. The first and easiest case is that of a bounded Rankine vortex in which \( d\tilde{\eta}/dr \) is identically zero within \( r_1 \leq r \leq r_2 \), where \( r_1 \) is an inner frictionless boundary and \( r_2 \) is an outer frictionless boundary (Carr and Williams 1989; henceforth CW). In this case the solutions are cylindrical analogues of plane-wave solutions describing sheared disturbances in rectilinear simple-shear flow (Thomson 1887; Farrell 1987). Physically pertinent properties of these solutions, such as the dependence of integrated kinetic energy on azimuthal wave-number, have been examined in CW and SM.

The second model for which explicit solutions have been constructed is the unbounded Rankine-vortex, wherein \( \tilde{\eta} \) experiences a finite jump at the radius of maximum winds, but is otherwise uniform inside and outside this radius (section 5 of SM). In this model the solution is a superposition of shear-wave (continuous spectrum) components and Rossby edge-wave (discrete spectrum) components that propagate azimuthally, at a slower speed than the vortex. However, because these waves do not propagate radially, they are not able to transport energy out of the vortex core.

A third model still permitting useful insight is the extension of the unbounded-vortex model of SM covering multiple discontinuities in \( \tilde{\eta} \). With more than one discontinuity, interference effects can arise. As an example, consider the simplest case of a three-region model in which the innermost vorticity \( \tilde{\eta}_1 \), is greater than the intermediate vorticity \( \tilde{\eta}_2 \), which is greater than the outermost vorticity \( \tilde{\eta}_3 \). Such a distribution can be regarded physically as a three-region approximation of a vortex monopole (or hurricane) possessing a finite transition region between its rapidly rotating core and its slowly rotating environment. Because the generalized radial gradient of vorticity is non-positive, the vortex is exponentially stable. If the initial condition consists of no exterior edge-wave but consists of smooth disturbance-vorticity in \( r_1 \leq r \leq r_2 \), some portion of the energy of the initial disturbance may be permanently transferred to the exterior edge-wave. Sheared disturbances play an essential role in this transfer process, for without them the disturbance energy as \( t \rightarrow \infty \) resides solely in the interior edge-wave. Keeping the limitations of the three-region model in mind, we expect that the continuous model may possess an analogous mechanism for transferring energy outwards*.

A formal solution to the continuous model (2) can be obtained following Case (1960) and SM. Unfortunately, this representation does not allow one to infer the transient nature of the solution, let alone the possibility of radially propagating waves. A formal solution, using somewhat different methods, has been obtained by Sutyrin (1989), though careful review of Sutyrin's work revealed no discussion of radially propagating vortex Rossby-waves. Since Sutyrin's solution is expressed in terms of an operator on an infinite function space, identification of outward or inward wave propagation is difficult. Fortunately, there is at least one instance in which an explicit solution is available.

* Linear contour dynamical models cannot predict such effects since the sheared disturbance-component residing between contours is neglected.
An exact solution

For azimuthal wave-number one, the initial-value problem (2) may be integrated exactly (Smith and Rosenbluth 1990). The solution is

$$\hat{\psi}_1(r, t) = -r \int_r^\infty dr' e^{-i\tilde{\Omega}(r') t} [1 + ir\tilde{\Omega}(r) - ir\tilde{\Omega}(r')] \tilde{h}(r'),$$

where

$$\tilde{h}(r) = \frac{1}{r^3} \int_0^r d\rho \rho^2 \tilde{\xi}^{(0)}(\rho),$$

and $\tilde{\xi}^{(0)}$ denotes the initial radial structure of relative vorticity for wave-number one. The corresponding Fourier vorticity amplitude

$$\hat{\xi}_1(r, t) = \tilde{\xi}^{(0)}(r) e^{-i\tilde{\Omega}(r'r')} - it \frac{d\tilde{\xi}}{dr} \int_r^\infty dr' e^{-i\tilde{\Omega}(r') t} \tilde{h}(r'),$$

where $\tilde{\xi} = \tilde{\eta} - f$ denotes the relative vorticity of the basic-state vortex. The derivation of (3), (4) and (5) exploits the fact that $r\tilde{\Omega}(r)$ (the pseudo-mode) is an exact solution of (2) that corresponds physically to a translation of the vortex. In Fourier–Laplace space the second-order radial structure equation can then be reduced to a first-order equation and integrated in closed form. For solutions that are bounded at the origin and which vanish at infinity, the inverse Laplace-transform furnishes (3), (4) and (5). Provided

$$\int_0^\infty d\rho \rho^2 \tilde{\xi}^{(0)}(\rho) < \infty,$$

the exact solution is valid for smooth profiles whose angular velocity is finite at the origin and whose behaviour when $r$ is large approaches a constant (possibly zero). This covers a variety of vortex profiles of geophysical interest.

Smith and Rosenbluth developed (3), (4) and (5) in the context of plasma physics. The solution technique has been rediscovered by Reznik and Dewar (1994) in an analytical study of vortex motion on a beta plane. In both investigations, however, emphasis was directed at understanding the behaviour when $r$ is large, and a thorough analysis of the near-field transient dynamics was not considered. The next subsection investigates the near-field transient dynamics of the exact solution.

Exact solution results

From the mathematical solution, it is difficult to identify radially propagating waves. The wave mechanics may nevertheless be revealed by considering the evolution of wave-number-one asymmetries on the non-dimensional basic-state swirl profile

$$\tilde{v}(r) = \frac{2r}{1 + r^2};$$

the corresponding angular velocity is

$$\tilde{\Omega}(r) = \frac{2}{1 + r^2},$$

* (Note added in proof) After this paper was accepted for publication, M.T.M. was made aware of another study which rediscovered the Smith and Rosenbluth (1990) exact solution and applied it to assess the linear stability of circular vortices to azimuthal mode-one disturbances (Llewellyn-Smith 1995). Although this study considered vortices possessing zero and non-zero circulation, it focused solely on the linear $t \to \infty$ asymptotics and again did not identify the nature of the transient dynamics considered here and in section 3.
and the corresponding relative vorticity is

\[ \tilde{\zeta}(r) = \tilde{\Omega}(r) + \frac{d\tilde{v}}{dr}, \]

where characteristic length- and velocity-scales correspond to the radius of maximum winds (RMW) and to the maximum tangential velocity respectively. Plots of \( \tilde{\Omega}(r) \), \( \tilde{v}(r) \), and \( \tilde{\zeta}(r) \) are presented in Fig. 1. Since the mean vorticity is a monotonic function of radius, the vortex satisfies Rayleigh's sufficient condition for exponential stability (Gent and McWilliams 1986). Even though this vortex has infinite integrated kinetic energy and angular momentum, the results shown below are not an artefact of these properties and have been verified for other swirl profiles with finite energy and angular momentum (see, for example, section 3).
Figures 2 and 3 show the evolution of an initial wave-number-one disturbance
\[ \tilde{\xi}(0) = (6\alpha)^{\frac{1}{3}} r \exp \left( \frac{1}{6} - \alpha r^6 \right), \]
where \( \alpha = 1/(6 \times 3^6) \). This initial condition has a maximum vorticity-amplitude of unity at \( r = 3 \). The left-hand column of panels displays the Fourier vorticity-amplitude as a function of \( r \), while the right-hand column displays maps plots of the asymmetric fields of vorticity. Negative values are enclosed in dashed lines. Contour values below and above \((-3.8/3.8)\) have been omitted for the sake of clarity. The solution displays both expected and unexpected features.

At a radius of 0.6, we see the first of the expected features: the rapid emergence of the pseudo-mode (section 5 of SM). As demonstrated in SM for the unbounded Rankine-vortex, the pseudo-mode emerges through a projection of the initial wave-number-one asymmetry onto a stationary mode. Physically, this mode is simply the mathematical representation of a displaced vortex viewed from a stationary system of coordinates. After the transients symmetrize away, one expects no subsequent vortex-motion for non-divergent dynamics on an \( f \)-plane. The exact solution confirms this expectation and asymptotically approaches a steady state at long times; (see appendix A). If one forms the total stream-function \( Re\{\tilde{\Psi}(r) + \tilde{\Psi}_1(r, t) e^{it}\} \), the vortex centre traces out a cycloidal trajectory; the amplitude of the trajectory vanishes as the initial asymmetry is symmetrized. Here the vortex centre is defined as the point of minimum total streamfunction. In this example, the final position of the vortex lies due east of the initial centre (not shown). The \( t \to \infty \) analysis of appendix A then furnishes the distance from the origin as the solution of
\[ \tilde{\varphi}(r) - \left\{ \frac{\varepsilon}{4} \int_0^\infty d\rho \rho^2 \tilde{\zeta}_0(\rho) \right\} \frac{d\tilde{\varphi}}{dr}(r) = 0, \]
where the initial data \( \tilde{\zeta}_0(r) \) is assumed real-valued, and \( \varepsilon \) is the magnitude of the asymmetric disturbance. Consistent with the assumption of linear theory, the final position is only a small fraction of RMW. Numerical quadrature of (3), (4) and (5) is consistent with this result. These findings are in accord with the simulations of hurricane motion of Smith et al. (1990; see their Fig. 15(a) and accompanying discussion), and also complement the track predictions of Reznik and Dewar (1994) who examined the particular solution associated with \( \beta \)-forcing rather than the homogeneous solutions associated with arbitrary wave-number-one perturbations.

The pseudo-mode is not the only interesting solution-feature found in the inner core. Superposed on the pseudo-mode is a cycle of transient growth and decay. Initially, the asymmetry is aligned radially. The associated asymmetric radial flow produces inner-core vorticity-anomalies at radii where the gradient of basic-state vorticity is large. The resulting geometry gives an effective upshear tilt and growth in disturbance energy (Farrell 1982). Since the interior asymmetry rotates faster than the exterior one, the geometry quickly reverses causing a downshear tilt and subsequent energetic decay. In the outer regions of the storm, the gradient of basic-state vorticity is small, and symmetrization proceeds as in the bounded Rankine-vortex (cf. SM).

Upon close inspection, one also observes outward-propagating waves of vorticity between the inner-core and outer-core asymmetries \((1.25 < r < 2.5)\). The wave crests emerge near \( r = 1 \) and travel outwards to a radius where their outward propagation ceases. The radial plots are especially useful for seeing this effect. Figure 3 subdivides the time interval between \( t = 20 \) and \( t = 40 \) in Fig. 2 so as to help the reader track individual wave-packets. To examine the outward propagation of waves further, radius-time plots are
Figure 2. Time series showing the radial plots of $|\xi|$ (left-hand panels) and horizontal plots of $\xi'$ (right-hand panels) for the Smith and Rosenbluth exact solution. The initial asymmetry is maximum at $r = 3$. For the sake of clarity, values of $|\xi'| > 3.8$ have not been contoured.
Figure 3. As Fig. 2, except for finer temporal resolution between $t = 20$ and $t = 40$. 
presented in Fig. 4 that track the radial location of wave crests in the Fourier vorticity-amplitude $\bar{\zeta}(r)$. Because these waves are superposed on the quasi-steady pseudo-mode, wave crests are identified by zeros in $d^3|\bar{\zeta}|/dr^3$. This analysis method purposely filters out individual phase features and is adequate for higher azimuthal wave-numbers as well. The filter method may be illustrated by considering a Hankel-function representation of an out-going cylindrical wave with given phase speed. The absolute value of the Hankel-function wave is time invariant and has no zeros in $d^3|\bar{\zeta}|/dr^3$. This method is therefore naturally suited to determining radial group-velocity.

Figure 4 is a radius-time plot for the exact solution that unquestionably shows outward-propagating wave-packets. Three other characteristics are also noteworthy. The first is the tendency of individual packets to slow as they move outwards. Secondly, the radial wave-number increases and the corresponding radial group-velocity decreases at long times. Thirdly, the radial group-velocity for individual packets exhibits a dependence on radial wavelength, indicating wave dispersion. This example reveals an important wave-mechanism operating in stable vortex-flows whereby disturbances may extract energy from the vortex at one radial band and deposit it at another. Whether such processes are operative for higher wave-numbers is the subject of the next subsection.

(e) Higher wave-numbers

Wave-number two exhibits symmetrization and radial-wave propagation without the complication of the pseudo-mode. As an illustration, consider an idealized wave-number-two vorticity-asymmetry forced by convection in the eye wall of a hurricane

$$\bar{\zeta}^{(0)} = r^2 \exp \left\{ \frac{1}{2} \left( 1 - r^4 \right) \right\}. \tag{7}$$

The evolution summarized in Fig. 5 shows what is, perhaps, the simplest case of outward-propagating shear-waves.

Initially, the leading wave-packet moves from $r = 1$ to $r = 1.3$ and, by approximately one-half of a circulation time, the initial disturbance has induced an inner-core asymmetry near the maximum of the basic-state vorticity-gradient ($r \approx 0.4$). This asymmetry then
Figure 5. Time series showing the radial plots of $\hat{\xi}$ (left panels) and horizontal plots of $\xi'$ (right panels) for the non-divergent numerical solution for wave-number two.
emits wave packets which propagate outward to a non-dimensional radius of \( r = 0.7 \). The fact that inner and outer wave-regions appear separated by the RMW is coincidental, however. If the centre of the initial condition (7) is placed at greater radius, two qualitative differences are noted. Firstly, the inner-core asymmetries are observed to propagate beyond the RMW; secondly, many more wave packets are observed. (Figure 11 presents examples in the context of a shallow-water model.) Since distant asymmetries symmetrize more slowly, they appear as quasi-steady forcing processes to the inner core. The result is a series of wave-shedding events which terminate when the outer disturbance has decayed sufficiently to force a negligible inner-core response. This example may have important meteorological applications since it raises the possibility that stationary forcing-processes in the environment (e.g. an environmental-straining flow) may continuously excite radially propagating shear-waves in the near-core region of a hurricane vortex.

For still higher wave-numbers \((n > 2)\), numerical experiments indicate that radial wave-propagation effects diminish. At very high wave-numbers \((n \gg 2)\), radial propagation is virtually non-existent and outer-core asymmetries induce almost no inner-core response. The latter property is easily traced to the wave-number-dependent scale of influence that relates vorticity to streamfunction. The former property will be explained in the next subsection. Taken together, these experiments indicate that for a stable vortex, asymmetric dynamics for high wave-numbers is qualitatively similar to the bounded Rankine-vortex which possesses no basic-state vortex gradient (cf. SM).

\[(f)\] **Wenzel-Kramers-Brillouin (WKB) analysis**

From the behaviour of the exact solution and the numerical results for higher wave-numbers, outward-propagating waves often possess radial length-scales \((l)\) that are small compared to the characteristic radial scale \((L)\) of the vortex. Under such conditions, it is justifiable to seek approximate solutions near \( r = R \) in the form

\[
\psi'(r, \lambda, t) \approx A(t) \exp\{i(n\lambda + k(t)(r - R) - \Lambda(t))\},
\]

where \( A(t) \) is a time-dependent amplitude, \( k(t) \) a time-dependent radial wave-number, and \( \Lambda(t) \) a time-dependent phase. Since (8) is expected to describe sheared disturbances in the tightly wound limit \( kR \gg 1 \), we may, without loss of generality, assume that \( A \), \( k \) and \( \Lambda \) are real-valued. Provided \( l \ll L \), basic-state variables can be assumed to be slowly varying and may be expanded in series:

\[
\tilde{\Omega}(r) = \tilde{\Omega}_0 + \tilde{\Omega}_0'\delta r + \cdots
\]

\[
\frac{d\tilde{\xi}}{dr}(r) = \tilde{\xi}_0' + \tilde{\xi}_0''\delta r + \cdots
\]

\[
\frac{1}{r} = \frac{1}{R} \left( 1 - \frac{\delta r}{R} + \cdots \right),
\]

where \( \delta r = r - R \), prime denotes differentiation with respect to radius, and the zero subscript on mean flow quantities denotes evaluation at \( r = R \). Substituting (8) and (9) into (2), neglecting terms of \( O(1/kR, \delta r/R) \) and equating real and imaginary parts to zero gives, respectively,

\[
\hat{A} \left( k^2 + \frac{n^2}{R^2} \right) + 2kkA = 0
\]

and

\[
\left( k^2 + \frac{n^2}{R^2} \right) (\hat{\Delta} - n\tilde{\Omega}_0) - n\tilde{\xi}_0' - \left( k^2 + \frac{n^2}{R^2} \right) (\hat{k} + n\tilde{\Omega}_0')\delta r = 0,
\]
where dot denotes differentiation with respect to time. The system (10) and (11) can be integrated as follows. Because (11) is valid for small, but otherwise arbitrary, \( \delta r \), we may set \( \delta r = 0 \) to furnish the instantaneous wave-frequency

\[
\dot{\Lambda}(t) = n \Omega_0 + \frac{n}{R} \frac{\Omega_0'}{\left( k^2 + n^2 / R^2 \right)}.
\]  

(12)

Since \( k^2 + n^2 / R^2 \) is never zero, (11) implies

\[
\dot{\Lambda}(t) = n \Omega_0',
\]

whose integration yields

\[
k(t) = k_0 - n t \Omega_0',
\]

(13)

where \( k_0 \) is an initial radial wave-number. Equations (10) and (13) then furnish

\[
A(t) = \frac{(k_0^2 + n^2 / R^2) A_0}{(k_0 - n t \Omega_0')^2 + n^2 / R^2},
\]

(14)

where \( A_0 \) is the initial wave-amplitude. Expressions (13) and (14) represent the cylindrical analogue of sheared disturbances in rectilinear simple-shear flow and the solutions are well known. The derivation of (12) in the context of axisymmetrizing disturbances, however, is believed to be new.*

Equation (12) represents the local dispersion relation

\[
\omega = n \Omega_0 + \frac{n}{R} \frac{\Omega_0'}{\left( k^2 + n^2 / R^2 \right)}
\]

for a spectrally localized wave-packet whose initial central wave-numbers are \( k_0 \) and \( n \) respectively (cf. Yamagata 1976; Tung 1983). This dispersion relation is analogous to the dispersion relation for non-divergent Rossby-waves on a beta-plane in a uniform zonal wind. The meridional derivative of planetary vorticity is replaced by the radial derivative of basic-state storm vorticity, while the Doppler-shifted frequency is replaced by the azimuthal wave-number multiplied by the basic-state angular velocity. For a cyclonic monopole whose basic state has, everywhere, a negative gradient of radial vorticity, the dispersion relation predicts that individual waves retrogress relative to the local angular velocity. Unlike unsheared Rossby-waves on a beta-plane, however, the radial wave-number is ever-changing because of the symmetrizing effect of the vortex. This has fundamental consequences on the kinematics and dynamics of vortex wave-packets.

Radial and azimuthal phase-velocities, defined by \( C_{pr} = \omega / k \) and \( C_{ph} = \omega R / n \), are given by

\[
C_{pr} = \frac{n}{k} \Omega_0 + \frac{n}{R k} \frac{\Omega_0'}{\left( k^2 + n^2 / R^2 \right)},
\]

(15)

* (Note added in proof) The WKB results (12) to (14) are easily generalized to include viscous effects. With \( \nu \cdot \nabla^2 \)-diffusion of perturbation vorticity, (12) and (13) are unaltered, but \( A(t) \) in (14) is multiplied by

\[
\exp \left\{ -\nu \int_0^t dt' (k^2 + n^2 / R^2) \right\}.
\]

In the limit of large times, the inviscid wave amplitude is multiplied by

\[
\exp \left\{ -\nu n^2 \Omega_0^2 t^2 / 3 \right\}.
\]
VORTEX ROSSBY-WAVES IN HURRICANES

\[ C_{p\lambda} = R \tilde{\Omega}_0 + \frac{\tilde{\zeta}_0'}{(k^2 + n^2/R^2)}. \]  \hfill (16)

The speed and direction of energy propagation, however, is controlled by the group velocity whose radial and azimuthal components, defined by \( C_{gr} = \partial \omega / \partial k \) and \( C_{g\lambda} = \partial \omega / \partial (n/R) \) are given by

\[ C_{gr} = \frac{-2kn\tilde{\zeta}_0'}{R(k^2 + n^2/R^2)^2}, \]  \hfill (17)

\[ C_{g\lambda} = R \tilde{\Omega}_0 + \frac{\tilde{\zeta}_0'}{(k^2 + n^2/R^2)^2} \left[ k_0^2 - \frac{n^2}{R^2} (1 + t^2 R^2 \tilde{\Omega}_0^2) \right]. \]  \hfill (18)

Note that in (15), (16), (17) and (18) \( k \) is given by (13).

To fix ideas, consider the simple case of a cyclonic monopole with a non-positive \( \tilde{\zeta}_0' \). A hypothetical spiral that spirals cyclonically inwards is sketched in Fig. 6. Portions of this spiral can be described by

\[ kr + n\lambda = \text{constant}, \]

and \( n \) may be assumed positive without loss of generality. Since a trailing spiral is associated with positive \( k \), its corresponding radial group-velocity is positive. Hence, symmetrizing disturbances on vortex monopoles always have an outwardly directed group velocity. This is quite unlike the behaviour of sheared disturbances in a uniform vorticity environment that have zero group-velocity in the cross-shear direction. As the packet propagates outwards it is continually slowed by the shearing effect that increases its radial wave-number. Since, for large \( k \), the radial group-velocity goes as \( O(k^{-3}) \), the shearing effect eventually dominates and radial propagation ceases. Such locations are henceforth referred to as stagnation radii. From the expression (B.1) for the instantaneous position of a wave packet derived in appendix B, the stagnation radius \( r_s \) follows, on letting \( t \to \infty \),

\[ r_s = R + \frac{\tilde{\zeta}_0'}{RS\tilde{\Omega}_0} \frac{1}{(k_0^2 + n^2/R^2)^2}. \]

The existence of a stagnation radius is an important result since it implies that these waves are confined to the near-vortex region and cannot radiate to infinity. Their dynamics
is thus distinct from that of gravity-inertia waves which, in the absence of critical levels, radiate to infinity (the vortex environment). In non-divergent dynamics on a beta-plane, vortices are 'isolated' provided that the integrated angular momentum vanishes (Flierl et al. 1983). On the $f$-plane, the WKB results show that vortices are always isolated, regardless of their integrated angular momentum.

Although the stagnation radius in the free-wave problem can be interpreted as a critical level in the local rectilinear approximation (cf. Tung 1983—Eq. (7.10)), the attendant absorption process should not be confused with the traditional idea of 'critical-level absorption' which requires viscosity (Tung 1983). A simple estimation of the neglected nonlinear terms in the horizontal momentum equations $u'/\partial r$, $u'/r \partial \lambda$, $u'/\partial \lambda$, and $v'/r \partial v'/\partial \lambda$ shows them to vanish faster than the corresponding linear terms. Thus despite the ever-increasing radial wave-number, the linear approximation remains uniformly valid in time at least for monopolar vortices.

Inferences about azimuthal kinematics can also be deduced. For $k_0 R \gg 1$, (18) shows that $C_{g_3} - R \hat{\Omega}_0$ is initially negative. As the packet propagates outward, however, $C_{g_3} - R \hat{\Omega}_0$ quickly becomes positive and ultimately approaches zero asymptotically at long times. The latter effect resembles the behaviour of the zonal group-velocity for unsheared Rossby-waves on a beta-plane. As the shearing effect predominates, the total wave-number becomes large. Instead of the packet always retrogressing, it temporarily moves faster than the mean wind until it sheared by the differential rotation of the vortex.

Similar reasoning suggests that leading spirals (i.e. disturbances possessing an upshear tilt $k_0 < 0$) have an inward-directed group-velocity. In this case, environmental asymmetries can excite inward-propagating waves until their radial wave-number changes sign, after which they would begin their outward propagation. Although highly tilted upshear and downshear initial conditions are equally likely, the $t \to \infty$ limit results in trailing spirals in the absence of discrete modes. Further discussion of this point is reserved for section 3.

Although conclusions drawn above may be of a general nature, the case of an unstable vortex requires us to understand how unstable growth and saturation interact with the wave processes examined here. For mean vorticity-profiles that are not monotonic, stable discrete modes may also exist. These topics remain for future work.

(g) WKB validation

To test the usefulness of the WKB approximation developed above, time integrations of WKB group-velocity and numerical radius-time plots are now compared. Since the radial length-scale of the initial condition used in the numerical integration has roughly the scale of the radius of maximum wind, $k_0 = 1$ is a natural choice*. Because the WKB approximation assumes $k R \gg 1$, $R = 1$ is the innermost radius for which we might hope the approximation to be valid.

The WKB method is a local approximation that ignores distant influences. From Fig. 2 we observe that the pseudo-mode is a quasi-steady phenomenon that influences the shape and evolution of radially propagating waves in the near-core region. Because of these limitations, we should expect only qualitative similarity between the WKB approximation and the observed radial propagation for wave-number one. WKB results for wave-number one (not shown) show a larger discrepancy with the numerical calculation

* Strictly speaking, radially aligned initial conditions spiral neither inwards nor outwards. Thus, in the plane-wave representation, $k_0 = 0$. On the other hand, a Hankel transform of these initial conditions yields a central radial wave-number of $k \approx 2$. As a first approximation in describing the entire life-cycle of radially aligned initial asymmetries in this local WKB formulation, we take the average of these two wave-numbers.
than that noted below for higher wave-numbers. The foregoing limitations may be manifesting themselves in the representation for \(k(t)\) since the observed group-velocities lie between group velocities calculated for \(k = k_0\) and \(k = k_0 - n i \Omega_c\). In addition, the results also show that wave-number-one features travel a significant radial distance, violating the local assumptions used to derive \(k(t)\).

In Fig. 7 we compare the WKB approximation and the numerical model for wave-numbers two and three for a wave packet initially centred at \(R = 1\). The initial condition for the wave-number-two run is the same as (7), while the wave-number-three initial condition is given by \(\tilde{\zeta}^{(3)} = r^3 \exp\{3(1 - r^4)/4\}\). The WKB approximation duplicates the character of radial propagation observed in the numerical model fairly well. Especially noteworthy is the replication of the asymptotic \((t \to \infty)\) behaviour of radially propagating wave-packets.

The numerical model shows that wave-numbers two and three asymptotically approach \(r = 1.3\) and \(r = 1.17\) respectively, while WKB theory approaches \(r = 1.4\) and \(r = 1.2\) respectively. The WKB plots show faster movement from the initial \(R\) to the asymptotic value of \(r\). This discrepancy may be attributed to the tightly-wound assumption \((kR \gg 1)\) of
the WKB theory. This condition is not met for the initial disturbance which must evolve for some time before it has become tightly wound. (See footnote in this subsection.) Another factor that may contribute to the discrepancy is that the radial displacement is $O(1)$, pushing the local assumptions used to derive $k(t)$. For higher wave-numbers (not shown) radial wave-propagation diminishes. The WKB predictions are consistent with this behaviour by virtue of the $n^{-3}$ dependence of $C_{gr}$.

The ability of the WKB method to forecast basic observed features of radial wave-propagation gives us confidence in the asymptotic formulation. The functional dependence of both phase and group velocities on the basic-state vorticity gradient clearly indicates that such wave features are indeed vortex Rossby-waves, confirming early speculations of MacDonald (1968).

The above results, in combination with the results of subsection 2(h), suggest a deeper interpretation of the axisymmetrization process in regions of variable vorticity than previously offered by McWilliams (1989—section 2.6). As an illustration, consider the axisymmetrization of a smooth elliptical vortex. In non-divergent vorticity dynamics, the angular-momentum invariant

$$\frac{\partial}{\partial t} \int_A r^2 \zeta \ dA = 0,$$

where $\zeta$ denotes total vorticity, implies that if the elliptical vortex symmetrizes, the ends of the ellipse must be transported to a greater radius. The WKB formulation goes still further by predicting the propagation rate and stagnation radius of symmetrizing vorticity disturbances. The outward-propagating vorticity-filaments of Melander et al. (1987—Fig. 8) may thus be identified with vortex Rossby-wave dynamics.

(h) Wave–mean-flow interaction

This section investigates the effects of radially propagating vortex Rossby-waves on the mean vortex. Provided the amplitudes of the asymmetries remain small, the mean-flow changes at second order in disturbance amplitude can be obtained from the divergence of the eddy-momentum flux

$$\frac{\partial \bar{v}}{\partial t} = \frac{-\partial}{r^2 \partial r} (r^2 u' v'),$$

where bar denotes azimuthal mean. If the momentum flux vanishes after some time $\tau$, the change in the mean tangential wind follows on integrating (19)

$$\delta \bar{v}(r) = -\int_0^{\tau} \frac{\partial}{r^2 \partial r} (r^2 u' v') \ dt.$$

Although asymmetries cannot change the area-integrated angular-momentum and circulation, they can change local values of these quantities. An equally valid representation for the change in the mean tangential wind is given by the integrated radial flux of vorticity

$$\delta \bar{v}(r) = -\int_0^{\tau} \bar{u} \zeta' \ dt.$$
of constant shear, wave packets will travel down the vorticity gradient until they reach their stagnation latitude. When these ideas are applied to the vortex problem, they suggest that, in order to move both the stagnation radius and the wave–mean-flow-interaction radius away from the location of the initial asymmetry, a vortex-β effect \( d\vec{\zeta} / dr \) is needed as opposed to a variable shear \( r \, d\vec{\zeta} / dr \). CW also noted that \( \delta \vec{v}(r) \) vanishes at the radius where \( |\vec{\psi}| \) is a maximum. These effects appear to be coincidental, however. Based on the results of section 2, the radius–time plots show that \( |\vec{\psi}|_{\text{max}} \) continues to move outwards and does not approach the stagnation radius asymptotically as the zero in \( \delta \vec{v}(r) \) does. Also, \( |\vec{\psi}|_{\text{max}} \to 0 \) as \( t \to \infty \), so \( |\vec{\psi}|_{\text{max}} \) is unlikely to be the explanation of a long-lived feature of the flow.

To investigate the consequences of a variable shear on \( \delta \vec{v}(r) \) without the complicating influences of boundaries, calculations of the acceleration-integral (20) were carried out using the non-divergent numerical model with horizontally-upright localized asymmetries placed at \( r = 6, 8 \) and 10 respectively. In this region of the vortex, \( d\vec{\zeta} / dr \) is negligible while changes in the effective shear \( r \, d\vec{\zeta} / dr \) are still significant. At these radii, \( \delta \vec{v}(r) \) exhibits a nearly symmetric reverse ‘S’ shape, with accelerating mean flow on the upshear side and decelerating mean flow on the downshear side (Farrell 1987). Unlike the results of CW, the calculations ‘asymptote to’ the analogous rectilinear case of constant shear that exhibits no radial shift in the reverse ‘S’ shape.

Implications of a variable vorticity-gradient on the wave–mean-flow dynamics will now be examined. Figure 8 shows \( \delta \vec{v}(r) \) and \( \delta \vec{\zeta}(r) = \vec{\zeta}(r) - \vec{\zeta}(\text{basic state}) \) for the wave-number-two initial condition of Fig. 7. Dimensionally, \( \delta \vec{v}(r) \) scales as \( O(e^2 V_{\text{max}}) \) and \( \delta \vec{\zeta}(r) \) scales as \( O(e^2 V_{\text{max}} / RMW) \), where \( e \) denotes the strength of asymmetry relative to the basic state, \( V_{\text{max}} \) denotes the maximum tangential velocity, and \( RMW \) denotes the radius of maximum winds. It is evident from Fig. 8(a) that the zero in \( \delta \vec{v}(r) \) occurs at the very radius shown to be the stagnation radius for both the numerical model and the WKB approximation. It is also significant that the maximum acceleration is larger than the deceleration and that it occurs outside the radius at which the initial asymmetry is a maximum. This is notably different from the rectilinear case with zero \( \beta \). In the rectilinear constant shear problem, the asymmetry decays in place and leaves a symmetric reverse ‘S’ pattern centred at the latitude of the initial asymmetry. This difference may be particularly significant in hurricanes and incipient tropical disturbances since it signifies an internal mechanism by which asymmetric shearing disturbances, excited in the inner-core region, can accelerate the basic-state tangential wind. For localized excitation centred near the radius of maximum winds, radial propagation causes the wave–mean-flow interaction to occur outside the radius of maximum winds.

From a meteorological standpoint, the above result is quite interesting and we briefly digress so as to speculate on its consequences. Imagine an incipient cyclonic vortex monopole embedded in a conditionally-unstable moist tropical atmosphere. If a weak horizontal straining-flow acts on the vortex, a wave-number-two moist-convective response will ensue in the near-core region, inducing positive PV-anomalies at low-levels in regions of buoyancy-induced convergence. Approximating the subsequent dynamics as dry adiabatic and nearly inviscid, vortex Rossby-waves will travel outwards and will accelerate the tangential winds outside the wave-excitation region. If the forcing continues for several eddy-turnover times without appreciably disrupting the outward propagation, the low-level circulation will encompass a larger area and the mean tangential winds will intensify. This intensification mechanism does not appear to have been identified until now. A thorough investigation of the nonlinear dynamics and eddy-forced secondary circulations associated with stochastically forced vortex Rossby-waves in both two- and three-dimensional flows is an exciting topic deserving of further study.
Returning to the simple quasi-linear calculation, the corresponding change in the basic-state vorticity $\delta \zeta (r)$ is also of interest. Figure 8(b) shows that the mean vorticity $(\bar{\zeta} + \delta \zeta)$ increases inside $r = 1$, is reduced slightly outside $r = 1$, and increases again near $r = 1.75$. While the reduction in mean-vorticity resembles 'vortex erosion', the vorticity increase inside $r = 1$ is associated with an up-gradient eddy-vorticity flux. The secondary maxima in $\delta \zeta$ shows some qualitative resemblance to observations of secondary PV-maxima in hurricanes evident in Fig. 11 of Shapiro and Franklin (1995). For monopolar vortices, quasi-linear dynamics evidently captures an encouraging amount of the wave–mean-flow physics.

Another consequence of a variable vorticity-gradient can be demonstrated with broader initial asymmetries. A representative example is furnished by Fig. 9, corresponding to the wave-number-two initial condition $\zeta^{(0)} = r^2 \exp(1 - r^2)$. In this case, the outward

* (Note added in proof) The quasi-linear predictions described here have been verified using fully nonlinear finite-difference and pseudo-spectral models and the details will be reported in a forthcoming publication. Pertinent to the vorticity budget is the fact that the up-gradient vorticity-flux corresponds to a preferential mechanism for ingesting like-sign vorticity inside the RMW. The down-gradient vorticity-flux corresponds to an outward transport of both positive and negative vorticity-anoanies.
Figure 9. (a) and (b): As Fig. 8 except for a broader initial asymmetry $z^{(0)} = r^2 \exp(1 - r^2)$, after $t = 30$ (4.7 circulation times) in the non-divergent numerical model. (c): Radius–time plot for the model run of (a) and (b). Note that the radial scale in (c) differs from that in (a) and (b).
shift of the reverse ‘S’ pattern (Fig. 9(a)) is more pronounced than for Fig. 8(a). From the radius–time plot of Fig. 9(c), the initially broad asymmetry provides a pronounced inner-core response that then propagates outwards near the stagnation radius. This outward-propagating asymmetry appears to provide an additional acceleration to the mean flow near the stagnation radius. With even broader-scale asymmetries, the zero in \( \delta \tilde{u}(r) \) no longer coincides with the stagnation radius. The interaction of the initial asymmetry with the basic state, and the subsequent interaction between the inner asymmetries and outer asymmetries, exaggerate the outward shift of the reverse ‘S’ imprint on the mean vortex.

3. APPLICATION TO HURRICANES

Having developed a basic theory for vortex Rossby-waves, we now proceed to apply it to hurricane vortices. There are several questions that must be answered before tackling the nonlinear problem. The first question is whether trailing spirals are dynamically more favoured than leading spirals in hurricane flows (cf. subsection 2(h)). The second question concerns whether a vortex Rossby-wave mechanics can be developed for rapidly rotating divergent flow. The third, and most important, question is whether observations provide evidence for the existence of vortex Rossby-waves in real hurricanes. The next subsections address these issues in turn. An investigation of the wave–mean-flow dynamics in regimes of large Rossby-numbers and Froude numbers of order unity awaits future work.

(a) Limitation of upshear tilt and algebraic growth

Initially, a migratory trough may be tilted horizontally upshear as it approaches a hurricane vortex. As with any synoptic-scale pattern, however, the approaching trough is typically larger than the vortex. This implies that large upshear-tilt geometry will exist only locally, with the rest of the trough being nearly upright relative to the horizontal shear. In the analogous plane-wave model on the other hand, perturbations are easily configured to have large upshear tilt throughout the domain, giving the potential for much greater growth (e.g. Farrell 1987). To produce large algebraic growth, vortex perturbations must be tightly wound, anticyclonic spirals. This is unlikely in the face of cyclonically spiralling inflow and large anticyclonic horizontal shear that forces cyclonic spirals asymptotically in the absence of discrete modes.

The \( r \to \infty \) limit is one reason downshear tilt is preferred, but another is the radially varying basic-state shear. If the trough is tilted upshear, a small portion of it will have a significant inward group-velocity. The inward-directed wave will then encounter ever-increasing shear, quickly changing the upshear configuration to downshear (cf. subsection 2(f)). This results in a fast reversal to outward motion. The conclusion is similar to that of Tung (1983) who considered the dynamics of planetary Rossby-wave packets in rectilinear simple-shear flow. Tung’s formulation, however, did not consider latitudinal variation of the basic-state shear which is significant in hurricanes. Since the shear increases rapidly with decreasing \( r \), we expect much less inward motion.

At smaller scales, eye-wall convection may produce asymmetries with any orientation relative to the horizontal shear. But there is no reason to expect upshear tilt to be preferred. Indeed, recent radar analyses by Tuttle and Gall (1995) show the opposite to be true, with trailing spirals predominating in the near-core region. This discussion does not rule out transient algebraic growth nor minimize its importance: we merely point out a realistic limitation of algebraic growth and inward propagation within hurricane-scale vortices.

(b) Adding divergence and a variable deformation-radius

To investigate the applicability of the ideas of section 2 to hurricanes further, a divergent model is briefly examined. James Franklin (personal communication) provides
observational evidence that asymmetric divergence is of the same order as asymmetric vorticity in the near-vortex region of hurricanes. This is particularly important for wave-number one as it may greatly influence storm motion. Asymmetric-Balance (AB) theory proposed by Shapiro and Montgomery (1993) allows for O(1) divergence in rapidly rotating flows. A shallow-water formulation of AB has been developed and tested by Kallenbach and Montgomery (1995—hereafter KM). For an exponentially-stable hurricane-like vortex, KM have validated the evolution of small-amplitude asymmetries for azimuthal wave-numbers one to seven. For wave-numbers greater than one, the local Rossby-number in the near-vortex region is large compared with unity, yet the balance model consistently evolves these asymmetries and exhibits good agreement with known analytical solutions for non-divergent dynamics. Key steps of the AB shallow-water formulation follow.

For small amplitude disturbances on a stationary circular vortex in gradient-wind balance, the linearized $f$-plane momentum and continuity equations in cylindrical coordinates are

\[
\begin{align*}
\frac{D}{Dt} u' - \xi v' & = -\frac{\partial \phi'}{\partial r}, \\
\frac{D}{Dt} v' + \bar{\eta} u' & = -\frac{\partial \phi'}{r \partial \lambda}, \\
\frac{D}{Dt} \phi' + \frac{\partial (r \Phi u')}{r \partial r} + \frac{\Phi}{r \partial \lambda} & = 0,
\end{align*}
\]

where

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{\bar{v}}{r} \frac{\partial}{\partial \lambda}
\]

denotes the substantial derivative following the mean tangential wind $\bar{v}$, $\xi = f + 2\bar{v}/r$ the inertia parameter, $\bar{\eta} = f + (1/r) d(r \bar{v})/dr$ the absolute vorticity, $\Phi$ the geopotential of the basic state swirl, and $\phi'$ the perturbation geopotential. Expanding in a local Rossby-number incorporating planetary and storm rotation, the horizontal winds $(u', v')$ are approximated by

\[
\begin{align*}
u' & = -\frac{1}{\bar{\eta} \bar{\xi}} \frac{\partial \phi'}{\partial \lambda} - \frac{1}{\bar{\eta} \bar{\xi} \bar{\bar{r}}} \frac{\partial}{\partial t} (\partial \phi'/\partial r), \\
u' & = \frac{1}{\bar{\xi} \partial r} - \frac{1}{\bar{\eta} \bar{\xi} \bar{\bar{r}}} \frac{\partial}{\partial t} (\partial \phi'/\partial \lambda).
\end{align*}
\]

The first terms on the right represent the generalization of the geostrophic wind, while the second terms represent the generalization of the isallobaric wind. The first terms are thus similar to Dickinson's galactostrophic eddy velocities (Dickinson (1964) and references therein). In hurricanes, however, the physical basis for such a formulation and the corresponding mathematical structure is quite different (Shapiro and Montgomery 1993; Montgomery and Shapiro 1995). The expressions for $u'$ and $v'$ are then substituted into the continuity equation, yielding a closed first-order evolution-equation for the geopotential tendency $\zeta = \partial \phi'/\partial t$, viz.

\[
\begin{align*}
\frac{\gamma^2}{r^2} \frac{\partial}{\partial r} \left( \frac{r \partial \zeta'}{\gamma^2 \partial r} \right) + \frac{\partial^2 \zeta'}{r^2 \partial \lambda^2} - \gamma^2 \zeta' & = -\bar{v} \frac{\partial}{\partial \lambda} \left[ \frac{\gamma^2}{r^2} \frac{\partial (r \partial \phi')}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi'}{\partial \lambda^2} - \gamma^2 \phi' \right] \\
& + \frac{\xi}{\bar{\eta}} \frac{d \bar{q}}{dr} \frac{\partial \phi'}{r \partial \lambda}.
\end{align*}
\]
where \( \tilde{q} = \tilde{\eta}/\Phi \) the basic-state PV, and \( \gamma^2 = \tilde{\eta}^2/\Phi \) the square of the inverse of the local Rossby-radius. The disturbance PV for the AB shallow-water model is

\[
q'_\xi = \frac{1}{\Phi} \left\{ \frac{\partial}{r \partial r} \left( \frac{r \partial \phi'}{\tilde{\xi}} \partial r \right) + \frac{1}{\tilde{\xi}} \frac{\partial^2 \phi'}{r^2 \partial \lambda^2} - \tilde{q} \phi' \right\}
\]

and can be shown to be conserved on fluid parcels in the linearized formulation. This balance system also admits pseudo-conservation laws for momentum, energy and vorticity which are exactly analogous to those of the linearized shallow-water primitive equations.

The numerical solution technique employed is a spectral azimuthal-modes model which uses grid points radially and Fourier modes azimuthally. Model prognosis is carried out using the highly accurate fourth-order Runge–Kutta method. For the numerical results shown below, a radial grid-spacing of 2.5 km and a time-step of approximately 3 minutes are employed. Mathematical and numerical details have been further described by KM.

The AB shallow-water model is initialized with an exponentially stable basic-state vortex possessing a maximum tangential wind of 36.8 m s\(^{-1}\) at a radius of 75 km, a resting depth of 1 km, and a Coriolis parameter corresponding to 20°N. Consistent with the requirements for a quasi-steady vortex (Riehl 1963; Pearce 1992), the basic-state tangential wind (see Fig. 10) decays approximately as \( r^{-1/2} \) in the near-vortex region: \( RMW \leq r < 500 \) km. Beyond 500 km, the hurricane blends smoothly into its environment. For the idealized model considered here, the environment is at rest, so the tangential wind decays exponentially as \( K_1 (r f / \sqrt{gh_{\infty}}) \) for \( r \gg \sqrt{gh_{\infty}/f} \); \( K_1 \) denotes the modified Bessel-function of the second kind of order one.

Figure 11 shows radius–time plots for the AB model initialized with wave-number-one and wave-number-two asymmetries. The wave-number-one initial condition (Fig. 11(a)) is

\[
\tilde{q}_\xi = A_0 (6\alpha)^{1/2} \left( \frac{r}{RMW} \right) \exp \left\{ \frac{1}{6} - \alpha \left( \frac{r}{RMW} \right)^6 \right\},
\]

where \( A_0 = 10^{-8} \) s m\(^{-2}\), \( \alpha = 1/(6(3)^5) \) and \( RMW = 75 \) km. The wave-number-two initial condition (Fig. 11(b)) is

\[
\tilde{q}_\xi = A_0 (2\alpha)^{1/2} \left( \frac{r}{RMW} \right)^2 \exp \left\{ \frac{1}{2} - \alpha \left( \frac{r}{RMW} \right)^4 \right\},
\]

where \( A_0 = 10^{-8} \) s m\(^{-2}\), \( \alpha = 1/(2(3)^4) \) and \( RMW = 75 \) km. Both initial conditions attain their maximum PV-amplitude at \( r = 3(RMW) \).

Upon comparing the exact solution (Fig. 4) with that of the AB model for wave-number one, we see that adding a variable deformation-radius does not alter the character of the pattern of outwardly propagating waves. Even fine details, such as the 'cusp' shape at \( t = 8 \) in the exact solution, appear at \( t = 5 \) hours in the AB simulation (1.4 circulation times in both models). In the exact solution, the dimensional group-velocity for the first wave-crest at a non-dimensional \((r, t) = (1.33, 15)\) is 1.5 m s\(^{-1}\). A similar calculation for the first wave-crest at \((r, t) = (84 \text{ km}, 6 \text{ hours})\) in the AB simulation yields 1.4 m s\(^{-1}\). As another check of model consistency, we note that both the exact solution and the AB simulation exhibit dispersion.

On repeating the analysis of subsection 2(f), the WKB dispersion relation for an isolated wave-packet is only slightly modified by the variable deformation-radius. Adhering
to the notation of subsection 2(f), the instantaneous wave-frequency becomes

\[
\omega = n\tilde{\Omega}_0 + \frac{n\tilde{\xi}_0}{R\tilde{q}_0} \frac{\tilde{q}_0'}{k^2 + (n^2/R^2) + \gamma_0^2}
\]

where \(k(r)\) is given by (13). Corresponding expressions for the radial and azimuthal group-velocities are

\[
C_{gr} = \frac{-2kn\tilde{\xi}_0}{R} \frac{\tilde{q}_0'}{\tilde{q}_0 \left(k^2 + (n^2/R^2) + \gamma_0^2\right)^2}
\]

\[
C_{gi} = R\tilde{\Omega}_0 + \frac{\tilde{\xi}_0}{\tilde{q}_0} \frac{\tilde{q}_0'}{k^2 + (n^2/R^2) + \gamma_0^2} \left(\frac{k^2}{k^2 + (n^2/R^2) + \gamma_0^2} - \frac{n^2}{R^2 (1 + t^2 R^2 \tilde{\gamma}_0^2)}\right)
\]
The stagnation radius follows upon integrating $C_{gr}$ in time and letting $t \to \infty$:

$$r_s = R + \frac{\xi_0 \tilde{a}'_0}{\tilde{a}_0 R \Omega_0} \left\{ \frac{1}{k_0^2 + (n^2 / R^2) + \gamma_0^2} \right\}.$$

The dependence of $r_s$ on $\gamma^2$ is consistent with the nonlinear quasi-geostrophic simulations of Melander et al. (1987—section 5.1) who noted, without explanation, the tendency for the vorticity bands to remain closer to the core with increasing $\gamma^2$.

Wave-number-two radius-time plots for the AB simulation are consistent with the wave-mechanics theory. The initial asymmetry moves from 225 km to its stagnation radius near 275 km. Asymmetries originating in the inner core find their stagnation radii near 210 km. The source for the near-core asymmetries is the wave crest at 40 km which is forced by the slowly symmetrizing initial condition. When the initial asymmetry reaches its stagnation radius, the forced response at 40 km terminates.
(c) Radar observations

Using a wavelet-analysis technique on radar-reflectivity fields, Tuttle and Gall (1995) studied the nature of small-scale spiral bands observed in two hurricanes. These bands are small-scale bands that are different from the O(100 km)-scale bands, called zones, that typically contain several small-scale bands (e.g. Staff members, Tokyo University 1969; Willoughby et al. 1984). The Hurricane Research Division of the USA National Oceanic and Atmospheric Administration provided Tuttle and Gall with several hours of continuous radar-coverage for hurricanes Andrew (1992) and Hugo (1989). Their analysis revealed detailed near-core structures with curious characteristics. These characteristics and their potential relationship to the current work are noted below.

Outward propagation: Areas in the near-core-region which had enhanced values of radar reflectivity were observed to propagate generally outwards. The radius–time plot (Tuttle and Gall 1995—Fig. 6) shows an outward velocity for these features of approximately 4 m s\(^{-1}\). This is consistent with velocity estimates calculated from Fig. 11 for wave-number two. For the first several wave-packets emanating from the inner core, the velocity is between 2.5 and 3.5 m s\(^{-1}\). Also noted in Tuttle and Gall’s Fig. 6 is a stationary-wave feature at 95 km, about 3 times the radius of maximum winds. For the inner-core wave-number-two asymmetries of Fig. 11, the stagnation radius is also approximately 3 times the radius of maximum winds.

Wave length: The wavelet-analysis wavelength used was 10 km, corresponding to the approximate scale of the reflectivity bands. This analysis tends to suppress scales smaller and larger than 10 km. Close examination of Figs. 5 and 6 of Tuttle and Gall (1995) shows that there are several regions of the storm where the radial wavelength of the inner bands may be of the order of 20 km, or at least different from 10 km. Figure 11 shows that much of the near-core wave-activity in the AB model exhibits a radial wavelength of between 10 and 25 km.

Possible cause: Tuttle and Gall (1995) speculated that the fine structure of the near-core region is consistent with boundary-layer-roll theory (Fung 1977), but also admit further detailed kinematic information is needed for a complete confirmation. Perhaps a more plausible explanation is vortex Rossby-waves. According to the WKB analysis, high azimuthal-wave-number asymmetries associated with small-scale convection in the eye-wall should exhibit very little radial propagation. In contrast, lower azimuthal-wave-number asymmetries (associated with an enhanced frictional stress for storms approaching land (Shapiro 1983) or with a linearly varying horizontal steering flow) should exhibit substantial radial propagation until the waves reach their stagnation radius. The associated vertical-velocity couplet, propagating with the PV wave, will alternately enhance and suppress convection in the moist unstable environment of the near-core region. Figures 2, 3 and 5 show that the long-trailing-spiral structure of the outward-propagating waves is qualitatively similar to the map plots of Fig. 5 of Tuttle and Gall (1995). One can imagine that, when there is spectrally-broad continuous forcing, the clean results exhibited in Figs. 2, 3 and 5 will become noisy (more like that shown in Tuttle and Gall’s Fig. 5).

4. Discussion and Conclusions

Early weather-surveillance radars revealed outward-propagating banded features throughout hurricanes (see, for example, Senn and Hiser 1959). More recent radar studies suggest the existence of outward-propagating small-scale (O(10 km)) bands near the eye wall (Tuttle and Gall 1995). In addition to observations that identify outward propagation, many observational studies lacking sufficient temporal resolution nonetheless show evolving fine structure throughout the vortex (e.g. Barnes et al. 1983; Black and Willoughby
A particularly noteworthy feature of Black and Willoughby's work was the observation that formation of a secondary eye-wall was accompanied by small-scale fluctuations in the tangential wind.

In an effort to understand the observations, we chose perhaps the simplest dynamical model for simulating outward-propagating waves. For azimuthal wave-number one, an exact solution of the linearized non-divergent vorticity equation on an $f$-plane (Smith and Rosenbluth 1990) was found to contain radially propagating vorticity-waves throughout the vortex region which had a non-zero vorticity-gradient. Although the exact solution does not generalize to higher wave-numbers, the numerical-model formulation for higher wave-numbers also exhibited radially propagating waves. Radially propagating PV-waves were further shown to be a robust feature in a shallow-water balance model possessing a variable deformation-radius consistent with real hurricanes. Because these waves appear in balance models, they are not gravity waves. Since these features are present in the simple models employed here, we suspect that radially propagating PV-waves have also been observed in primitive-equation models, but may have been mistaken for gravity waves.

To better understand the potential importance of vortex Rossby-waves in vortex and structure changes, an inviscid wave-mechanics was developed that included group velocity and wave-mean-flow interaction calculations. Using simple WKB theory, expressions for phase and group velocity were derived and radial wave-packet trajectories were consistent with numerical simulations in regions of the vortex where the WKB assumptions were valid. Their dispersive nature and their dependence on the vortex vorticity-gradient justifies their designation as vortex Rossby-waves.

The kinematics of individual wave-packets shows the existence of a stagnation radius which provides a site for wave-mean-flow interaction. Calculation of $\partial \tilde{v}/\partial r$ for vortex monopoles shows that the maximum acceleration generally occurs outside the radius of the initial asymmetry. This effect is found to be more pronounced for radially broad asymmetries. In the presence of convective forcing, this process represents an asymmetric spin-up mechanism that could be operative in the formative stages of tropical cyclones. At later stages in the hurricane life-cycle, this process is consistent with the formation of secondary wind-maxima. Further understanding of the impact of small-scale PV-bands requires an exploration of their eddy-forced secondary circulations as well as their coupling to the boundary layer and convection.

To sum up, the wave mechanics developed here describes spiral bands as vortex Rossby-waves. Within the vortex region, the theory is naturally suited for describing both inner and outer bands. Although freely propagating gravity-inertia waves certainly account for some of the wave structures in a hurricane vortex, their dynamical significance is diminished in the absence of strong damping or critical levels, since they individually satisfy the conditions of the 'non-acceleration' theorem (Schubert 1985). For non-stationary gravity-inertia waves with low vertical-mode number and horizontal wavelengths comparable to the radius of maximum winds, critical levels are unlikely when considering realistic hurricane-vortices. By contrast, as shown here, vortex Rossby-waves are tied to the vortex and must eventually interact with it. This suggests an intimate link between spiral bands and the evolution of hurricanes. Future work will employ fully nonlinear two- and three-dimensional models to further investigate the quasi-linear predictions developed here.

ACKNOWLEDGEMENTS

This research was partially supported by ONR N00014-93-0456 and NSF ATM-9312655. R.T.K. was supported through the AFIT program. M.T.M. thanks Mr Stefan
Llewellyn-Smith for pointing out the Reznik and Dewar (1994) solution during a workshop at Cambridge University in December 1994. M.T.M. also thanks Dr Ed Schneider for pointing out the Dickinson reference at the recent Generations Symposium in Boston, Massachusetts, USA. Assistance of Mr John Persing in integrating the exact solution, supportive feedback from Drs Alexander Khain, Lloyd Shapiro, Hugh Willoughby, Frank Marks and James McWilliams, and many stimulating conversations with Dr Wayne Schubert are gratefully acknowledged.

Appendix A

$t \to \infty$ asymptotics

This appendix examines the $t \to \infty$ asymptotics for the wave-number-one exact solution (3), (4) and (5) for stable swirl-profiles of the form used in subsections 2(d) to 2(h). The asymptotic properties prove useful in determining the final location of the vortex centre. Although Smith and Rosenbluth (1990) considered the case where $\Omega$ had stationary points in the interior of a wall-bounded vortex ($0 < r < a$), their analysis did not address the geophysically relevant case where $\Omega$ has the origin as its only stationary point. Hereafter, we assume that $|\Omega(r)|$ decreases monotonically with $r$, $d\Omega(0)/dr = 0$, and $d^2\Omega(0)/dr^2 \neq 0$. For the purposes of elucidating the large-time behaviour, it is convenient to express the exact solution (3) as

$$\hat{\psi}_1(r, t) = I_1(r, t) + I_2(r, t),$$

where

$$I_1(r, t) = -r \int_r^\infty dr'e^{-i\Omega(r')t}\hat{h}(r'),$$

and

$$I_2(r, t) = -itr \int_r^\infty dr'e^{-i\Omega(r')t}[\Omega(r) - \Omega(r')]\hat{h}(r').$$

Although $\hat{h}(r)$ is indeterminate at $r = 0$, application of L'Hospital's rule yields $\hat{h}(0) = \xi_0(0)/3$. Since $\hat{\psi}_1(0, t) = 0$, $I_1(0, t) = I_2(0, t) = 0$. Therefore, we need consider only $r \neq 0$. The integrals $I_1$ and $I_2$ are now analysed in turn.

$I_1$: Since $\hat{\Omega}$ is non-stationary in the interval $r \ll r' < \infty$, integration by parts with

$$u = \frac{-r\hat{h}(r')}{-it\left(d\Omega/dr'\right)}$$

and

$$dv = -itr\frac{d\hat{\Omega}}{dr'}e^{-i\hat{\Omega}(r')t}dr',$$

yields, as $t \to \infty$,

$$I_1(r, t) = O\left(\frac{1}{t}\right).$$

$I_2$: Integration by parts with

$$u = \frac{-itr[\hat{\Omega}(r) - \hat{\Omega}(r')]\hat{h}(r')}{-it\left(d\hat{\Omega}/dr'\right)}$$

and

$$dv = -itr\frac{d\hat{\Omega}}{dr'}e^{-i\hat{\Omega}(r')t}dr',$$

yields, as $t \to \infty$,
and
\[ \text{d}v = -it \frac{\text{d}\tilde{\Omega}}{\text{d}r'} e^{-i\tilde{\Omega}(r')t} \text{d}r', \]
yields
\[ I_2(r, t) = \left[ \frac{r[\tilde{\Omega}(r) - \tilde{\Omega}(r')]\hat{h}(r') e^{-i\tilde{\Omega}(r')t}}{(\text{d}\tilde{\Omega}/\text{d}r')} \right]_{r' = r}^{r' = \infty} - \int_r^\infty \text{d}r' e^{-i\tilde{\Omega}(r')t} \frac{\text{d}}{\text{d}r'} \left\{ \frac{r[\tilde{\Omega}(r) - \tilde{\Omega}(r')]\hat{h}(r')}{(\text{d}\tilde{\Omega}/\text{d}r')} \right\}. \quad (A.1) \]

The braced term is well behaved for all \( r' \gg r \). Since \( \int_0^\infty \text{d}\rho \rho^2 \tilde{\zeta}_0(\rho) \) is assumed finite (cf. (6)), it follows that for \( \tilde{\Omega} \) profiles behaving like \( \tilde{\Omega}(r') = (\lambda/r^2)(1 + \mathcal{O}(1/r^2)) \) as \( r' \to \infty \) (i.e. vortices possessing finite circulation at infinity),
\[ \frac{\hat{h}(r')}{(\text{d}\tilde{\Omega}/\text{d}r')} = \left( \frac{-1}{2\lambda} \int_0^\infty \text{d}\rho \rho^2 \tilde{\zeta}_0(\rho) \right) \left\{ 1 + \mathcal{O}\left(\frac{1}{r'^2}\right) \right\}, \quad (A.2) \]
whereas
\[ \frac{\tilde{\Omega}(r')\hat{h}(r')}{(\text{d}\tilde{\Omega}/\text{d}r')} = \mathcal{O}\left(\frac{1}{r'^2}\right). \quad (A.3) \]
The integral in (A.1), therefore, converges and the argument used to show that \( I_1 \) vanishes for large times also applies to this integral. At large times then
\[ I_2(r, t) = r\tilde{\Omega} \lim_{r' \to \infty} \left( \frac{\hat{h}(r')}{(\text{d}\tilde{\Omega}/\text{d}r')} e^{-i\tilde{\Omega}(r')t} \right) - \lim_{r' \to \infty} \left( \frac{\tilde{\Omega}(r')\hat{h}(r')}{(\text{d}\tilde{\Omega}/\text{d}r')} e^{-i\tilde{\Omega}(r')t} \right) + \mathcal{O}\left(\frac{1}{t}\right) \quad (A.4) \]
Inserting (A.2) and (A.3) into (A.4) gives, as \( t \to \infty \),
\[ \tilde{\psi}_1(r, t) = \left\{ -\frac{1}{2\lambda} \int_0^\infty \text{d}\rho \rho^2 \tilde{\zeta}_0(\rho) \right\} r\tilde{\Omega}(r), \quad (A.5) \]
confirming one's expectation that the wave-number-one solution for \( \tilde{\psi}_1 \) asymptotes to the pseudo-mode \( r\tilde{\Omega} \), corresponding to a translation of the vortex. The asymptotic behaviour displayed in Fig. 2 has been verified to be consistent with (A.5) using \( \lambda = 2 \) (see section 2(d)).

**APPENDIX B**

**Numerics**

This appendix summarizes the numerical techniques employed to obtain the results described in section 2.

* (Note added in proof) An alternative derivation of (A.5) is given by (Llewellyn-Smith 1996) using the method of steepest descent. The steepest descent technique also permits a \( t \to \infty \) analysis of circular vortices with zero circulation. In this case, the streamfunction amplitude grows slowly with time (see Llewellyn-Smith (1996) for details). No such behaviour was observed in the shallow-water model results reported here, however. The slow growth predicted by the non-divergent model is thus likely to be an artefact of its possessing an infinite Rossby-radius of deformation which enables the negative vorticity flow at infinity to influence the positive vorticity flow of the near-field. The vortex then moves like a dipole initially.
Exact solution: The exact solution was evaluated using trapezoidal quadrature. Convergence was verified by doubling and tripling the number of points. The removable singularity in $\hat{h}(r)$ at $r = 0$ was handled by incorporating two terms in the Taylor-series representation near the origin. The radial interval required by the Taylor series was selected to provide the smoothest transition between the integral and series representation of $\hat{h}(r)$. This interval was found to be of the order of three to six grid points. An outer radius of $r_{\text{max}} = 40$ was determined adequate for the initial conditions chosen. For the results shown in subsection 2(d), the radial grid increment $\delta r = 1/60$.

Higher wave-numbers: The solution to (2) for higher azimuthal wave-numbers ($n > 1$) was obtained by time-stepping the Fourier vorticity-amplitude and inverting the corresponding Poisson equation for the streamfunction. Since the streamfunction must behave equidimensionally at both small and large $r$, boundedness requires that $\psi \approx r^n$ for small $r$, while $\psi \approx r^{-n}$ for large $r$. The transformed Poisson-equation was discretized using second-order centred differences in the radial direction and was solved with a tri-diagonal matrix solver. A minimum radius of $r_{\text{min}} = 10^{-3}$ and a maximum radius of $r_{\text{max}} = 20$ were determined adequate for the results presented in section 2. The results were verified to be insensitive to these values. The vorticity was forecast using the highly accurate fourth-order Runge–Kutta method. Typical time and space increments used were $\delta t = 1/20$ and $\delta r = 1/60$ respectively. These grid parameters fall well below the observed Courant–Friedrichs–Lewy (CFL) threshold and adequately resolve the enstrophy cascade for the duration of the simulations shown. Consequently, explicit diffusion was unnecessary. As a test of the numerical model, the exact solution results of subsection 2(d) were reproduced.

WKB wave-packet trajectories: The radial trajectory of an isolated wave-packet is obtained from a time integral of the radial group-velocity (17) (cf. Tung 1983):

$$ r(t) = R + \frac{\xi_0'}{R \Omega_0} \left[ \frac{1}{k_0^2 + (n^2/R^2)} - \frac{1}{(k_0 - nt \Omega_0)^2 + (n^2/R^2)} \right]. \tag{B.1} $$

In (B.1) the notation is as defined in subsection 2(f), and $r(t)$ denotes the instantaneous radial position of the wave packet. As long as $kR \gg 1$, (B.1) was found to predict, quite adequately, the radial trajectories and stagnation radii for high azimuthal wave-numbers ($n \gg 3$) and sufficiently localized initial conditions. For lower azimuthal wave-numbers, however, (B.1) systematically predicted stagnation radii outside those observed in the numerical results. In these circumstances, radial trajectories are obtained through numerical quadrature by taking into account the changing basic-state variables. All results shown in section 2 employ the latter method.

References

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Title</th>
<th>Journal/Proceedings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flierl, G. R., Stern, M. E. and</td>
<td>1983</td>
<td>The physical significance of modons: Laboratory experiments and general integral constraints.</td>
<td><em>Dyn. of Atmos. and Oceans</em>, 7, 233–263</td>
</tr>
<tr>
<td>Whitehead, J. A.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fung, I. Y.</td>
<td>1977</td>
<td>The organization of spiral rainbands in a hurricane.</td>
<td>Massachusetts Institute of Technology, USA. Ph.D. Thesis</td>
</tr>
<tr>
<td>Kallenbach, R. J. and</td>
<td>1995</td>
<td>'Symmetrization and hurricane motion in an asymmetric balance model'.</td>
<td><em>Proceedings of 21st Conference on Hurricanes and Tropical Meteorology</em>, Miami, Florida, USA</td>
</tr>
<tr>
<td>Montgomery, M. T.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MacDonald, N. J.</td>
<td>1968</td>
<td>The evidence for the existence of Rossby-like waves in the hurricane vortex.</td>
<td><em>Tellus</em>, XX, 138–150</td>
</tr>
<tr>
<td>May, P. T.</td>
<td>1995</td>
<td>'The organization of convection in the rainbands of tropical cyclone Laurence'.</td>
<td><em>Proceedings, 21st Conference on Hurricanes and Tropical Meteorology</em>, Miami, Florida, USA</td>
</tr>
<tr>
<td>McWilliams, J. C.</td>
<td>1989</td>
<td>Geostrophic vortices.</td>
<td>*Proc. Int. School of Physics 'Enrico Fermi', Italian Physical Society</td>
</tr>
<tr>
<td>Yavneh, I.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Melander, M. V., McWilliams, J. C.</td>
<td>1987</td>
<td>Axisymmetrization and vorticity-gradient intensification of an isolated two-dimensional vortex through filamentation.</td>
<td><em>J. Fluid Mech.</em>, 178, 137–159</td>
</tr>
<tr>
<td>and Zabusky, N. J.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1990b</td>
<td>Boundary layer structure and dynamics in outer hurricane rainbands. Part II: Downdraft modification and mixed layer recovery.</td>
<td><em>J. Atmos. Sci.</em>, 118, 918–938</td>
</tr>
<tr>
<td>Schubert, W.</td>
<td>1985</td>
<td>'Wave, mean-flow interactions and hurricane development'.</td>
<td><em>Tellus</em>, 37A, 140–141</td>
</tr>
<tr>
<td>Montgomery, M. T.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dietachmayer, G.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
VOXET ROSSBY-WAVES IN HURRICANES


