Broadening of convective cells

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SUMMARY

The turbulent convection in a dry Boussinesq fluid above a heated surface is studied in a domain of large aspect ratio and long duration. Large-eddy simulations are used to investigate the influence of the dynamical character of the lower boundary and the effect of cooling at the top on the growth of convective cells. The aim is to identify physical mechanisms that lead to the broadening of convective cells even under conditions where moisture processes play no role. Cooling at the top produces a vertically uniform heat-flux, and it acts to enhance the total kinetic energy of the whole flow. Essentially, there are two competitive effects: first, the prescribed heat-flux at the upper (lower) boundary produces very warm (very cool) fluid that is forced to rise (sink). The resulting large temperature-fluctuations are transported by large-scale motions and cause broad temperature-variances independent of height. On the other hand, the strong turbulent mixing (mainly near the boundaries where the turbulent kinetic energy is maximum) tries to homogenize the flow structure in such a way that the resulting temperature-distribution is uniform. Except directly close to the walls, horizontal and vertical temperature-gradients are reduced. Additionally, a trend to form an organized large-scale horizontal drift in one direction near the bottom and in the opposite direction near the top was found for runs without surface friction. This horizontal streaming motion has two effects: firstly, the turbulent mixing is enhanced due to larger shear and, secondly, separated thermals approach faster and are able to merge more easily, forming gradually growing cells. An adiabatic upper boundary condition leads to a heat-flux profile that is linear and decreasing with height, and to a moderate temporal growth of thermal structures. A considerable scale-reduction of the temperature structures occurs because of friction at the lower surface.

KEYWORDS: Large-eddy simulation Radiative cooling Turbulent convection

1. INTRODUCTION

There is enormous interest in expanding large-eddy simulations (LESs) from the classical application in the dry convective boundary-layer (CBL), as by Mason (1989) and Schmidt and Schumann (1989), to other, more realistic, cases. These include the treatment of cloud processes and inhomogeneous lower boundaries produced by heterogeneous land surfaces; they may involve interactions between the CBL and dynamical processes in the overlying free atmosphere (such as internal gravity-waves) or attempt to predict the correct gradual transition from microscale turbulence to mesoscale flow regimes as, for example, in cold air outbreaks over oceans. In order to simulate these processes, it is necessary to know how the CBL evolves in larger domains and for longer times than in previous LESs.

Fiedler and Khairoutdinov (1994), hereafter FK94, investigated the cell broadening of dry turbulent convection in a fluid which was confined between frictionless and poorly conducting boundaries in a domain with aspect ratio 16:1. Their main conclusion was that broad convection cells dominate the temperature field but are not present in the vertical-velocity field. Approximately 100 convective time units $t_*$ are required for the dominance of one convection cell in the domain. FK94 applied their LES results to the growth of mesoscale convective cells in cold air flow over oceans. They estimated a mean diameter of simulated cells after 100 $t_*$ that corresponds to observations of mesoscale convective cells in the atmosphere. Unfortunately, FK94 gave no physical causes for the observed cell-growth or for the different behaviour of the velocity and the temperature fields.

In the present paper, large-eddy simulations are used to investigate the influence of the dynamical character of the lower boundary and the effect of cooling at the top of the domain on the growth of convective cells. The aim is to highlight the differences between the runs and to identify the physical processes responsible for the broadening of convective cells (even under conditions where moisture processes play no role). The

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main technical difference between FK94 and the present simulations is in the choice of subgrid-scale (SGS) model. Whereas FK94 used a gradient relation for parametrized SGS fluxes with a rather large Smagorinsky constant \( C_{Sm} = 0.4 \), the present work uses an SGS model based on an equation of the SGS kinetic energy as described by Krettenauer and Schumann (1992). The effective Smagorinsky constant of this closure is estimated to be 0.13 (Andrén et al. 1994).

Section 2 briefly introduces the numerical model including the SGS scheme, and discusses the boundary conditions. Sections 3 and 4 present the results and discuss them. Section 5 draws conclusions.

2. MODEL FORMULATION

LESs are used to investigate the broadening of convective cells in a Boussinesq fluid which is confined between two horizontal rigid boundaries. The numerical scheme integrates the full primitive equations of motion in their non-hydrostatic form, together with the thermodynamic equation, in three dimensions and as a function of time (see Eqs. (2) to (4) in Schmidt and Schumann (1989)). Here, the subgrid-scale fluxes are determined by means of a first-order closure according to

\[
F_{ij} = -K_M \left( \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right) + \frac{2}{3} \delta_{ij} e, \quad Q_i = -K_H \frac{\partial \theta}{\partial x_i},
\]

where \( u_i \) are the velocity components in the three coordinate directions \( x_i \) \((i = 1, 2, 3)\), \( \delta_{ij} \) is the Kronecker symbol and \( \theta \) denotes the temperature. The diffusivities \( K_M \) and \( K_H \) are functions of the mixing length \( \ell \) and the square root of the SGS kinetic energy \( e \) according to

\[
K_M = c_v e^{1/2} \ell \quad \text{and} \quad K_H = c_r e^{1/2} \ell.
\]

The SGS kinetic energy \( e \) is calculated by a separate transport-equation (Eq. (5) of Schmidt and Schumann (1989)) and the mixing length is \( \ell = \text{min}\{(\Delta x + \Delta y + \Delta z)/3, 0.854 z\} \). The model constants in Eq. (2) are \( c_v = 0.057 \) and \( c_r = 0.136 \). The results presented in the next sections are not dominated by the choice of the SGS parameters. In the following, all results are normalized by the scales of height, temperature, time and convective velocity,

\[
H, \quad \theta_s = Q_s/w_s, \quad t_s = H/w_s, \quad w_s = (\beta g Q_s H)^{1/3},
\]

respectively. Here, \( g \) is the acceleration under gravity, \( Q_s \) is the prescribed vertical flux of heat at the surface and \( \beta \) is the volumetric expansion coefficient. Details of the numerical implementation are those used by Krettenauer and Schumann (1992). The side length \( L \) of the domain is large in terms of the model depth \( H \) and is the same in the \( x \) and \( y \) directions. Table 1 lists essential parameters of three simulations. Run A simulates the CBL heated from below by a constant heat-flux \( Q_s \) and with an adiabatic and frictionless rigid lid. This is an idealized representation of the classical CBL as investigated by Mason (1989) and Schmidt and Schumann (1989). In run B, the friction at the lower surface is neglected but all other parameters are the same as those in run A. Run C is the same as simulation B, except that the top of the domain is cooled at the same rate as the bottom is heated. This simulation repeats FK94's case number 1 with an enhanced vertical resolution. For all runs, cyclic boundary conditions are used at the lateral boundaries. The simulation time is 100 convective time-units \( t_s \).
TABLE 1. ESSENTIAL PARAMETERS OF THE RUNS

<table>
<thead>
<tr>
<th>Run</th>
<th>$L/H$</th>
<th>$\Delta x/H$</th>
<th>$\Delta z/H$</th>
<th>Bottom boundary</th>
<th>Top boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16</td>
<td>1/16</td>
<td>1/16</td>
<td>heating at constant $Q_s$, no-slip</td>
<td>adiabatic and free-slip</td>
</tr>
<tr>
<td>B</td>
<td>16</td>
<td>1/8</td>
<td>1/16</td>
<td>heating at constant $Q_s$, free-slip</td>
<td>adiabatic and free-slip</td>
</tr>
<tr>
<td>C</td>
<td>16</td>
<td>1/8</td>
<td>1/16</td>
<td>heating at constant $Q_s$, free-slip</td>
<td>cooling at constant $Q_s$, free-slip</td>
</tr>
</tbody>
</table>

The domain length $L/H$ and grid spacing $\Delta x/H$ are the same in the $x$ and $y$ directions. All simulations run until $t = 100t_*$. The time-step $\Delta t = 0.005t_*$.  

3. RESULTS

Figures 1 and 2 show horizontal sections of the fields of vertical velocity and temperature, which allow the structure of the flows in runs A, B, and C at $10t_*$ and $70t_*$ to be compared. The fields of vertical velocity and temperature are shown as deviations of local values from their horizontal mean and normalized as dimensionless fluctuations $w'/w_*$ and $\theta'/\theta_*$.  

(a) Cell broadening

In Fig.1(a) the average cell-size and the typical width of individual updraughts are larger in run C than in run A. The $w'$ field of all runs is organized in cells where cold compensating downdraughts are surrounded by warm upward motions. Mostly, these cells are closed, but in runs B and C some of the structures are open and a few of them are organized in straight elongated coherent patterns. At this time, the corresponding positive $\theta'$ field is strongly correlated with the upward motion and possesses the same geometrical pattern as the $w'$ field.

At $t = 70t_*$ (Fig. 2), the cell structures of the $w'$ and $\theta'$ fields are destroyed. Although some cells are still recognizable, the majority of the $w'$ field consists of small-scale, randomly distributed updraughts and coherent elongated streaks, extending over large parts of the $xy$ plane. The orientation of these streaks depends on the direction of the local and instantaneous horizontal velocity field (see discussion of Fig. 9). In contrast to the vertical velocity field, the $\theta'$ structures have become large-scale. The warm and cold regions have merged and extend coherently over large portions of the domain. The individual runs differ in characteristic cell size. Compared with run C, runs A and B have narrower updraughts (less than one $H$) and smaller positive $\theta'$ patterns. In run C, the plane $z = 0.5H$ seems to be equally divided between warm and cold regions as expected by the symmetry of forcing. The broadening is less pronounced in runs A and B: although the separation of the $\theta'$ regions becomes larger, the structures themselves remain small-scale.

Typical length-scales of the flow regime can be estimated by using the autocorrelations

$$\rho_f(r, z, t) = \frac{\int dx_H f'(x_H, z, t) f'(x_H + r, z, t)}{\int dx_H f'(x_H, z, t) f'(x_H, z, t)}, \quad x_H = (x, y),$$  \hspace{1cm} (4)$$

where $f$ denotes $w$ or $\theta$. We use two length scales to characterize the scale of the flow structures: $\ell_f$ is derived by estimating the distance over which $\rho_f$ falls to $1/e$ of its value, and measures the typical width of the structures (microscale), whereas $r_0^f$ (the first zero-crossing of $\rho_f$) gives the characteristic separation distance of these structures (macroscale).
Figure 1. Horizontal cross-sections of positive values of updraughts $w'$ and potential temperature $\theta'$ at mid-height, normalized by $w_*$ and $\theta_*$ respectively, for $t = 10t_*$. Minimum values of the contour lines are $0.2w_*$ and $0.5\theta_*$, and increments $0.4w_*$ and $1.0\theta_*$. (a) $w'/w_*$, run A; (b) $\theta'/\theta_*$, run A; (c) $w'/w_*$, run B; (d) $\theta'/\theta_*$, run B; (e) $w'/w_*$, run C; (f) $\theta'/\theta_*$, run C. The dimensionless fluctuations $w'/w_*$ and $\theta'/\theta_*$ are calculated as deviations of local values from the horizontal means.
Figure 2. As Fig. 1, but for $t = 70t_s$. (a) $w'/w_s$, run A; (b) $\theta'/\theta_s$, run A; (c) $w'/w_s$, run B; (d) $\theta'/\theta_s$, run B; (e) $w'/w_s$, run C; (f) $\theta'/\theta_s$, run C.
Figure 3. Evolution of the microscale $r_0$ and macroscale $\ell$ estimated from the autocorrelation functions $\rho_w$ and $\rho_\theta$; (see Eq. (4) in text). The different curves denote height levels: $z = 0.125H$ (full lines), $z = 0.25H$ (short dashes), $z = 0.50H$ (long dashes) and $z = 0.75H$ (dash-dots): (a) $w$, run A; (b) $\theta$, run A; (c) $w$, run B; (d) $\theta$, run B; (e) $w$, run C; (f) $\theta$, run C.

Figure 3 presents the time histories of $\ell$ and $r_0$ for the vertical velocity and temperature fields. Consider first the behaviour of the $w'$ field. Generally, $\rho_w$ falls to zero in a short spatial lag $r_0$. In an initial period, $r_0^{\omega}$ increases in time and attains typical values of $H$ for runs A and B, and about $2H$ for run C. For separation distances larger than $r_0^{\omega}$, the autocorrelation $\rho_w$ is nearly zero and, therefore, the vertical velocity field behaves irregularly, in a way typical of turbulent processes. The microscale $\ell_w$ is much shorter ($<0.5H$) than $r_0^{\omega}$ and its magnitude does not show similar temporal oscillations. It increases with height up to mid-height of the domain and decreases above it. The $\ell_w$ values of run C are about 40% larger than those for runs A and B.

The corresponding length-scales of the temperature field are generally larger than those of the vertical velocity field. The microscale $\ell_\theta$, as well as the macroscale $r_0^{\theta}$, shows a strong temporal increase. In the period up to 50 $t_*$, the strongest growth in $r_0^{\theta}$ occurs for run C (up to 6 $H$), whereas runs A and B reach values between 3.5 and 4 $H$. By 100 $t_*$, $r_0^{\theta}$ reaches nearly time- and height-independent values ($\approx 3.5H$ and 6 $H$) for runs A and C, whereas run B shows a late increase in $r_0^{\theta}$ at 85 $t_*$. By contrast to the macroscale $r_0^{\theta}$, $\ell_\theta$ increases with increasing height (strongly for runs A and B, weakly for run C) and, thus,
Figure 4. Power spectra as functions of dimensionless horizontal wave-number $k = 2\pi/(\lambda/H)$ for heights in the domain of $z = 0.125H$ (full lines), $z = 0.25H$ (short dashes), $z = 0.5H$ (long dashes) and $z = 0.75H$ (dash-dots). The lower set of curves in each panel is for a starting time of $t = 20t_*$ and the upper set for a starting time of $t = 90t_*$; for clarity, the lower set of curves in each panel has been displaced by multiplying values by 0.1. Values in all curves are temporal averages over ten convective time units. (a) Total kinetic energy $\Phi(k)$, run A; (b) total kinetic energy $\Phi(k)$, run C; (c) field of temperature $\Phi_\theta(k)$, run A; (d) field of temperature $\Phi_\theta(k)$, run C.

its growth rate depends on altitude. The largest values of $\ell_\theta$ are obtained for run C (about 3.8 $H$ to 4 $H$ for times greater than 70 $t_*$); the smallest are produced by run A (between 0.5 $H$ and 2 $H$).

Another remarkable feature of the autocorrelations is the strong similarity between the shape of $\rho_\theta$ and $\rho_u$ at early times $t < 50t_*$ in run C (not shown). In particular, the spatial oscillations of $\rho_u$ near the lower boundary occur at the same wavelength and phase, and the same zero-crossing lag $r_0$, as those for $\rho_\theta$. Additionally, values of $\ell_\theta$ near the boundaries grow at the same rate as $\ell_\theta$. In the middle of the domain, the length-scale $r_0^\prime$ remains small and amounts to approximately 2 $H$.

To complete the discussion of the cell broadening, Fig. 4 shows the power spectra of the total kinetic energy $\Phi(k)$ and of the temperature field $\Phi_\theta(k)$; $k = 2\pi/(\lambda/H)$ is the dimensionless horizontal wave-number. Results of runs A and C are plotted as temporal averages at two stages of the flow evolution and at different heights. The characteristics of $\Phi(k)$ of run A in both periods are those of a flow field in a domain large enough to cover the essential dynamical scales and with a resolution fine enough to obtain an energy decay at high wave-numbers close to the $k^{-5/3}$ law. Most of the energy is at scales between 1 $H$ and 3.5 $H$. Scales larger, and smaller, than this are less energetic. In run C, the overall
amplitude of $\Phi(k)$ is much higher and the spectra show a pronounced peak at $k \approx 0.8$ (a wavelength close to $8H$) as a result of the predominance of the vertical velocity field. In contrast to run A, the energy content at large scales increases drastically in time. Whereas spectra of $\Phi(k)$ show an energy maximum at scales smaller than half the box length, spectra of $\Phi_\sigma(k)$ are maximum at the smallest possible wave-number for $t > 40t_\ast$ (run C) and $t > 70t_\ast$ (run A). Interestingly, for smaller scales ($\lambda < 2H$), the shape and the height dependence of $\Phi_\sigma(k)$ remains temporally the same for both simulations.

Summarizing the cell broadening documented in Figs. 1, 2 and 3, we see that the most rapid increase of cell size occurs for run C. The final cell size is about 50% larger than in run A.

(b) Turbulent mixing

The different boundary conditions between the runs result in different amounts and distributions of turbulent kinetic energy (TKE). Table 2 lists statistical properties derived from the time histories of the domain-averaged total TKE for each run. They characterize the mean value $\langle E \rangle$ in five selected segments, the temporal change of $E(t)$ and the strength of the TKE fluctuations.

Generally, we can distinguish two different periods. The initial period is characterized by a steep increase of $E(t)$ until $t \approx 5t_\ast$, when a first maximum in $E(t)$ occurs. At this time, rising thermals have reached the rigid top. There, the vertical motion is converted into horizontal motions and the thermals return down into the bulk of the fluid. This process retards other warm updraughts and leads to a temporal decrease in $E(t)$. The subsequent increase resulting from further rising energetic thermals is monotone and $E(t)$ achieves a quasi-steady state after approximately $20t_\ast$. In this second period, $E(t)$ fluctuates as a result of the combined action of up- and downdraughts. As expected, run C shows twice as much TKE as runs A and B, since it is forced twice as strongly. Although the mean values $\langle E \rangle$ of all runs increase slightly with time (5%, 1%, and 8% from segment 2 to 5 for runs A, B and C respectively) the temporal change $\Delta E/\Delta t$ is generally weak in all runs and has no preferred sign at late times. The r.m.s. value $\sqrt{\langle E^2 \rangle}$ is smallest in run A; stronger fluctuations of $E(t)$ occur in runs B and C. The volume averaged SGS-energy does not vary with time and its ratio to the total TKE is approximately 20% in each run.

A direct consequence of the higher TKE of run C is the amplification of the subgrid-scale diffusion, as $K_M$ and $K_H$ are proportional to the SGS kinetic energy (see Eq. (2)).

| TABLE 2. STATISTICAL ESTIMATES OF THE TEMPORAL DEVELOPMENT OF THE DOMAIN-AVERAGED TOTAL TKE FOR ALL RUNS |
|---|---|---|---|
| $t_\ast$ | $\langle E \rangle$ | $\sqrt{\langle E^2 \rangle}$ | $\Delta E/\Delta t$ |
| run A | run B | run C |
| 1 | 0.531 | 3.4 | 10.5 | 0.475 | 3.7 | 12.0 | 0.944 | 5.9 | 19.6 |
| 7 | 0.565 | 2.2 | 1.6 | 0.539 | 1.3 | 4.3 | 1.011 | 3.5 | 1.3 |
| 3 | 0.588 | 1.3 | 1.2 | 0.556 | 0.9 | 0.0 | 1.069 | 4.3 | 2.7 |
| 4 | 0.596 | 1.6 | 0.3 | 0.543 | 1.9 | 3.3 | 1.074 | 2.5 | 1.1 |
| 5 | 0.599 | 1.0 | 1.3 | 0.545 | 2.5 | 1.1 | 1.185 | 2.9 | 9.3 |

The individual time histories are divided into five segments, each $\tau = 10t_\ast$ long, and linear regression analysis using least squares is carried out in these segments. $\langle E \rangle$ denotes the arithmetic mean value of $E$ in each period $i = 1, ..., 5$. $E(t)$ is the calculated linear regression line, whereby its variance and slope are made dimensionless by $\langle E \rangle$ and by the length of the sample period $\tau$ and they are given in %. The periods start at $t_1 = 10t_\ast$, $t_2 = 30t_\ast$, $t_3 = 50t_\ast$, $t_4 = 70t_\ast$, and $t_5 = 90t_\ast$ respectively.
Thus, the (subgrid- and resolved-scale) turbulent exchange is enhanced near the upper and lower boundaries of the domain where TKE is maximum. Furthermore, enhanced lateral entrainment of differently buoyant air into the active up- and down-draughts leads to an erosion of their edges and to a reduction of horizontal temperature-gradients. If we take the velocity variances as a measure of the intensity of turbulent exchange (mixing), we can define its anisotropy $\alpha$ as the ratio of the total horizontal-velocity variances to the vertical-velocity variances where $\alpha = 0.5 \left( \bar{u}^2 + \bar{v}^2 \right) / \bar{w}^2$. For isotropic turbulence, $\alpha$ is 1; for predominantly horizontal (vertical) mixing $\alpha > 1 \left( < 1 \right)$. Figure 5 shows $\alpha$ for the three individual runs as time evolution at four heights (two in the bulk of the domain, the other two directly above/beneath the lower/upper boundary). Vertical mixing is dominant in the initial period at all heights and for all runs. For $t > 20 \, t_\ast$, runs A and B (which behave similarly) show pronounced horizontal mixing near the boundaries, whereas, in the bulk of the domain, vertical exchange processes are dominant. The physical picture is different in run C: here, the horizontal mixing dominates at all heights except in the middle of the
domain where the turbulent mixing is nearly isotropic. It should be noted that the diffusion coefficient \( K_H \) in the transport equation of \( \theta' \) is a factor of 2.4 (= \( c_r/c_c \)) larger than that for momentum and it leads therefore to a stronger reduction in gradients of \( \theta' \). Eventually, as a result of these turbulent mixing processes, horizontal gradients of \( \theta' \) are reduced, and broader coherent warm and cold areas with less heterogeneity appear.

The double forcing of run C also enhanced fluctuations of velocity and temperature. These lead to broader variances of the probability density functions (p.d.f.s), and finally to broader structures as plotted in Figs. 1 and 2. Table 3 lists the variance \( \sigma^2_f = \bar{f^2}/f_s^2 \), the skewness \( S_f = \bar{f^3}/(f^2)^{1.5} \), and the flatness factor \( F = \bar{f^4}/(f^2)^2 - 3 \) of the p.d.f.s for the vertical velocity and the temperature at \( z = 0.5 \) \( H \). Here, we concentrate on the differences between runs A and C since the results of runs A and B are very similar.

The p.d.f.s of \( w' \) in run A are positively skewed with a negative mode for all heights. The positive (negative) portion under the p.d.f. curves gives the fractional area that rising (sinking) fluid occupies. Thus, \( S_w > 0 \) means narrower updrafts and broader downdrafts. The skewness increases with height up to \( z \simeq 0.5 \) \( H \) as a result of the increasing importance of the turbulence generation by buoyancy effects (vertical mixing and no influence of near-wall shear) and is nearly constant above. The variances of \( w' \) are parabolic in shape and have a peak value of \( \sigma_w^2 \approx 0.42 \cdots 0.45 \) at \( z = 0.5 \) \( H \). The flatness factor, weighting the contribution of the p.d.f. in the tails beyond \( \sigma_w \), is small but increases in time.

The corresponding p.d.f.s of run C are different in many ways. First, the distributions are broader; the magnitude of \( \sigma_w^2 \) is roughly 20% larger below and 70% larger above \( z = 0.5H \) as compared with run A. Second, the distributions are more symmetric. Therefore, the resulting small skewness (generally one order of magnitude smaller than in run A) must be caused by an areal equipartition between upward and downward motions. As in case A, the temporal evolution is weak: the p.d.f.s of \( w' \) remain self-similar, only the maximum value grows; i.e., the distribution becomes narrower. The variances of the p.d.f.s of \( \theta' \) in all runs are larger than those of \( \sigma_w \) and they increase drastically in time. Table 3 shows that the distributions of \( \theta' \) in run C are essentially broader than those in run A. As it was of \( w' \), the skewness of the p.d.f.s of \( \theta' \) is very small.

In order to get an insight into the mechanisms of heat transport, consider the vertical profiles of the components of the resolved-scale heat-flux of runs A and C at \( t = 50 \tau_z \) in Fig. 6(a,b). We distinguish four classes: warm and upward (\( \theta' > 0, w' > 0 \)), cold and upward (\( \theta' < 0, w' > 0 \)), cold and downward (\( \theta' < 0, w' < 0 \)), and finally warm and downward (\( \theta' > 0, w' < 0 \)). Figure 6 (c,d) also shows the relative frequency with which each of

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**Table 3. Moments for the p.d.f.s of \( w' \) and \( \theta' \) at \( z = 0.5 \) \( H \)**

<table>
<thead>
<tr>
<th>Period</th>
<th>( w' )</th>
<th>( \theta' )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>run A</td>
<td>run B</td>
</tr>
<tr>
<td></td>
<td>variance</td>
<td>skewness</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>0.454</td>
<td>0.396</td>
</tr>
<tr>
<td></td>
<td>0.621</td>
<td>0.617</td>
</tr>
<tr>
<td></td>
<td>0.045</td>
<td>0.089</td>
</tr>
<tr>
<td>( t_2 )</td>
<td>0.425</td>
<td>0.375</td>
</tr>
<tr>
<td></td>
<td>0.599</td>
<td>0.609</td>
</tr>
<tr>
<td></td>
<td>0.149</td>
<td>0.188</td>
</tr>
</tbody>
</table>

Values are averages over two periods: \( t_1 \), from 20 to 30 \( \tau_z \); \( t_2 \), average from 80 to 90 \( \tau_z \).
the four classes occurred in runs A and C. The frequency with which a class is observed during a run is denoted by \( n_i \); the relative frequency is expressed as the fraction \( n_i/n \) of the total frequency of occurrence \( n \) of all four classes in each run. The fractions \( n_i/n \) for each class of up- or down-draught are calculated for each run separately and may be regarded as the fraction of horizontal area in that run occupied by warm or cold up- or down-draughts comprising each of the four classes. From now on these fractions \( n_i/n \) will be termed 'area fractions'.

The total heat-flux decreases linearly with height for run A. Positive resolved-scale contributions to \( \overline{w'\theta'}_{tot} \) are from warm updraughts and compensating sinking of cold fluid. The magnitude of the negative contributions (cold updraughts and warm downdraughts) is small but increases with height. Because downdraughts occupy more than half the horizontal area, it follows that ascending flow has a larger \( w' \) on average than the descending flow and, hence, the dominating convection regime is organized into narrow quickly-rising
thermals and broad areas of slower subsidence as discussed by Schmidt and Schumann (1989). The total heat-flux of run C is uniform with height and the individual contributions of the four classes to $\overline{w'\theta'_\text{tot}}$, as well as their respective area fractions, show a weak height-dependence. The positive heat-flux components are dominated by positively and negatively buoyant thermals with an individual average magnitude of 0.6 $Q_s$. The magnitude of each negative contribution is approximately $-0.1 Q_s$; i.e. the sum over the four classes gives approximately $Q_s$ at all levels. The area fraction is roughly 0.3 for positive and 0.2 for negative components of $\overline{w'\theta'_\text{tot}}$. Obviously, there is an equipartition between warm and cold regions whereby active thermals (rising warm and sinking cold air) cover roughly 60% and negative components of $\overline{w'\theta'_\text{tot}}$ cover roughly 40% of the total horizontal area.

The co-spectra of the vertical-velocity and temperature fluctuations (vertical heat-flux spectra $\Phi_{w\theta}(k)$ in Fig. 7) reveal the essential scales responsible for the heat transport in both simulations. For run A, the functional shape of $\Phi_{w\theta}(k)$ and its height dependence agree with observations (Kaimal et al. 1976—Fig. 8) as well as with other LES results (Schmidt and Schumann 1989—Fig. 12). The largest values of $\Phi_{w\theta}$ occur near the lower surface, at wavelengths between $0.7 \ H$ and $1.5 \ H$. With increasing height, the spectral amplitude becomes smaller and the maximum peak is shifted towards larger scales of motion. Hence, heat is mainly carried upwards by small-scale motions near the bottom and larger scales in the upper half of the domain. The scale distribution of the heat transport in run C is quite different from that in run A, and changes dramatically with time. At early times, the maximum spectral amplitude occurs at $k = 0.8$ ($\approx 8 \, \text{H}$) independent of altitude. This maximum is produced mainly by peaks in the spectra of $w'$ and $\theta'$ at the same wave-number. The spectral amplitude decays exponentially at smaller scales ($k > 8$). At later times, the peak value at $k = 0.8$ has disappeared and $\Phi_{w\theta}(k)$ shows a different dependence on the scale of motion for different heights. Large scales ($k < 2$) contribute most to $\Phi_{w\theta}$ in the bulk of the domain, whereas smaller-scale motions dominate near the walls. The exponential decay and its height dependence for larger wave-numbers do not change with time. $\Phi_{w\theta}(k)$ is nearly constant up to $k \approx 4 \ldots 5$, i.e. all wavelengths $\lambda > H$ have nearly the same contribution to $\Phi_{w\theta}(k)$ for run C.
Figure 8. Horizontally averaged vertical profiles for successive instants during run C \( t = 47 \tau_s \) — full lines, \( t = 48 \tau_s \) — short dashes, \( t = 49 \tau_s \) — long dashes, and \( t = 50 \tau_s \) — dash-dots. (a) Magnitude of the horizontal velocity vector \( V_h \), normalized by \( u_* \), see section 2 of text; (b) flow direction \( \alpha_h \), see subsection 3(c) of text; 180° and -180° denote flow in the positive direction of \( y \); 0° denotes flow in the negative direction of \( y \); -90° denotes flow in the positive direction of \( x \); +90° denotes flow in the negative direction of \( x \).

(c) Large-scale drift

Figure 8 shows the horizontally averaged vertical profiles of the magnitude of the horizontal velocity vector \( V_h = \sqrt{\overline{u'^2} + \overline{v'^2}} \) and the flow direction \( \alpha_h = \arctan(\overline{v}/\overline{u}) \) for a sequence of times ending at \( t = 50 \tau_s \) for run C. The velocity profiles in Fig. 8 are different from those expected for a well mixed CBL: there is a large vertical shear with typical values of 0.4 \( u_* / H \) across the whole depth of the boundary layer. The maximum shear occurs approximately 0.25 \( H \) away from each wall and can reach local values up to 0.6 \( u_* / H \). Furthermore, there is a strong directional shear of the velocity field. The mean-flow direction \( \alpha_h \) just below the top boundary is opposite to that just above the ground. Interestingly, the mean-flow direction at all levels changes systematically with time; i.e. between times \( t \) and \( t + dt \) the direction of the horizontal flow near the upper and lower wall has changed by the same magnitude and in the same direction.

To clarify this phenomenon, Fig. 9 shows the instantaneous vertical \( (w) \) and horizontal \( (u) \) velocity fields near the bottom surface, at mid-height and just beneath the top boundary at \( t = 50 \tau_s \). The field of \( u \) near the upper and lower boundaries is characterized by large-scale coherent regions of either positive or negative sign. The regions of positive \( u \) near the top of the domain correspond to regions of negative \( u \) near the bottom. The patterns of \( u \) at mid-height occur at smaller scale and are aligned perpendicular to the main fronts near the boundaries. In this way, large-scale shear acts in two directions. Formerly-nearby thermals (see, for example, the updrafts in the \( u' \) field at \( z = 0.5 H \) of Fig. 9) are separated, leading to a broadening of convective cells. Simultaneously, some thermals approach each other much faster, so that the shear facilitates their merging and supports the mixing near the boundaries. These effects are especially pronounced in runs C and B where the walls are frictionless. In run A, these large-scale streaming motions do not exist. There, the mean shear is less than 0.2 \( u_* / H \), and maximum values of \( V_h \) are much smaller than in run C and they occur mainly at the frictionless upper boundary (maximum shear: 0.4 \( u_* / H \) at \( t = 30 \tau_s \)). The effect of the shear in run A is simply to stretch and to disrupt originally connected regions in such a way that no coherent patterns of convective cells appear after
Figure 9. Horizontal sections of instantaneous fields of horizontal velocity $u$ and vertical velocity $w$ in run C at $t = 50 \Delta t$; (a) $u$ and (b) $w$ near the top of the domain; (c) $u$ and (d) $w$ at mid-height in the domain; (e) $u$ and (f) $w$ near the bottom surface of the domain.
Figure 10. Evolution of the difference $\Delta \alpha_h$ between the directions of the mean flow in the upper and lower parts of the domain: (a) run A; (b) run C. Open circles represent differences between flows close to the bottom and top of the domain and black discs represent differences between flows at $z = 0.75H$ and $z = 0.25H$. The mean value of $\Delta \alpha_h$ over all data points is $180^\circ$ for each run.

$t \approx 30 t_a$. Figure 10 shows the evolution of the difference $\Delta \alpha_h$ between the mean flow direction near the top and near the bottom of the domain, illustrating the coherence of the large-scale drift in run C as compared to run A. The mean value over all data points is $180^\circ$ for each run, but the variance of $\Delta \alpha_h$ is much larger for run A. This means that the instantaneous deviations are of more random nature in run A whereas the large-scale organized drift in run C causes values of $\Delta \alpha_h$ to be much more coherent and very close to $180^\circ$.

4. Discussion

Generally, statistical quantities, such as turbulent variances and fluxes of run A at early times (up to $t \approx 15 t_a$), agree well with those of previous LES studies (Schmidt and Schumann 1989; Mason 1989) and with atmospheric observations. Previous LES studies, however, have shown smaller values of horizontal velocity variance than those observed. In run A, the horizontal velocity variance grows in time and steady-state values ($t > 50 t_a$) are much closer to atmospheric data. Thus, a possible explanation of this frequently observed deviation between atmospheric measurements and LESs could be the growing scale of convective cells in larger domains and for longer times. The four classes of the resolved-scale components of the total heat flux $w' \theta'_h$ largely resemble findings of Mahrt and Paumier (1984—Figs. 6 and 7), although our curves differ for heights $z > 0.6 H$ because of the missing zone of entrainment in run A. The area fraction of updraughts is approximately 0.5 near the bottom and has minimum values of $\approx 0.4$ in the middle of the mixed layer, values very close to LES results of Schumann (1989). Although run A is representative of
the classical CBL over land, there is a marked broadening of convective cells up to typical diameters of 3.5 $H$ until 50 $r_t$, which was not seen in previous LES studies (Schmidt and Schumann 1989; Mason 1989). We have seen that $\ell_0$ increases with height, i.e. the dominant scales of the thermal field grow with height because small-scale thermals produced near the lower surface merge while rising upwards. This height dependence of the length scale was also found in laboratory experiments (Deardorff and Willis 1985) and in LES (Mason 1989—Fig. 3).

Note that, as compared to run A, the scale of the thermal field in runs B and C is significantly larger and that the cells broaden much faster. The differences in scale between these runs are produced only by physical processes at the lower wall and by the double forcing of run C. Applying the linear theory to the onset of convective instability under various boundary conditions, Jakeman (1968) found that larger critical wavelengths arise for stress-free boundaries, and that surface friction diminishes the final scale of convective cells. This is consistent with run A in which wall friction caused thermal elements of smaller horizontal sizes. It is well known (see, for example, Ishiwatari et al. 1994) that convective cells with large horizontal sizes are formed where the heat flux, but not the temperature, is fixed at the boundary. This means that the temperature at the wall can reach very large absolute values. Furthermore, because thermals move unimpeded over the frictionless boundary in runs B and C, they are able to absorb more internal energy (i.e. become hotter) compared with those in run A where dissipation acts to reduce the horizontal width of thermal elements.

In run C, the convective circulations are forced to reach a state where $\overline{w'\theta'}_{tot}$ is vertically positive and uniformly distributed. In this way, the buoyancy term in the TKE equation contributes twice to the energy production and this leads to a TKE which is twice as big as that in run A or B. Because the active thermals (the warm up- and the cold down-draughts) positively contribute to $\overline{w'\theta'}_{tot}$ (and both are larger than one), some other motions must supply the negative counterparts. Rising warm (sinking cold) fluid parcels forced to reverse their flow direction at the upper (lower) wall should be considered. Indeed, near-wall maxima of the respective negative heat-flux components in Fig. 6 support this possibility. However, the magnitude of the negative heat-flux contributions is similar to that in the flow interior. Let us hypothesize that these components are caused by lateral entrainment of differently buoyant air into the active up- and down-draughts because the horizontal diffusivity dominates run C as shown by the anisotropy $\alpha$ in Fig. 5. Eventually, this increase of the effective horizontal diffusion leads to broader coherent warm and cold areas with less heterogeneity. Using a simple analytical model, Ray (1965) found that if “horizontal transfer coefficients of any kind” are large compared with the vertical one, convective cells will flatten, i.e. the aspect ratio (cell depth to width) will increase with increasing horizontal exchange processes. This is in agreement with the findings of the present work.

We have seen that the heat transport in run C occurs at scales near the boundaries which differ from those in the flow interior. The small-scale eddies near the walls are responsible for the strong turbulent mixing, whereas the large-scale structures in the interior pump warm fluid from bottom to top and cold fluid from top to bottom. Thus, an increase in the size of the scale on which cells are organized in the middle of the domain is the result of efficient transport of very warm and very cool fluid from the walls through the interior of the fluid by large scales, and of small-scale turbulent mixing. In addition to these effects, the formation of organized, large-scale horizontal streaming motions supports the turbulent mixing as separated thermals approach faster and are able to merge more easily, forming gradually growing cells. The trend to form a large-scale streaming motion in one direction near the bottom and in the opposite direction near the top was found experimentally by
Krishnamurti and Howard (1981). They observed that with the onset of this *organized* horizontal flow, the dominant scales of convective cells grow up to the dimension of the domain size.

A simple analytical-flow model explains the differences between the individual runs. Runs A, B and C differ in the friction at the surface (zero for runs B and C, non-zero for run A) and in the cooling at the top surface (zero for runs A and B, non-zero for run C). Any differences between the results for these runs must be related to these causes. The friction at the surface for run A contributes to turbulent dissipation, whereas the top cooling in run C leads to more vigorous downdraughts, enhancing the horizontal mixing. Let us assume that the convection adjusts its structure so that the given heat-flux is achieved by a minimum dissipation rate $\epsilon$ of kinetic energy. In order to estimate $\epsilon$, consider a single convection cell of diameter $D$ and height $\mathcal{H}$ that is described by a horizontal velocity $u$ and a vertical velocity $w$ in such a way that the continuity equation

$$ w D = u \mathcal{H}. \tag{5} $$

holds, to a first approximation. The dissipation can be estimated to be controlled by horizontal and vertical diffusivities $K_h$ and $K_v$ in the flow interior, and vertical diffusivity $K_s$ at the surface, causing momentum fluxes for gradients of the order $w/D$ (horizontally) and $u/\mathcal{H}$ (vertically),

$$ \epsilon = K_h (w/D)^2 + K_v (u/\mathcal{H})^2 + K_s (u/\mathcal{H})^2. \tag{6} $$

Because of continuity, $w/D = u/\mathcal{H} (\mathcal{H}/D)^2$. So

$$ \epsilon = \{K_h (\mathcal{H}/D)^4 + K_v + K_s\} (u/\mathcal{H})^2. \tag{7} $$

To first order, let us assume that $w$ varies less with the boundary conditions than $u$. Therefore, we express $u$ in terms of $w$ using the continuity Eq. (5) which yields

$$ \epsilon = \{K_h (\mathcal{H}/D)^4 + K_v + K_s\} (w/\mathcal{H})^2 (D/\mathcal{H})^2. \tag{8} $$

For fixed $w$, this takes a minimum value for

$$ \frac{D}{\mathcal{H}} = \left( \frac{K_h}{K_v + K_s} \right)^{1/4}. \tag{9} $$

This shows that the cell diameter $D/\mathcal{H}$ grows for smaller $K_s$ (i.e. for a frictionless bottom-surface) and for larger ratios $K_h/K_v$ (i.e. for cooling at the top surface). This explains why runs B and C produce larger horizontal scales than run A.

Any LES must have a domain size $L$ which is large enough to ensure that the most energetic eddies are much smaller than $L$. In particular, this is necessary to obtain sufficient data for reliable statistics. In run A, this requirement is achieved: the vertical velocity field is small-scale and is typically of the order of $\mathcal{H}$ and the magnitude of the dominant thermal scales is smaller than $2 \mathcal{H}$ for $t < 30 t_s$. At later times, these structures become larger (typical scales of $3 \mathcal{H}$), but they are still small compared with the domain size of $16 \mathcal{H}$. On the other hand, the dominant thermal scales of run C are roughly $3 \mathcal{H}$ at $t = 30 t_s$, and grow to $6 \mathcal{H}$ for times $t \geq 70 t_s$. This scale is nearly as large as half the box side, and thus the domain size becomes too small. In order to follow the subsequent evolution, a numerical technique similar to that proposed by Müller and Chlond (1996) could be applied.
5. Conclusions

We have considered turbulent convection in a dry Boussinesq fluid, heated from below between two horizontal solid boundaries, in a domain with an aspect ratio of 16:1, and for 100 convective timescales. We have investigated the influence of the dynamical character of the lower boundary, and the effect of cooling at the top of the domain, on the growth of convective cells by means of three LESs. The aim was to identify physical processes that lead to the broadening of convective cells even under conditions where moisture plays no role. The results of the numerical simulations and of the simple flow model presented in section 4 are:

(i) The cooling at the top of the domain produces a vertically uniform heat-flux and this double forcing enhances the total kinetic energy of the flow. Essentially, there are two competitive effects: firstly, the prescribed heat-flux at the upper (lower) boundary produces very warm (very cool) fluid that is forced to rise (sink); the resulting large temperature-fluctuations are transported by large-scale motions and cause broad temperature variances independent of height. Secondly, and contrariwise, the strong turbulent mixing (mainly near the boundaries where the TKE is a maximum) tries to homogenize the flow structure in such a way that the resulting temperature-distribution is uniform; except directly close to the walls, horizontal and vertical temperature-gradients are reduced.

(ii) The adiabatic upper boundary-condition leads to a heat-flux profile that is linear and decreasing with height. In this case, we observe thermal structures of smaller scale, and a pronounced increase of this scale with height as a result of merging thermals.

(iii) An organized, large-scale horizontal drift, in one direction near the bottom and in the opposite direction near the top, was generated in runs without friction at the lower surface. This horizontal streaming has two effects: firstly, because of the larger shear, the turbulent mixing is enhanced, and, secondly, separated thermals approach more quickly and are able to merge more easily, forming gradually growing cells.

(iv) Friction at the lower surface reduces the scales of structures in the fields of $w'$ and $\theta'$ near the surface. At higher altitudes, the thermal structures grow much more slowly than in the frictionless runs, and the final scale is significantly smaller. Additionally, the no-slip boundary condition suppresses the generation of large-scale horizontal streaming motions.

These results imply that additional heating in the bulk of the CBL (in this study by cooling the top, in reality e.g. by latent-heat release or by radiative cooling) lead to a broadening of convective cells. The generation of broader thermal structures is certainly sustained over smooth lower surfaces.

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References


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