Parametrization of momentum transport by convection. I: Theory and cloud modelling results

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SUMMARY

The theory of the mass-flux approach to parametrization of convective momentum-transport is presented. A cloud-resolving model is used to simulate momentum transport by ensembles of deep convective clouds in two very different regimes: a mid-latitude cold-air outbreak, and tropical convection forced by convergence. Idealized, unidirectional, wind-profiles are used to simplify interpretation of the results. Diagnostics relevant to the parametrization problem are presented, and it is shown that, for both regimes, the approximations inherent in the parametrization equations are reasonable. The pressure gradients inside clouds play an important role in determining horizontal velocities in the clouds. For the cold-air outbreak with linear shear, the results suggest that these pressure gradients are proportional to the shear and to the up/downdraught mass-fluxes. Sensitivity studies suggest that the results are not very sensitive to resolution or parametrization of subgrid-scale processes, giving some confidence that the results are reasonably accurate. For the tropical case with a low-level jet, the pressure gradients change sign with the wind shear. In part II of this paper, these results will be used to develop and validate a mass-flux parametrization of convective momentum-transport which is tested in single column and global versions of the Meteorological Office Unified Model.

KEYWORDS: Cloud-resolving model Cross-cloud pressure-gradient Deep convection Momentum flux

1. INTRODUCTION

During the past twenty years, both modelling and observational studies have demonstrated the importance of convective momentum-transports to the general circulation of the atmosphere. Houze (1973), in a study of the budget of atmospheric angular momentum, found that the contribution due to the vertical transport of horizontal momentum by convection was of similar magnitude to other terms. Stephens (1979), analysing GATE† easterly-wave data, showed that significant residual existed in the observed momentum-budgets of such systems; he attributed this residual to convective activity. Early modelling studies (Helfand 1979) suggested that convective momentum-transports contribute to the intensification of the Hadley circulation, and the recent study of Zhang and McFarlane (1995) has shown that such vertical transports have a large impact upon the zonal and meridional circulation as simulated by the Canadian Climate Centre model.

Several previous authors have attempted to develop parametrizations for use in large-scale models, making various assumptions regarding the evolution of the horizontal-wind field within the cloud. The earliest scheme, suggested by Schneider and Lindzen (1976), assumed that ascent would be sufficiently rapid to ensure that velocities in the cloud would remain constant at the inflow value. However, observational studies of deep convective

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systems (by LeMone (1983) for example), have demonstrated that the cloud pressure-gradients play an important role in modifying the horizontal wind inside the cloud as air ascends. More recent parametrizations (Zhang and Cho 1991a,b) have included this effect, calculating the effect of the cloud pressure-gradients from a simple model of flow around a cloud. König and Ruprecht (1989) also took account of internal circulations within the cloud, but they worked in terms of vorticity rather than momentum.

The momentum transports from such schemes were validated against observational estimates resulting from budget studies—with varying success. However, such methods do not allow details of the cloud model used in the parametrization to be validated well. For example, the variation of in-cloud wind within the updraughts and downdraughts cannot be estimated from the observations, nor can the important pressure-gradient term. Moreover, there are few sources of observational validation, and there are large errors associated with observational estimates of momentum sources derived as residuals of a momentum budget (Stephens 1979).

In the present paper and its companion part II (Gregory et al. 1997), the authors develop a parametrization of the vertical transport of horizontal momentum by convection. However, a different methodology is followed from that of previous studies. In this present paper, a cloud-resolving model is used to estimate the momentum transports caused by deep convection, for differing flow-regimes and cloud-organizations. In part II, these results will be used to develop and validate a parametrization of convective momentum-transport based upon the mass-flux convection-scheme of Gregory and Rowntree (1990). This will then be tested in single-column and global versions of the Meteorological Office Unified Model. Such a methodology has several advantages over using purely observational data. As well as providing estimates of the net transport of momentum, the cloud-resolving-model simulations provide details of the internal cloud-structure, such as the variation of horizontal wind with height within the cloud. Also, the structure and magnitude of the cloud pressure-gradient can be estimated directly, without appealing to a simple cloud-model, allowing representations for use in the convection scheme to be better validated.

Such an approach is consistent with that adopted by GCSS (the GEWEX* Cloud Systems Study (Browning 1994)), with the aim of improving cloud-parametrizations for large-scale models. It assumes that current cloud-resolving models are able to represent the main features of convective storms correctly. This will become clearer over the next few years as detailed comparisons of cloud models with each other and with observational data are made in projects such as the European Cloud-Resolving Modelling project (EUCREM). However, numerous cloud-modelling studies over the past decade have already shown that such models have some skill in simulating a wide variety of cloud structures. This gives some confidence that there is value in using these data to develop a parametrization scheme for use in large-scale models.

2. Theory

Consider the momentum budget of a grid square of a typical numerical weather-prediction (NWP) or climate model, within which subgrid-scale convection is occurring in the presence of shear. Such a domain would cover an area ranging from, say, (50 km)$^2$ to (200 km)$^2$ or more. In order to isolate the effects of convection on momentum, we omit those terms in the momentum equation which describe the resolved large-scale processes of horizontal and vertical advection, and acceleration by pressure gradients and

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Coriolis terms. (These terms will all be zero in our cloud-model simulations.) With these assumptions, the horizontal-momentum budget for the domain reduces to

$$\frac{\partial (\rho \bar{u})}{\partial t} = - \frac{\partial (\rho \bar{u} \bar{w}')}{\partial z}$$  \hspace{1cm} (1)

where \(u\) and \(w\) are the zonal and vertical components of the wind, and \(\rho\) is the air density. A similar equation holds for the meridional component \(v\). The overbar denotes an average over the horizontal domain and the prime denotes a departure from that average. So \(\bar{u}\) is resolved by the NWP model, whereas \(u'\) and \(w'\) are subgrid-scale (though they will be resolved by our cloud model). We make the further approximation (consistent with our cloud model) that \(\rho\) is solely a function of height \(z\), and define the apparent momentum-source Q3 by

$$Q3 = \frac{\partial \bar{V}}{\partial t} = \left( - \frac{1}{\rho} \frac{\partial (\rho \bar{u} w')}{\partial z}, - \frac{1}{\rho} \frac{\partial (\rho \bar{v} w')}{\partial z} \right) .$$  \hspace{1cm} (2)

The problem we seek to solve is to determine \(Q3\), the acceleration of the resolved flow. The problem is essentially solved once we have obtained the vertical profile of the flux of subgrid-scale momentum.

Following the mass-flux approach to cumulus parametrization (Ooyama 1971), this is achieved by partitioning the flux over the domain into contributions from convective updraughts (superscript \(u\)), convective downdraughts (superscript \(d\)), and the remainder of the domain, the 'environment' (superscript \(e\)):

$$\rho \bar{u} \bar{w}' = \sigma^u \rho \bar{u} \bar{w}^u + \sigma^d \rho \bar{u} \bar{w}^d + \sigma^e \rho \bar{u} \bar{w}^e ,$$  \hspace{1cm} (3)

where \(\sigma^u\) is the fractional area covered by convective updraughts, and \(\sigma^d\) and \(\sigma^e\) are similarly defined. The contribution from the environment is usually neglected in convective parametrization schemes (e.g. Gregory and Miller 1989); we shall demonstrate later that this is reasonable using our cloud-model results. The updraught contribution may be simplified by the following procedure. First replace \(u'\) by \(u - \bar{u}\) so that

$$\rho \bar{u} \bar{w}' = \rho (u - \bar{u}) \bar{w}' = \rho \bar{u} \bar{w}^u - \rho \bar{u} \bar{w}^d .$$  \hspace{1cm} (4)

Then within the updraughts we define \(u^*\) by \(u^* = u - \bar{u}\), obtaining

$$\rho \bar{u} \bar{w}^u = \rho (u^* + \bar{u}) \bar{w}^u - \rho \bar{u} \bar{w}^u = \rho \bar{u} \bar{w}^u - \rho \bar{u} \bar{w}^u + \rho \bar{u} \bar{w}^u .$$  \hspace{1cm} (5)

We will neglect the last term \(\rho \bar{u} \bar{w}^u\), assuming that intra-updraught correlations between \(u\) and \(w\) are small. Again, our cloud-model results will justify this. Hence

$$\rho \bar{u} \bar{w}^u = \rho (\bar{u}^* - \bar{u}) \bar{w}^u .$$  \hspace{1cm} (6)

and so we obtain the following parametrization equation for the convective momentum-flux

$$\rho \bar{u} \bar{w}^u = \sigma^u \rho (\bar{u}^* - \bar{u}) \bar{w}^u + \sigma^d \rho (\bar{u}^d - \bar{u}) \bar{w}^d .$$  \hspace{1cm} (7)

This is a natural extension of the expression given by Schneider and Lindzen (1976), allowing us to consider the effects of updraughts and downdraughts separately. Thus, the essence of the problem of parametrizing convective momentum-flux is to predict the mass fluxes \(\rho \sigma^u \bar{w}^u\) and \(\rho \sigma^d \bar{w}^d\) in the updraughts and downdraughts, and the mean horizontal velocities there.
In cumulus parametrizations, these quantities are often obtained from one-dimensional cloud-models, based on the idea of entraining plumes. The mass flux is given by the vertical integration of the mass continuity equation, with horizontal convergence and divergence terms replaced by entrainment \((E)\) and detrainment \((D)\) rates

\[
E - D = \frac{1}{\rho} \frac{\partial (\sigma^u \rho \bar{w}^u)}{\partial z}. \tag{8}
\]

The vertical variation of the updraught velocity can be obtained from the momentum equation averaged over cloud updraughts, following Gregory and Miller (1989)

\[
\sigma^u \frac{\partial \bar{u}^u}{\partial t} = -\sigma^u \frac{\partial \bar{u}^u}{\partial x} - \sigma^u \frac{\partial (\rho \bar{u}^w)}{\partial y} - \frac{1}{\rho} \sigma^u \frac{\partial (\rho \bar{w}^u)}{\partial z} - \frac{1}{\rho} \sigma^u \frac{\partial \bar{p}^u}{\partial x}. \tag{9}
\]

Assuming that the convection averaged over the domain is steady, so that time derivatives are zero, that the horizontal-divergence terms may be approximated by entrainment and detrainment terms, and that the vertical divergence may be approximated as before, this is simplified to

\[
\frac{1}{\rho} \frac{\partial (\sigma^u \rho \bar{u}^w)}{\partial z} + D \bar{u}^u - E \bar{u} = -\frac{1}{\rho} \sigma^u \frac{\partial \bar{p}^u}{\partial x}. \tag{10}
\]

Note that we have assumed here that the entraining air has the environmental momentum and the detraining air has the updraught momentum. Equation (10) and analogous equations can be integrated to determine \(\bar{u}^w, \bar{u}^d, \bar{v}^u,\) and \(\bar{v}^d.\)

The vertical evolution of the horizontal velocity in the updraughts thus depends on the amount of entrainment and detrainment specified, and on the horizontal pressure-gradient across the updraughts. The pressure-gradient term is difficult to specify, far more so than the condensational-heating term which replaces it in the equivalent parametrization-equations for \(\theta\) and \(q.\) This is what makes the parametrization of momentum transport particularly difficult.

In the remainder of this paper, we use a cloud-resolving model to evaluate the momentum transport caused by convection, and the quantities such as \(\bar{w}^u, \bar{v}^u,\) and \(\frac{\partial \bar{p}^u}{\partial y}\) which are required for parametrization, as outlined above. We use idealized wind-profiles (to simplify interpretation of the results) and two very different convective regimes: a mid-latitude cold-air outbreak forced by surface fluxes of heat and moisture, and tropical convection forced by convergence.

3. DESCRIPTION OF CLOUD MODEL

We used the Meteorological Office cloud-resolving model described by Shutts and Gray (1994), which solves the anelastic equations for a three-dimensional, rectangular Cartesian grid over a flat surface. The model predicts the time variation of the three velocity-components \(u, v,\) and \(w,\) pressure \(p,\) and thermodynamic variables \(T_i\) and \(q_i,\) which are conserved except when precipitation is formed:

\[
T_i = T + \frac{g z}{c_p} - \frac{L_v q_i}{c_p} \tag{11}
\]

\[
q_i = q_v + q_i, \tag{12}
\]

where \(T\) is the absolute temperature, \(g\) the acceleration under gravity, \(z\) the height above the surface, \(c_p\) the specific heat of dry air at constant pressure, \(L_v\) the latent heat of condensation.
of water, \( q_i \) the cloud (liquid) water mixing ratio, and \( q_e \) the water-vapour mixing ratio. The model also predicts \( q_r, q_s, q_g, \) and \( q_i \), the mixing ratios of rain, snow, graupel, and ice cloud, and \( N_i \), the number concentration of ice crystals. The ice processes can be switched off if not required.

The domain is a box with cyclic lateral-boundary conditions and no flux of air through the top or bottom. There is no slip condition at the surface and the surface fluxes of heat and moisture are specified and held constant throughout the integration. The model has a positivity-preserving advection-scheme and subgrid parametrization schemes for turbulence (first order, stability-dependent closure) and microphysics (either Kessler-type warm rain or three-phase bulk-water scheme based on Lin et al. (1983) and described by Swann (1994)). Although the model runs on an \( f \)-plane, \( f = 0 \) in these runs: rotational effects were not thought likely to be important at the horizontal scales considered here, and non-zero \( f \) complicates the model initialization. This assumption is reasonable unless the convection is organized on the mesoscale.

4. Description of cold-air-outbreak experiments

For the majority of the experiments, we used a domain 50 km square by 15 km deep, and grid lengths of 1000 m horizontally and 500 m vertically, though some experiments were also carried out at higher resolution to test the robustness of the results. We used horizontally uniform initial conditions, specified in terms of potential temperature \( \theta \), humidity mixing ratio, and the \( v \)-component of the wind (see Fig. 1). Figure 1 also shows how \( \theta \) and \( u \) varied linearly with height through prescribed layers. The other wind-components \( u \) and \( w \) were initially zero, as were the microphysical variables \( q_i, q_r, q_s, q_g, q_l \). The temperature profile was dry adiabatic to 1500 m, conditionally unstable to 4 km, and isothermal above the tropopause at 9 km. Convection was initiated by small (< 0.05 K) random temperature-perturbations at each grid point of the lowest model-level (height 250 m), and maintained by the surface fluxes: 123 W m\(^{-2}\) sensible heat and 492 W m\(^{-2}\) latent heat. The surface moisture-flux was doubled or tripled in some experiments to vary the intensity of the convection. These highly idealized initial conditions represent a mid-latitude cold-air outbreak, with deep, precipitating shower-clouds developing in varying unidirectional shear.

A wide range of experiments was run for two hours, to investigate the dependence of the momentum fluxes on the intensity of convection and on vertical shear. Several experiments were then extended for ten hours, for comparison with the performance of a convective parametrization in a single-column model (Gregory et al. 1997). These extended experiments were also used for studies of the sensitivity to resolution and representation of subgrid-scale processes. The statistical diagnostics, such as the momentum fluxes, were averaged over the horizontal domain and over each hour. Most experiments were run with a damping layer above 10 km, but this does not appear to have had any significant impact on the results below that level. Results from some of these experiments were reported by Kershaw (1995), but he concentrated on the momentum transport by waves generated above the convection. Here we focus on the transport by the convection itself.

5. Cold-air-outbreak results

(a) Preliminary tests

The first set of experiments is listed in Table 1, which also gives some statistical diagnostics from the second hour of each experiment. To help explain these diagnostics, we present some results from the control simulation [180]. Figure 2 shows the simulated
Figure 1. Initial profiles in cold-air outbreak experiments: (a) $\theta$ (solid) and $\theta_e$ (stars); (b) $q_t$ (solid) and $q_{sat}$ (crosses); (c) $v$. See Tables 1 and 2 for details.
TABLE 1. List of cloud-model experiments and results

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Shear (m s(^{-1})(6 km(^{-1}))</th>
<th>Latent-heat flux (W m(^{-2}))</th>
<th>Ice (on/off)</th>
<th>(\sqrt{w_{\text{max}}^2}) (m s(^{-1}))</th>
<th>(\rho u w_{\text{max}}) (N m(^{-2}))</th>
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</thead>
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<tr>
<td>180</td>
<td>10</td>
<td>492</td>
<td>off</td>
<td>0.98</td>
<td>-0.26</td>
</tr>
<tr>
<td>181</td>
<td>20</td>
<td>492</td>
<td>off</td>
<td>0.90</td>
<td>-0.47</td>
</tr>
<tr>
<td>183</td>
<td>10</td>
<td>985</td>
<td>off</td>
<td>1.30</td>
<td>-0.33</td>
</tr>
<tr>
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<td>985</td>
<td>off</td>
<td>1.21</td>
<td>-0.67</td>
</tr>
<tr>
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<td>off</td>
<td>1.19</td>
<td>-1.05</td>
</tr>
<tr>
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<td>10</td>
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<td>off</td>
<td>1.58</td>
<td>-0.42</td>
</tr>
<tr>
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<td>off</td>
<td>1.47</td>
<td>-0.86</td>
</tr>
<tr>
<td>169</td>
<td>30</td>
<td>1477</td>
<td>off</td>
<td>1.40</td>
<td>-1.23</td>
</tr>
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<td>40</td>
<td>1477</td>
<td>off</td>
<td>1.43</td>
<td>-1.88</td>
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</table>

Figure 2. Horizontal section through control simulation [180] showing vertical velocity at 4 km, after 2 hours. Contour interval 1 m s\(^{-1}\), dashed contours negative.

vertical-velocity field in the middle of the convective layer after 2 hours. We see an ensemble of updraughts, the strongest having a peak velocity of more than 8 m s\(^{-1}\), surrounded by much weaker descending motion. The strongest downdraughts (\(-2\) m s\(^{-1}\)) at this level are in the ring of compensating subsidence which surrounds the most active updraughts. Figure 3 shows the perturbation, the difference from the mean, in the \(v\)-component of the wind at the same level, for comparison with the vertical velocities. The updraughts tend to be associated with negative \(v\)', implying negative momentum-fluxes. At the lowest model-level, the vertical-velocity field (not shown) is much less skewed, indeed the peak downdraughts (driven by evaporative cooling and water loading) are slightly stronger (\(-2.7\) m s\(^{-1}\)) than the peak updraughts (2.5 m s\(^{-1}\)). Positive (negative) perturbations in the \(v\)-component (not shown) at the same level tend to occur downstream (upstream) of the downdraughts, as these spread out on approaching the surface. There is no very clear indication of the likely sign of the momentum flux at this level, though we shall see later that the downdraughts tend to act to oppose the updraughts: they have positive momentum flux. Figure 4 shows the vertical profile of the standard deviation of \(w\) over the domain, which provides one measure of the intensity of convective activity in the model. We use the peak value of this diagnostic (\(\sqrt{w_{\text{max}}^2}\)), marked with a star in the figure, to compare the convective intensity in the different experiments. Figure 5 shows the vertical profile of the momentum flux,
Figure 3. Horizontal section through control simulation [180] showing $v'$ at 4 km, after 2 hours. Contour interval 1 m s$^{-1}$, dashed contours negative.

Figure 4. Vertical profile of r.m.s. vertical velocity, averaged over second hour of control simulation [180].

Figure 5. Vertical profile of momentum flux, averaged over second hour of control simulation [180].
Figure 6. Scatter plot of momentum flux plotted against its product of the shear and the r.m.s. of $w$, averaged over second hour of the numerical experiments.

which is negative except at the lowest model-level and has maximum magnitude in the middle of the convective layer. The convection (in all the cold-air-outbreak experiments) is transporting momentum down-gradient (the vertical shear and the flux have opposite signs throughout most of the convecting layer) and is acting to reduce the shear in the middle of the layer. Shear is intensified at the top and bottom of the layer, though, because higher-momentum air is being transported towards the surface (where there is a no-slip condition), and lower-momentum air is transported towards cloud top (above which the undisturbed flow retains most of its original high momentum). We use the peak value ($\rho \overline{v w_{\text{max}}}$), marked with a star, to compare the convective momentum-transport in the experiments.

The diagnostics in Table 1 show that increasing the surface flux of latent heat increases the intensity of convection and the convective momentum-flux. Increasing the shear tends, in general, to decrease the convective intensity slightly, but increase the momentum flux. This result is summarized in Fig. 6, which shows that there is an approximately linear relationship between the convective momentum-flux and the product of the shear $\Delta V$ and the convective intensity $\sqrt{w_{\text{max}}^2}$. The simplicity of this relationship is encouraging because it suggests that successful parametrization of the process of convective momentum-transport is possible, at least when the convection is not organized on the mesoscale. The linearity of the relationship also suggests that it is necessary to compare the parametrization only with a subset of the experiments listed in Table 1.

(b) Simulations for comparison with single-column model

Table 2 shows details of the experiments which were run for comparison with the single-column model [232, 233, 236, 237]. The new control-experiment [232] is identical to experiment [180], save that it includes the microphysical parametrization of ice processes which became available during the course of this research. The other three experiments were chosen in order to validate the parametrization of Gregory et al. (1997) over a range of shear and with differing intensity of convective activity.

The cloud-model simulations are not useful only because they provide an estimate of the net effect of convection on the wind profile, they also enable us to diagnose those quantities which are intrinsic to the parametrization equation (7) and to test some of the underlying assumptions made in deriving and applying it. Figure 7, for example,
shows the updraught and downdraught mass-fluxes averaged over the ten hours of two of the simulations. (For the purpose of calculating partitioned diagnostics such as these, an updraught is taken to occur at a grid point if the vertical velocity is greater than +1 m s\(^{-1}\) and the total condensed water-content exceeds 0.1 g kg\(^{-1}\). A downdraught is indicated by negative vertical velocity and precipitating water-content exceeding 0.1 g kg\(^{-1}\).) Note that the updraught fluxes (\(\rho \sigma^v \vec{w}^v\)) peak at 2 km, implying net detrainment above this level. The downdraught fluxes (\(\rho \sigma^d \vec{w}^d\)) peak at the lowest model-level, achieving 60–70% of the updraught magnitude and implying net entrainment over most of their depth. All four experiments show this same structure. The mass fluxes are almost identical in those pairs of experiments in which only the vertical shear was changed, [232, 233] and [236, 237]. Hence, for clarity, only mass fluxes for experiments [232] and [236] are included in the figure. Comparison of these two experiments shows the effect of doubling the surface moisture-flux: the peak updraught-mass-flux is increased by about 70% and deepened; the downdraught flux is also increased.

Using the model, the pressure-gradient forces across the updraughts and downdraughts may be diagnosed directly. Figure 8 shows that parcels of air in the updraughts must be accelerated by pressure gradients across the cloud as they ascend. Similarly, except at the lowest model-level, the downdraughts are decelerated by pressure gradients as they descend. Figure 9 shows the horizontal velocity in the draughts (up or down) and the domain
average. Note that the difference $\bar{v}^u - \bar{v}$ is always negative, implying negative momentum transport, and the difference $\bar{v}^d - \bar{v}$ is also negative, implying positive momentum transport. Thus the downdraughts are mainly composed of air which has previously ascended in an updraught. If they were transporting (anomalously)-high-momentum air downwards we would expect this difference to be positive, which would imply negative momentum flux. That this is not the case is confirmed by Fig. 10; the downdraught momentum-flux is of opposite sign to the updraught flux. This is true of all four experiments, but it will not always be so; it will depend on the structure of the convection. Note that the updraught transports are by far the most important; they account for nearly all the total transport. Again, this is true in all four experiments. Figure 10 also shows a comparison of the updraught and downdraught momentum transports with the estimates of those quantities obtained from the parametrization equation (7), the product of the mass flux and the velocity anomaly. The agreement is very good, indicating that the terms neglected in (7) are not important. This is so in all four experiments.
Figure 10. Vertical profile of partitioned momentum-fluxes, averaged over ten hours of control simulation [232] (solid line, domain average; stars, updraught; diamonds, downdraught) plus estimates based on Eq. (7) (dashed, updraught estimate; dash-dotted, downdraught estimate).

Figure 11. Change in $\bar{u}$ over ten hours of simulation (plus signs, [232]; stars, [233]; diamonds, [236]; crosses, [237]).

Figure 11 shows the net effect of convection (and subgrid-scale turbulence) on the $v$-component of the wind (effectively $Q_3$) in all four experiments. The down-gradient transports have eroded the shear in the middle of the convective layer, accelerating the flow below 4 km and decelerating it above. $Q_3$ is enhanced by shear and by increased convective activity. Increasing the shear (in [233]cf.[232] and [237]cf.[236]) increases the momentum transport by increasing $\bar{u}^u - \bar{v}$. (Remember that the mass flux is largely unchanged by increased shear.) Increasing the mass flux (in [236]cf.[232] and [237]cf.[233]) actually decreases $\bar{u}^u - \bar{v}$ slightly, so the effect on $Q_3$ is not quite proportional to the mass flux. The pressure-gradient force across the updraught increases with shear and with updraught mass-flux. This will be presented and discussed in part II of this paper.
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Figure 12. Change in $\bar{v}$ over ten hours of simulation (plus signs, [237]; stars, [242]; diamonds, [246]; triangles, [247]; crosses, [248]).

(c) Sensitivity tests

Table 2 also shows details of the sensitivity tests which were carried out to aid assessment of the accuracy of the cloud-model results: [242] cf. [237] shows the effect of increasing horizontal resolution; [248] cf. [246] shows the effect of increasing vertical resolution in the boundary layer; [247] cf. [246] shows the sensitivity to turbulence parametrization; [237] cf. [246] shows the effect of including a representation of the ice phase. Most of these changes are self-explanatory, however the change in turbulence parametrization requires some explanation. The parameter $l$ is a mixing length which is usually related to the horizontal grid-length in our simulations by the relationship $l = c_s \Delta x$ where $c_s = 0.23$. Thus, in the simulation with double the horizontal resolution [242], $l$ is halved. In simulation [247] a smaller value $c_s = 0.15$ is used. As the eddy viscosity is proportional to $l^2$ this has the effect of reducing the turbulent (subgrid-scale) fluxes of heat, moisture and momentum, and increasing the resolved fluxes. Using a much smaller value than $c_s = 0.15$ would be likely to lead to noisy simulations, whereas using a much larger value than $c_s = 0.23$ would result in excessively smooth simulations. We have thus tested the sensitivity to the acceptable range of values of $c_s$. Further details of the turbulence parametrization are given by Shutts and Gray (1994) (who denote $l$ by $\lambda_0$) and a comparison with parametrizations in other large-eddy-simulation codes may be found in a paper by Andren et al. (1994).

The results of the sensitivity tests are summarized in Fig. 12. They show that the momentum transport in the simulations is not very sensitive to any of these details, giving us confidence that our estimates of $Q_3$ for these experiments are accurate to, say, $\pm 10\%$.

Although the net momentum-transport is insensitive to these changes, some aspects of the simulations are, nevertheless, sensitive to the details of the model. In particular, cloud amounts in the upper troposphere are doubled when the ice phase is included, and ice processes (presumably melting) enhance the downdraught mass-flux by 40%. Increasing the horizontal resolution has some impact on the momentum transports because the updraughts are better resolved. Hence the pressure-gradient force across the updraughts is stronger, producing a smaller updraught–environment velocity difference ($\bar{v}' - \bar{v}$). Thus, there is slightly less momentum-transport in the model at higher resolution even though the mass fluxes are greater.
6. Description of tropical experiments

Next, we present results for a case study in which the convective regime is quite different. Whereas the cold-air-outbreak case is for a very unstable airstream in which convection is strongly forced by surface fluxes, here we choose a tropical case with only weak surface-fluxes in which the convection is forced by large-scale convergence. The initial profiles are shown in Fig. 13. The temperature and moisture are based on the 20-hour profiles from the composite GATE easterly-wave data-set (Thompson et al. 1979), as used by Gregory and Miller (1989). Similar numerical experiments have also been reported by Soong and Tao (1984), Tao and Soong (1986), Tao et al. (1987), Krueger (1988), and Tao and Simpson (1989). The novel feature of our experiments is that the initial \( v \) is again an idealized profile, a low-level jet, chosen (as in the cold-air-outbreak case) to simplify interpretation of the results. We ran two experiments, detailed in Table 3, which differed only in that they had different horizontal domains.

As in the cold-air-outbreak experiments, the other wind-components \( u \) and \( w \) were initially zero, as were the microphysical variables \( q_1, q_2, q_2, q_3, q_4 \). Convection was again initiated by small (\( < 0.05 \) K) random temperature-perturbations at each grid point of the lowest model-level (height 250 m), but the surface fluxes were much weaker: 12 W m\(^{-2}\) sensible heat and 145 W m\(^{-2}\) latent heat. The convection was driven by imposed cooling and moistening due to domain-scale ascent. The forcing rates were as shown in Fig. 14, smoothed versions of those used in Gregory and Miller (1989). They were applied throughout the 10 hours of each experiment, uniformly over the horizontal domain.

7. Tropical results

The convective mass-fluxes averaged over the 10 hours of both experiments are shown in Fig. 15. Note that, although they are weaker than those in the cold-air outbreak simulations, they share the same structure. The peak updraught mass-flux is near cloud base, implying net detrainment at all levels above. For the GATE case, there is observational evidence to support this result. Thompson et al. (1979) reported a maximum in the average vertical (pressure) velocity at around 800 hPa (2 km) during phase III of the experiment and deduced from the associated vertical profile of horizontal divergence that convective clouds tended to have tops near 800 hPa, 500 hPa (5 km) and 250 hPa (10 km). Cheng (1989) calculated updraught and downdraught mass-fluxes diagnosed from phase-III mean-data; these have the same structure as our modelled mass-fluxes. Previous work by Nitta (1975) on data from BOMEX, by Yanai et al. (1976) on Marshall Islands data, and by Soong and Tao (1984) on a particular rainband during GATE, also showed updraught mass-fluxes which decreased with height. Using profiles similar to those used here, Gregory and Miller (1989) also found that updraught mass-flux decreased above 700 hPa, implying a deep layer of detrainment. The magnitude of their simulated mass-flux was similar to that reported here.

Increasing the domain size from 50 km to 100 km has little impact on the simulations. Neither experiment shows any sign of meso-scale or two-dimensional organization of the convection (such as squall-line formation). This is also true of additional experiments which
Figure 13. Initial profiles in tropical experiments: (a) $\theta$ (solid) and $\theta_e$ (stars); (b) $q_e$ (solid) and $q_{sat}$ (crosses); (c) $v$. 
were run with different initial perturbations (cold and warm pools of various intensities and shapes). Furthermore, in all these experiments, the momentum transports were very similar. Consequently, in the remainder of this section we shall concentrate on the results from experiment [235].

The pressure gradients in cloud are shown in Fig. 16. Note that the sign seems to depend on the sign of the wind shear, and that the force across the downdraught opposes the force across the updraught at each level. The convective momentum-fluxes are shown in Fig. 17. As in the cold-air outbreak, the updraughts account for most of the momentum transport and the downdraughts act in the opposite sense to the updraughts. The convection is transporting momentum down gradient, away from the jet, because updraughts contain air of anomalously low $v$ below jet level, and anomalously high $v$ above the jet. The pressure-gradient forces are acting, as in the cold-air-outbreak case, to maintain $\bar{v}^u$ close to $\bar{v}$, thus moderating the convective momentum-transport. Figure 17 also shows the
estimates of the momentum transports obtained from the parametrization equation (7), the product mass-flux times velocity anomaly. The agreement is very good for the updraught component, though less good for the downdraught. As the downdraught term is not important, we conclude that, in this experiment too, the terms neglected in (7) are not important. The net effect of the convection (Fig. 18) is to weaken the jet and to accelerate the flow near the surface and above 6 km.

8. CONCLUSIONS

We have used a cloud-resolving model to simulate momentum transport by ensembles of deep convective clouds in two very different regimes: a mid-latitude cold-air outbreak forced by surface fluxes of heat and moisture, and tropical convection forced by convergence. We used idealized, unidirectional, wind profiles to simplify interpretation of the
results. (The inclusion of directional shear would be a natural extension of this work.) We have shown that, for the cold-air outbreak, the momentum transport is, to a good approximation, proportional to the product of the shear and the convective intensity, and that, for the two regimes studied here, the approximations inherent in the parametrization equation (7) are reasonable. Also, it is clear that the pressure gradients in clouds play an important role in determining the horizontal velocities there, confirming previous studies. For the cold-air-outbreak case with linear shear, our results suggest that these pressure gradients are proportional to the shear and to the up/downdraught mass-fluxes. For the tropical case with the low-level jet, the pressure gradients change sign with the wind shear. The convection appears to be randomly organized, consisting of relatively short-lived storms, and the momentum is transported down gradient.

In a companion paper (Gregory et al. 1997), these results will be used to develop and validate a mass-flux parametrization of convective momentum-transport which is tested in single-column and global versions of the Meteorological Office Unified Model. There is, of course, an inherent danger in using a cloud model to provide validation data. However, sensitivity studies suggest that the results presented here are not very sensitive to resolution, domain size, initial perturbation, or parametrization of subgrid-scale processes, giving us some confidence that the results are reasonably accurate.

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