A physically based scheme for the treatment of stratiform clouds and precipitation in large-scale models. I: Description and evaluation of the microphysical processes

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SUMMARY

A stratiform-cloud and precipitation scheme, incorporating prognostic variables for cloud liquid water and cloud ice, has been developed for the CSIRO global climate model (GCM). The scheme includes physically based treatments of key microphysical processes, turbulent mixing and semi-Lagrangian advection of cloud-water species and interactive cloud radiative properties. Objectives in the development of the scheme were to improve upon the physical realism of parametrizations used in earlier schemes, whilst also trying to provide a scheme with moderate computational overheads.

The parameterized microphysical processes are evaluated in relation to observations and theory, and are compared to treatments used in earlier schemes in a series of short GCM experiments. It is argued that the treatment of precipitation formation in warm, and mixed-phase, stratiform clouds is more realistic in the present scheme than in earlier schemes, which used crude methods for the parametrization of autoconversion, and did not treat key ice-processes in a consistent way. In the present scheme, accretion processes are more important, whereas autoconversion is less important than in earlier schemes.

To determine whether the cloud scheme requires the use of a reduced (split) time-step, the sensitivity of the various terms to the time-step is evaluated in another series of short GCM experiments. It is shown that the various terms are not very sensitive to the time-step, so the scheme can be efficiently implemented without the use of a split time-step. Overall, analytical or time-centred treatments perform better than implicit or explicit schemes, especially in the calculation of the precipitation of cloud ice, where only an accurate analytical treatment is found to perform satisfactorily at large time-steps.

As a preliminary validation of the scheme, zonal-mean fields from a six-year model-run are presented for the month of July. The results generally agree well with observations; in particular, the modelled cloudiness and long-wave cloud-forcing fields are more realistic than those obtained with the standard version of the CSIRO GCM.

KEYWORDS: Clouds, Global climate models, Microphysics, Precipitation

1. INTRODUCTION

Global climate models (GCMs) are the most important tools currently in use for the study of anthropogenically induced climate change, but suffer from large uncertainties in their response to increased levels of greenhouse gases. Cess et al. (1990) identified the treatment of clouds as the major cause of the large differences in climate sensitivity found in 19 atmospheric GCMs. Recently, there has been a move towards more physically based treatments of clouds in GCMs and numerical weather prediction (NWP) models (Sundqvist 1978; Le Treut and Li 1988; Sundqvist et al. 1989; Smith 1990; Roeckner et al. 1992; Ose 1993; Ricard and Royer 1993; Tiedtke 1993; Le Treut et al. 1994; Boucher et al. 1995; Fowler et al. 1996). Most of these schemes have incorporated one or more prognostic cloud-variables, since the traditional approach, in which clouds are diagnosed as a function of relative humidity and other model variables, does not allow the clouds to be fully integrated into the model’s hydrological cycle, and provides limited opportunity to improve the physical realism of the modelled clouds. The use of prognostic cloud-schemes is not a panacea, since there is considerable uncertainty about how to treat a number of key processes, and validation of cloud schemes remains a problem, despite the increased availability of satellite observations. Prognostic schemes also have the potential to consume larger amounts of computer time than simple diagnostic schemes; this can be a serious

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problem for climate studies which require lengthy integrations and for NWP where limited time is available to complete a forecast.

The treatment of cloud microphysical processes in earlier prognostic schemes was generally very simple (Sundqvist 1978; Le Treut and Li 1988). Later schemes (Sundqvist et al. 1989; Smith 1990; Roeckner et al. 1992; Ose 1993; Ricard and Royer 1993; Tiedtke 1993; Le Treut et al. 1994) employed more complex, though still highly idealized parametrizations. Although these schemes included some treatment of the microphysics of both warm and cold clouds, the formation of rainfall, for example, was based on intuition. More recent schemes (Boucher et al. 1995; Fowler et al. 1996) have included more physically-based parametrizations, similar to bulk parametrizations that have been in use for some years in the mesoscale-cloud-modelling community (Tripoli and Cotton 1980; Cotton et al. 1982, 1986; Lin et al. 1983; Rutledge and Hobbs 1983). These parametrizations are based on concepts—such as observationally based raindrop- (and snowflake-) size distributions, the continuous-collection equation for the accretion of cloud liquid water by falling raindrops (or snowflakes), and the equations which describe the diffusional growth or decay of raindrops or ice particles—that have been developed over several decades by cloud physicists (e.g. Rogers and Yau (1988) hereafter RY88), so they should be more physically realistic than the simple approaches used previously in large-scale models. However, there are large differences between the spatial and temporal scales treated by global models and those treated by mesoscale models; the small time-steps and degree of complexity employed by the microphysics schemes used in mesoscale models render them computationally too expensive for use in extended GCM simulations on current supercomputers. With this in mind, Ghan and Easter (1992) evaluated approximations to the Colorado State University (CSU) cloud-microphysics scheme (Tripoli and Cotton 1980; Cotton et al. 1982, 1986) that would permit a tenfold increase in the allowable time-step from a few tens of seconds to a few minutes. Fowler et al. (1996) implemented a complex microphysics scheme, based largely on Lin et al. (1983) and Rutledge and Hobbs (1983), in the CSU GCM, using a time-splitting approach with a 2 minute time-step in the microphysics scheme. This time-step is small compared to those used for the physics in most global models (typically 10 to 60 minutes) and could be expected to result in a large overhead that might be considered unacceptable in NWP models or in GCMs that are to be used in long integrations. Indeed, Fowler et al. (1996) found that inclusion of their cloud-microphysics scheme resulted in a doubling of the computer time required to run the CSU GCM. Boucher et al. (1995) included physically based parametrizations of warm-cloud precipitation processes in the Laboratoire de Météorologie Dynamique (LMD), using a time-splitting approach with a 6 minute time-step. Authors of the earlier schemes made no mention of time-splitting, so it seems reasonable to assume that they have used the usual physics time-step of their model for the cloud microphysics. Despite the large range of time-steps implied by this discussion, the sensitivity of the simulated processes to the time-step was not investigated in any of the studies mentioned above, other than by Ghan and Easter (1992) in the context of single-column tests. Also, both Fowler et al. (1996) and Boucher et al. (1995) treated rainfall as a diagnostic quantity, in contrast to earlier schemes, which treated it as a diagnostic quantity that was assumed to fall through the atmosphere in a single time-step. Fowler et al. (1996) stated that a time-split prognostic treatment of rainfall is "by far the simplest and most rigorous" approach, but provided no concrete results to support their assertion. The treatment of rain as a diagnostic quantity was one of the approximations evaluated and found to be satisfactory by Ghan and Easter (1992).

The philosophy underlying the approach described here is to attempt to capture the 'essential' physics, without resorting to a highly complex scheme which is difficult to
understand and could have unforeseen effects in the model. It continues the recent trend towards treatments of microphysical processes which are more physically based, but is less complex (in terms of the number of processes treated) than the scheme of Fowler et al. (1996). It adds two prognostic variables (cloud liquid water and cloud ice) to the GCM used by the Australian Commonwealth Scientific and Industrial Organization (CSIRO), but, in contrast to the schemes of Boucher et al. (1995) and Fowler et al. (1996), rain is treated as a diagnostic quantity. Key objectives were to provide for the CSIRO GCM a cloud scheme which was more physically realistic than earlier prognostic schemes, and which also produced cloudiness and cloud radiative-forcing fields which were in better overall agreement with observations than those produced by the model's standard diagnostic cloud-scheme. At the same time, it was considered essential that this should be achieved with an 'acceptable' level of computational overheads. With this in mind, considerable effort was put into an evaluation of the sensitivity of the simulated microphysical processes to the choice of time-step, since the use of a reduced (split) time-step for the microphysical processes would obviously result in additional computational overheads. The present paper shows that the simulated processes are not very sensitive to the time-step; on this basis, it was decided not to use a split time-step for the cloud microphysics. This was a key factor in the implementation of the scheme with moderate computational overheads (see section 7 for details).

An outline of the remainder of the present paper is as follows. Section 2 contains an overview of the CSIRO atmospheric GCM which is used in this study. Section 3 contains a brief description of the cloud scheme, including the treatment of cloud-radiative properties. Section 4 contains a more detailed description and evaluation of the rainfall processes in the scheme, including sensitivity of the processes to the time-step and comparison of the parameterizations with others that have been used in GCMs. Section 5 contains a similar analysis for the frozen precipitation (snow) processes in the scheme. In section 6, zonal-mean results from a six-year run of the model are compared with observations as a preliminary validation of the scheme—a more detailed validation of the large-scale fields produced by the scheme will be given in a subsequent paper (Part II). Section 7 contains a summary and concluding discussion. All quantities referred to in the text have SI units, except where it is explicitly stated otherwise.

2. Overview of the CSIRO Atmospheric GCM

The CSIRO atmospheric GCM is a spectral model which utilizes the flux form (Gordon 1981) of the primitive equations; an earlier version (Mark 1) has been described in detail by McGregor et al. (1993). Mark 1 had nine vertical levels and horizontal spectral resolution of R21, corresponding to a grid of 56 latitudes and 64 longitudes. The current version (Mark 2) of the CSIRO GCM differs from the earlier Mark 1 in several major respects. It has:

- Variable horizontal resolution, so that the model can be run with spectral resolution of R21, R42 or T63. The use of nine vertical levels has been retained in the standard Mark 2 version of the model. The experiments described in the present paper were run at R21, but with 18 levels, in order to achieve a better representation of physical processes in the vertical. The 18-level R21 version requires a time-step of 24 minutes; the leapfrog scheme in the model means that the time-step effectively 'seen' by the physical parameterizations is 48 minutes. The 18 levels are at \( \sigma = 0.9955, 0.9784, 0.9458, 0.8999, 0.8426, 0.7761, 0.7023, 0.6235, 0.5415, 0.4585, 0.3765, 0.2977, 0.2239, 0.1574, 0.1001, 0.0542, 0.0216 \) and 0.0045 where \( \sigma = p/p_s \) is the model's vertical coordinate, \( p \) is pressure and \( p_s \) is surface pressure.
A new soil-canopy scheme (Kowalczyk et al. 1991) which includes spatially varying data-sets of soil type, albedo, roughness length, canopy resistance and dominant vegetation-type. Physical processes parametrized by the scheme include canopy interception of moisture, runoff (including deep soil percolation) and snow accumulation and melting.

A semi-Lagrangian scheme for moisture advection, instead of the pseudo-spectral scheme used in Mark 1. The semi-Lagrangian scheme includes an economical scheme for the calculation of the departure points of the trajectories (McGregor 1993) and uses cubic Lagrangian interpolation to calculate the field values at the departure points. A quasi-monotone scheme (Bermejo and Staniforth 1992) is used in the vertical-advection routine to prevent the generation of spurious oscillations and ensure non-negativity of the interpolated moisture values. Any negative values generated by the horizontal-interpolation procedure are simply truncated to zero. Conservation of total global moisture is enforced by an a posteriori adjustment to the increments of the water-vapour mixing ratio.

A new sea-ice scheme, based on the dynamical model of Flato and Hibler (1990) and the thermodynamic model of Semtner (1976). The new scheme allows the fraction of leads (open water) to vary with the prevailing winds and ocean currents, and allows calculation of the surface fluxes of heat, moisture and momentum through the leads, whereas the simple thermodynamic model that was used in Mark 1 assumed that sea-ice points were totally covered with ice.

Minor changes to the model dynamics: inclusion of a modified thermodynamic variable (Simmons and Jiabin 1991) which reduces noise in the surface pressure field that had become apparent at the higher horizontal resolutions, and an implicit treatment of the vorticity equation (Simmons et al. 1989), adapted to the flux form of the equations used by the CSIRO GCM.

Other physical parametrizations in the model include:

A radiation scheme based on Schwarzkopf and Fels (1991) for the long wave and Lacis and Hansen (1974) for the short wave. The radiation scheme includes absorption by water vapour, carbon dioxide and ozone, and Rayleigh scattering of short-wave radiation, but does not include absorption by trace gases or scattering by aerosols. There are three layers of diagnostic clouds, loosely following Slingo (1987). The cloud radiative properties are fixed, with short-wave reflectivity and absorptivity specified as a function of height, and emissivity set to 1.

A moist convective adjustment (MCA) scheme which generates a mass flux, based on the ideas of Arakawa (1972). Katzfey (1994) compared this scheme with the Betts–Miller, Kuo and Arakawa–Schubert schemes in simulations of an Australian east-coast low using a limited-area model, and found that the performance of the MCA scheme was comparable to that of the Arakawa–Schubert scheme, and superior to that of the Betts–Miller and Kuo schemes.

Treatment of large-scale precipitation by removal of supersaturation. Evaporation of falling rain is included.

A turbulent-mixing scheme based on stability-dependent K-theory. The diffusion coefficients for vertical turbulent mixing of heat, momentum and moisture are specified as functions of the Richardson number following Louis (1979). The calculation of the Richardson number includes virtual-temperature effects but not the effect of liquid water.

Treatment of shallow convection following either Geleyn (1987) or Tiedtke (1988), a version of the latter scheme having been adopted for this study. In this approach, shallow convection is assumed to occur when the lifting condensation-level (LCL) for near-surface air is lower than \( \sigma = 0.9 \). When this criterion is satisfied, the diffusion coefficients in the turbulent-mixing scheme are increased by 6 \( \text{m}^2\text{s}^{-1} \) between the LCL and cloud top, and
by 2 m$^2$s$^{-1}$ at cloud top. Cloud top is assumed to occur at the level at which near-surface air becomes neutrally buoyant, or at the layer interface closest to $\sigma = 0.75$, whichever is lower.

3. **Brief description of the cloud scheme**

   (a) **Sources and sinks of cloud water**

   The new cloud-scheme replaces the diagnostic cloud-scheme and treatment of large-scale precipitation in the standard Mark 2 version of the CSIRO GCM (described above). It incorporates prognostic variables for the cloud-liquid-water mixing ratio ($q_l$) and the cloud-ice mixing ratio ($q_i$). The prognostic equations governing the evolution of the new variables may be written as

   \[
   \frac{\partial q_l}{\partial t} = (\dot{q}_l)_{C/E} + (\dot{q}_l)_{F/M} + (\dot{q}_l)_P + (\dot{q}_l)_{AV} + (\dot{q}_l)_{TM} + (\dot{q}_l)_{CV} \tag{1}
   \]

   and

   \[
   \frac{\partial q_i}{\partial t} = (\dot{q}_i)_{C/E} + (\dot{q}_i)_{F/M} + (\dot{q}_i)_P + (\dot{q}_i)_{AV} + (\dot{q}_i)_{TM} + (\dot{q}_i)_{CV} \tag{2}
   \]

   respectively. Here, C/E denotes formation or dissipation of stratiform cloud due to condensation or evaporation, F/M denotes freezing or melting, P denotes formation of precipitation, AV denotes advection by the large-scale flow, TM denotes vertical turbulent mixing and CV denotes convection. The convection scheme in the model does not detrain cloud liquid water or cloud ice. However, a simple diagnostic treatment of convective cloudiness is included, as described below.

   Figure 1 contains a schematic overview of the microphysical processes included in the scheme. Processes not shown are advection and turbulent mixing, which transfer cloud liquid water and cloud ice between grid boxes, but do not result in conversion among
the variables within a grid box. For simplicity, a single variable is used to represent all forms of falling ice, so that no distinction is made between falling ice-crystals, snowflakes (aggregates of ice crystals) and graupel (heavily rimed ice-particles). Rain and falling ice are not prognostic variables of the scheme—they are diagnostic quantities which are not carried from one time-step to the next. Rain (with fall speeds of 4–5 m s\(^{-1}\)) is assumed to leave the atmosphere in a single time-step. This assumption is difficult to justify for ice, which falls at less than 1 m s\(^{-1}\) in the scheme, so ice which enters a grid box from above is permitted to act as a source term for cloud ice. Precipitation of cloud liquid water in the scheme occurs as the result of three processes, i.e.

\[
(\dot{q}_l)_P = (\dot{q}_l)_{AC} + (\dot{q}_l)_{AU} + (\dot{q}_l)_{CO},
\]

where AC denotes accretion by falling ice, AU denotes autoconversion (i.e. collision and coalescence of cloud droplets) and CO denotes collection by falling rain. Accretion is a source term for falling ice, whereas autoconversion and collection are source terms for rain (see Fig. 1). Precipitation of cloud ice is treated differently, because of the different physical processes occurring in cloud ice. Ice-forming nuclei (IFN) are relatively rare in the atmosphere (RY88), so ice clouds consist of relatively small numbers of ice crystals many of which grow quickly by diffusion to sufficient size to acquire appreciable fall-speeds. The rate of precipitation of cloud ice is therefore calculated by the use of an observationally based fall-speed. The processes shown in Fig. 1 are treated by three subroutines: the first calculates the formation or dissipation of cloud and the freezing or melting of cloud water, the second calculates frozen precipitation, including sublimation and melting of falling ice and accretion of cloud liquid water by falling ice, and the third calculates the formation of rain by autoconversion and collection by raindrops, and the evaporation of falling rain. The treatment of precipitation is described in detail in sections 4 and 5, whereas the other components of the scheme are described in the remainder of this section.

\[(b) \text{ Formation and dissipation of stratiform cloud}\]

The formation and dissipation of stratiform cloud is treated by a simple statistical condensation scheme (Smith 1990, hereafter referred to as S90) which uses an assumed triangular probability-density function (PDF) for the sub-grid distribution of the moisture about its grid-box-mean value. S90 expressed the PDF in terms of sub-grid fluctuations of a ‘generalized cloud-water’ variable

\[Q_c = a_L(q_t - q_s(T_L, \rho)),\]

which represents the cloud-water mixing ratio\(^*\) when positive, or the amount of subsaturation when negative. Here, \(q_t = q_v + q_l + q_i\) is the total-water mixing ratio and \(T_L = T - (L_v/c_p)q_t - (L_s/c_p)q_i\) is the liquid-frozen water temperature (i.e. the temperature that would be obtained by evaporation of all the ice and liquid water in the grid box). The factor

\[a_L = \left(1 + \frac{L}{c_p} \left(\frac{\partial q_s}{\partial T}\right)_{T_L}\right)^{-1}\]

accounts for latent heating which increases the temperature inside the cloud above \(T_L\); \(\partial q_s/\partial T\) is obtained from the Clausius–Clapeyron equation; \(L = L_v\) when \(T < 0\,^\circ\text{C}\), and \(L = L_s\) otherwise.

\(^*\) Strictly, S90 described his scheme in terms of specific humidities, whereas the present scheme uses mixing ratios, to be consistent with the treatment of water vapour in the CSIRO GCM. The only difference is in the formula used to relate \(e_s\) to \(q_s\).
A mathematically identical but physically simpler formulation is obtained by expressing the PDF $f(q_i)$ in terms of the sub-grid variation of $q_i$, while assuming that $T_L$ does not vary within a grid box. In this formulation, the total-water mixing ratio at a point within a grid box (denoted by $q_i$, to distinguish it from the grid-box-mean value $q_i$) is assumed to vary between $q_i - \Delta q$ and $q_i + \Delta q$ as shown in Fig. 2, and cloud is assumed to form in the portion of the grid box which is supersaturated, i.e. the shaded part in Fig. 2. (The two types of shading in Fig. 2 denote regions that are supersaturated with respect to ice ($q > q_{sl}$) and liquid water ($q > q_{sl}$) respectively in mixed-phase clouds—the significance of these regions is explained below.) The stratiform cloud fraction is given by

$$C = \int_{q_{sl}}^{\infty} f(q_i) \, dq_i$$

where $q_s = q_{sl}$ if $T < 0$ °C and $q_s = q_{sl}$ otherwise. The cloud-water mixing ratio $q_c = q_i + q_i$ is calculated on the assumption that sufficient condensation occurs to remove any supersaturation, i.e.

$$q_c = \int_{q_s}^{\infty} a_L(q - q_s(T_L, p)) f(q_i) \, dq_i.$$  

This assumption is reasonable for liquid-water clouds (in which supersaturations are generally small) but is more difficult to justify for ice clouds, in which air can be significantly supersaturated with respect to ice (Heymsfield and Miloshevich 1995). S90 expressed the width of the PDF in terms of $\sigma_s$, the standard deviation of the sub-grid fluctuations of the variable $Q_c$, and parametrized $\sigma_s$ as a function of an arbitrary critical relative humidity $RH_{CR}$ at which clouds begin to form. In the present scheme, the half-width $\Delta q$ of the triangular distribution $f(q_i)$ is given by

$$\Delta q = q_s(1 - RH_{CR})$$

and is related to the variable $\sigma_s$ defined by S90 as $\Delta q = \sqrt{6}\sigma_s/a_L$. With this formulation, the present scheme is mathematically identical to the S90 scheme—the equations required
to calculate $C$ and $q_c$ are given in appendix C of S90. The only difference is the simpler physical picture, which clarifies the treatment of mixed-phase clouds described below. In the present scheme, the critical relative humidity $RH_{CR}$ is set to 0.8 at land points and 0.85 at non-land points—this is a crude way to allow for greater sub-grid variability at land points. The modelled cloud-water contents show realistic variation with temperature, as shown in Fig. 3 where the cloud water contents from a single model time-step are plotted along with the median and 95 percentile values of a large number of observations collected by Mazin (1994). Both in the model and in nature, the variation of cloud water content with temperature is primarily due to the increase of $\partial q_c/\partial T$ with temperature.

(c) Freezing of cloud liquid water and melting of cloud ice

Freezing of cloud liquid water is assumed to occur instantaneously at temperatures less than $-30^\circ C$, while melting of cloud ice occurs instantaneously at temperatures greater than $0^\circ C$. Between these two temperatures, liquid water and ice are allowed to coexist in the model, so an approach is needed to determine the fraction of cloud water that is liquid. This is difficult to treat realistically in large-scale models, since the physical processes involved in mixed-phase clouds are very complex and not completely understood (RY88). Ideally, the treatment adopted should be able to simulate the life cycle of real mixed-phase clouds, which consist mainly of liquid water initially (because of the relative abundance of cloud condensation nuclei (CCN) compared to IFN), but contain increasing amounts of ice as they evolve, since ice crystals grow at the expense of liquid water as a result of the difference between the saturation vapour pressures with respect to ice and liquid water (often referred to as the Bergeron–Findeisen mechanism). It is also desirable that the treatment should ensure that, at a given temperature, deep clouds (represented by multilayer clouds in the model) are more likely to be glaciated than thin clouds (Ryan 1996), since the lower levels of deep clouds are ‘seeded’ by ice falling from above. This ‘seeder-feeder’ mechanism is thought to be responsible for most of the precipitation produced by mid-latitude stratiform clouds (e.g. Rutledge and Hobbs (1983), hereafter RH83).
Most schemes used in GCMs (S90; Ose 1993; Tiedtke 1993; Kristjánsson 1994; Boucher et al. 1995) have used some form of interpolation to specify the liquid fraction at temperatures below freezing, although the form of the interpolation and the lower limit (at which all cloud water is assumed frozen) differ between schemes. For example, S90 used a quadratic function of temperature to specify the liquid fraction at temperatures between 0 °C and −15 °C, while Ose (1993) used linear interpolation between −5 °C and −40 °C for stratiform cloud. Although cloud liquid water has been observed at temperatures as low as −40 °C (Heymsfield 1993), Ryan (1996) has argued, based on a review of a wide range of observations of stratiform clouds, that at temperatures below −15 °C and away from regions of embedded convection, the incidence of liquid water is low. This suggests that stratiform clouds in large-scale models should contain relatively little liquid water at temperatures below −15 °C, provided that a separate treatment of convective clouds is included to represent the embedded convection. The interpolation approach, while simple to implement, has a number of drawbacks. For example, ice falling into a sub-freezing model-layer can be forced to ‘melt’ in order to achieve the prescribed fraction of the liquid at that temperature. Also, the interpolation approach does not model the glaciation of deep clouds realistically. The alternative approach of explicitly parametrizing the processes that result in glaciation of clouds is appealing, but suffers from various uncertainties, especially (but not exclusively) on the spatial and temporal scales resolved by current GCMs. An approach of this type is outlined in appendix A, along with a brief discussion of some of the difficulties.

In view of these difficulties, the following relatively simple approach is adopted here for the treatment of mixed-phase clouds (i.e. clouds at temperatures between −30 °C and 0 °C).

1. The cloud fraction $C$ and the total-cloud-water mixing ratio $q_c$ are calculated, following S90 and using $q_s = q_{st}$ in (6) and (7). This calculation implies that cloud can exist in the portion of the grid box that is supersaturated with respect to ice (i.e. in the shaded area in Fig. 2), and that sufficient condensation occurs to remove any supersaturation with respect to ice.

2. The updated-cloud-liquid water mixing ratio $q_l$ is calculated using (7), but with $q_s = q_{si}$; i.e. it is assumed that supercooled liquid water can coexist with ice in the part of the grid box which is supersaturated with respect to liquid water (i.e. in the heavily shaded area in Fig. 2), while only ice can exist in the part of the cloud that is subsaturated with respect to liquid water (i.e. in the lightly shaded area in Fig. 2). This is the maximum amount of liquid water that can exist under the assumed PDF.

3. The updated cloud-ice mixing ratio $q_i$ is calculated as the difference between the total-cloud-water mixing ratio and the liquid-water mixing ratio, i.e. $q_i = q_c - q_l$.

4. Since ice should not spuriously melt at sub-freezing temperatures, a further condition is imposed. The updated $q_i$ is adjusted so that it is not less than its value before the condensation calculation, unless the total amount of condensate is found to decrease, when $q_l$ is permitted to decrease through sublimation, in proportion to the decrease in the total amount of condensate. If this condition is invoked, the updated $q_i$ is also adjusted so that the total cloud water $q_c$ remains consistent with that calculated previously.

Though simple, this calculation does contain some of the key physics described in appendix A: it is driven by the relative difference between the saturation mixing ratios with respect to ice and liquid water, by the amount of ice pre-existing in the cloud, and also by the total amount of condensate. The last factor enters the calculation because, in a very moist area, a large fraction of the grid box may exceed saturation with respect to liquid water, while in a less moist area some ice cloud may form when no part of the grid box
is saturated with respect to liquid water (see Fig. 2). The relative difference \((q_{sl} - q_{si})/q_{si}\) decreases monotonically with decreasing temperature and is the key parameter driving the calculation described above. It behaves differently from the absolute difference \(q_{sl} - q_{si}\), which reaches a maximum around \(-12 \, ^\circ\text{C}\). Figure 2 shows that it is the relative (rather than the absolute) difference which drives the above calculation, since the relevant quantity is the ratio of \(q_{sl} - q_{si}\) to the half-width \(\Delta q\), where \(\Delta q\) is a linear function of \(q_{si}\) given by (8). This relative difference also appears as a key parameter in the physically based treatment discussed in appendix A.

Freezing of cloud liquid water can also occur by riming, the accretion of liquid water by ice which falls from above into a grid box containing supercooled liquid water, thus representing the increased likelihood of glaciation in deep clouds (see section 5). Freezing of cloud liquid water through deposition on ice falling from above has been omitted from the present scheme, as it is unclear how to parametrize this process in a manner which is consistent with the treatment of mixed-phase clouds just described (see appendix A). The liquid-water fractions generated by the scheme during a single time-step are plotted against temperature in Fig. 4, and may be compared with observations made by the Meteorological Research Flight in frontal and other stratiform clouds (see Fig. 5 which is reproduced from Ryan (1996) by permission of the American Meteorological Society). The liquid-water fractions generated by the scheme look realistic when compared with the observations, especially in view of the uncertainty in the observed data, although the modelled liquid-water fractions tend to be somewhat higher than those observed at higher temperatures. Note also that the scheme contains no mechanism which might generate larger liquid-water fractions in continental clouds than in maritime clouds, which the observations suggest; this is thought to be related to the broader droplet-size distributions found in maritime clouds, but the mechanism is uncertain (Rangno and Hobbs 1994). However, the fairly realistic variation of liquid-water fraction with temperature shown in Fig. 4 suggests that, in the mean, the relative difference between \(q_{sl}\) and \(q_{si}\) is the single most important factor which determines the liquid-water fraction in real clouds. The observations in Fig. 5 may also be compared with those shown for frontal clouds alone by Bower et al. (1996); the
points with low liquid-water fractions at relatively high temperatures correspond mainly with the deep frontal clouds, rather than shallower clouds such as stratocumulus.

(d) Other sources and sinks of cloud water

Advection of the cloud-water variables $q_l$ and $q_i$ is handled by the semi-Lagrangian scheme that is used for advection of the water-vapour mixing ratio $q_v$ in the standard version of the model (described above). For the cloud-water variables, the quasi-monotone scheme of Bermejo and Staniforth (1992) is applied to both the vertical and horizontal advection, thereby suppressing the generation of negative values or spurious oscillations in regions of sharp gradients. The advection of cloud water makes only a slight difference to the modelled cloud-fields (consistent with the findings of others, such as Fowler et al. (1996)). It is retained, however, for completeness and in the expectation that, when a more realistic treatment of anvil cirrus is included, the advection term will become more important, since anvil cirrus clouds have long lifetimes and can be advected over large distances (Randall 1989).

Vertical turbulent mixing of cloud liquid water and cloud ice is implemented along similar lines to the model’s usual turbulent-mixing scheme. The inclusion of vertical mixing of the cloud-water species results in a slight decrease in low cloud and a slight increase in middle-level cloud. Also included is a modification to the Richardson number which accounts for the latent heating and cloud-water loading in cloudy air, similar to that described by S90. The overall effect of the modified Richardson number is to slightly increase the instability of cloudy layers, as the destabilizing effect of the latent heating tends to outweigh the stabilizing effect of the cloud-water loading. This results in a more vigorous hydrological cycle with less low-cloud cover and slightly more middle-level cloud.

(e) Diagnostic treatment of convective clouds

A simple diagnostic treatment of convective cloudiness, similar to the Slingo (1987) scheme used in the standard Mark 2 version of the CSIRO GCM, has been retained for
now. Following Hack et al. (1993), the cover of convective cloud is set to

\[ \overline{C}_{CV} = a_{CV} + b_{CV} \ln(1 + R_{CV}) \]  \hspace{1cm} (9)

where \( R_{CV} \) is the convective rainfall rate at cloud base and \( a_{CV} \) and \( b_{CV} \) are tunable constants (currently set to 0.2 and 0.07 respectively, with \( R_{CV} \) in mm d\(^{-1}\)). With this formulation, \( a_{CV} \) is the convective cloud cover for non-precipitating (shallow) convection. The convective cloud is then assumed to be randomly overlapped, so that the fraction of convective cloud at each model level within the convectively active region is given by

\[ C_{CV} = 1 - (1 - \overline{C}_{CV})^{1/N_{CV}} \]  \hspace{1cm} (10)

where \( N_{CV} \) is the number of convectively active levels. The water content of convective clouds is specified as 0.2 g m\(^{-3}\), based on observations given by Gayet et al. (1993). This water content is assumed to be in the form of liquid water at temperatures above 0 °C and in the form of ice at temperatures below -35 °C. Between 0 °C and -35 °C, the fraction occurring as liquid water is specified by linear interpolation in temperature. When convective cloud is diagnosed in a grid box, the stratiform condensation-scheme is applied only in the environmental air outside the convective cloud. This avoids 'double counting' of cloud when convective and stratiform cloud coexist in a grid box. A total cloud-amount consisting of the sum of the convective and stratiform cloud amounts, with cloud-water content given by the weighted mean of the convective and stratiform values, is then computed for input to the model's radiation scheme.

(f) Cloud radiative properties

The radiation scheme in the CSIRO GCM assumes random overlap between cloud layers and requires the specification of the bulk cloud radiative properties, i.e. the short-wave reflectivity and absorptivity and the broadband long-wave emissivity. In the standard version of the model, the radiative properties of clouds are prescribed, varying as a function of height only. In the version with prognostic cloud-water, the short-wave properties are calculated for warm clouds following Slingo (1989) and using a similar scheme for ice clouds (Francis et al. 1994). Both schemes use the delta-Eddington approximation to calculate the short-wave properties for four bands, which are then averaged to give the properties for the two bands used by the model's short-wave radiation scheme. Both schemes require as input parameters the liquid-(or ice-)water path and the effective radius \( r_e \). The liquid- and ice-water paths are provided by the cloud scheme—the values passed to the radiation scheme are taken as averages of those generated by the cloud scheme before and after the calculation of precipitation, and are also averaged over several time-steps, since the radiation scheme is called just once every two model hours. For warm clouds, \( r_e \) is specified as a function of liquid-water content and droplet concentration following Martin et al. (1994), with a prescribed droplet-concentration of \( 1 \times 10^8 \) m\(^{-3}\) for maritime clouds and \( 5 \times 10^8 \) m\(^{-3}\) for continental clouds. For simplicity, continental clouds are taken to be those at land grid-points and maritime clouds are taken to be those at non-land grid-points. As in the study by Jones et al. (1994), the liquid-water content at cloud top is used in the calculation of \( r_e \). For ice clouds, \( r_e \) is diagnosed from a relation between ice content \( W_i \) and visible volume extinction coefficient \( \beta_{ext} \) deduced from observations by Platt (1994), viz.

\[ \beta_{ext} = 0.032 W_i^{0.333} \]  \hspace{1cm} (11)

if \( \beta_{ext} \) and \( W_i \) are both given in SI units (m\(^{-1}\) and kg m\(^{-3}\) respectively). Equation (11) was derived on the basis that ice crystals can be approximated as spheres—a more sophisticated
parametrization, which accounts for ice-crystal morphology, is given by Platt (1997). Since $r_e = 3W_i/(2p_i\beta_{ext})$, (11) implies that the effective radius increases with $W_i$ according to

$$r_e = 0.051 W_i^{0.667}$$

(12)

in contrast to many current schemes which specify a fixed value for $r_e$ in ice clouds. According to (12), $r_e$ increases from about $5 \times 10^{-6}$ m when $W_i = 10^{-6}$ kg m$^{-3}$ to about $110 \times 10^{-6}$ m when $W_i = 10^{-4}$ kg m$^{-3}$. $W_i$ is calculated from the in-cloud ice mixing ratio, i.e. $W_i = \rho q_i/C_i$, where $C_i$ is the stratiform ice-cloud fraction. For ice clouds, the asymmetry parameter $\delta_i$, which affects the ratio of forward-scattered to back-scattered short-wave radiation, is set to 0.8 in the control version of the scheme. This is the value suggested by Francis et al. (1994), although, as pointed out by these authors (and others), there is considerable uncertainty regarding the specification of $\delta_i$. Stephens et al. (1990) used $\delta_i = 0.7$ (a value which would result in increased backscatter and hence more reflective clouds), while the value for spheres (about 0.85) serves as an upper limit on the reasonable range of values for $\delta_i$.

It is now well known that the treatment of clouds as plane-parallel by the radiation schemes used in GCMs results in an overestimate of cloud albedo (e.g. Harshvardhan and Randall 1985), so that GCMs have to predict artificially low cloud-water paths to achieve realistic albedos. Cahalan et al. (1994), in a study of marine stratocumulus during the First ISCCP (International Satellite Cloud Climatology Project) Regional Experiment (FIRE), found a 15% reduction in cloud albedo (compared to the plane-parallel values) because of horizontal liquid-water variability, and pointed out that the bias will be larger for other types of clouds which are less plane-parallel. They also suggested that reasonable estimates of the albedo could be achieved in models by multiplying cloud optical depths by a factor $\xi = 0.7$. Tiedtke (1996) included this modification in the model used by the European Centre for Medium-Range Weather Forecasts (ECMWF) and found improved agreement between the model’s top-of-the-atmosphere short-wave fluxes and satellite observations. Kogan et al. (1995), based on three-dimensional large-eddy simulations of the evolution of a marine cloud-topped boundary-layer, found that $\xi \approx 0.5$ for cumulus and $\xi \approx 0.8$ for layer cloud. Guided by their results, the present scheme uses $\xi = 0.5$ for convective clouds, whereas for layer clouds $\xi$ is increased linearly with stratiform-cloud fraction $C$, from a minimum value of 0.6 when $C = 0.2$ to a maximum value of 0.9 when $C = 0.8$.

The cloud emissivity is calculated as a function of infrared optical depth, and thus is a function of effective radius as well as liquid-(or ice-) water path (C. M. R. Platt, personal communication, 1994). Given the cloud visible optical depth $\delta_v = \beta_{ext}\Delta z$, with $\Delta z$ the depth of the layer, the infrared optical depth is calculated as $\delta_a = 0.4\delta_v$ for water clouds or $\delta_a = 0.5\delta_v$ for ice clouds. The emissivity is then given by

$$\epsilon = 1 - \exp(-3\mathcal{K}\delta_a)$$

(13)

with the optical depth diffusivity factor $\mathcal{K}$ approximated as 1.6 if $\delta_v > 0.4$ or 1.8 otherwise (derived from curves given by Platt and Stephens (1980)).

At temperatures between $-30$ °C and $0$ °C, cloud ice and cloud liquid water can coexist in the model, so it is necessary to specify the way in which ice and liquid water are mixed within a grid box. Sun and Shine (1994) showed that the radiative properties of mixed-phase clouds are sensitive to the method by which the phases are mixed, and pointed out that the choice of method could have a significant effect on the climate sensitivity of models that incorporate mixed-phase clouds. The three configurations they considered were ‘uniform’ (with ice and liquid water coexisting throughout the cloud), ‘stratified’ (with ice layers above the liquid-water layers) and ‘adjacent’ (with the cloud consisting of
two separate, vertically homogeneous clouds of single phase). They found that, for given amounts of cloud liquid water and cloud ice, the uniform method resulted in significantly higher cloud-albedos than the adjacent method. The adjacent method has been adopted in the scheme described here; this was also the method used by Mitchell et al. (1989). A number of other schemes have used the uniform method—there is currently no theoretical or observational basis for choosing between these two methods.

(g) Fractional cloudiness and vertical overlap assumptions

It is highly desirable that parametrizations for use in a GCM take into account fractional cloudiness, so that, for example, the 'in-cloud' values (rather than the grid-box-mean values) of the cloud-water mixing ratios are used in the calculation of precipitation. This avoids the problem encountered by Fowler et al. (1996), who had no treatment of fractional cloudiness and therefore had to use relatively low thresholds for the autoconversion of cloud water to precipitation. The parametrizations described here all use the cloud fraction calculated by the S90 condensation scheme to evaluate the mean in-cloud values of the required quantities. Clouds are assumed to be strictly randomly overlapped in the vertical, an assumption chosen for its simplicity and its consistency with the radiation scheme, although there is some observational evidence that maximum overlap is a better assumption for vertically adjacent cloud layers (Tian and Curry 1989). The parametrizations of precipitation and related processes are also developed on the assumption that all clouds completely fill their layers in the vertical, and that mixed-phase clouds are comprised of two horizontally adjacent clouds of single phase, again consistent with the methods used for calculation of the cloud radiative properties. (Thus, \( C = C_1 + C_1 \), where \( C_1 \) and \( C_1 \) are the ice and liquid-water stratiform cloud fractions respectively.) It is also desirable that the parametrizations related to falling precipitation take into account the fraction of the grid box into which the precipitation falls, hereafter referred to as the rainy or snowy fraction. The calculation of this fraction is described in appendix C.

4. Parametrization of rain formation and related processes

In this section the modelled processes of rain formation (autoconversion, collection of cloud liquid water and evaporation of rain—see Fig. 1) are described and are compared with other schemes that have been used. The collection and evaporation parametrizations require assumptions about the fall speed and size distributions of raindrops—these are presented, together with observational evidence for the chosen formulations. The sensitivity of the rainfall processes to the time-step and numerics is also evaluated, by using a time-splitting approach in which the time-step used for the rainfall processes is varied independently of that used for other processes in the model in a series of 10-day experiments forced by climatological sea-surface temperatures (SSTs). The initial condition for each of these experiments was taken as 0000 UTC on July 1 in the second year of a model run performed using a version of the scheme very similar to the control version described here. The control experiment (hereafter referred to as 'CNTRL') used the version of the scheme described here, with no time-splitting, i.e. the usual model leapfrog time-step of 48 minutes was used for all the precipitation processes. Another experiment referred to is 'RAIN6', which was identical to CNTRL, except that the time-step used for the rainfall processes was reduced to six minutes. For experiments designed to evaluate the sensitivity of the rainfall processes to the time-step and numerics, differences from the RAIN6 run are shown, i.e. the RAIN6 run is used as the benchmark. For experiments designed to compare different parametrizations, differences from the CNTRL run are shown, since all of these experiments use the 48 minute time-step.
(a) Autoconversion of cloud liquid water

S90 parametrized the conversion of cloud liquid water to precipitation as the sum of an autoconversion term (Sundqvist 1978) and an accretion term which increases the conversion rate when precipitation (liquid or frozen) falls into the layer from above, viz.

\[
(q_{1})_P = -C \left( c_T \left[ 1 - \exp \left\{ -\left( \frac{q_1/C}{c_w} \right)^2 \right\} \right] + c_A P \right) \frac{q_1}{C} .
\]  
(14)

Here, using the notation of S90, \( c_T \) and \( c_A \) are rate constants for the autoconversion and accretion terms respectively, \( c_w \) is a critical mixing ratio at which the autoconversion process begins to be efficient and \( P \) is the rate at which precipitation falls into the layer from above. The parameters were given the values \( c_T = 10^{-4} \) s\(^{-1} \), \( c_A = 1 \) m\(^2\) kg\(^{-1} \) and \( c_w = 8 \times 10^{-4} \) kg kg\(^{-1} \), although more recent versions of the scheme have employed a reduced value of \( c_w = 2 \times 10^{-4} \) kg kg\(^{-1} \) over the oceans, in order to improve the agreement between the liquid-water paths in the Meteorological Office model and satellite observations (Smith 1993). This parametrization has been widely used in large-scale models (Roeckner et al. 1992; Tiedtke 1993; Ricard and Royer 1993; Le Treut et al. 1994) although neither Sundqvist (1978) nor S90 gave any physical justification for its form.

An autoconversion parametrization with a stronger physical basis was derived by Manton and Cotton (1977), who used physical and dimensional arguments to improve upon the simple scheme of Kessler (1969), which has been used by Ose (1993) and Fowler et al. (1996). (See also Tripoli and Cotton (1980), hereafter TC80, in which the Manton and Cotton scheme was used in a cloud-resolving model in a study of cumulus clouds over Florida.) According to detailed calculations by J. B. Jensen (personal communication, 1995), the Manton and Cotton scheme gives more realistic rates of autoconversion than the Kessler scheme. In contrast to the Kessler scheme, which uses a simple formulation of the threshold, the Manton and Cotton scheme takes the cloud droplet concentration as an input parameter. The cloud droplet concentration can be parametrized as a function of the CCN concentration (e.g. Jones et al. 1994), which means that it can distinguish between the microphysics of maritime and continental clouds, and that it can be used to study the effects of anthropogenic production of aerosols on cloud microphysics. The parametrization, modified to allow for fractional cloudiness, takes the form

\[
(q_{1})_{AU} = -C_1 \frac{0.104 g E_{AU} \rho^{4/3}}{\mu (N_d \rho_w)^{1/3}} \left( \frac{q_1}{C_1} \right)^{7/3} H \left( \frac{q_1}{C_1} - q_{CR} \right)
\]  
(15)

where \( C_1 \) is the liquid water cloud fraction, \( \mu \) is the dynamic viscosity of air, \( \rho_w \) is the density of water, \( N_d \) is the droplet concentration, \( g \) is the acceleration under gravity, \( E_{AU} \) is the mean collection efficiency and \( \rho \) is the air density. \( H \) is the Heaviside unit step function which suppresses autoconversion until \( q_1/C_1 \) reaches \( q_{CR} \). The critical mixing ratio at which autoconversion begins is given by

\[
q_{CR} = \frac{4}{3} \pi \rho_w r_{CR}^3 N_d / \rho
\]  
(16)

where \( r_{CR} \) is the critical mean droplet radius at which autoconversion begins. The present scheme uses \( E_{AU} = 0.55 \) (the same as the value used by TC80), \( r_{CR} = 9 \times 10^{-6} \) m (slightly smaller than the value of \( 1 \times 10^{-5} \) m used by TC80), \( N_d = 5 \times 10^8 \) m\(^{-3} \) for continental clouds and \( N_d = 1 \times 10^8 \) m\(^{-3} \) for maritime clouds (compared to \( N_d = 3 \times 10^8 \) m\(^{-3} \) as used by TC80). The slightly smaller value of \( r_{CR} \), compared to that given by TC80, can be
The RAIN6 and CNTRL experiments are described in the text, while the columns labelled implicit and explicit refer to experiments identical to CNTRL, except that each individual term is treated implicitly or explicitly in turn.

justified by the much coarser spatial scales resolved by the GCM—autoconversion starts to become efficient when some droplets grow by diffusion to radii of around 20 \( \mu \text{m} \) (e.g. Mason 1975), and it seems reasonable to assume that this occurs at a smaller mean droplet-radius in a coarser resolution model such as a GCM. The use of different values of \( N_d \) for continental and maritime clouds is a simple way of allowing for the larger numbers of CCN in continental airmasses—a more sophisticated approach would involve parametrization of \( N_d \) as a function of the CCN concentration, the latter quantity probably being obtained from a chemical-transport model which provides the concentration of sulphate (and preferably other) aerosols as an output (e.g. Taylor and Penner 1994). A parametrization similar to (15) has been used recently by Boucher et al. (1995), who have described the sensitivity of their scheme to parameters such as \( N_d \) and \( r_{\text{CR}} \). Note that the values of \( q_{\text{CR}} \) implied by (16) with \( r_{\text{CR}} \) and \( N_d \) as specified here are somewhat larger than the values used by Smith (1993) and many other existing schemes, especially over land.

Early experiments using a simple explicit evaluation of (15) showed a tendency for the scheme to overestimate the autoconversion rate, in the sense that negative liquid-water mixing ratios would be generated if not explicitly suppressed in the code. This was due to the large GCM time-step which is much greater than the time-steps for which the scheme was originally designed. A form more suitable for use with large time-steps is obtained by analytical integration of (15) with respect to time (see appendix B). This form of the parametrization prevents the generation of negative values and results in lower conversion rates than a simple explicit implementation of (15). A further modification is introduced to make the scheme more suitable for use with large time-steps: the autoconversion process is prevented from reducing the in-cloud liquid-water mixing ratio below \( q_{\text{CR}} \). However, accretion of cloud liquid water by the raindrops generated by the autoconversion process is allowed to contribute to the conversion process (see appendix B), so that \( q_{l}/C \) can be reduced to less than \( q_{\text{CR}} \). This approach is consistent with the finding of Berry and Reinhardt (1974a, b) that autoconversion initiates the process of conversion from small droplets to large drops, after which the conversion rate is dominated by accretion processes.

The zonal-mean autoconversion rates from the CNTRL run are shown in Fig. 6(a), with the differences of the CNTRL run from the RAIN6 run shown in Fig. 6(b). It can be seen that some increase in the autoconversion rates resulted from the increase of time-step from six minutes to 48 minutes. This was presumably because the autoconversion parametrization is invoked prior to the collection parametrization at each time-step. The globally averaged, vertically integrated values of the various conversion terms are shown in Table 1, which shows that the autoconversion term is, on average, roughly 16% larger in the CNTRL run compared to the RAIN6 run. As is shown below, the impact of this increase on the simulation is small, since the autoconversion term is quite small compared
Figure 6. Time-averaged zonal-mean autoconversion rates (10^{-9} \text{ kg kg}^{-1} \text{s}^{-1}) from various ten-day model experiments performed under July conditions. (a) CNTRL run; (b) differences of CNTRL run from RAIN6 run; (c) as (a), but autoconversion parametrized using (14) (differences from CNTRL run shown). Negative contours are dashed, and the depth of shading indicates the magnitude of the plotted values.
with those representing the other processes (collection, accretion) which deplete cloud liquid water. The globally averaged value for the explicit scheme given in Table 1 is just slightly larger than that from the CNTRL run. This experiment included the condition mentioned in the previous paragraph, preventing reduction of the in-cloud liquid-water mixing ratio below $q_{\text{CR}}$; without this condition, the explicit treatment gives much worse results.

Shown in Fig. 6(c) are the differences from the CNTRL run of the zonal-mean autoconversion rates from a run identical to CNTRL, except that autoconversion was parametrized using Sundqvist's (1978) scheme. In this run, the parameters were given the values used by S90, except that the critical mixing ratio $q_{\text{CR}}$ at non-land grid-points was reduced to $2 \times 10^{-4}$ kg kg$^{-1}$, as used by Smith (1993). Clearly, Sundqvist's scheme gave much larger autoconversion rates than the present scheme, with the parameters chosen as specified above. The globally averaged autoconversion-rate using Sundqvist's scheme was 0.182 mm d$^{-1}$, roughly 2.3 times larger than that in the CNTRL run. The mean autoconversion-rates are controlled both by the choice of $q_{\text{CR}}$ and by the functional form of the parametrization—Sundqvist's scheme gave autoconversion rates somewhat more similar to the present scheme when used with values of $q_{\text{CR}}$ based on (16) at each grid-point, with a globally averaged rate of autoconversion of 0.132 mm d$^{-1}$. These results are discussed in section 7.

(b) Fall speed and size distribution of raindrops

In this study, raindrops of diameter $D_t$ are assumed to fall with terminal velocity

$$V_t(D_t) = k_t D_t^{1/2} \left( \frac{\rho_0}{\rho} \right)^{1/2}$$

(17)

where $k_t = 141.4$ m$^{1/2}$ s$^{-1}$ and $\rho_0 = 1.2$ kg m$^{-3}$ is a reference air density (RY88). This fall speed provides a good approximation to the observational data of Gunn and Kinzer (1949) in the range $1.2 \times 10^{-3}$ m $< D_t < 4 \times 10^{-3}$ m.

There is a considerable body of observational evidence and some theoretical arguments (RY88) which suggest that the size distributions of raindrops (and ice particles) can be approximated by a negative exponential distribution, first suggested by Marshall and Palmer (1948). In this approximation, the number concentration of raindrops with diameters between $D_t$ and $D_t + dD_t$ is $N_t(D_t) \ dD_t$, where

$$N_t(D_t) = N_{0_t} \exp(-\lambda_t D_t).$$

(18)

The use of (18) is very convenient for the development of parametrizations, since it has the property that, for any non-negative real number $x$,

$$\int_0^\infty D_t^x N_t(D_t) \ dD_t = \frac{N_{0_t}}{\lambda_t^{x+1}} \Gamma(x + 1)$$

(19)

where $\Gamma(\cdot)$ denotes the gamma function. Marshall and Palmer found that the slope factor depends on the local rainfall rate $R_t^l$ (kg m$^{-2}$s$^{-1}$) according to

$$\lambda_t(R_t^l) = 734 (R_t^l)^{-0.21}$$

(20)

and that the intercept parameter is a constant, given by $N_{0_t} = 8 \times 10^6$ m$^{-4}$. The present scheme uses this constant value for $N_{0_t}$, in common with many other schemes (e.g. RH83; Lin et al. 1983; Fowler et al. 1996; Gregory 1995). The assumptions made here regarding
the fall speed and drop-size distribution imply a relation between the slope factor and the rainfall intensity, similar to (20). As shown in appendix B, this relation is

\[ \lambda_r = 714 \left( \frac{\rho_0}{\rho} \right)^{1/9} (R^4_r)^{-0.22}, \]

which is similar to (20), deduced observationally by Marshall and Palmer (1948). This provides a useful check of the consistency of the parametrizations described in this subsection. Note that the alternative assumption of a constant slope-factor and variable intercept-parameter adopted by Tripoli and Cotton (1980) and Boucher et al. (1995) is more justifiable when very heavy rainfall rates, more typical of convective storms than large-scale precipitation, are being considered (e.g. Sauvageot and Lacaux 1995).

(c) Collection of cloud liquid water by rain

The rate of collection of cloud liquid water by falling rain is obtained by integration of the continuous-collection equation (which gives the rate of collection by a single raindrop) over the Marshall–Palmer distribution (18), together with the use of (17) for the fall speed of raindrops (see appendix B). After some minor simplifications (to save computer time) and evaluation of parameters, the parametrization takes the form

\[ (q_l)_C = -0.24 f_r E_C (R^4_r)^{3/4} q_l \]

where \( f_r \) is the rainy fraction of the grid box, \( E_C \) is the mean collection efficiency and \( R^4_r = R_r / f_r \) is the local rainfall rate. Measured collection-efficiencies are less than unity, and are generally an increasing function of both cloud-droplet and raindrop sizes (RY88). TC80 parametrized \( E_C \) as a function of the Stokes number, based on an assumption of potential flow around the falling raindrop. A trial of their scheme in the CSIRO GCM yielded collection efficiencies that were generally in excess of 0.9, i.e. considerably larger than the measured values given by RY88. The present scheme uses the simpler approach of prescribing \( E_C = 0.7 \), a value which is close to the measured data for 8 \( \mu \)m radius cloud droplets and essentially all raindrop sizes.

The control version of the scheme uses a centred-in-time method to evaluate \( q_l \) on the right-hand side (RHS) of (22) (see appendix B). With the centred-in-time approach, the modelled collection rates were quite insensitive to the time-step over the range tested, as shown in Fig. 7(a) (CNTRL run) and (b) (difference of CNTRL run from RAIN6 run). The differences shown in Fig. 7(b) can be partly attributed to the effect of the increased rates of autoconversion found in the CNTRL run compared to the RAIN6 run—in mid-latitudes, regions of decreased collection exist aloft (where collection and autoconversion compete for the available liquid water), with regions of increased collection below, where the rainfall released by autoconversion at higher levels collects liquid water. The globally averaged rates of collection, given in Table 1, are almost identical in the two runs. The effect of the choice of numerical scheme on the time truncation errors was also evaluated, via two other experiments. These were identical to the CNTRL run, except that \( q_l \) on the RHS of (22) was evaluated implicitly in one run and explicitly in the other. As is shown in Table 1, implicit evaluation of \( q_l \) resulted in collection rates which were, on average, 10% lower than those in the RAIN6 run. Conversely, explicit evaluation of \( q_l \) resulted in collection rates which were roughly 14% larger than those in the RAIN6 run. Analytical integration of (22), treating \( R^4_r \) as a constant, gave results similar to the centred-in-time approach, but the latter was preferred in order to save computer time.

Shown in Fig. 7(c) are the differences from the CNTRL run of the collection rates from another run identical to CNTRL, except that collection was parametrized according
Figure 7. Time-averaged zonal-mean rates of collection of cloud liquid water by rain (10^{-9} \, \text{kg kg}^{-1}\text{s}^{-1}) from various ten-day model experiments performed under July conditions. (a) CNTRL run; (b) differences of CNTRL run from RAIN6 run; (c) as (a), but with collection parametrized using (14) (differences from CNTRL run shown).

Negative contours are dashed, and the depth of shading indicates the magnitude of the plotted values.
to the scheme used by S90 (i.e. using the collection term in (14)). This parametrization resulted in somewhat lower collection rates than those generated by the present scheme, with a globally averaged value of 0.152 mm d\(^{-1}\), roughly 29% lower than in the CNTRL run. (See discussion in section 7.)

\[(d)\] **Evaporation of rain**

The rate of change of water-vapour mixing ratio due to evaporation of rain is obtained by integration of the equation for the evaporation of a single raindrop of diameter \(D_r\) over the Marshall–Palmer distribution (18), together with the use of (17) for the fall speed of raindrops (see appendix B). After some minor simplifications and evaluation of parameters, the parametrization takes the form

\[
(q_v)_{EV} = f_r \frac{8.7 \times 10^2 (q_{sl} - q_v)}{\rho^{1/2} (A' + B') q_{sl} (R_t)^{0.61}}
\]

(23)

where \(q_v\) is the water-vapour mixing ratio, \(A'\) and \(B'\) are temperature-dependent terms representing heat conduction and vapour diffusion respectively and other symbols are as defined previously. The experiments described in this paper use a computationally cheaper form of (23), in which \(8.7 \times 10^2 (R_t)^{0.61}\) is approximated as \(3.8 \times 10^2 (R_t)^{1/2}\).

A centred-in-time method is used to evaluate \((q_{sl} - q_v)\) on the RHS of (23) (see appendix B). With the centred-in-time approach, the modelled rates of evaporation were very insensitive to the time-step over the range tested, as shown in Fig. 8(a) (CNTRL run) and (b) (difference of the CNTRL run from the RAIN6 run). The globally averaged value, given in Table 1, is just slightly (2.4%) larger in the CNTRL run. This increase can probably be attributed to the increase in the amount of rain available for evaporation with the larger time-step (see Table 1). Two other runs, identical to the CNTRL run, except that \((q_{sl} - q_v)\) was evaluated implicitly in the first and explicitly in the second, were also performed. Compared to the RAIN6 run, the implicit scheme gave slightly (2.9%) reduced evaporation-rates, whereas the explicit scheme gave somewhat (8%) larger evaporation-rates (see Table 1). Although the magnitude of the average time-truncation errors obtained with the implicit scheme is not much different from that obtained with the time-centred scheme, the latter appears to be the better choice, as the errors obtained are in the direction expected from the slight increase in the amount of rainfall available for evaporation with the larger time-step. Further, theory suggests that the time-centred scheme will have smaller time-truncation errors. However, the differences are quite small compared to those resulting from the use of different parametrizations.

Shown in Fig. 8(c) are the differences from the CNTRL run of a run identical to the CNTRL run, except that evaporation of rain was parametrized using the scheme proposed by Gregory (1995). Gregory’s scheme aimed to improve upon the simple formulation of Kessler (1969), and has been adopted in various forms in more recent versions (Smith et al. 1995) of the S90 scheme, which originally treated the evaporation of rain in a very simple manner. Gregory’s scheme gives somewhat larger evaporation rates than the present scheme, with a globally averaged value of 0.252 mm d\(^{-1}\), roughly 18% larger than that obtained in the CNTRL run. The evaporation rates obtained with Kessler’s scheme (which has been used in large-scale models by Roeckner et al. (1992), Ose (1993) and Tiedtke (1993)) were larger again, with a globally averaged value of 0.260 mm d\(^{-1}\). The version of Gregory’s scheme mentioned above assumed a Marshall–Palmer drop-size distribution, as in the present scheme. Shown in Fig. 8(d) are the evaporation rates obtained using a modified version of Gregory’s scheme, based on a gamma distribution of order 1. This version gave lower evaporation rates, which were quite similar to those produced by the
Figure 8. Time-averaged zonal-mean rates of evaporation of rain \((10^{-9} \text{ kg kg}^{-1} \text{s}^{-1})\) from various ten-day model experiments performed under July conditions. (a) CNTRL run; (b) differences of CNTRL run from RAIN6 run; (c) as (a) but using Gregory's (1995) evaporation scheme with a Marshall–Palmer drop-size distribution (differences from CNTRL run shown); (d) as (c), but using a gamma drop-size distribution (differences from CNTRL run shown). Negative contours are dashed, and the depth of shading indicates the magnitude of the plotted values.

The globally averaged evaporation rate for this run was 0.220 mm d\(^{-1}\), just slightly larger than that obtained in the CNTRL run. (See discussion in section 7.)

(e) Sensitivity of the modelled cloud-water and rainfall amounts to the time-step

As shown above, the various terms related to large-scale rainfall (autoconversion, collection and evaporation) are quite insensitive to the time-step, provided that the numerical schemes used to calculate these terms are chosen carefully. This suggests that the modelled cloud-liquid-water contents and amounts of rainfall will also be insensitive to the time-step, justifying the decision to avoid using a time-splitting approach with a smaller time-step for the rain processes. Shown in Fig. 9(a) are the zonal-mean stratiform cloud-liquid-water mixing ratios from the CNTRL run, with the differences of the CNTRL run from the RAIN6 run in Fig. 9(b). The differences shown in Fig. 9(b) are small, especially in view of the uncertainties in the various parametrizations. The globally averaged,
vertically integrated cloud-liquid-water path is just slightly reduced due to the increase in time-step, from 0.0668 to 0.0665 kg m\(^{-2}\). Also, the globally averaged large-scale rainfall differed by only 0.01 mm d\(^{-1}\) between these runs—the increase from 0.70 to 0.71 mm d\(^{-1}\) resulting from the increase in time-step can be attributed to the slightly increased autoconversion rates obtained with the larger time-step. (Note that the figure of 0.71 mm d\(^{-1}\) for the CNTRL run can be recovered from Tables 1 and 2 as the sum of autoconversion plus collection plus flux divergence minus evaporation minus sublimation.) There was also a small corresponding decrease in the global-mean forcing of clouds by short-wave radiation, or short-wave cloud forcing (SWCF) of 0.1 W m\(^{-2}\)—these differences are small compared to the effects of replacing one parametrization by another.

There appears to be excessive cloud at very low levels in the model—a possible reason is that the model may be too moist at these levels. Although the modified Richardson number introduced with this cloud scheme has resulted in somewhat more vigorous vertical turbulent mixing of moisture, it may still be insufficient, since the model’s turbulent-mixing-scheme is similar to a scheme which has been shown to lead to an excessively moist lowest level in the United States’ National Center for Atmospheric Research (NCAR)
Figure 9. Time-averaged zonal-mean cloud-liquid-water mixing ratios (g kg$^{-1}$) from two ten-day model experiments performed under July conditions. (a) CNTRL run; (b) differences of CNTRL run from RAIN6 run. Negative contours are dashed, and the depth of shading indicates the magnitude of the plotted values.

<table>
<thead>
<tr>
<th>TABLE 2. GLOBALLY AVERAGED, VERTICALLY INTEGRATED CONVERSION TERMS FOR FROZEN-PRECIPITATION PROCESSES (mm d$^{-1}$).</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNOW6</td>
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<tr>
<td>-------</td>
</tr>
<tr>
<td>flux divergence</td>
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<tr>
<td>accretion</td>
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<tr>
<td>sublimation</td>
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</table>

The SNOW6 and CNTRL experiments are described in the text, while the columns labelled implicit and explicit refer to experiments identical to CNTRL, except that each individual term is treated implicitly or explicitly in turn.
Community Climate Model (Holtslag and Boville 1993). Another possible reason is the use in the condensation scheme of a critical relative humidity $RH_{CR}$ which does not vary with height—see the discussion in section 7.

5. Parametrization of Frozen Precipitation and Related Processes

In this section, the modelled frozen-precipitation processes (flux divergence of falling ice, accretion of cloud liquid water and sublimation of falling ice—see Fig. 1) are described and compared with other schemes that have been used in GCMs. The assumptions regarding fall speeds and size distributions of ice particles are presented and evaluated against observations. The sensitivity of the ice processes to the choice of time-step and numerics is also evaluated, by comparison of the results from the ten-day CNTRL experiment described above with those from an otherwise identical run in which the time-step used for the ice processes was reduced to six minutes. This experiment is referred to as the ‘SNOW6’ run. For experiments designed to evaluate the sensitivity of the ice processes to the time-step and numerics, differences from the SNOW6 run are shown, i.e. the SNOW6 run is used as the benchmark. For experiments designed to compare different parametrizations, differences from the CNTRL run are shown, since all these experiments use a 48-minute timestep.

(a) Precipitation of cloud ice

A detailed treatment of precipitating ice (e.g. Lin et al. 1983) would include separate variables for pure ice crystals, snowflakes (aggregates of ice crystals) and graupel (heavily rimed ice particles). Hail can occur as an extremely dense form of graupel when the freezing of the accreted cloud-droplets is not immediate, and is generally produced only in the strong updraughts found in convective clouds. According to data shown by RY88, ice crystals typically fall at about 0.4–0.5 m s$^{-1}$, dry snowflakes at about 1 m s$^{-1}$ and graupel particles at speeds greater than 1 m s$^{-1}$. In view of the computational demands of more sophisticated treatments, as well as the uncertainties associated with some of the terms in these schemes (especially the conversion of cloud ice to snow), a simpler approach is adopted in this study, in common with several other schemes (S90; Roeckner et al. 1992; Le Treut et al. 1994; Jakob and Morcrette 1995) that have been used in large-scale models. A single variable is used to represent all forms of atmospheric ice so that no real distinction is made between falling ice-crystals, snowflakes and graupel. Instead, the rate of precipitation of ice is parametrized using an empirical relation between the fall speed of ice and cloud ice content, derived from measurements made by Heymsfield (1977). Allowing for fractional cloudiness, this fall speed can be written as

$$\bar{V}_f = 3.23 \left( \frac{\rho q_i}{C_i} \right)^{0.17}. \quad (24)$$

The rate of change of cloud ice as a result of the flux divergence of falling ice is given by

$$q_{i}(t) = \frac{R_{f}}{\rho \Delta z} - q_{i}(t) \frac{\bar{V}_f}{\Delta z} \quad (25)$$

where $R_{f}$ is the rate at which ice falls into the layer from above (see appendix B) and $\Delta z$ is the layer thickness. Explicit evaluation of (25) with the time-steps and vertical resolutions typical of current global models is liable to violate the Courant–Friedrichs–Levy criterion. Although an artificial limit to the flux leaving a layer can prevent values
of $q_i$ becoming negative, to use one results in an underestimate of the precipitation rate (whenever $V_i \Delta t > \Delta z$) and a consequent build-up of ice. In some recent studies, the problem has been addressed by adoption of a 'fall-through' assumption, (by ignoring the first term on the RHS of (25) (Senior and Mitchell 1993; Le Treut et al. 1994), or by making some other ad hoc assumptions (e.g. Jakob and Morcrette 1995). As is shown below, the 'fall-through' assumption leads to an overestimate of the precipitation rate and a corresponding underestimate of the cloud ice content, relative to that predicted by a more accurate evaluation of (25). Smith et al. (1995) used an implicit scheme for the evaluation of (25). This has the virtue of preventing the generation of negative values, and also allows the ice to fall through more than one model layer in a time-step, in contrast to explicit treatments. However, it is shown below that the implicit scheme results in an underestimate of the precipitation rate and a corresponding upward shift of the modelled cloud ice, compared to a more accurate analytical treatment.
The modelled cloud-ice mixing ratios are effectively controlled by the treatment of (25), rather than by the treatment of the accretion or sublimation terms described below, so cloud-ice amounts are shown as here as a proxy for \( \dot{q}_i \). (The globally averaged values of the flux divergence \(-\langle \dot{q}_i \rangle \) are given in Table 2.) Figure 10(a) shows the zonal-mean cloud ice mixing ratios from the CNTRL run, with the differences of the CNTRL run from the SNOW6 run shown in Fig. 10(b). The use of the larger time-step resulted generally in a slight increase in the mean cloud-ice mixing ratios, although there are smaller regions of reduced cloud ice aloft and at low levels in mid-latitudes. The global-mean long-wave cloud radiative forcing (LWCF) for the CNTRL run was 29.8 W m\(^{-2}\), in close agreement with the 29.7 W m\(^{-2}\) obtained in the SNOW6 run. Figure 10(c) shows the differences from the SNOW6 run of a run identical to the CNTRL run, except that the ‘fall-through’ approximation was made. The average cloud-ice mixing ratios obtained in this run were substantially lower than those in the SNOW6 and CNTRL runs and the global-mean LWCF was reduced to 27.6 W m\(^{-2}\). Figure 10(d) shows the differences from the SNOW6 run of a run identical to the CNTRL run, except that (25) was evaluated implicitly. This run gave increased cloud ice aloft, with reduced cloud ice below, i.e. there was a systematic upward
shift of cloud ice associated with the implicit scheme, although the globally averaged value
of the flux-divergence term is only slightly less than in the CNTRL run (see Table 2). A
useful diagnostic of this shift is the global-mean LWCF, which increased to 32.0 W m$^{-2}$
in the implicit run. Explicit evaluation of (25) was also found to be unsatisfactory at large
time-steps (Table 2). Some improvement was obtained using a time-centred treatment, but
the results (not shown) were still less satisfactory than those obtained with the analytically
integrated scheme, which is also the best theoretically. Note that each of the numerical
treatments of (25) implies that a certain fraction of the ice falling into a grid box during a
time-step is allowed to fall through (see appendix B). So the use of this form is a natural
way to determine the fraction of ice that falls through, rather than attempting to prescribe
this fraction in some way.

(b) Fall-speed and size distribution of falling ice particles

The simplifying assumption that all ice particles fall with the mass-weighted mean
velocity (24) is made in this study. Fall velocities of ice particles are a strong function of
particle morphology as well as diameter (Locatelli and Hobbs 1974), but in general the
exponent in a fall-speed relation analogous to (17) would be expected to be less than the
value of 0.5 used for raindrops, because the density of ice particles tends to decrease with
size. For example, RH83 used an exponent of 0.11 in their assumed relation between fall
speed and particle diameter, while Lin et al. (1983) used an exponent of 0.25.

As with raindrops, the distribution of falling ice particles with size is assumed to
follow a Marshall–Palmer distribution, so that the number concentration of falling ice
particles with diameters between $D_t$ and $D_t + dD_t$ is

$$N_t(D_t) = N_{0f} \exp(-\lambda_t D_t).$$

(26)

The assumption of a constant value for the intercept parameter $N_{0f}$ is more difficult to justify
for ice than for rain. Ryan (1996) showed plots of intercept parameter and slope factor
as a function of temperature based on observations from a number of sources. Although
there is considerable scatter, the data suggest that both $N_{0f}$ and $\lambda_t$ tend to increase with
decreasing temperature and that $\lambda_t$ can be parametrized as a function of temperature by

$$\lambda_t = 1.6 \times 10^3 \cdot 10^{0.023(T_0 - T)}.$$  

(27)

By analogy with the arguments given in appendix B for rain, (27) implies a relation for
the intercept parameter $N_{0f}$, viz.

$$N_{0f} = \left(\frac{q_t f_t}{\pi \rho_t}\right)^{\lambda_t^4},$$  

(28)

where $q_t$ is the mixing ratio of falling ice, $f_t$ is the snowy fraction of the grid box (see
appendix C) and $\rho_t$ is the bulk density of ice particles (discussed below). Use of (28) in
the model to diagnose $N_{0f}$ from $\lambda_t$, gives values which are plotted against temperature in
Fig. 11 and are broadly consistent with the data shown by Ryan (1996). The form of (27)
was chosen to give reasonable agreement with these data, as well as with the observed
values of $\lambda_t$. In the absence of suitable data at temperatures less than about $-30^\circ$C, (27)
is used at these lower temperatures also.

The bulk density of falling ice particles $\rho_t$ is very uncertain. Ryan et al. (1976) found
ice-crystal densities in cloud to lie in the range 400 kg m$^{-3}$ to 900 kg m$^{-3}$, while Locatelli
and Hobbs (1974) found a wide range of mass–diameter relationships for different types
of precipitating ice particles, with the bulk density tending to decrease with increasing
size of particle, except for graupel, which had roughly constant density with size. Clough and Franks (1991) used several of these mass–diameter relationships in a one-dimensional model and found the corresponding mass-weighted mean particle densities to vary from around 20 kg m\(^{-3}\) to 400 kg m\(^{-3}\). For parametrization purposes, \(\rho_l\) has typically been given a constant value of 100 kg m\(^{-3}\) (RH83; Lin et al. 1983; Gregory 1995), a value which is adopted for this study.

Falling ice is assumed to melt instantaneously to form rain when it enters a model layer which has a temperature of 2 °C or more, since observations show that snow usually melts in a layer of no more than a few hundred metres below the freezing level (Mason 1971). The assumption of instantaneous melting of snow has been made in most schemes used in large-scale models.

(c) Accretion of cloud liquid water by falling ice

The rate of accretion of cloud liquid water by falling ice is obtained by integration of the continuous-collection equation (which gives the rate of collection by a single ice particle) over the Marshall–Palmer distribution (26) (see appendix B). It has the conveniently simple form

\[
(\dot{q}_l)_{AC} = -\frac{E_{AC} \lambda_I R_I q_I}{2\rho_l},
\]

where \(E_{AC}\) is the mean value of the collection efficiency and other symbols are as defined previously. The slope factor \(\lambda_I\) is parametrized as a function of temperature using (27). Real efficiencies for the accretion of cloud droplets by falling ice are again less than unity (Pitter and Pruppacher 1974), but are not well understood. Cotton et al. (1982) used an approach based on the Stokes number, similar to that used by TC80 for collection by raindrops. In the absence of a sufficiently simple and rigorous method for the parametrization of \(E_{AC}\), the present scheme uses \(E_{AC} = 0.7\). There is an additional source of uncertainty, because, as discussed above, the bulk density \(\rho_l\) of the ice particle is not as well defined as is the density of water \(\rho_w\).
The control version of the scheme uses a centred-in-time method to evaluate \( q_i \) on the RHS of (29) (see appendix B). Shown in Fig. 12(a) are the zonal-mean rates of accretion from the CNTRL runs, with the differences of the CNTRL run from the SNOW6 run shown in Fig. 12(b). The differences resulting from the increase in time-step are quite strongly correlated with the differences in the downward flux of ice available for accretion of liquid water (not shown), with the choice of numerical scheme for the calculation of (29) having only a secondary effect. The globally averaged accretion rates are given in Table 2; the decrease of roughly 6% resulting from the increase of time-step from six minutes to 48 minutes is qualitatively consistent with the decrease in the flux-divergence term, which is the source of falling ice. Explicit evaluation of \( q_i \) on the RHS of (29), in conjunction with the 48-minute time-step, resulted in accretion rates that were slightly larger than those obtained in the CNTRL run, and in somewhat better agreement with the SNOW6 run. Conversely, an implicit scheme resulted in smaller accretion rates that were in worse agreement with the SNOW6 run. The variation with time-step of the downward flux of ice available to accrete liquid water makes it difficult to assess the relative merits of the schemes from these results, so the time-centred scheme has been chosen on theoretical grounds, and also because of the results obtained for the analogous collection term in section 4.

Shown in Fig. 12(c) are the differences from the CNTRL run of the accretion rates from a run identical to the CNTRL run, except that accretion of cloud liquid water by falling ice was parametrized using the S90 scheme (i.e. using the accretion term in (14)). This scheme resulted in much lower accretion rates than those obtained with the present scheme, with a globally averaged value of just 0.209 mm d\(^{-1}\), i.e. 57% lower than in the CNTRL run. (See discussion in section 7.)

\((d)\) Sublimation of falling ice

The rate of change of water-vapour mixing ratio due to sublimation of falling ice is obtained by integration of the equation for the sublimation of a falling ice particle of diameter \( D_t \) over the Marshall–Palmer distribution (26) (see appendix B). It has the form

\[
(q_v)_{SB} = \frac{4(q_{si} - q_v)}{\rho (A'' + B'')q_{si}} \left( \frac{R_t}{\pi \rho_l V_t} \right) \left\{ 0.65 \lambda_t^2 + 0.493 \lambda_t^{3/2} \left( \frac{V_t \rho_l}{\mu} \right)^{1/2} \right\},
\]

(30)

where \( A'' \) and \( B'' \) are temperature-dependent terms representing heat conduction and vapour diffusion respectively, \( V_t \) is given by (24) and other symbols are as defined above. The slope factor \( \lambda_t \) is again parametrized as a function of temperature using (27).

As with the evaporation of rain, a time-centred scheme is used for the evaluation of \( (q_{si} - q_v) \) on the RHS of (30). The zonal-mean rate of sublimation from the CNTRL run are shown in Fig. 13(a), with the differences of the CNTRL run from the SNOW6 run in Fig. 13(b). There was generally a slight decrease in the sublimation rates resulting from the increase in time-step, with the globally averaged value some 3% less in the CNTRL run (see Table 2). This is, again, qualitatively consistent with the reduction in the downward flux of ice resulting from the increase in time-step. The use of an explicit scheme gave sublimation rates which were in better agreement with the SNOW6 run at lower levels but which were too large in the tropical upper troposphere, whereas an implicit scheme gave sublimation rates which were lower than those from the SNOW6 run everywhere (not shown). Both theory and the globally averaged values given in Table 2 suggest that the time-centred scheme is again the best choice, bearing in mind the sign of the change expected because of the reduction in the available downward flux of ice.
Figure 12. Time-averaged zonal-mean rates of accretion of cloud liquid water by falling ice ($10^{-9} \text{ kg kg}^{-1} \text{s}^{-1}$) from various ten-day model experiments performed under July conditions. (a) CNTRL run; (b) differences of SNOW6 run from CNTRL run; (c) as (a), but with accretion parametrized using (14) (differences from CNTRL run shown). Negative contours are dashed, and the depth of shading indicates the magnitude of the plotted values.
Figure 13. Time-averaged zonal-mean rates of sublimation of falling ice \( (10^{-9} \text{ kg kg}^{-1}\text{s}^{-1}) \) from various ten-day model experiments performed under July conditions. (a) CNTRL run; (b) differences of CNTRL run from SNOW6 run; (c) as (a), but using Gregory’s (1995) scheme (differences from CNTRL run shown); (d) as (a), but treating falling ice as rain (differences from CNTRL run shown). Negative contours are dashed, and the depth of shading indicates the magnitude of the plotted values.

Shown in Fig. 13(c) are the differences from the CNTRL run of the sublimation rates from a run identical to the CNTRL run, except that Gregory’s (1995) scheme was used for the sublimation of falling ice. Compared to the present scheme, Gregory’s scheme gave generally larger rates of sublimation (as much as 100% larger at low levels), in qualitative agreement with the earlier result regarding the evaporation of rain. However, at very low temperatures, Gregory’s scheme gave lower sublimation rates than the present scheme. Figure 13(d) shows the differences from the CNTRL run of the sublimation rates obtained in a run identical to the CNTRL run, except that sublimation was calculated using (23), i.e. treating ice as rain. Much lower sublimation-rates were obtained in this run, with a globally averaged value of 0.132 mm d\(^{-1}\), about 40% less than that from the CNTRL run. (See discussion in section 7.)
6. PRELIMINARY VALIDATION OF LARGE-SCALE CLOUD FIELDS

In this section, a preliminary validation of a six-year run of the model, forced by climatological SSTs and using the control version of the scheme, is presented: zonal-mean fields averaged over July from the last five years of the run are shown and compared to appropriate observational data for July. This run will be analysed in more detail in Part II. Also shown, for comparison, are zonal-mean cloudiness and cloud radiative forcing from a 10 year run which used the standard Mark 2 version of the model (described in section 2). Other than inclusion of the prognostic cloud scheme and the increase of vertical resolution, the only significant difference between the two versions of the model is the treatment of shallow convection, which follows Geleyn (1987) in the standard model and Tiedtke (1988) in the version with the prognostic cloud scheme. The two model experiments are referred to as the ‘PROG’ and ‘DIAG’ runs respectively.

Figure 14 shows July zonal-mean total cloudiness from the two model runs, along with the D2 data from ISCCP (Rossoow et al. 1996) averaged over the period 1990 to 1992 and surface observations (land and ocean combined) from Warren et al. (1986, 1988). The
global-mean cloudiness from the DIAG run is 47%, less than the observed values of 66% from ISCCP and 63% from Warren et al., whereas the PROG run has a more realistic value of 57%. The zonal-mean distribution of cloudiness from the PROG run also shows improved agreement with the observed data. The peaks in cloudiness associated with mid-latitude storm tracks are captured more realistically in the PROG run, and the cloudiness over Antarctica appears to be more realistic, even bearing in mind the uncertainty in the observations at high latitudes. However, cloud cover in the model is still too low in the subtropics and mid-latitudes (in common with many other models). Preliminary experiments (not shown) with a 24-level version of the model (including an additional 2 levels below $\sigma = 0.8$) have considerably more cloud cover in the subtropics, suggesting that a lack of vertical resolution at the boundary layer at least partly causes the deficiency of subtropical cloud in the 18-level version.

Figure 15 shows the zonal-mean variation of the liquid-water paths from the PROG run averaged over ocean points for July, together with Special Sensor Microwave/Imager (SSM/I) observations retrieved by the algorithms of Greenwald et al. (1993) and Weng and Grody (1994). The data of Greenwald et al. are averaged over the period 1987 to 1991, whereas those of Weng and Grody are averaged over the period 1987 to 1995. Unfortunately, the observed liquid-water paths are very uncertain at present—the liquid-water paths retrieved by Weng and Grody are substantially lower than those of Greenwald et al., with a global-mean value over oceans of 0.062 kg m$^{-2}$ compared to 0.084 kg m$^{-2}$, and much lower values in the subtropics. In the southern hemisphere, the modelled liquid-water paths agree quite well with the values of Weng and Grody, but are too low compared to those of Greenwald et al. In the northern hemisphere, the modelled values agree quite well with those of Greenwald et al., but are too large compared to those of Weng and Grody. The modelled values are somewhat too large in mid-latitudes, compared to both sets of observations. In the tropics, the peak in the modelled values appears to be too broad. The global-mean liquid-water path over oceans in the model is 0.076 kg m$^{-2}$, in between the global-mean values from the two sets of observations.

The zonal-mean LWCF (calculated using method 2 of Cess et al. 1992) from both model runs is shown in Fig. 16, together with observed data from the Earth Radiation
Budget Experiment (ERBE) averaged over the period 1985 to 1989. The ERBE cloud-forcing fluxes are considered to be quite uncertain poleward of about 60° latitude, and should be regarded with caution there. The global-mean LWCF is 28.9 W m⁻² in the DIAG run and 29.9 W m⁻² in the PROG run, both in close agreement with the ERBE value of 28.4 W m⁻². However, the zonal-mean distribution in the PROG run is more realistic than in the DIAG run, particularly in the tropics and over Antarctica. Even in the PROG run, some deficiencies are evident: the modelled LWCF is somewhat weaker than the LWCF from ERBE in northern hemisphere mid-latitudes, and the peak in the tropics again appears to be too broad.

The zonal-mean (method 2) SWCF from both model runs is shown in Fig. 17, together with data from ERBE averaged over the period 1985 to 1989. The global-mean SWCF is
−43.5 W m$^{-2}$ in the DIAG run and −45.6 W m$^{-2}$ in the PROG run, both in reasonable agreement with the ERBE value of −47.9 W m$^{-2}$. The zonal-mean distributions from the two runs do not differ greatly, and agree well with the observations, except in the northern hemisphere mid-latitudes, where the modelled SWCF is too weak in both runs (in common with most other models). In this region, the SWCF from the PROG run is noticeably more realistic than that from the DIAG run. Polewards of about 65°N, the SWCF from the PROG run appears to be slightly worse, but this turns out to be the result of excessive snow and sea-ice that is retained in July in the PROG run, and which reduces the modelled SWCF because it provides a highly reflective surface underneath the clouds. The SWCF from both runs is also somewhat too strong in the tropics, again in common with most other models. Some possible explanations for the deficiencies in the SWCF from the PROG run are suggested in the next section.

7. SUMMARY AND DISCUSSION

(a) Summary of scheme

The preceding sections have described a prognostic scheme for the treatment of stratiform clouds and precipitation, intended for use in GCMs and other large-scale models. The scheme incorporates prognostic variables for cloud liquid water and cloud ice, and includes the following components:

- a simple ‘statistical’ scheme for the formation and dissipation of cloud—the equations for calculation of the cloud fraction and the amount of condensed water are given by Smith (1990);
- a novel approach to the determination of the liquid-water fraction in mixed-phase clouds, driven primarily by the relative difference between the saturation mixing ratios with respect to ice and water (section 3(c));
- semi-Lagrangian advection and vertical turbulent mixing of cloud liquid water and cloud ice—these terms have a relatively minor effect on the simulation (section 3(d));
- initiation of rain by autoconversion, using a parametrization which can distinguish between the microphysics of maritime and continental clouds (Eq. (15));
... an observationally based fall-speed for ice particles (Eq. (24)), which is used to calculate the rate of precipitation of cloud ice as the flux divergence of falling ice (Eq. (25));

- accretion of cloud liquid water by falling rain and ice, derived from the continuous-collection equation using observationally based (Marshall–Palmer) size distribution for raindrops and ice particles (Eqs. (22) and (29));

- evaporation of rain and falling ice, parametrized by integration of the equation for evaporation of a single raindrop (or ice particle) over the appropriate Marshall–Palmer size-distribution (Eqs. (23) and (30));

- parametrization of cloud effective radius as a function of liquid water content and cloud droplet concentration for warm clouds (Martin et al. 1994) or as a function of ice water content for cold clouds (Eq. (12));

- calculation of cloud radiative properties from the liquid- (or ice-) water path and effective radius, following Slingo (1989) for the short-wave properties and using (Eq. (13)) for the emissivity.

The scheme has been implemented and tested in an R21, 18-level version of the CSIRO, combined with a simple diagnostic treatment of convective clouds. During the development of the scheme, considerable attention was paid to the physical principles underlying the various parametrizations, especially those related to precipitation, while simultaneously trying to provide a scheme with moderate computational overheads.

(b) Choice of time-step and computational overheads

The sensitivity of the precipitation processes to the time-step used for their calculation was evaluated in a series of short GCM experiments, to determine whether it is necessary to use a reduced (split) time-step of six minutes for the cloud microphysics, or whether the usual 48-minute leapfrog time-step is sufficiently small. (Note that the scheme which calculates the formation and dissipation of cloud does not use a time-step.) The experiments showed that the various terms are not very sensitive to the time-step, provided that some of the numerical schemes used for their evaluation are chosen with care. Theoretically, time-centred differencing was expected to give smaller time-truncation errors than implicit or explicit differencing in the evaluation of the collection, evaporation, accretion and sublimation terms. For the rainfall processes (i.e. collection and evaporation), these expectations were clearly borne out by the GCM experiments, whereas for the ice processes (i.e. accretion and sublimation) it was more difficult to assess the relative merits of the various schemes, because the available downward flux of ice varied with the change of time-step. For the calculation of autoconversion, an analytical treatment gave better results than a simple explicit treatment. The choice of numerical scheme was found to be important in the calculation of the flux divergence of falling ice, where the use of an implicit scheme for the calculation of the flux divergence resulted in an increase of more than 2 W m$^{-2}$ in the global-mean LWCF, compared to a more accurate analytical scheme, which showed little variation (0.1 W m$^{-2}$) over a wide range of time-steps. Another scheme that has been used in global models (the ‘fall-through’ approximation) resulted in a decrease of more than 2 W m$^{-2}$ in the global-mean LWCF. The excellent result obtained with the analytical scheme suggests that it is the natural way to handle the calculation of the flux-divergence of ice implied by (24), especially at large Courant numbers. Overall, the results support the conclusion that the cloud microphysics does not require the use of a split time-step, which would entail considerable computational overheads. Higher resolution (R42 or T63) versions of the GCM would use a time-step half that of the low-resolution version used here and would improve the results obtained for the frozen precipitation, which showed
some sensitivity to the time-step. One question this study has not addressed is whether the diagnostic treatment of rainfall is satisfactory, or whether the results would be substantially altered by treating rain as a prognostic variable, as is done in some other schemes. With this qualification, the present results suggest that it is not necessary to use a split time-step in the relatively simple microphysical schemes which are typically used in current GCMs. This conclusion may not hold for more complex schemes (Ghan and Easter 1992; Fowler et al. 1996).

Inclusion of the control version of the prognostic cloud-scheme in the R21, 18-level model increases the computer time needed from about 221 seconds to about 305 seconds per model day on a single processor of a Cray J916 mini-supercomputer. Of this increase of 38%, a little less than half (18%) is the result of the cloud microphysical processes shown in Fig. 1, with the bulk of the remainder resulting from increased overheads in the radiation scheme, including the increased number of cloud layers and interactive calculation of cloud radiative properties (10%) and the semi-Lagrangian advection of cloud water (5%). If a six-minute time-step is used for the precipitation processes, the overhead caused by microphysical processes increases from 18% to 65%. The overhead of 38% for the entire scheme is regarded as reasonable, especially in view of the modest computational demands of the standard version of the CSIRO GCM relative to some other GCMs. Also, with increased horizontal resolution, the cloud scheme is expected to account for a lower percentage of the model's total requirement than it does at R21, since the spectral transforms will become relatively more expensive, while the number of time-steps between calls to the radiation scheme will increase. The overhead of 38% also compares very favourably with the scheme of Fowler et al. (1996) which caused a doubling of the computer time required to run the CSU GCM, from 110.5 to 220.2 seconds per model day on a Cray C-90. The CSU GCM has 17 vertical levels and horizontal resolution of 4° by 5°, i.e. slightly lower resolution overall than the 18-level R21 model used in the present study. Based on tests performed with the CSIRO GCM, the processors on the C-90 are approximately four times faster than those on the J916, so the overhead for the present scheme of 84 seconds per model day on the J916 would translate to about 21 seconds per model day on the C-90, i.e. more than five times faster than the figure of 110 seconds given by Fowler et al. (1996). Alternatively, inclusion of the Fowler et al. scheme in the CSIRO GCM would roughly triple the computer time required to run a model day, relative to the version with the diagnostic cloud scheme.

(c) Comparisons with other schemes

In a further series of short GCM experiments, the parametrizations of the processes which remove cloud liquid water were compared with (14), which was used by Smith (1990) and subsequently in a number of other models. Compared to (14), the present scheme gave considerably lower rates of autoconversion, slightly higher rates of collection by falling rain, and considerably higher rates of accretion by falling ice. The lower autoconversion rates resulting from the present scheme arose from both the larger critical mixing ratios used and the differing functional form of the parametrization. The larger accretion rates can be explained by the lower density of falling ice particles compared to raindrops—this difference is not accounted for in the Smith scheme, which treats all falling precipitation identically. Putting the density of ice particles lower than that of raindrops in (29) results in larger accretion rates for a given downward flux of ice. The combination of a smaller autoconversion term and a larger accretion term, as used in the present scheme, is more consistent with the accepted view (e.g. Rutledge and Hobbs 1983) that ice processes are responsible for most of the precipitation produced in frontal clouds. The method used here to determine the relative amounts of liquid water and ice in mixed-phase clouds also
appears to be more realistic than the simple interpolation in temperature used in earlier schemes, since the present method does not allow ice to melt spuriously at sub-freezing temperatures, and generates liquid-water fractions that agree fairly well with observations. However, the treatment of mixed-phase clouds used here is still quite simple—some deficiencies are noted below.

The remaining experiments were designed to permit comparison of the rates of evaporation of precipitation produced by the present scheme with those produced by other schemes (Kessler 1969; Gregory 1995) that have been used in large-scale models. The present scheme gave lower rates of evaporation of rainfall than the other schemes, and lower rates of sublimation of falling ice than Gregory’s scheme. (Kessler’s scheme did not include ice processes.) According to Gregory (1995), his scheme gave rainfall evaporation rates which were too large when compared to the detailed microphysical calculations of Clough and Franks (1991) when a Marshall–Palmer drop-size distribution was used in the derivation of the scheme. An explanation proposed by Gregory involved the form of the drop-size distribution, since he obtained better agreement with the results of Clough and Franks when a gamma distribution of order 1 was used. When tested in the CSIRO GCM, this version of Gregory’s scheme gave rates of evaporation similar to those obtained with the present scheme. Comparison of the physical basis of the present scheme with that of Gregory’s scheme suggests a possible explanation for the lower evaporation rates produced by (23), despite the use of a Marshall–Palmer distribution. In the derivation of his scheme, the temperature at the surface of an evaporating drop is approximated by the wet-bulb temperature, whereas (23) is based on the (approximate) simultaneous solution of the equations for diffusion of vapour away from and heat towards the surface of an evaporating drop (Mason 1971). Use of the wet-bulb temperature is known to result in an overestimate of the evaporation rate (J. D. Kepert, personal communication, 1995), so this is another possible reason for the larger evaporation rates produced by Gregory’s scheme. Kessler’s (1969) scheme is based on an even simpler approach, in which the variations with temperature of the diffusivity and other quantities are ignored. The differences between the results obtained for the sublimation of falling ice using Gregory’s scheme and the present scheme may possibly be explained similarly in terms of the use of the ice-bulb temperature in the development of his parametrization, although further investigation would be required to explain the interesting variation of the differences with temperature. A final experiment tested the effect of treating the sublimation of falling ice as evaporation of rain, which resulted in considerably reduced sublimation rates. This result is consistent with the finding of Clough and Franks (1991) that most forms of ice evaporate more efficiently than rain, primarily due to the lower density of ice particles. This is not accounted for in some schemes (e.g. Sundqvist et al. 1989; Ose 1993; Tiedtke 1993) which treat the evaporation of all precipitation identically.

(d) Some uncertainties in the scheme

The method used in the present scheme to determine the liquid-water fraction in mixed-phase clouds is driven primarily by \((q_{sl} - q_{dl})/q_{sl}\), i.e. by the relative difference between the saturation mixing ratios with respect to liquid water and ice, which increases with decreasing temperature. This quantity also appears as a key parameter in the physically based parametrization discussed in appendix A. The encouraging agreement between the modelled liquid-water fractions and observations shown in Fig. 5 suggests that, in the mean, this relative difference is the primary factor that controls the observed variation of liquid-water fraction with temperature, although in individual clouds other factors (such as IN concentration and droplet-size distribution) will be important too. It was argued in section 3 that the present approach is more realistic than the interpolation in temperature
used in earlier schemes, but it still has some deficiencies. While it includes a physically based treatment of accretion of liquid water by falling ice particles, the other process by which falling ice-particles grow, namely by vapour deposition in regions of supercooled water, is not explicitly treated in the scheme, since an obvious method of parametrizing this process (appendix A) was found to be inconsistent with assumptions made in the cloud-formation scheme. The somewhat higher liquid-water fractions generated by the simple scheme used here (as compared with aircraft observations in stratiform cloud) suggest that inclusion of further processes that convert liquid water to ice would improve the results, particularly in deep frontal clouds, which are more likely to be glaciated (Bower et al. 1996). It may also be helpful to include some treatment of ice-multiplication processes, which can result in explosive growth of the number concentration of ice crystals, but are not fully understood (Rogers and Yau 1988). Another issue is the assumption that sufficient condensation occurs to remove any supersaturation with respect to ice; this assumption is made in the cloud formation scheme and carried through to the treatment of mixed-phase clouds. The assumption (which is made in many GCMs) is questionable, since the lack of IN in the real atmosphere means that the air in ice clouds can be significantly supersaturated with respect to ice (Heymsfield and Miloshevich 1995). One effect of the assumption in the present scheme is that, at temperatures just below 0 °C, the model may produce pure ice clouds if the atmosphere is supersaturated with respect to ice but subsaturated with respect to water, whereas, in nature, ice production at these temperatures generally occurs via the freezing of water droplets. This is less of a problem in practice than in theory, because at these temperatures \((q_{HI} - q_{SI})/q_{SI}\) is small, and it is only in grid boxes with very small cloud fractions that this situation can occur (i.e. the cloud fraction would have to be smaller than the light grey area in Fig. 2). In practice, the modelled liquid-water fractions (see Fig. 5) are generally large at temperatures close to 0 °C, and, if anything, are larger than the observed values. The treatment of mixed-phase clouds in large-scale models remains very uncertain, and more work is needed to establish improved parametrizations of the key processes.

Another uncertainty in the present scheme is the formation and dissipation of cloud, treated using a simple statistical condensation scheme (Smith 1990). The scheme in its present form (using an arbitrary critical relative humidity to specify the sub-grid variability of the moisture distribution) is a rather simple approach to the problem, and lacks a clear physical basis for the selection of the critical relative humidity. The use of a fixed \(R_{HC}\) which does not vary with height was also identified as a possible cause of the excessive cloud generated by the scheme in the lowest one or two layers of the model. The use of larger values of \(R_{HC}\) in the lowest one or two layers would ameliorate this problem, but this approach was avoided in view of the lack of a sound basis for the choice of \(R_{HC}\) and the intention to implement a more sophisticated treatment in a future version of the scheme. More sophisticated versions of the condensation scheme have been tried, for example the scheme of Ricard and Royer (1993), in which the sub-grid variance of moisture is parametrized as a function of the Richardson number calculated by the model's turbulent-mixing scheme. Ideally, in a large-scale model, the sub-grid moisture variance should also depend on unresolved mesoscale motions, which could possibly be parametrized as a function of the model's grid spacing (Ek and Mahrt 1991). The possibility of upgrading the scheme as indicated remains an attractive possibility, as a more explicit treatment of condensation and evaporation of cloud water on the scale of a GCM grid-box is a difficult problem, because of uncertainties in how to calculate the cloud fraction and morphology, the amount of mixing between cloudy and environmental air and, at sub-freezing temperatures, the number concentration of ice crystals. Alternative condensation schemes which also allow fractional cloudiness have been presented by Sundqvist (1978)
and Tiedtke (1993), although neither of these schemes completely avoids the use of an arbitrary critical relative humidity. Another question regarding the condensation scheme is whether it is appropriate when coupled to a convection scheme that includes a more realistic treatment of anvil cirrus clouds—these clouds have long lifetimes and can be advected over long distances, yet the scheme assumes instantaneous evaporation of cloud when the relative humidity drops below the critical value.

The critical relative humidity RH_{CR} used by the condensation scheme is one of several parameters in the present scheme which are very uncertain; other such parameters are the critical droplet-radius r_{CR} used by the autoconversion parametrization and the density of falling ice \rho_{I}. The treatment of cloud radiative properties also contains some highly uncertain parameters, such as the asymmetry parameter for ice clouds φ_{i} and the factor ξ which reduces the optical depth to compensate for the treatment of clouds as plane-parallel. The sensitivity of the model simulation to some of these parameters will be examined in Part II.

(e) Validation of the large-scale cloud fields

A preliminary validation of the large-scale cloud-fields focused on July global-mean and zonal-mean results from a six-year run using the control version of the scheme. The results generally agreed well with observations, and showed a marked improvement in the modelled cloudiness and LWCF, as compared with a run using the standard Mark 2 version of the CSIRO. Deficiencies noted were similar to those which have been found in many other GCMs, in particular, insufficient cloud-cover in the subtropics (probably due to insufficient vertical resolution in the boundary layer), excessive SWCF in the tropics, and insufficient SWCF in the mid-latitude storm tracks.

A possible cause of the deficiencies in the SWCF is the coarse horizontal resolution of the model—at R21 it is difficult for the model to resolve the mid-latitude storm tracks adequately or the inter-tropical convergence zone (ITCZ). There is some evidence that the modelled liquid-water paths in the ITCZ are too large and that the peak values spread over too broad an area. This could explain the excessive SWCF in the tropics, but it is unclear at present to what extent the liquid-water paths can be blamed on inadequate resolution and to what extent the model's treatment of physical processes (such as convection) is at fault. Another possible cause of the deficiencies in the model's SWCF is the treatment of cloud radiative properties, which assumes that clouds are homogeneous and plane-parallel, except for the reduction of cloud optical depth by the factor ξ described in section 3. The factor ξ used here is based on large-eddy simulations by Kogan et al. (1995), but they considered only cases with a solar zenith angle of zero. There is both theoretical and observational evidence (Welch and Wielicki 1984; Loeb and Davies 1996) to suggest that the variation of cloud reflectivity with solar zenith-angle is not captured realistically by the plane-parallel approximation. These studies have suggested that the reflectivity of real clouds increases more strongly with increasing zenith-angle than does that of plane-parallel clouds. Moreover, the deficiencies in the model's SWCF noted here are qualitatively consistent with this finding, since the average solar zenith-angle in the mid-latitude storm tracks is larger than in the tropics. The modelled liquid-water paths over mid-latitude oceans of the northern hemisphere are larger than the observed (Fig. 15), also suggesting that the cloud radiative properties are a part of the problem, since it is the low-level liquid-water clouds which are the primary source of SWCF (although, as noted in section 6, the observed liquid-water paths are very uncertain at present). More detailed comparison of the model's large-scale cloudiness and cloud radiative forcing with observations is deferred until Part II.
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APPENDIX A

Physically based parametrization of mixed-phase clouds

Mixed-phase clouds generally consist mainly of supercooled water initially, because of the relative abundance of CCN compared to IFN. As the cloud evolves, ice crystals grow at the expense of liquid-water droplets, because the saturation vapour pressure is lower with respect to ice than with respect to water. This effect is commonly referred to as the Bergeron–Findeisen mechanism. The cloud model of RH83 included a parametrization for deposition on cloud ice in which the ice-crystal number concentration \( N_i \) (to which the calculation is sensitive) was assumed to follow the relation given by Fletcher (1962) for the concentration of IFN active at temperature \( T \), viz.

\[
N_i = N_{i0} \exp(\eta(T_0 - T))
\]

(A.1)

where \( N_{i0} = 10^{-2} \text{ m}^{-3} \) and \( \eta = 0.6 \text{ K}^{-1} \). However, observed ice-crystal number-concentrations can be several orders of magnitude greater than the IFN concentration due to ice-multiplication effects (RY88). This suggests that a reasonably rigorous approach might require the inclusion of ice-crystal number-concentration as a prognostic variable (e.g. Cotton et al. 1986) resulting in a substantial increase in computational complexity, while retaining a degree of uncertainty due to the currently incomplete understanding of the processes responsible for ice multiplication. An adaptation of the approach of RH83 would involve parametrizing the rate of growth of cloud ice (assumed to consist of monodispersed, hexagonal, plate-like crystals) by vapour deposition at the expense of supercooled water as

\[
(q_i)_{DE} = \frac{65.2N_i^{1/2}q_i^{1/2}(e_{sl} - e_{si})}{\rho^{1/2}(A'' + B'')e_{si}},
\]

(A.2)

obtained by combining (A.17) and (A.18) of RH83 and assuming that the air is saturated with respect to liquid water. Here, \( A'' \) and \( B'' \) are temperature-dependent terms representing heat conduction and vapour diffusion respectively (defined in appendix B). Analytical integration of (A.2) with respect to time (treating \( q_i \) as the dependent variable) yields a version more suitable for implementation with large time-steps typical of GCMs. The rate of generation of cloud ice given by (A.2) varies as a function of temperature according to the square root of the prescribed ice-crystal concentration \( N_i \), which increases with decreasing temperature, the term \( 1/(A'' + B'') \), which decreases with decreasing temperature and the relative difference between \( e_{si} \) and \( e_{sl} \) (or, equivalently, \( q_{sl} \) and \( q_{si} \)), which increases with decreasing temperature. The net effect is that the glaciation rate increases monotonically with decreasing temperature. The rate of glaciation also depends on the square root of the mixing ratio of cloud ice \( q_i \), giving increased rates of glaciation in regions where ice is already present. It does not include a dependence of the glaciation rate on the observed
variation of ice-crystal habit with temperature and supersaturation (RY88) or on realistic values of the ice-crystal concentration. The resultant liquid fraction will also depend on the amount of liquid water present before the calculation (A.2) is performed, so that very moist clouds (e.g. in regions of strong uplift) will favour relatively high liquid fractions. Experiments with schemes based on (A.2) (not shown) yielded very low rates of deposition on cloud ice at temperatures about about $-15 \, ^{\circ}C$, probably because of the underestimate of $N_i$ entailed by the use of (A.1). In addition to these problems, it is uncertain how to organize the coexisting ice and liquid water within a grid box, and even how to specify the saturation mixing ratio $q_s$ which is required by the condensation scheme.

It is also possible to calculate deposition on falling ice according to (30) when ice falls into a cloud containing supercooled liquid water, if the latter is assumed to be at liquid saturation. In this calculation, deposition on falling ice is treated in identically the same way as the sublimation of falling ice, except that $q_s > q_{sl}$. This is an important mechanism in the 'seeder–feeder' process mentioned in section 3, in addition to the growth of ice particles by accretion (riming) of supercooled cloud droplets, parametrized in the scheme using (29). Calculation of deposition on falling ice in this manner has been omitted from the present scheme, as the assumption of saturation with respect to liquid water is incompatible with the assumptions used in the condensation scheme (see section 3).

### APPENDIX B

**Derivations and numerical details of the parametrizations of precipitation and related processes**

The precipitation processes described in this appendix are applied in a split manner, i.e. sequentially. Where the numerical scheme used for each process is described, a superscript $n$ denotes the value of a variable before application of that process, and a superscript $n + 1$ denotes the updated value after application of the process.

**Relation between the slope factor and rainfall rate**

The local mixing ratio of rain $q^+_r = q_r / f_r$ is given by

$$q^+_r = \frac{1}{\rho} \int_0^\infty \rho_w \pi \frac{D^3}{6} N_i(D_i) \, dD_i,$$

which implies, after substitution of (19), that

$$\lambda_r = \left( \frac{\pi \rho_{w} N_{0r}}{\rho q^+_r} \right)^{1/4}. \quad (B.1)$$

The mixing ratio of rain is not directly available, since it is not a prognostic variable of the scheme, but it can be estimated as

$$q^+_r = \frac{R^+_r}{\rho \overline{V}_r}, \quad (B.2)$$

where $\overline{V}_r$ is the mass-weighted-mean fall-speed of rain, i.e.

$$\overline{V}_r = \frac{\int_0^\infty N_i(D_i) M(D_i) \overline{V}_r(D_i) \, dD_i}{\int_0^\infty N_i(D_i) M(D_i) \, dD_i}.$$
This gives, after substitution of (17) and (18),
\[
\overline{V}_r = 1.94 k_r \left( \frac{\rho_0}{\rho} \right)^{1/2} \lambda_r^{-1/2}.
\]  
(B.3)

The use of (B.2) and (B.3) in (B.1) gives
\[
\lambda_r = \left( \frac{1.94 \pi \rho_0 N_{ww} k_r (\rho_0/\rho)^{1/2}}{R_t^1} \right)^{2/9},
\]  
(B.4)

which gives (21) after evaluation of parameters.

*Autoconversion of cloud liquid water*

Analytical integration of (15) with respect to time yields
\[
q_l(t) = C_1 \left\{ \left( \frac{q_l(0)}{C_1} \right)^{-4/3} + \frac{4}{3} c_{AU} t \right\}^{-3/4},
\]  
(B.5)

where \( c_{AU} = 0.104 E_{AU} \rho_r^{4/3} / (\mu N_{ww}^{1/3}) \) is a constant at a given grid-point and time-step. This is implemented in the model as
\[
q_l^{n+1} = C_1 \max \left[ q_{CR}, \left\{ \left( \frac{q_l^n}{C_1} \right)^{-4/3} + \frac{4}{3} c_{AU} \Delta t \right\}^{-3/4} \right],
\]  
(B.6)

with \( q_{CR} \) given by (16).

*Collection of cloud liquid water by rain*

The rate at which a raindrop of diameter \( D_r \) falling with speed \( V_r(D_r) \) grows by collection of cloud liquid water is given by the continuous-collection equation (RY88)
\[
\frac{dM(D_r)}{dt} = E_{CO} \frac{\pi}{4} D_r^2 V_r(D_r) \rho q_l
\]  
(B.7)

where \( M(D_r) \) is the mass of a raindrop of diameter \( D_r \) and \( E_{CO} \) is the mean collection efficiency. Multiplication of (B.7) by (18), elimination of \( V_r(D_r) \) using (17) and integration over all drop sizes gives the collection rate as
\[
\rho(\hat{q}_l)_{CO} = -\frac{E_{CO} \pi k_r}{4} N_{w} \left( \frac{\rho_0}{\rho} \right)^{1/2} \frac{\Gamma(3.5)}{\lambda_r^{3.5}} \rho q_l.
\]

Elimination of \( \lambda_r \) using (B.4) and evaluation of parameters gives
\[
(\hat{q}_l)_{CO} = -0.305 f_r E_{CO} \left( \frac{\rho_0}{\rho} \right)^{1/9} (R_t^1)^{7/9} q_l,
\]

where the rainy fraction \( f_r \) has been included. Ignoring the weak dependence on \( \rho/\rho_0 \) and approximating \( 0.305 (R_t^1)^{7/9} \) as \( 0.24 (R_t^1)^{3/4} \) gives the computationally cheaper form (22) which yields similar results. The evaluation of \( q_l \) on the RHS of (22) is centred-in-time as \( 0.5(q_l^n + q_l^{n+1}) \) to reduce time truncation errors; this gives
\[
q_l^{n+1} = q_l^n \frac{1 - 0.5 c_{CO} \Delta t}{1 + 0.5 c_{CO} \Delta t},
\]
where $c_{CO} = 0.24 f_c E_{CO}(R^4)^{3/4}$. This approach gives similar results to analytical integration of (22) but uses a little less computer time. This parametrization is also used for the collection of cloud liquid water by convective rain, and for the collection of cloud liquid water by raindrops generated within a grid box by autoconversion; in the latter case, the collection rate is multiplied by a factor of 0.5 because, in the mean, the raindrops are generated at the midpoint of the layer.

**Evaporation of rain**

The rate of evaporation of a raindrop of diameter $D_r$ can be written as (RY88)

$$- \frac{dM(D_r)}{dt} = \frac{2\pi D_r(1 - S_i)F_r}{A' + B'},$$  \hspace{1cm} (B.8)

where $S_i = e/e_{sl}$ is the saturation ratio;

$$A' = \frac{L_V}{K_a T} \left( \frac{L_V}{R_v T} - 1 \right),$$

and $B' = R_v T /\chi e_{sl}$ are terms representing heat conduction and vapour diffusion respectively. The inverse dependence of the diffusivity $\chi$ on pressure is taken into account. The ventilation factor is given by $F_r = 0.78 + 0.31 S c^{1/3} R e^{1/2}$ (Beard and Pruppachner 1971), where $Sc = 0.6$ is the Schmidt number, and $Re = V_r(D_r)D_\rho /\mu$ is the Reynolds number. Multiplication of (B.8) by (18), use of (17) for the fall speed and integration over all drop sizes gives

$$\rho (\dot{q}_v)_{EV} = \frac{2\pi N_0 (1 - S_i)}{A' + B'} \left\{ \frac{0.78}{\lambda^2} + \frac{0.31 S c^{1/3}}{\lambda^2 75} \left( \frac{k_f \rho}{\mu} \right)^{1/2} \Gamma(2.75) \left( \frac{\rho_0}{\rho} \right)^{1/4} \right\}. \hspace{1cm} (B.9)

This equation is similar to equation (A.12) of RH83, but using the relation (17) for the fall speed rather than their linear relation, and including the factor $Sc^{1/3}$ erroneously omitted from their expression. Since $Re \gg 1$ for raindrops, we can approximate $F_r = 0.31 S c^{1/3} R e^{1/2}$ with little loss of accuracy, so that the first term in the square brackets in (B.9) drops out. With this simplification, use of the approximation $S_i = q_v /q_{sl}$, and use of (B.4) to eliminate $\lambda$, we obtain

$$(\dot{q}_v)_{EV} = \frac{1.04 N_0 7^{1/8} S c^{1/3}}{\rho^{1/2} (A' + B') R_0^{1/9} \mu^{1/2} \rho_{\infty}^{11/18} q_{sl}^{11/18}} (R^4)^{11/18},$$

where a factor $(\rho /\rho_0)^{1/18}$ has been ignored. After evaluation of parameters and inclusion of the rainy fraction $f_r$, (23) is obtained. Note that (23) can be made implicit (with respect to $q_{sl} - q_v$) by using the first-order terms in a Taylor series to obtain $q_{sl}^{n+1}$ in terms of $q_v^n$ (Smith et al. 1995). If we write $(\dot{q}_v)_{EV} = c_{EV}(q_{sl} - q_v)$, this gives (after a little manipulation)

$$q_v^{n+1} = q_v^n + (c_{EV} q_{sl} - q_v^n),$$

where

$$b_{EV} = 1 + \gamma c_{EV} \Delta t \left\{ 1 + \frac{L_v}{c_p} \left( \frac{\partial q_v}{\partial T} \right) \right\}$$

and $\partial q_v /\partial T$ is obtained from the Clausius–Clapeyron equation. The parameter $\gamma$ ($0 \leq \gamma \leq 1$) controls the amount of ‘implicitness’, so that, for example, $\gamma = 1$ gives
a fully implicit version whereas $\gamma = 0.5$ gives a time-centred version which has been adopted for the present scheme. The fully implicit scheme ensures that a grid box cannot become supersaturated through the evaporation of rain, whereas an extra line of code is required to enforce this condition if the time-centred scheme is used. Based on the vertical random-overlap assumption, the total evaporation ($q^n_{i}^{n+1} - q^n_{i}^{n}$) is not allowed to exceed $(1 - C_1)R_i \Delta t/(\rho \Delta z)$.

**Flux divergence of falling ice**

Equation (25) is integrated analytically, treating $R_i$ as a constant. This gives

$$q_i(t) = q_i(0) \exp\left(-\frac{\bar{V}_i t}{\Delta z}\right) + \frac{R_i}{\rho \bar{V}_i} \left\{ 1 - \exp\left(-\frac{\bar{V}_i t}{\Delta z}\right) \right\}$$

which is implemented in the model as

$$q_{i}^{n+1} = q_{i}^{n} \exp(-\alpha) + \frac{R_i \Delta t}{\rho \Delta z} (1 - \exp(-\alpha))/\alpha,$$  \hspace{1cm} (B.10)

where $\alpha = \bar{V}_i \Delta t/\Delta z$ is the Courant number*, and $R_i \Delta t/(\rho \Delta z)$ is the mixing ratio of ice which falls into the layer from above during the time-step. The parameter $R_i$ is calculated as the average rate at which ice falls into the layer from above during the current time-step, less any sublimation, and is available since (B.10) has already been calculated for higher layers. Equation (B.10) shows clearly how the analytically integrated version determines the fraction of ice entering the layer from above which falls through, and the fraction of the ice present at the start of the time-step which falls out. The other schemes mentioned in the text (explicit, implicit, time-centred) can be expressed similarly.

**Accretion of cloud liquid water by falling ice (snow)**

The rate at which a (spherical) ice particle of diameter $D_i$ falling with speed $V_i(D_i)$ grows by accretion of cloud liquid water is given by the continuous-collection equation

$$\frac{dM(D_i)}{dt} = E_{AC} \frac{\pi}{4} D_i^2 V_i(D_i) \rho q_i,$$  \hspace{1cm} (B.11)

where $M(D_i)$ is the mass of a particle of diameter $D_i$ and $E_{AC}$ is the mean collection efficiency. Multiplication of (B.11) by (26) and integration over all particle sizes gives the accretion rate as

$$\rho(q_i)_{AC} = -\frac{E_{AC} f_i \pi \bar{V}_i N_{t} \Gamma(3)}{4 \lambda_i^3} \rho q_i$$

if all ice is assumed to fall with the mass-weighted-mean fall-speed $\bar{V}_i$ and $f_i$ is the snowfall fraction of the grid box. Elimination of $N_{t}$ using (28), and substitution of the frozen-

* Here, the Courant number represents the number of model layers of depth $\Delta z$ traversed during one time-step by a particle falling at $\bar{V}_i$. 
precipitation rate \( R_t = \rho q_t \bar{V}_t \), yields (29). As with the term for collection of cloud liquid water by rain, evaluation of \( q_t \) on the RHS of (29) is centred in time.

**Sublimation of falling ice (snow)**

The rate of sublimation of a falling ice particle of diameter \( D_t \) is given by (RY88):

\[
- \frac{\mathrm{d}M(D_t)}{\mathrm{d}t} = 4\pi \mathcal{C}_t (1 - S_t) F_t \frac{A''}{A'' + B''}
\]

(B.12)

where \( \mathcal{C}_t \) is the capacitance, \( S_t = e/e_{si} \) is the saturation ratio and

\[
A'' = \frac{L_S}{K_s T} \left( \frac{L_{SB}}{R_s T} - 1 \right)
\]

and \( B'' = R_v T / \chi e_{si} \) are terms representing heat conduction and vapour diffusion respectively. The ventilation factor is \( F_t = 0.65 + 0.445 e^{1/2} R_t^{1/2} \) (Thorpe and Mason 1966), where \( R_t = V_t(D_t) D_t \rho / \mu \). Assuming that the particles are in the form of plate-like crystals for which \( \mathcal{C}_t \approx D_t / \pi \), using the approximation \( S_t = q_v / q_{si} \), multiplying (B.12) by (26) and integrating over all particle sizes, we obtain

\[
(\dot{q}_v)_{SB} = f_t \frac{4(q_{si} - q_v) N_{0t} \lambda_t^2}{\rho (A'' + B'') \lambda_t} \left\{ \frac{0.65}{\lambda_t^2} + \frac{0.493}{\lambda_t^{3/2}} \left( \frac{\bar{V}_t \rho}{\mu} \right)^{1/2} \right\}
\]

(B.13)

if all ice particles are assumed to fall with the mass-weighted fall-speed \( \bar{V}_t \) and sublimation occurs only in the snowy fraction \( f_t \) of the grid box. An alternative form of (B.13) in which the dependence on the snowfall rate is made explicit, is obtained by substitution of (28) to eliminate \( N_{0t} \) and use of the rate of frozen precipitation \( R_t = \rho q_t \bar{V}_t \), which yields (30). This calculation is discretized in a time-centred manner, analogous to that used for the evaporation of rain.

**APPENDIX C**

**Calculation of the fraction of a grid box into which precipitation falls**

A number of the parametrizations described above require an estimate of the rainy or snowy fraction (i.e. the fraction of a grid box into which precipitation falls). Similar approaches are used for rain and falling ice (snow). In the following description, model levels are specified by superscripts, so that a superscript \( k \) denotes a quantity at the level under consideration and a superscript \( k + 1 \) denotes a quantity at the level immediately above.

**Treatment of rain**

For stratiform rainfall, the rainy fraction is calculated using the random-overlap assumption, as follows. The rainy fraction \( f_{rk}^k \) in a grid box at level \( k \) is calculated as the sum of

- the fraction \( f_{rk}^k \) of the grid box into which rain falls from a cloud at level \( k + 1 \), and
- the fraction \( f_{ro}^k \) of the grid box into which rain falls from clear sky at level \( k + 1 \).

Rain which falls from a cloud at level \( k + 1 \) either originates in that cloud (by autoconversion) or at a higher level (by collection by stratiform or convective rain). Rain which
originates at level $k + 1$ is assumed to fall from the entire (liquid water) cloud, so that $f_{IC}^k = C_{IC}^{k+1}$ if autoconversion occurs at level $k + 1$. Rain which forms at level $k + 1$ from collection by rain falling from above is assumed to fall from a fraction of the cloud equal to the rainy fraction $f_r^{k+1}$, so that $f_{IC}^k = C_{IC}^{k+1} f_r^{k+1}$ if autoconversion does not occur at level $k + 1$. For simplicity, the scheme sets $f_{IC}^k = (1 - C_{IC}^{k+1}) f_r^{k+1}$, unless all rain entering the clear portion of the grid box at level $k + 1$ was found to evaporate, in which case $f_{IC}^k = 0$.

Since collection of cloud liquid water by convective precipitation is included in the scheme, an estimate of the rainy fraction for convective precipitation is also required. This is assumed to equal the randomly-overlapped convective-liquid cover above level $k$, calculated according to (10).

**Treatment of falling ice**

The fraction of a grid box into which ice falls is calculated in a manner similar to that described above for stratiform rain. The snowy fraction $f_t^k$ in a grid box at level $k$ is calculated as the sum of

- the fraction $f_{IC}^k$ of the grid box into which ice falls from a cloud at level $k + 1$, and
- the fraction $f_{IO}^k$ of the grid box into which ice falls from clear sky at level $k + 1$.

The first quantity is assumed to equal the ice cloud fraction at level $k + 1$, i.e. $f_{IC}^k = C_{IC}^{k+1}$. The second quantity comes from ice which falls through layer $k + 1$ during the time-step; the scheme sets $f_{IC}^k = (1 - C_{IC}^{k+1}) f_r^{k+1}$, unless all ice entering the clear portion of the grid box at level $k + 1$ is found to evaporate, in which case $f_{IC}^k = 0$.

**Appendix D**

**List of symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
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<tr>
<td>$A'$</td>
<td>Term representing heat conduction in evaporation equation</td>
<td></td>
<td>m s kg$^{-1}$</td>
</tr>
<tr>
<td>$A''$</td>
<td>Term representing heat conduction in sublimation equation</td>
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<td>Factor accounting for latent heating in cloud</td>
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</tr>
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<td>$B'$</td>
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</tr>
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<td>Capacitance of a falling ice particle</td>
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<td>F</td>
</tr>
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<tr>
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<td>$C_L$</td>
<td>Stratiform liquid-water cloud fraction</td>
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<td>$c_A$</td>
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<tr>
<td>$c_{CG}$</td>
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<td>$c_p$</td>
<td>Specific heat of dry air at constant pressure</td>
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<td>J kg$^{-1}$K$^{-1}$</td>
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</tbody>
</table>
$c_T$ Tunable parameter in precipitation parametrization used by S90 $10^{-4}$ s$^{-1}$

$C_W$ Tunable parameter in precipitation parametrization used by S90 $8 \times 10^{-4}$ kg kg$^{-1}$

$D_r$ Diameter of a raindrop m

$D_t$ Diameter of a falling ice particle m

$E_{AU}$ Autoconversion collection efficiency 0.55

$E_{CO}$ Rain/cloud-liquid-water collection efficiency 0.7

$E_{NC}$ Falling-ice/cloud-liquid-water collection efficiency 0.7

$e$ Water-vapour pressure Pa

$e_a$ Saturation vapour pressure with respect to ice Pa

$e_l$ Saturation vapour pressure with respect to liquid water Pa

$F_I$ Ventilation factor for falling ice-particles

$F_r$ Ventilation factor for raindrops

$f(q)$ PDF for sub-grid distribution of total-water mixing ratio

$f_t$ Rainy fraction of grid box

$f_c$ Rainy fraction resulting from rain falling from cloud at the level above

$f_{to}$ Rainy fraction resulting from rain falling from clear sky at the level above

$f_t$ Snowy fraction of grid box

$f_c$ Snowy fraction resulting from ice falling from cloud at the level above

$f_{to}$ Snowy fraction resulting from ice falling from clear sky at the level above

$g$ Acceleration under gravity 9.806 m s$^{-2}$

$Q_h$ Asymmetry parameter for ice clouds 0.8

$H()$ Heaviside unit step function

$\mathcal{K}$ Cloud optical depth diffusivity factor 1.6 or 1.8

$K_h$ Thermal conductivity of air $2.40 \times 10^{-2}$ J m$^{-1}$s$^{-1}$K$^{-1}$

$k_r$ Constant in raindrop fall-speed relation 141.4 m$^{1/2}$s$^{-1}$

$L$ Latent heat (either $L_V$ or $L_V + L_S$) J kg$^{-1}$

$L_S$ Latent heat of sublimation of water $2.834 \times 10^6$ J kg$^{-1}$

$L_V$ Latent heat of vaporization of water $2.501 \times 10^6$ J kg$^{-1}$

$M(D_r)$ Mass of a raindrop of diameter $D_r$ kg

$M(D_t)$ Mass of a falling ice particle of diameter $D_t$ kg

$N_{ol}$ Intercept parameter in size distribution of falling ice particles m$^{-4}$

$N_{or}$ Intercept parameter in size distribution of raindrops $8 \times 10^6$ m$^{-4}$

$N_{CV}$ Number of convectively active levels

$N_d$ Cloud-droplet concentration $1 \times 10^8$ (ocean) m$^{-3}$

$N_d$ Cloud-droplet concentration $5 \times 10^8$ (land) m$^{-3}$

$N_{r}(D_r)dD_t$ Number of falling ice particles with diameters between $D_r$ and $D_r + dD_r$ m$^{-3}$

$N_{r}(T)$ Ice-crystal concentration at temperature $T$ m$^{-3}$

$N_{r}(T_0)$ Ice-crystal concentration at temperature $T_0$ m$^{-3}$

$N_{r}(D_t)dD_r$ Number of raindrops with diameters between $D_r$ and $D_r + dD_r$ m$^{-3}$

$P$ Rate at which precipitation enters a layer from above kg m$^{-2}$s$^{-1}$

$p$ Air pressure Pa
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_x$</td>
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<td>Pa</td>
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<tr>
<td>$Q_c$</td>
<td>Generalized cloud-water mixing ratio</td>
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<td>$q$</td>
<td>Total-water mixing ratio at a point within a grid box</td>
<td>kg kg$^{-1}$</td>
</tr>
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<td>Cloud-water mixing ratio</td>
<td>kg kg$^{-1}$</td>
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<td>Critical cloud-liquid-water mixing ratio</td>
<td>kg kg$^{-1}$</td>
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<td>Mixing ratio of falling ice</td>
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<tr>
<td>$q_i$</td>
<td>Cloud-ice mixing ratio</td>
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<td>$(\dot{q}<em>i)</em>{AV}$</td>
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<td>$(\dot{q}<em>i)</em>{C/E}$</td>
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<td>$(\dot{q}<em>i)</em>{DE}$</td>
<td>Rate of change of $q_i$ because of vapour deposition</td>
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<td>$(\dot{q}<em>i)</em>{F/M}$</td>
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<td>( \bar{V}_i )</td>
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<td>Model vertical coordinate (( p/p_* ))</td>
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<td>Standard deviation of the sub-grid fluctuations of ( Q_r )</td>
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<td>( \chi )</td>
<td>Diffusivity of water vapour in air at 1000 hPa</td>
<td>( 2.21 \times 10^{-5} ) m(^{2} ) s(^{-1} )</td>
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</table>

**Superscripts**
- \( k \): Denotes a quantity evaluated at model level \( k \)
- \( l \): Denotes the local value of a quantity (as distinct from the grid-box-mean value)
- \( n \): Denotes a quantity evaluated at time-step \( n \).

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