Modelling fluxes of momentum, sensible heat and latent heat over heterogeneous snow cover

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Summary

A boundary-layer model is used to simulate turbulent fluxes of momentum, sensible heat and latent heat over heterogeneous surfaces. Results for heterogeneous snow cover are compared with results for heterogeneous snow-free vegetation, and effective parameters characterizing the surfaces are discussed. A tile model, which calculates area-average fluxes from given area-average windspeeds, temperatures and humidities at a reference height in the atmosphere, is assessed in comparison with the boundary-layer model. The performance of the tile model is found to be degraded for surfaces with regions of both upward and downward sensible-heat flux, as can occur over heterogeneous snow cover. Better results can be obtained by using different reference heights for heat flux and momentum flux calculations.

Keywords: Boundary layer Heterogeneity Parametrization of surface fluxes Snow-covered surfaces Vegetation

1. Introduction

Most current General Circulation Models (GCMs) assume that land-surface properties are homogeneous throughout each gridbox or can be characterized by effective parameters. Gridbox-average fluxes of momentum, sensible heat and latent heat are generally calculated from gridbox-average vertical gradients of temperature, humidity and wind speed, using parametrizations similar to those used to relate local fluxes to local gradients over homogeneous surfaces. Local fluxes, however, depend nonlinearly on local gradients, and average fluxes are not simply related to average gradients. Variations in local stability over a surface can, for example, lead to the average sensible-heat flux being counter to the average temperature gradient. It has been suggested that the problem of calculating gridbox-average surface fluxes can be addressed by gathering distinct surface types within a gridbox into homogeneous 'tiles' and calculating fluxes separately over each tile (Avissar and Pielke 1989; Claussen 1990, 1991b).

Snow cover can be heterogeneous on many subgrid length-scales, introducing marked heterogeneities in surface properties such as albedo, roughness, thermal characteristics and moisture availability. To illustrate differences in fluxes over partial snow cover, Fig. 1 shows net radiation and sensible-heat fluxes over a pine forest and a snow-covered frozen lake in Saskatchewan, measured on 3 March 1994 as part of the Boreal Ecosystem Atmosphere Study (Harding and Pomeroy 1996). The two measurement sites were 10 km apart. The snow-free forest canopy absorbed more solar radiation than the high-albedo snowpack on the lake, and there was a large upward (positive) sensible-heat flux over the forest around midday while the flux over the lake was smaller and downward. Analogous situations, modelled by Claussen (1991a) and Vihma (1995), occur for broken sea ice; large upward sensible-heat fluxes over even small areas of open water can dominate the average heat flux.

Mason (1988) studied momentum fluxes over surfaces of heterogeneous roughness in neutral conditions using a 2D boundary-layer model; this solves the Boussinesq equations with first-order turbulence closure and a second-order accurate energy conserving numerical scheme. Wood and Mason (1991) extended this to consider heat transfer and the

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influence of stability, and Blyth et al. (1993) added a surface-moisture flux parametrization to study effective resistances to sensible- and latent-heat fluxes from heterogeneous vegetation, partitioning a fixed available energy. The same boundary-layer model is used in this paper, but the available energy at the surface is allowed to vary with albedo and temperature to simulate the large differences in radiation balance between snow-free and snow-covered surfaces. The parametrizations used to calculate surface fluxes are described in the next section. Model results obtained for surfaces with heterogeneous snow cover are presented in section 3 and compared with results for heterogeneous vegetation. Effective roughness lengths characterizing the surfaces are discussed in section 4, and the performance of a tile model is assessed in comparison with the boundary-layer model and an effective-parameter model in section 5. Finally, section 6 summarizes conclusions and discusses GCM applications. Preliminary results obtained by coupling a tile model of heterogeneous snow cover to an atmospheric model are discussed by Essery (1997).

2. THE BOUNDARY-LAYER MODEL

The boundary-layer model grid has a horizontal spacing of 31.25 m and 20 vertical levels (5 in the lowest 10 m) extending up to 5000 m. Zero-flux conditions and a constant horizontal pressure gradient are imposed at the upper boundary, and periodic boundary conditions are used in the horizontal direction.

Incoming fluxes of solar and long-wave radiation (\(SW_L\) and \(LW_L\)), an albedo (\(\alpha\)), a surface resistance to moisture transfer (\(r_s\)) and a momentum roughness length (\(z_0\)) are specified for each point on the surface. Temperature roughness lengths (\(z_{0T}\)) and moisture roughness lengths (\(z_{0S}\)) are generally less than \(z_0\) for naturally occurring conditions over vegetation (Garratt and Hicks 1973) and snow (Andreas 1987), and are simply set to \(z_0/10\) here. Changes in prescribed surface parameters are smoothed over four grid points to avoid numerical instability.

The net radiation at a point with surface temperature \(T_0\) is

\[
R = (1 - \alpha)SW_L + LW_L - \sigma T_0^4,
\]

where \(\sigma\) is the Stefan–Boltzmann constant and the surface emissivity has been assumed to be equal to 1.0. Given potential temperatures (\(\theta\)), specific humidities (\(q\)) and wind speeds
(U) on the lowest model level (at height z1 = 0.5 m), fluxes of momentum, sensible heat and moisture are calculated as

\[
\tau = \frac{\rho}{r_m} U(z),
\]

\[
H = \frac{\rho c_p}{r_h} [\theta_0 - \theta(z)]
\]

and

\[
E = \frac{\rho}{r_q + r_s} [q_{sat}(T_0) - q(z)],
\]

where \(\theta_0\) is the surface potential temperature, \(\rho\) and \(c_p\) are the density and heat capacity of air and \(q_{sat}(T_0)\) is the saturation humidity at the surface temperature. The aerodynamic resistance for momentum transfer in Eq. (2) is

\[
r_m = \frac{1}{\kappa^2 U(z)} \left[ \ln \left( \frac{z + z_0}{z_0} \right) - \psi_m \left( \frac{z}{L_M} \right) \right]^2,
\]

where \(\kappa\) is the von Kármán constant and \(L_M\) is the Monin-Obukhov length. Vertical coordinates are chosen such that the wind speed is zero at \(z = 0\). The scalar aerodynamic resistances are

\[
r_h = \frac{1}{\kappa} \left( \frac{\tau}{\rho} \right)^{-1/2} \left[ \ln \left( \frac{z + z_0}{z_0} \right) - \psi_h \left( \frac{z}{L_M} \right) \right]
\]

and

\[
r_q = \frac{1}{\kappa} \left( \frac{\tau}{\rho} \right)^{-1/2} \left[ \ln \left( \frac{z + z_0}{z_0q} \right) - \psi_h \left( \frac{z}{L_M} \right) \right].
\]

The Monin-Obukhov length is

\[
L_M = -\frac{\rho}{\kappa g} \left( \frac{\tau}{\rho} \right)^{3/2} \frac{H}{c_p \theta_0 + (\epsilon - 1) E}^{-1},
\]

where \(g\) is the acceleration due to gravity and \(\epsilon\) is the molecular-weight ratio of water to dry air. The stability functions \(\psi_m\) and \(\psi_h\) are obtained from Businger-Dyer similarity functions for unstable conditions \((L_M < 0)\) and log-linear similarity functions for stable conditions \((L_M > 0)\), as discussed by Wood and Mason (1991). The log-linear form and first-order turbulence closure are only considered adequate for moderately stable conditions (Högström 1988). Strongly stable conditions can occur over snow, but are not considered here.

After calculating the net radiation and latent-heat flux from Eqs. (1) and (4), surface temperatures are found by inverting Eq. (3) using the surface energy balance,

\[
R = H + \lambda E + G.
\]

The ground heat flux, \(G\), is often much smaller than the radiative and turbulent fluxes, and is neglected here. \(\lambda E\) is the latent-heat flux, \(\lambda\) being taken to be the latent heat of sublimation at snow-covered points or vaporization at snow-free points. Snow-melt and sublimation of suspended snow are not represented; advection over heterogeneous melting snow has been studied by Liston (1995) using a similar modelling strategy.

Since the aerodynamic resistances used to calculate the surface fluxes depend on stability through \(L_M\), which is itself determined by the fluxes, Eqs. (1) to (9) are solved
iteratively. The model is run until it reaches a near-steady state, starting from homogeneous initial conditions supplied by a 1D version of the model. Twenty minutes of model time was found to be sufficient in all cases, giving temperatures varying by less than 0.005 °C, humidities varying by less than 0.001 g kg⁻¹ and wind speeds varying by less than 0.1 m s⁻¹ per minute throughout the boundary layer. The nature of the boundary conditions prevent the model from reaching a true equilibrium, so the final state should be interpreted as a snapshot of a slowly varying boundary layer (Blyth et al. 1993).

3. Boundary-layer model results

Fourteen runs of the boundary-layer model were performed with surface parameters intended to represent forest with grass- or snow-covered clearings (Table 1). Snow and grass are given the same roughness lengths; these are much larger than values measured over deep uniform snow, but could be appropriate for snow-covered vegetation. Fractions of each surface type and domain sizes (L) for the model runs are given in Table 2. Homogeneous downward fluxes were prescribed for solar radiation (400 W m⁻²) and long-wave radiation (300 W m⁻²), and the geostrophic wind was 10 m s⁻¹ in all runs. Results for homogeneous snow, forest and grass are supplied by runs 1, 7 and 13. The surface in each of the other runs has a single patch of snow or grass, the remainder of the surface being forest. The periodic boundary conditions make these surfaces equivalent to infinite lengths of alternating surface types, repeating over distance L.

There is a strong contrast between the albedos of snow and forest, so there is a large variation in the radiation absorbed by surfaces with different fractions of snow cover. Crosses on Fig. 2 show temperatures and fluxes averaged over the surface for runs 1 to 7.

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<th>TABLE 1. Surface Parameters</th>
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<th>TABLE 2. Descriptions of Model Runs</th>
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The energy available to be partitioned into latent- and sensible-heat fluxes decreases from 300 W m\(^{-2}\) for homogeneous forest to 80 W m\(^{-2}\) for homogeneous snow, and the surface as a whole becomes smoother and wetter with increasing snow cover. The surface stress (momentum flux) decreases as the surface becomes smoother, and the sensible-heat flux decreases as the available energy decreases. The latent-heat flux initially increases due to the decreasing surface resistance, but eventually reaches a maximum and then decreases due to the decreasing available energy.

![Graphs showing surface temperature, surface stress, sensible heat flux, and latent heat flux against snow fraction.](image)

Figure 2. Average surface temperatures and fluxes for forest and snow from boundary-layer model runs 1 to 7 (crosses), a tile model with a single reference height (solid lines), a tile model with separate reference heights for stress and heat flux calculations (dotted lines), and an effective-parameter model (dashed lines).

Forest and grass are given the same albedo, and the available energy is nearly homogeneous in runs 8 to 12, similar to situations studied by Blyth et al. (1993). The surface becomes smoother and wetter with increasing grass fraction, but the available energy remains nearly constant, only decreasing from 300 W m\(^{-2}\) for homogeneous forest to 287 W m\(^{-2}\) for homogeneous grass. As shown in Fig. 3, as the grass fraction increases the surface stress decreases, the latent-heat flux increases and the sensible-heat flux decreases.

Local surface temperatures and fluxes from runs 4 (50% forest, 50% snow) and 10 (50% forest, 50% grass) are shown as solid lines on Figs. 4 and 5. Both runs have the same distribution of surface roughness, and produce similar stress distributions with marked peaks at the windward edge of the forest (the wind blows from left to right). The latent-heat fluxes are upward, but smaller over the forested patches and least where moist air
crosses the windward edge of the forest. Higher latent-heat fluxes over snow and grass have to be balanced by lower sensible-heat fluxes. The available energy is nearly the same for forest and grass in run 10, and the sensible-heat flux is upward everywhere. The available energy is much less for snow, however, which requires the sensible-heat flux to be downward locally and gives a much larger variation in sensible-heat flux across the surface in run 4.

Vertical profiles of wind speed, temperature and humidity from runs 4 and 10 are shown in Figs. 6 and 7. Solid lines are horizontal averages over the domain on each model level, broken lines are local profiles near the downwind edge of each surface, and marker symbols show equilibrium profiles required to support the same local fluxes over homogeneous surfaces. For example, the equilibrium wind speed at height $z$ above a point with surface roughness $z_0$, given $\tau$ and $L_M$, is found from Eqs. (2) and (5) as

$$U_{eq}(z) = \frac{1}{\kappa} \left( \frac{\tau}{\rho} \right)^{1/2} \left[ \ln \left( \frac{z + z_0}{z_0} \right) - \psi_m \left( \frac{z}{L_M} \right) \right] .$$

Average profiles in run 4 are dominated by the influence of the forest fraction, and the local temperature and humidity profiles are only in equilibrium with the snow surface up to a height of a few metres. The average temperature profile is unstable ($\theta$ decreases with height), but a shallow stable internal boundary layer forms over the snow. The equilibrium
Figure 4. Local surface temperatures and fluxes from boundary-layer model run 4 (solid lines), a tile model with a single reference height (dashed lines) and a tile model with separate reference heights for stress and heat flux calculations (dotted lines).

humidity profile over the snow has a larger lapse rate than the actual local profile, because moisture from a surface source would be less well mixed into a homogeneous stable boundary layer; unphysical negative humidities are required to maintain the equilibrium gradient above 40 m.

Run 14 was performed with the same snow fraction as run 4, but with a domain size of 8 km rather than 1 km, to investigate the influence of heterogeneity length-scales. Figure 8 compares results from run 14 (dotted lines) with results shown previously in Fig. 4 for run 4 (solid lines). Dashed lines are for homogeneous snow and forest from runs 1 and 7, showing the values that local temperatures and fluxes should approach over long homogeneous fetches. Extrema in $\tau$, $H$ and $\lambda E$ due to advection at the windward edge of the forest are more marked in run 14, because the wind speed is higher and the air is colder and wetter after traversing a longer snow-covered fetch. The heat fluxes only approach equilibrium slowly with downwind distance over the snow, which is smoother and less well coupled to the atmosphere than the forest. If it is the rough areas of the surface that are wet and the smooth areas dry, then the enhancement in evaporation at dry to wet transitions is sharper, and also the decrease in average latent-heat flux, as the number of transitions per unit length decreases with increasing length-scale, is greater (noted by Blyth 1995).
Figure 5. Local surface temperatures and fluxes from boundary-layer model run 10 (solid lines) and a tile model with a single reference height (dashed lines).

4. EFFECTIVE ROUGHNESS LENGTHS

In practice, effective parameters relating the horizontally averaged structure of the boundary layer to average surface fluxes can only be used to calculate average fluxes from average gradients if they can be determined from surface-roughness distributions without reference to atmospheric stability.

Putting spatial averages (denoted by angle brackets) in Eqs. (2) and (5), an effective momentum roughness length, $z_{0}^\text{eff}$, can be defined such that the effective momentum resistance

$$r_m^\text{eff} = \frac{1}{\kappa^2 (U(z))} \left[ \ln \left( \frac{z + z_{0}^\text{eff}}{z_{0}^\text{eff}} \right) - \psi_m \left( \frac{z}{L_M^\text{eff}} \right) \right]^2,$$

gives the correct average stress from the average wind speed at height $z$, where $L_M^\text{eff}$ is an effective Monin–Obukhov length calculated using average fluxes in Eq. (8). Invoking the no-slip condition that the wind speed is zero at the surface and choosing a reference height sufficiently far above the surface that $U(z)$ is approximately homogeneous, averaging momentum resistances is equivalent to adding resistances in parallel, giving $(r_m^\text{eff})^{-1} = (r_m^{-1})$. The effective momentum roughness length is thus dominated by the rougher (i.e. lower resistance) parts of the surface. Given $z_{0}^\text{eff}$, effective scalar roughness lengths $z_{dH}^\text{eff}$ and $z_{ly}^\text{eff}$.
can similarly be defined as the values that satisfy

\[
(H) = \frac{\rho C_p}{r_{h,\text{eff}}} \left[ (\theta_h) - (\theta(z)) \right]
\]

and

\[
(E) = \frac{\rho}{r_{q,\text{eff}} + (\eta_h)} \left[ q_{\text{sat}}((T_h)) - \langle q(z) \rangle \right],
\]

where

\[
r_{h,\text{eff}} = \frac{1}{\kappa} \left( \frac{\langle \tau \rangle}{\rho} \right)^{-1/2} \left[ \ln \left( \frac{z + z_{0\text{eff}}}{z_{0\eta,q}} \right) - \psi_h \left( \frac{z}{L_M} \right) \right].
\]

Local values of \( z_{0h} \) and \( z_{0q} \) are generally taken to be equal, but their effective values over heterogeneous terrain need not be. There are no analogues of the no-slip condition for heterogeneous surface temperatures or humidities. If the surface heat fluxes are approximately homogeneous, however, scalar resistances combine in series \( (r_{h,q,\text{eff}} = (r_{h,q})) \), giving effective scalar roughness lengths dominated by the smoother parts of the surface.

Temperature, humidity and wind speed profiles close to the surface are dominated by local equilibrium, but there may be a range of heights, above the near-surface layer, over
which average profiles can be described by similarity theory; effective roughness lengths will be nearly independent of height in this range. Wood and Mason (1991) studied effective roughness lengths for momentum and temperature in model simulations with prescribed surface sensible-heat fluxes, either homogeneous across the surface or doubled over the smoother areas to simulate a variable Bowen ratio. They found $z_{0\text{eff}}$ to be greater than the log average roughness length, $\exp(\langle \ln z_0 \rangle)$, and to have little dependence on stability, whereas $z_{0\text{fr}}$ was less than $\exp(\langle \ln z_0 \rangle)$ and had a stronger dependence on stability. Effective momentum and temperature roughness lengths for runs 8 to 12 (triangles on Fig. 9) show the same qualitative behaviour, and effective moisture roughness lengths are close to $z_{0\text{fr}}$. In runs 2 to 6 (crosses on Fig. 9), however, the sensible-heat flux changes direction from upward over the forest to downward over the snow, giving much larger variations in stability than those considered by Wood and Mason (1991). This has little impact on $z_{0\text{eff}}$ and $z_{0\text{fr}}$, but effective temperature roughness lengths are much larger than log averages for runs 4 to 6 (shallow inversions in the average temperature profiles mean that well defined values of $z_{0\text{fr}}$ cannot be found for runs 2 and 3). For a surface with both stable and unstable regions that balance to give a small upward average heat flux, counter to a stable average temperature gradient, $r_{0\text{fr}}$ tends to $-\infty$ and $z_{0\text{fr}}$ diverges as $\langle H \rangle$ approaches zero from above. Similar problems can be anticipated in the calculation of $z_{0\text{fr}}$ if there are regions of both upward and downward latent-heat flux. Effective scalar roughness lengths are very sensitive to stability in these situations and cannot be specified a priori.
Figure 8. Local surface temperatures and fluxes from boundary-layer model runs 4 (solid lines) and 14 (dotted lines). Dashed lines are from run 1 for homogeneous snow and run 7 for homogeneous forest.

5. TILE MODELS AND BLENDING HEIGHTS

GCMs require parametrizations which calculate gridbox-average surface fluxes given gridbox-average wind speeds, temperatures and humidities at a reference height in the atmosphere, and incoming radiation fluxes at the surface. In a tile model, distinct surface types within a gridbox are gathered into separate tiles. Surface temperatures and fluxes are calculated for each tile from Eqs (1) to (9) using the roughness lengths, surface resistance and albedo characteristic of that tile. Average fluxes are found from summing the fluxes from the tiles, weighted by the fractions of the gridbox which they cover. Detailed information on the distribution of surface types within the gridbox is not required, but consideration has to be given to the length-scales over which the surface is heterogeneous. Perturbations due to heterogeneity on length-scales of the order of 10 km and greater will extend through the entire depth of the boundary layer (Shuttleworth 1988). At the other extreme, air at the height of a sparse vegetation canopy, heterogeneous on length-scales less than the canopy height, will be well mixed; single aerodynamic resistances can then be used to calculate fluxes between the canopy height and a reference height. ‘Sparse canopy’ and tile strategies have been compared by Koster and Suarez (1992) and unified in a generalized tile model by Blyth (1995). Partial snow covers can have fractal characteristics (Shook et al. 1993), giving heterogeneities on many length-scales.
A tile model and a simple effective-parameter model, both driven by area-average data from the boundary-layer model at a height of 19 m (typical of the lowest atmospheric level in a GCM), have been assessed in comparison with the boundary-layer model. The effective-parameter model uses average albedos, average surface resistances and log average roughness lengths.

Compared with runs 8 to 12 (heterogeneous forest and grass) in Fig. 3, the effective-parameter model underestimates the average stress as a result of using too small a value for $z_0^{ef}$, but this compensates for the use of overestimated scalar roughness lengths in the scalar resistances to give good surface temperature and heat flux values. The tile model does not represent the details of advective effects, such as the peak in surface stress seen at the windward edge of the forest in Fig. 5, but gives good predictions of average surface temperature, heat fluxes and stress (solid lines on Fig. 3).

For runs 2 to 6 (heterogeneous forest and snow), the effective-parameter model performs poorly, overestimating temperatures and latent-heat fluxes but underestimating stresses and sensible-heat fluxes (dashed lines on Fig. 2). The tile model still gives good values for the average surface temperature and stress but overestimates the sensible-heat flux and underestimates the latent-heat flux (solid lines on Fig. 2). The dashed lines on Fig. 4 reveal that this is largely due to errors over the snow-covered fraction; the upward latent-heat flux and the downward sensible-heat flux calculated for the snow tile are both
too small. Nevertheless, the tile model captures the change in stability between forest and snow.

The tile-model concept relies on the existence of a 'blending height' at which horizontal advection and vertical diffusion of perturbations due to surface heterogeneity balance, so that the atmospheric structure is both nearly independent of horizontal position and close to equilibrium with the local surface. By setting the reference level at the blending height, fluxes from homogeneous tiles can be calculated using local surface parameters. Blending heights will, in general, depend on surface roughness, stability and heterogeneity length-scale, and differ for momentum, temperature and humidity perturbations. Wood and Mason (1991) argued that momentum and temperature blending heights, \( l_b \) and \( l_{bh} \), can be approximated by solutions of

\[
I_b \left[ \ln \left( \frac{l_b + z_0^{\text{eff}}}{z_0^{\text{eff}}} \right) - \psi_m \left( \frac{l_b}{L_M^{\text{eff}}} \right) \right]^2 = \frac{\kappa^2 L}{\pi},
\]  

(10)

and

\[
I_{bh} \left[ \ln \left( \frac{l_{bh} + z_0^{\text{eff}}}{z_0^{\text{eff}}} \right) - \psi_m \left( \frac{l_{bh}}{L_M^{\text{eff}}} \right) \right] \ln \left( \frac{l_{bh} + z_0^{\text{eff}}}{z_0^{\text{eff}}} \right) = \frac{\kappa^2 L}{\pi},
\]  

(11)

where \( L \) is the length-scale of the surface heterogeneity. Extending the scalar argument, a humidity blending height \( l_{bh} \) is given by substituting \( z_0^{\text{eff}} \) for \( z_0^{\text{eff}} \) in Eq. (11).

The calculation of blending heights from Eqs. (10) and (11) is an iterative procedure unsuitable for GCM applications and, in any case, only provides order of magnitude estimates. Neutral blending heights, which may give adequate estimates, can be calculated \textit{a priori} by neglecting the stability functions in Eqs (10) and (11), with effective roughness lengths found from

\[
\left[ \ln \left( \frac{l_b + z_0^{\text{eff}}}{z_0^{\text{eff}}} \right) \right]^2 = \left( \left[ \ln \left( \frac{l_b + z_0}{z_0} \right) \right]^2 \right)
\]

and

\[
\ln \left( \frac{l_{bh} + z_0^{\text{eff}}}{z_0^{\text{eff}}} \right) \ln \left( \frac{l_b + z_0^{\text{eff}}}{z_0^{\text{eff}}} \right) = \left( \ln \left( \frac{l_b + z_0}{z_0} \right) \ln \left( \frac{l_{bh} + z_0}{z_0} \right) \right).
\]

Neutral blending heights are shown by solid lines on Fig. 10. Momentum blending heights are greater than scalar blending heights, and both decrease as the smooth fraction of the surface increases.

Blyth \textit{et al.} (1993) suggested neglecting stability effects and differences between momentum, temperature and humidity blending heights to set the tile-model reference level at the neutral momentum blending height for heterogeneous vegetation. Claussen (1990 and 1991b) used a similar approach, but set the reference level at the diffusion height, \( l_d \), given by

\[
l_d \left[ \ln \left( \frac{l_d + z_0^{\text{eff}}}{z_0^{\text{eff}}} \right) \right] = \kappa c_1 L_C,
\]

where \( c_1 \) is an O(1) constant. Taking \( L_C \) to be the fetch length, Claussen (1991b) found that using values of \( l_d \) obtained with \( c_1 = 1.75 \) gave small flux errors. Diffusion heights are typically an order of magnitude greater than blending heights; the dashed line on Fig. 10 shows \( l_d/10 \), calculated using an average fetch length of 500 m.
Figure 10. Blending heights for momentum, \( l_b \), temperature, \( l_{bb} \), and humidity, \( l_{bh} \), comparing boundary layer model runs 2 to 6 (+) and 8 to 12 (\( \Delta \)) with neutral blending heights (solid lines) and \( l_d/10 \) (dashed line). See text for explanation of symbols.

Blending heights calculated from Eqs. (10) and (11) for boundary-layer model runs with heterogeneous forest and grass (runs 8 to 12) are shown by triangles on Fig. 10, and can be seen to be close to the neutral blending heights. The influence of the chosen reference height on the performance of the tile model, compared with run 10, is shown in Fig. 11. Fractional errors in the average stress (Fig. 11(a)) are positive (overestimated) for reference heights below 35 m, and negative for greater reference heights (\( l_b = 6.2 \) m and \( l_d = 65 \) m in this case). The variation in calculated surface stress with reference height is greatest for the forest tile; below 35 m the average wind speed is greater than the local equilibrium wind speed over the forest (Fig. 7) and the stress is overestimated, but above 35 m the average wind speed is less than the equilibrium wind speed and the stress is underestimated. Fractional errors in latent- and sensible-heat fluxes (Fig. 11(b)) are small across the range of reference heights considered, and disappear around 10 m and 3 m, respectively. The errors do not change greatly with height above these levels, and good results can be obtained from a tile model using a single reference height for stress and heat flux calculations.

Blending heights for boundary-layer model runs with heterogeneous forest and snow (runs 2 to 6) are shown by crosses on Fig. 10. Momentum and humidity blending heights are close to neutral blending heights. Temperature blending heights, however, are much larger for runs 4 to 6 owing to the large values of \( z_{0h} \), and the iterative calculation does
not converge for runs 2 and 3. Taking run 4 as an example, Fig. 6 shows that the air temperature is far from equilibrium with the snow surface at heights where it is close to horizontal homogeneity and, strictly, the concept of a temperature blending height breaks down. Vihma (1995) noted similar conditions in simulations of the flow over heterogeneous sea ice. Compared with boundary-layer-model run 4, the tile-model stress errors shown in Fig. 12(a) are similar to those for run 10 (Fig. 11(a)), but the heat flux errors shown in Fig. 12(b) are much larger (cf. Fig. 11(b)). Latent-heat fluxes are underestimated and sensible-heat fluxes are overestimated at all heights, with minimum errors of around 12% for reference heights in the range 10–20 m.

The calculation of scalar resistances from Eqs. (6) and (7) requires stresses for each tile. The average temperature and humidity, used with the scalar resistances in Eqs. (3) and (4) to calculate tile heat fluxes, are far from equilibrium with the snow tile at a reference height chosen to give good stress values. A compromise can be achieved by using different reference heights for stress and heat flux calculations. Figure 12(c) shows errors in sensible- and latent-heat fluxes calculated using average temperatures and humidities at a range of reference heights, with stresses found from the average wind speed at 19 m. The errors do not vanish at any height, but they are significantly less than errors obtained using a single reference height (cf. Fig. 12(b)) and are less than 7% for reference heights below 5 m. Dotted lines on Fig. 4 show tile-model results obtained using average temperatures and humidities on the boundary-layer-model level at 2.75 m, close to the neutral scalar blending height for run 4. This has a slightly detrimental effect on the temperature and heat fluxes for the forest tile but gives significant improvements in the partitioning of the available energy over the snow tile and hence improves the calculation of the average fluxes (dotted lines on Fig. 2). Even better results can be obtained using a lower scalar reference height over the snow than over the forest, but it is not clear that optimal reference heights can be identified a priori.

6. Conclusions and discussion

The boundary-layer simulations show that effective temperature roughness lengths and temperature blending heights can be ill-defined over surfaces with mixed stable and unstable regions. The performance of a tile model in calculating area-average heat and moisture fluxes is degraded as a result. Better results can be obtained using separate reference heights for heat flux and stress calculations, but further consideration will have to be
Figure 12. Fractional tile-model errors, compared with boundary-layer model run 4. (a) Errors in average surface stress, (b) Errors in average sensible-heat flux (solid line) and latent-heat flux (dashed line) and (c) Errors in average heat fluxes calculated with fixed stress values.

given to the development of robust parametrizations of fluxes over strongly heterogeneous surfaces such as patchy snow.

In a GCM, data on a reference level that does not coincide with a model level would be obtained by interpolation, as discussed by Claussen (1991b). In addition to a method for calculating average fluxes, a complete parametrization of heterogeneous snow cover would require a closure relating fractional coverage to parameters either calculated within a GCM, such as grid-box averages of snow mass, temperature, wind speed and precipitation, or made available from observations, such as simple statistical characterizations of subgrid-scale topography (Walland and Simmonds 1996) and vegetation cover (Donald et al. 1995).

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