Linear stability and single-column analyses of several cumulus parametrization categories in a shallow-water model

By JUN-ICHI YANO, MITCHELL W. MONCRIEFF and JAMES C. MCWILLIAMS

1Monash University, Australia
2National Center for Atmospheric Research, USA
3UCLA, USA

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SUMMARY

Using a mass-flux based approach, the thermodynamic cumulus parametrization problem is reformulated in a simple atmospheric model, which is an analogue of the shallow-water equations. The objective is to investigate basic effects of elementary representations of several parametrization categories. In particular, a linear stability analysis and a single-column experiment are performed to infer the characteristics of each parametrization as regards its ability to simulate the large-scale organization or coherence of tropical convection.

The moisture-convergence closure (MC) scheme, which assumes that the ensemble of cumulus convection is controlled by the low-level moisture convergence as in Kuo-type schemes, predicts the largest growth at the smallest scale. Hence, although it ensures the generation of a coherent propagating structure, its scale always corresponds to the grid size. Furthermore, the MC tends to produce a catastrophic positive feedback of moist convection to the large-scale convergence.

In contrast, the statistical equilibrium scheme, which assumes an instantaneous adjustment of the large-scale environment to a quasi-equilibrium state, such as Arakawa–Schubert and moist convective adjustment schemes, asymptotes to a constant growth rate at small scales. Hence, this type of parametrization tends to generate a field like white noise with no large-scale coherence. The lagged-adjustment (LA) schemes, which have a short time-lag for the cumulus growth, as in the Betts–Miller scheme, feature a finite scale selection in the linear growth rate. This ensures a smooth large-scale coherence that is independent of the grid size, and is consistent with the scale-separation principle.

A new type of parametrization is also tested. This convective life-cycle (CLC) scheme represents the life cycle of a common type of convective system made up of deep precipitating convection and a subsequent mesoscale response. It uses a buoyancy-based closure. The growth-rate curve is similar to the other LA schemes, but the behaviour in the zero-dimensional (single-column) version of the model is qualitatively different. Although the CLC scheme does not automatically satisfy the scale-separation principle, its grid-size dependence can be treated by a re-normalization principle.

The results are used to interpret some reported general-circulation-model results regarding the impact of different parametrization schemes on the tropical atmosphere at large scales.

KEYWORDS: Convection, Mesoscale processes, Parametrization, Tropical dynamics

1. INTRODUCTION

Cumulus convection is well known to be a key process in the tropical atmosphere. An appropriate incorporation of cumulus processes into a large-scale model by means of a cumulus parametrization is, for this reason, crucial in simulating tropical atmospheric phenomena such as El Niño, intraseasonal variability and monsoonal circulations. However, the choice of which cumulus parametrization to use still remains a controversial issue (Frank 1983; Arakawa 1993; Emanuel 1994 Chapter 16).

A recent review by Emanuel et al. (1994) has stoked this continuing debate (cf. Smith 1997; Stevens et al. 1997). They promoted the concept of 'statistical equilibrium' originally proposed by Arakawa and Schubert (1974), and strongly criticized the use of methods based on the moisture budget as in the wave–CISK theory (Lindzen 1974), as well as in the Kuo-type scheme (Kuo 1974). The motivation for the present paper is to provide a quantitative baseline for this debate in an idealized setting.

The present work stems from our own efforts (Yano et al. 1995, 1996) to simulate the convective hierarchy associated with tropical intraseasonal variability (Nakazawa 1988) in a simple way. To facilitate a high horizontal resolution, only one vertical degree of...
dynamical freedom is retained, which results in an analogue of the shallow-water equation system. Even with such a drastic simplification, much of the freedom to experiment with cumulus parametrizations remains. In fact, simple forms of the basic types of cumulus parametrization methods can be examined in this framework.

In our previous work, the focus has been on investigating the ability of cumulus parametrizations to generate large-scale convective coherence in the tropical atmosphere. This ability is indispensable in order to simulate the Madden–Julian oscillation (MJO) and other observed coherence in a general circulation model (GCM). The advantage of the simplified approach is the reduction of the problem into its basic elements, in contrast to the use of full GCMs to study the impact of cumulus parametrization (e.g. Hack 1994; Slingo et al. 1994; Colman and McAvaney 1995; Baik and Takahashi 1995). The basic philosophy of our approach originates from the aqua-planet GCM experiments by Hayashi and Sumi (1986), followed by Numaguti and Hayashi (1991a, b) and others. Chao and Lin (1994) further simplified this problem into a horizontally one-dimensional configuration. In this spirit, we adopt a maximal simplification of the model.

The model formulation is given in the next section. In section 3, we introduce five cumulus parametrizations: a moisture-convergence closure (MC) scheme akin to that of Kuo (1974), a statistical equilibrium (SE) scheme under a strict statistical-equilibrium assumption, as proposed by Arakawa and Schubert (1974), and three types of the lagged-adjustment (LA) scheme conceptually following Betts (1986) and Betts and Miller (1986). For physical clarity the acronyms MC and LA are intended to replace the expressions ‘Kuo-like’ and ‘prognostic’ used in Yano et al. (1995, 1996), respectively. For the same reason we also replace specific expressions for the two LA schemes previously used; namely, we rephrase ‘dynamic adjustment’ by ‘Lagrangian parcel acceleration’ (LPA) and ‘grid column’ by ‘convective life cycle’ (CLC). The final LA scheme is the boundary-layer entropy equilibrium (BEE) scheme.

Simple analyses are performed, for example, linear stability (section 4) and the zero-dimensional analyses (section 5). The former is a simplified model version of the analysis by Brown and Bretherton (1995) and by Neelin and Yu (1994). The latter constitutes a single-column test in the present model framework. However, unlike the standard single-column test (e.g. Krishnamurti et al. 1980; Betts and Miller 1986), the large-scale vertical velocity is explicitly predicted in the model. Those analyses are suggestive of a way to interpret the behaviour of more sophisticated parametrizations in GCMs (section 6).

2. Model formulation

We follow Yano et al. (1995; hereinafter YM²E) for the model formulation, which is summarized in the present section. Definitions and values of the model constants are listed in Table 1.

(a) Dynamics

The dynamical framework (see Fig. 1) is a one-and-a-half-layer model, namely a dynamically active free troposphere and a dynamically passive boundary layer. The wind \( \mathbf{v}_H \) defined at the top of the boundary layer is driven by the pressure gradient, surface friction represented by a decay law with a constant drag coefficient \( C_D \), and a Rayleigh wind-forcing:

\[
\frac{D}{Dt} \mathbf{v}_H = - \nabla \phi - \frac{C_D}{h} |\mathbf{v}_H| \mathbf{v}_H - \frac{1}{\tau_D} (\mathbf{v}_H - \mathbf{v}_0). \tag{1}
\]
TABLE 1. Model constants

Thermodynamical

\( C_p = 10^5 \text{ J K}^{-1} \text{ Kg}^{-1} \): specific heat with constant pressure

\( N = 10^{-2} \text{ s}^{-1} \): Brunt-Väisälä frequency

\( \gamma = \Gamma_d/\Gamma_m = 1.7 \)

\( \Gamma_d = 10^{-2} \text{ K m}^{-1} \): adiabatic lapse rate

\( \Gamma_m = 6 \times 10^{-3} \text{ K m}^{-1} \): moist lapse rate

\( \theta_0 = 300 \text{ K}\): the reference potential temperature

\( \varepsilon = (T_b - \bar{T})/T_b = 0.1 \)

\( T_b = 300 \text{ K}\): temperature at the top of the boundary layer

\( \bar{T}\): mean temperature over the troposphere

\( C_b = 1.2 \times 10^{-3}\): surface-flux rate by wind

Radiative

\( \tau_R = 50 \text{ days}\): long-wave radiative relaxation time (constant for Newtonian cooling term)

\( Q_{R0} = -1 \text{ K day}^{-1}\): mean radiative heating rate

Cloud physics

\( B_c = 6 \times 10^{-3} \text{ K}\): critical buoyancy (\( B = 8.8 - \gamma \theta \) is necessary to initiate cumulus convection in the CLC scheme)

\( c_p\): precipitation-efficiency coefficient (\( c_p = 1.0 \) for the MC and the CLC schemes, \( c_p = 0.9 \) for the SE, the LPA, and the BEE schemes)

\( \sigma_c = 0.01\): fractional area occupied by cumulus clouds in a single grid box

\( \sigma_d\): fractional area occupied by the downdraughts in a single grid box

\( \tau_b = 1 \text{ hour}\): relaxation time-scale of the boundary layer in the BEE scheme

\( \tau_s = 1.8 \text{ hours}\): decaying time-scale of mesoscale downdraughts (CLC scheme)

Dynamical

\( C_D = 1 \times 10^{-3}\): surface drag coefficient

\( \tau_D = 150 \text{ days}\): Rayleigh-forcing time-scale

\( u_0 = -10 \text{ m s}^{-1}\): Rayleigh-forcing wind

Vertical scales

\( h = 500 \text{ m}\): depth of boundary layer

\( H = 8 \text{ km}\): depth of troposphere (density-weighted scale in log-\( p \) coordinate, where \( p \) is pressure)

\( H_m = 5 \text{ km}\): height of middle-level troposphere (a level of minimum moist potential temperature, which is given by \( \theta_{em} \))

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Figure 1. The model configuration; see text and Table 1 for definitions of the variables.
The Lagrangian time-derivative is defined by
\[ \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \frac{\partial}{\partial x} + \mathbf{v} \cdot \frac{\partial}{\partial y}. \]

The Rayleigh wind-forcing relaxes to a constant velocity \( \mathbf{v}_0 = (u_0, 0) \) on a time-scale \( \tau_D \). The effect of planetary rotation is neglected in the present paper.

The perturbation geopotential, \( \delta \phi \), is related to the potential-temperature anomaly, \( \theta \) (characterizing the mid-troposphere temperature), by
\[ \delta \phi = -C_p \gamma \epsilon \theta, \]
following Emanuel (1987) and Yano and Emanuel (1991). Continuity of mass is given by the incompressible-approximation integrated from the top of the boundary layer to the middle troposphere over the depth \( H_m \):
\[ \nabla \cdot \mathbf{v}_H + \frac{w}{H_m} = 0, \]
where \( w \) is a representative mean vertical velocity of the troposphere.

(b) Thermodynamics

We describe the thermodynamics of the system by \( \theta \), and the moist potential temperature anomaly, \( \theta_e \), which is conserved following the motion of a moist parcel with the precipitation efficiency, \( \epsilon_p \), introduced below. Both variables are defined as deviations from reference values. The moist convective processes are described in terms of a bulk mass-flux formulation (Yanai and Johnson 1993). The thermodynamic equation for predicting \( \theta \) is given by
\[ \frac{D}{Dt} \theta + \frac{N^2}{g} \theta_0 w = Q_c + Q_R. \]
Here, the cumulus convective heating, \( Q_c \), is assumed to be proportional to the cumulus mass-flux, \( M_c \):
\[ Q_c = M_c \frac{N^2}{g} \theta_0, \]
and the radiative heating, \( Q_R \), is represented by a Newtonian law
\[ Q_R = Q_{R0} - \frac{\theta}{\tau_R}, \]
with a constant cooling rate \( Q_{R0} = -1 \) K day\(^{-1} \), and a radiative relaxation time \( \tau_R = 50 \) days.

We assign the moist potential temperature in the boundary layer, \( \theta_{eb} \), and in mid-troposphere, \( \theta_{em} \), by considering the effect of surface flux on the moist-potential-temperature budget. Because the surface heat flux, \( E \), is supplied primarily to the boundary layer, and the radiative heating, \( Q_R \), is contributed mainly to the free troposphere, the equation for the moist potential temperature in each layer is approximately:
\[ H \frac{D}{Dt} \theta_{eb} = -D + E \]
\[ H \frac{D}{Dt} \theta_{em} = D + H Q_R \]
where $h$ and $H$ are the depth of the boundary layer and the troposphere, respectively. Here, $E$ and the convective downdraught effect, $D$, are given by

$$E = C_d |\nabla H| (\theta_{eb}^* - \theta_{eb}), \quad (9)$$

$$D = - (\tilde{M}^\dagger - M_d)(\theta_{eb} - \theta_{em}). \quad (10)$$

The quantity $\theta_{eb}^*$ is the saturated potential temperature in the boundary-layer, $M_d$ is the convective downdraught mass-flux, and $\tilde{M}^\dagger$ is the environmental subsidence defined by

$$\tilde{M}^\dagger = \begin{cases} \tilde{M} & \tilde{M} < 0, \\ 0 & \tilde{M} > 0, \end{cases} \quad (11)$$

where $\tilde{M} \equiv w - M_c$.

Finally, we measure the convective buoyancy, $B$, by

$$B = \theta_{eb} - \theta_{em}^*, \quad \text{in units of temperature. Hence, the buoyancy force is given by} \quad C_p \Gamma_m B/\theta_0.$$  

Here, the saturated moist potential temperature, $\theta_{em}^*$, is related to the potential temperature by

$$\theta_{em}^* = \gamma \theta,$$

and the ratio of the dry lapse rate to the moist lapse rate is $\gamma \equiv \Gamma_d/\Gamma_m$.

3. CUMULUS PARAMETRIZATION CATEGORIES

It transpires that under the mass-flux formulation, the cumulus parametrization problem reduces to one of defining a closed expression for the vertical mass fluxes, namely $M_c$ and $M_d$. The vertical structure of $M_c$ and $M_d$ is usually complicated; for example, Arakawa and Schubert (1974) assume a spectrum of plume-like cumuli. Herein, this procedure is greatly simplified in the one-and-a-half layer formulation. Nevertheless, various choices for the closure remain at our disposal, each of which conceptually corresponds to a major category of existing convective parametrization schemes.

Five categories of parametrization are considered. Since the first four are identical to those in YM$^2$E, they are described very concisely. The fifth is a new type based on Emanuel (1995).

(a) Moist-convergence closure (MC)

A popular method for determining the closure is to use the low-level moisture convergence, as originally proposed by Kuo (1965, 1974). For this reason, this scheme was formerly called ‘Kuo-like’ in YM$^2$E and in our subsequent papers. We set

$$M_c = w,$$  

whenever the conditions $B > 0$ and $w > 0$ are satisfied simultaneously. We also replace Eq. (5) by

$$Q_c = M_c \left( \frac{N^2}{g} \theta_0 + \frac{\theta_{eb} - \theta_{em}^*}{H} \right),$$  

closely following the formulation of Krishnamurti et al. (1980). This revised the original Kuo (1965, 1974) formulation in order to satisfy a Galilean invariance. Also, the convective downdraught (Eq. 10) is replaced by

$$D = (\theta_{eb} - \theta_{em}^*) w.$$  

(14)
(b) Statistical equilibrium (SE)

We set

$$B = 0 \quad (15)$$

whenever the system has a tendency to become unstable. This assumption of strict or instantaneous statistical equilibrium directly follows Arakawa and Schubert (1974). The constraint Eq. (15) diagnostically defines the cumulus updraught $M_c$. We relate the convective downdraught $M_d$ to the cumulus mass-flux $M_c$ by

$$M_d = \frac{1 - \epsilon_p}{\epsilon_p} M_c, \quad (16)$$

with a constant precipitation efficiency $\epsilon_p$.

The moist convective adjustment (MCA: e.g. Manabe et al. 1965; Miyakoda et al. 1969) also assumes an instantaneous adjustment as the closure, albeit to a different vertical profile. However, this difference is minimal because of the restricted vertical resolution. For this reason we expect MCA and SE schemes to behave in a similar manner.

(c) Lagrangian parcel acceleration (LPA)

The cumulus vertical velocity $M_c/\sigma_c$ is predicted by a parcel acceleration due to buoyancy:

$$\frac{D}{Dt} \frac{M_c}{\sigma_c} = \frac{C_p \Gamma_m}{\theta_0} B - \frac{M_c^2}{2\sigma_c^2 H}, \quad (17)$$

except that we set $M_c = 0$ when $M_c < 0$. The simple diagnostic relation of Eq. (16) between $M_d$ and $M_c$ is also assumed. The prognostic Eq. (17) produces a gradual growth of the cumulus updraught in response to convective instability. Hence, this contains a time-lag effect akin to the Betts and Miller (1986) scheme. We call this general category the ‘lagged-adjustment’, or LA scheme to distinguish it from the concept of ‘moist convective adjustment’. We prefer this prefix over the standard distinction of ‘hard’ and ‘soft’ (cf. Arakawa 1993) because of its clearer physical implication. We also consider the following two other LA schemes.

(d) Convective life-cycle (CLC)

This is similar to the LPA scheme except that it adopts an Eulerian description. Hence,

$$\frac{\partial}{\partial t} \frac{M_c}{\sigma_c} = \frac{C_p \Gamma_m}{\theta_0} B, \quad (18)$$

when $B \geq B_c = 6 \times 10^{-3}$ K. This finite threshold is imposed because otherwise the zero-dimensional result in section 4 will be much noisier. It turns out that the full numerical integration is not sensitive to this threshold (cf. YM9E). Cumulus convection is terminated when $B < 0$ and is replaced by a mesoscale convective downdraught: i.e. when we set $M_c = 0$, say, at $t = t_0$ then:

$$M_d = [M_c + M_d](t = t_0 - )e^{(t_0 - t)/\tau_d}, \quad (19)$$

where $t = t_0 -$ designates the value just before the termination. By disregarding the scale-separation principle (cf. Arakawa 1993), this scheme approximates a typical life cycle of a mesoscale convective system at each grid column.

Note that in these two LA schemes, the time-lag effect is implicitly represented in the prognostic Eqs. (17) and (18) for the cumulus updraught. In contrast, we consider the time-lag effect more explicitly in the next scheme.
Finally, a scheme is derived based on the effect of cumulus processes on boundary-layer moist entropy, which we call the boundary-layer entropy equilibrium (BEE) scheme. A tendency seen from Eq. (7) is that the wind-induced surface flux increases the boundary-layer moisture, whereas the cumulus processes dry and cool the boundary layer through the combined action of downdraughts and environmental subsidence. In this regard the effect of cumulus processes is to dry the boundary layer and oppose the moistening due to wind-induced surface flux. This idea was called ‘boundary-layer quasiequilibrium’ by Raymond (1995) and ‘subcloud-layer entropy equilibrium’ by Emanuel (1995).

In our BEE scheme, which closely follows that of Emanuel (1995), we additionally assume that an equilibrium convective updraft $M_c^{(e)}$ exactly maintains the boundary-layer moist potential-temperature, i.e.

$$ E - D^{(e)} = 0, $$

where $D^{(e)}$ is obtained by replacing $M_c$ by $M_c^{(e)}$ in Eq. (10), along with Eqs. (11) and (16), whenever $B > 0$ is satisfied (otherwise, we set $M_c^{(e)} = 0$). We force the actual cumulus mass flux towards this equilibrium mass flux $M_c^{(e)}$ with a relaxation time $\tau_b$, so that

$$ \frac{D}{Dt} M_c = - \frac{M_c - M_c^{(e)}}{\tau_b}. \quad (20) $$

The relaxation time is set at $\tau_b = 1$ hour.

4. LINEAR STABILITY

Our linear stability analysis infers that the preferred scale and structure of the largescale disturbances arise from the action of parametrized convection. We assume a homogeneous basic state with a zonal wind $\bar{u}$ and a radiative cooling $Q_{R0}$, but there is no mean vertical motion $\bar{w} = 0$. We perturb this basic state by a linear disturbance. The derivation, briefly described below, is similar to that of Yano and Emanuel (1991: hereinafter YE91).

The homogeneous basic state is given by

$$ \bar{M}_c = -\bar{\theta} = -\frac{g}{N^2 \theta_0} Q_{R0}, \quad (21) $$

$$ \theta_{eb} - \bar{\theta}_{eb} = -\theta_0 \frac{H Q_{R0}}{C_0 |\bar{u}|}, \quad (22) $$

$$ \bar{\theta}_{eb}^{*} - \bar{\theta}_{en} = \epsilon_p \frac{N^2 H}{g} \theta_0, \quad (23) $$

where

$$ N^2 = N^2 + \epsilon_k \frac{g}{H \theta_0} \bar{B}, $$

and the overbar designates the mean state. The switching parameter is $\epsilon_k = 1$ for the MC scheme but $\epsilon_k = 0$ otherwise. The mean buoyancy $\bar{B}(\equiv \bar{\theta}_{eb} - \gamma \bar{\theta})$ for the LPA scheme is given by

$$ C_p \Gamma_m \bar{B}/\theta_0 = \bar{M}_c^2 / 2 \alpha_c^2 H \quad (24) $$

from Eq. (17), but remains indefinite for other schemes. We set

$$ \bar{B} = 0 \quad (25) $$

in these cases for convenience. We also set $\bar{\theta} = 0$ without loss of generality.
In the basic state, the net radiative cooling is balanced by adiabatic heating due to environmental subsidence, which conversely defines a mean cumulus updraught (Eq. 21). The moist potential temperature of the boundary layer is defined by a balance between the surface flux and the downdraughts (Eq. 22). The tropospheric moist potential temperature is the result of a balance between the action of downdraughts and the radiative cooling (Eq. 23). In defining the basic state, we have assumed that Eq. (16) is satisfied even in the case of CLC schemes.

The perturbation equations for Eqs. (1), (3), (4), (7) and (8) are given by

\[
\left( \frac{\hat{D}}{Dt} + F_x \right) u' = \epsilon_{ih} \frac{\partial}{\partial x} \theta',
\]

\[
\left( \frac{\hat{D}}{Dt} + F_y \right) v' = \epsilon_{ih} \frac{\partial}{\partial y} \theta',
\]

\[
\nabla \cdot \vec{v}'_H + \frac{w'}{H_m} = 0,
\]

\[
\left( \frac{\hat{D}}{Dt} + \frac{1}{\tau_R} \right) \theta' = -\tilde{N} \tilde{M}' + \frac{\epsilon_k}{H} (\tilde{B} M' + \tilde{M}_c B'),
\]

\[
\left( h \frac{\hat{D}}{Dt} + \alpha_e + \alpha_m \right) \theta_{eb}' = \alpha_o u' + \lambda (\tilde{M}' - M'_d) + \alpha_m \theta_{em}' - \epsilon_k (\tilde{B} M'_c + \tilde{M}_c B'),
\]

\[
\left( H \frac{\hat{D}}{Dt} + \alpha_m \right) \theta_{em}' = -\lambda (\tilde{M}' - M'_d) + \alpha_m \theta_{eb}' - \frac{H}{\tau_R} \theta' + \epsilon_k (\tilde{B} M'_c + \tilde{M}_c B'),
\]

and

\[
B' = \theta_{eb}' - \gamma \theta',
\]

\[
\tilde{M}' = w' - M'_c,
\]

where the parameters are defined by

\[
\epsilon_{ih} = C_p y \epsilon,
\]

\[
F_x = 2 \frac{C_D}{h} |\bar{u}| + \frac{1}{\tau_D},
\]

\[
F_y = \frac{C_D}{h} |\bar{u}| + \frac{1}{\tau_D},
\]

\[
\alpha_e = C_o |\bar{u}|,
\]

\[
\alpha_m = \tilde{M}_d + \tilde{M}_c,
\]

\[
\alpha_o = C_o (\theta_{eb}' - \tilde{\theta}_{eb}) \text{ sgn}(\bar{u}),
\]

\[
\lambda = \tilde{\theta}_{eb}' - \tilde{\theta}_{em},
\]

\[
\tilde{N} = \frac{N^2}{g} \theta_0,
\]

and

\[
\frac{\hat{D}}{Dt} = \frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x}.
\]
Because of the switch conditions involved in cumulus parametrizations, a formal linearization of the equations for the cumulus updraught is not possible. In order to simplify the problem, we replace all the variables in the equations for the cumulus updraught by the perturbation values. In the MC case, we simply set

$$M_c' = w'. \quad (42)$$

This assumption replaces a 'positive-only heating' property in a fully nonlinear scheme (cf. Lau and Peng 1987) by the wave–CISK closure (cf. Lindzen 1974). The condition of convective neutrality, $B = 0$, (cf. Eq. 15) in SE is replaced by the condition of the overall convective neutrality,

$$B' = 0, \quad (43)$$

as assumed in YE91. Similarly, Eqs. (17), (18) and (20) for LPA, CLC, and BEE schemes are linearized to

$$\frac{\partial M_c'}{\partial \tau} = \frac{\partial M_c'}{\partial \tau} = C_p \Gamma_m \frac{B'}{\theta_0}, \quad (44)$$

$$\frac{\partial M_c'}{\partial \tau} = C_p \Gamma_m \frac{B'}{\theta_0}, \quad (45)$$

$$\frac{\partial M_c'}{\partial \tau} = \frac{M_c' - M_c^{(e)'} }{\tau_b}, \quad (46)$$

respectively, where

$$M_c^{(e)'} \equiv \epsilon_p \left[ \frac{w' - \alpha_c \theta_{eb}'}{\lambda} \right]. \quad (46)$$

Finally, the linear relation Eq. (16), or

$$M'_d = \frac{1 - \epsilon_c}{\epsilon_p} M'_c$$

is also assumed for CLC, as well as for SE, LPA and BEE schemes. We examine the stability of a propagating mode $\sim \exp[\text{i}k(x - ct) + \text{i}ly + \sigma \tau]$ along the $x$-axis with a phase speed $c$ and a growth rate $\sigma$. We refer to the positive $x$-direction as eastward in the following text. We present a particular case with $l = 0$, because it gives a maximum growth rate for a given $k$, but the result is not sensitive to the choice of $l$. Also the result with $l = 0$ is identical to the analysis of Kelvin-like modes on an equatorial $\beta$–plane by YE91.

In Fig. 2 we show the result for five cumulus parametrization schemes with a mean wind $\tilde{u} = -2$ m s$^{-1}$ and assuming other standard values. The upper panel shows the growth rate $\sigma$ (day$^{-1}$) and the lower panel the phase speed $c$ (m s$^{-1}$) for the fastest growing mode as a function of the wavelength $2\pi / k$, which is normalized by the equatorial circumference (40 000 km).

The growth rate of MC (chain-dash curve) drastically increases at smaller scales, as in other wave–CISK stability analyses. With SE (solid curve), the growth-rate curve flattens at smaller scales due to a suppression of thermodynamic instabilities by downdraughts (cf. YE91). A precise scale selection of linear modes does not exist for exact SE, although the grid-scale singularity of MC is avoided. As shown by Emanuel (1993), an introduction of the effect of finite time-scales on cumulus growth brings a maximum growth rate at a finite horizontal scale ($\sim 10^3$ km: shown by the long-dashed curve for LPA, and the
short-dashed curve for CLC). Finally, BEE (double-chain-dash) shows instability only at the larger scales (>3 x 10^3 km), implying that any meso-β disturbance (~10^2 km) will be damped. All the growing modes are expected to propagate upwind, but MC has a much slower phase velocity.

The LA schemes recover the SE results in the limit of $\tau_b \to 0$ and $\sigma_c \to 1$. These two limits are physically equivalent, because a larger fractional area $\sigma_c$ makes the convective adjustment more efficient. On the other hand, in the opposite limit (i.e. an increase of $\tau_b$ and a decrease of $\sigma_c$), the maximum growth rate decreases with the selection of larger preferred scales. In particular, with $\tau_b > 1$ day for BEE no disturbance grows with time. Physically the boundary-layer process is decoupled from the free atmosphere in this limit, and no adjustment counteracts the build-up of convective instability.

We show the phase structure of the fastest-growing linear modes with a wavelength $2\pi/k = 1000$ km for MC and SE in Figs. 3 and 4, respectively. Note that the phase relation between the variables depends only on the complex frequency $kc + i\sigma$ in Eqs. (26) to (33). Because the complex frequency of the two LA schemes (LPA and CLC) is close to that of SE for $2\pi/k = 1000$ km, very similar phase relations are obtained for SE, LPA and CLC at this scale. We assume a zonal wind structure $u' = \sin(kx)$, and plot the structure of each
variable as a function of $kx/2\pi$. The amplitudes of $\theta'$ and $u'$ are normalized to unity in frames (b) and (c), respectively, of Figs. 3 and 4.

In the MC case (Fig. 3), due to the absence of subsidence $\tilde{M}'$ (long-dashed curve in (c)), the wind-induced surface flux produces a strong anomaly of boundary-layer moist temperature $\theta'_s$ (the long-dashed curve in (b)). This is illustrated by dividing the actual value by 10 (compare with the amplitude of the potential temperature $\theta'$, the solid curve in (b)). As a result, it is in phase with the easterly wind anomaly $u'$ ($<0$; solid curve in (a)) and out of phase with the convective motion $M'_c$. The latter agrees with the large-scale vertical motion $w'$ (solid curve in (c)) by assumption. The phase of $\theta'$ lags the wind by about $\pi/2$, as a result of convective heating. Therefore, its positive correlation with cumulus activity $M'_c$ converts convective available potential energy (CAPE) into the large-scale motion. This positive correlation increases as the wavelength decreases, because the lag of $\theta'$ to $u'$ is insensitive to the wavelength, and, by mass continuity, $M'_c$ ($= w'$) becomes larger for a fixed eddy wind amplitude. Hence, a larger growth rate occurs at smaller scales as shown in Fig. 2.

With SE (Fig. 4), the boundary-layer moisture $\theta'_s$ (long-dashed curve, in (b)) slightly lags an easterly wind anomaly $-u'$. The temperature $\theta'$ (solid curve in (b)) is exactly in
phase with $\theta'_e$ through the convective neutrality assumption (Eq. (15)). The total subsidence (convective downdraught + environmental subsidence, $-M'_d + \tilde{M}'$: chain-dashed curve in (c)) is also in phase with these thermodynamic variables to maintain convective neutrality. Both the amplitude and the phase of cumulus updraught, $M'_c$, relative to $w'$ is a prerequisite for this downdraught profile. The resulting $M'_c$ (short-dashed curve in (c)) is of slightly smaller amplitude than $w'$, which it slightly leads (solid curve in (c)).

The cumulus activity is almost in phase with the low-level convergence, because only a weak subsidence is sufficient to maintain convective neutrality. This phase difference between the cumulus updraught $M'_c$ and the large-scale vertical velocity $w$, further decreases with a decrease of wavelength, because $w$ increases through mass continuity for a fixed eddy wind amplitude. As a result, the phase lag of the environmental subsidence, $\tilde{M}'$, to $w'$ also decreases. Furthermore, because $\theta$ tends to be $\pi/2$ ahead of $\tilde{M}'$ by Eqs. (4) and (5), the phase lag of both $\theta'$ and $\theta'_e$ to the easterly-wind anomaly $-u'$ decreases for the smaller scales. This phase matching between $\theta'_e$ and $-u'$ increases the instability due to the wind-induced surface heat exchange. This increase of instability is exactly counterbalanced by the downdraught stabilization, so that a flattening of the growth rate is obtained with SE
with a finite downdraught effect (cf. YE91). Once finite time-scales for cumulus growth are taken into account as in the LA schemes (LPA and CLC), the anomaly of both the CAPE, or buoyancy $B'$, and $\theta'_o$ increases with a decrease of scale. This results in stronger downdraughts. This is a simple demonstration of the moist convective damping (MCD) proposed by Emanuel et al. (1994), which is that cumulus convection per se has a damping effect on large-scale disturbances in the tropics.

5. **Zero-dimensional (single-column) model**

The single-column model is a way to analyse the response of a parametrized cumulus to large-scale vertical motion. Furthermore, it can be used to verify a parametrization against observed heating and moisture source by using the observed large-scale vertical motion (e.g. Betts and Miller 1986). This model reduces to the zero spatial dimension in the present formulation because of the minimal vertical resolution. As a major difference from the conventional vertical-column model, the large-scale vertical velocity is consistently computed by combining the thermodynamic equation with the momentum equation and assuming a fixed horizontal scale. A similar idea has been applied to a full primitive-equation system by B. E. Mapes (personal communication).

The zero-dimensional system is derived as follows. We linearize the momentum equation (1), neglecting the Rayleigh wind-forcing term:

$$\frac{\partial}{\partial t} v_H = \epsilon_{th} \nabla \theta,$$  

(47)

where $\epsilon_{th} = C_p \gamma \epsilon$. By applying the horizontal divergence operator on Eq. (47) with the help of the continuity equation (3), we obtain

$$\frac{\partial}{\partial t} w = -\epsilon_{th} H_m \Delta_H \theta,$$  

(48)

where $\Delta_H$ is the horizontal Laplacian. A combination of Eq. (48) with a linearized dry thermodynamic equation (cf. Eq. (4)):

$$\frac{\partial}{\partial t} \theta = -\frac{N^2}{g} \theta_0 w,$$

leads to gravity-wave dynamics with phase velocity $c_g = N(\epsilon_{th} \theta_0 H_m / g)^{1/2} \approx 50 \text{ m s}^{-1}$, and a dispersion relation:

$$\omega = c_g K,$$  

(49)

where $\omega$ is a frequency, and $K$ is a total wavenumber. (Note that a longitudinally propagating wave with longitudinal and latitudinal wavenumbers $k_x$, $k_y$ has a phase speed $c_p = (K / k_x) c_g$, where $K^2 = k_x^2 + k_y^2$.)

Consequently, in the dry limit, the response of the system to an initial horizontal convergence would be a standing gravity-wave oscillation with a period $T = 2\pi / K c_g$. Based on this observation, we replace the Laplacian by a fixed total wavenumber, i.e. $\Delta_H = -K^2$ to remove a spatial dependence in Eq. (48), so that

$$\frac{\partial}{\partial t} w = \epsilon_{th} K^2 H_m \theta.$$  

(50)
Finally, we replace the total time derivatives $D/Dt$ in the thermodynamic equations (4), (7) and (8) by

$$\frac{D}{Dt} \simeq \frac{\partial}{\partial t} - \frac{w}{H_m},$$

which invokes the continuity equation (3). Since all the vertical-transport terms in the thermodynamic equations are defined in terms of local variables, this reduction enables us to compute the evolution of the system totally in terms of local variables. With this formulation, we examine the evolution of the system at the centre of convergence. We retain the wind speed $|v_H|$ in the surface flux term $E$ (Eq. 9) as a constant parameter, which gradually destabilizes the system.

For demonstration, we set the parameters: $|v_H| = 0.2\ m\ s^{-1}$, $K^{-1} = 65\ km$, $\theta_{eb}^* = 12\ K$. We turn off the constant radiative cooling $Q_{R0} = 0$ in order to simplify the physics. The initial conditions are $\theta = \theta_{eb} = 0$, $\theta_{ea} = -1.2\ K$. A very weak negative anomaly is assumed for $\theta_{ea}$, because otherwise with this simple configuration the environmental downdraughts strongly stabilize the atmosphere. We also set $w = -7.7 \times 10^{-2}\ m\ s^{-1}$, so that the system is initiated from weak divergence, which turns into convergence with a period of the gravity-wave oscillation. The results are shown in Figs. 5–9.

In the MC case cumulus convection does not occur until low-level convergence starts, even when convective instability already exists. In our particular experiment (Fig. 5), the initial cumulus (d) dies two hours after onset as the large-scale motion turns from convergence (solid curve in (c): $w > 0$) to divergence, even though the environment remains convectively unstable (b). Once the large-scale vertical motion coincides with a buoyancy anomaly the moisture convergence and the cumulus motion enhance each other to grow without limit. This is the simplest demonstration of a grid-point catastrophe inherent in MC (Kuo-type schemes), although in more realistic situations this catastrophe will be regulated by a depletion of moisture.

When an instantaneous adjustment of cumulus to the convective neutrality is assumed, as in the SE scheme (Fig. 6), the system is stabilized after the several cumulus episodes; cumulus convection works as a stabilizer on the large-scale environment in contrast to the MC case.

The two LA schemes (LPA, Fig. 7; BEE, Fig. 8) behave similarly albeit more smoothly than the SE case: parametrized cumulus starts to grow after several gravity-wave periods. The cumulus updraught $M_c$ reaches a maximum in $\sim 10$ hours and then gradually decays. In both cases the boundary-layer potential temperature $\theta_{eb}$ tends to follow a similar curve as the cumulus updraught, and the buoyancy $B$ gradually approaches neutrality. $\theta_{ea}$ approaches a quasi-steady state more rapidly with the LPA scheme than with BEE; this is a major difference between the two.

With CLC (Fig. 9), parametrized convection is characterized by a single cumulus event, and a subsequent decaying mesoscale convective downdraught which stabilizes the atmosphere. This crudely mimics a typical life cycle of a tropical convective system as a grid-scale process. Since such a life cycle is independently experienced at each grid column, we expect that the generated convection field contains grid-scale structure, which turns out to be the case (Yano et al. 1995, 1996).

6. Comparison with GCM results

The behaviour of the several different cumulus parametrizations was investigated by the linear stability analysis and the zero-dimensional (single-column) model using the shallow-water atmosphere model. Each parametrization is a simple form of a major category of existing thermodynamic cumulus parametrizations used in GCMs. Fundamentally
different behaviour was demonstrated with different parametrizations in our present analyses, as well as in our earlier numerical modelling experiments (Yano et al. 1995, 1996). In this section we attempt a generalization of our results to qualitatively compare them with reported GCM results.

Because the SE scheme predicts a constant growth rate for the disturbance with horizontal scales less than ~1000 km, we infer that schemes based on instantaneous adjustment by cumulus convection, such as the MCA schemes (e.g. Manabe et al. 1965), are least likely to generate large-scale convective coherence in GCMs. The large-scale disturbance generated using this type of parametrization will result in white noise; convective adjustment will be achieved at each grid column without forming any coherency with neighbouring
grid columns (cf. Yano et al. 1995, 1996). For example, the MCA scheme produces a noisy precipitation distribution due to such an instantaneous grid-scale adjustment in the NCAR* CCM2 (J. J. Hack, personal communication). We anticipate that a relatively strong horizontal diffusion will smooth such a noisy field into a diffusive non-propagating feature. For example, Numaguti and Hayashi (1991a, b) found such behaviour in their aqua-planet GCM experiment.

The schemes based on the low-level MC, such as the Kuo scheme, produce disturbances whose fastest growth is at the smallest scales, which results in a singular grid-scale disturbance. It does, however, tend to produce coherence with a surrounding large-scale circulation, presumably due to its explicit coupling to low-level convergence. Due to this capacity to produce coherence, the Kuo scheme and wave–CISK have been used to simu-

* The National Center for Atmospheric Research.
late a coherence akin to MJO in GCMs (e.g. Hayashi and Sumi 1986) as well as in simpler models (e.g. Lau and Peng 1987). However, those simulated MJO-like coherencies are always on the grid scale. The chosen grid size artificially generates the elementary scale (1500–3000 km at T21–T42 spectrum truncation) of the MJO (i.e. super-clusters) in most of those idealized experiments. A GCM experiment by Kuma (1994) showed that eastward propagating super-clusters were replaced by smaller-scale westward propagating features when the resolution was increased from T42 to T63. Those westward propagating features also became less defined with a further increase of the resolution to T159. The result is consistent with a shallow-water experiment by YM²E.

Our zero-dimensional experiment with MC shows that the coupling of surface moisture convergence and cumulus convection can lead to a catastrophic self-enhancing feed-
back. The Kuo scheme is known to behave in this way. The scheme cannot produce precipitation in regions having substantial convective instability if there is no moisture convergence. When convection is eventually initiated by moisture convergence the instability is rapidly removed, and erroneously high precipitation rates occur. The self-enhancement problem can be further exacerbated by low vertical resolution.

Our linear analysis of the LA schemes confirms the result of Emanuel (1993), namely that finite time-scales for cumulus growth are crucial for large-scale convective organization based on the quasi-equilibrium hypothesis. With this category of parametrization the overall result is independent of the horizontal resolution. For example, the Betts–Miller scheme is in this category, so that the resulting cloud disturbance will be smooth and lack grid-scale structure, in contrast to the Kuo-type scheme. Slingo et al. (1994) provided a di-
rect test of these two schemes in UGAMP* UGCM; they found that the higher-frequency (less than 10 day period) smaller-scale transient modes were dominant when the Kuo scheme was used. Conversely, the MJO was better represented by the Betts–Miller scheme.

The interpretation of the behaviour of a model containing the Arakawa–Schubert scheme is considerably more complicated. It is reasonable to expect that it follows the characteristic SE behaviour due to its adherence to the SE principle. However, it is also known that the scheme can be reduced to a wave–CISK formulation under certain linearizations (Stark 1976; Crum and Stevens 1983). Therefore, its behaviour can even be akin to MC, for example the Kuo scheme. The issue can be further complicated because

* The UK Universities Global Atmospheric Modelling Programme.
of the various ways the scheme may be implemented (cf. Arakawa and Chen 1993). In particular, both the relaxed Arakawa–Schubert scheme (Moorthi and Suarez 1992) and the prognostic version (Randall and Pan 1993) are more likely to behave like a LA scheme. Finally, the Emanuel (1991) scheme can be considered as a sophisticated version of LA. Unfortunately, little is reported on the behaviour of those two schemes in GCMs, especially as regards their treatment of MJO-like phenomena.

The present results are, in general, consistent with the reported GCM tests and provide simple physical interpretations. However, due to the various free parameters contained in a full GCM parametrization, the behaviour of a particular parametrization in a particular GCM can be substantially modified through interactions with the other physical representations. For example, Chao and Lin (1994) report that the MCA scheme produced a more distinct convective coherence than either that of Emanuel or the relaxed Arakawa–Schubert scheme, contrary to our expectation. The cause of such unusual behaviour needs to be traced back to particular model configurations.

7. CONCLUSIONS

We recasted the thermodynamic convective parametrization problem in a simple way using the shallow-water analogue of large-scale tropical dynamics. Major categories of existing convective parametrizations for GCMs could be recovered within this simplified formulation in order to elucidate their basic characteristics.

Some basic characteristics of the parametrizations were best demonstrated by the zero-dimensional (single-column) analysis. The schemes based on the moisture-convergence closure (MC), such as those of Kuo-type, tend to accumulate the convective instability through a positive feedback between moisture convergence and cumulus convection, which can be catastrophic. The linear stability analysis, furthermore, implies that it can lead to persistent ‘grid-scale storms’, a behaviour that often occurs in both GCMs and regional models. This tendency contradicts the scale-separation principle upon which cumulus parametrization is based. This quantifies the unphysical basis of the Kuo-type scheme as pointed out by Emanuel et al. (1994).

Those schemes based on the convective quasi-equilibrium hypothesis, such as Arakawa–Schubert and moist convective adjustment, remove the convective instability through the action of parametrized convection. In contrast to the Kuo scheme, no additional feedback leading to a grid-scale catastrophe occurs with this parametrization category. However, the linear stability analysis implies that schemes which strictly apply the quasi-equilibrium principle (i.e. the instantaneous adjustment) are incapable of generating large-scale convective coherence akin to the MJO. Ironically the Kuo-type schemes can produce more realistic MJO behaviour, but only in a surrogate sense because the grid length has to be appropriately chosen, as demonstrated by Numaguti and Hayashi (1991a, b) and others. Short time-scales for cumulus growth are required to produce large-scale convective coherence based on the quasi-equilibrium principle. A smooth large-scale coherence is obtained if the cumulus scheme contains convective downdraughts (Lagrangian parcel-acceleration scheme, or LPA), and is consistent with the scale-separation principle.

On the other hand, if an exponential decay of mesoscale convective downdraughts that commonly occur in association with deep convection, especially organized precipitating systems, is included as in our new parametrization (convective life-cycle scheme, or CLC), then grid-scale substructures occur within the large-scale convective coherence (Yano et al. 1995, 1996). This was inferred from our zero-dimensional experiment. Additional experiments imply that the Galilean invariance assumed in LPA is less important in this type of coherence. Although the inclusion of the mesoscale downdraught violates the
scale-separation principle upon which present parametrization techniques rely, it has a more robust physical basis than MC. This is because the overall structure of the large-scale convective coherence is invariant to a change of grid length.

Although classical approaches, which are based on the scale-separation principle, insist that a cumulus parametrization must not represent grid-scale processes, this is at odds with both observations and theoretical analysis. For example, Moncrieff (1995) raised several issues involving the large-scale role of mesoscale convective processes that cast doubt on the physical robustness of the scale-separation principle. Other doubts are revealed, albeit more abstractly, by fractality in the horizontal pattern of the cloud system (Yano and Takeuchi, 1987; Yano et al. 1996). In particular, Moncrieff and Klinker (1997) showed that superclusters occurring during westerly-wind bursts in the tropical western Pacific were partly resolved in a T213 global weather forecasting model. This clearly violated the scale-separation principle, arising through the explicit treatment of large tropical cloud clusters, and caused substantial errors in the large-scale fields. The CLC scheme embarks on a new approach that could potentially address this type of problem because it treats mesoscale convective downdraughts in a grid-scale way. The re-normalization principle proposed by Yano et al. (1996) is an alternative basis for this new approach.

Although our simple model analysis illustrates the various characteristics of existing cumulus parametrizations, we recognize its limitations. For example, due to its anomaly description of the thermodynamic effects, it cannot consider the vertical structure of the heat and moisture sources, or the mean climate state. Both are important aspects of the cumulus parametrization problem that are left to a more sophisticated analysis.

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CUMULUS PARAMETRIZATION CATEGORIES


