The frequency spectrum of mountain waves

By R. M. WORTHINGTON* and L. THOMAS

University of Wales, UK

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Summary

Mountain waves are often assumed to remain steady with respect to time, but measurements of their vertical wind velocity by radar are known to show a clear time dependence. Specifically, the vertical wind $W$ obeys a power law, where the spectral density varies as $\omega^{-5/2}$ over at least three orders of magnitude, and $\omega$ is the frequency. Power laws in the frequency and wave-number spectra of parameters such as horizontal wind have been widely studied, but the frequency spectrum of mountain waves remains, by comparison, poorly understood. It is found that errors arising from the VHF radar measurement technique are not sufficient to explain the power law, and a previous model invoking vertical modes or two-dimensional turbulence does not, on its own, explain entirely the $W$ data. However, even a surprisingly simple model, in which the mountain-wave pattern drifts upwind or downwind in a random walk, gives a similar time dependence to the observed $W$ data. The radar data are contrasted with other observations and models, where $W$ time variations have been interpreted in terms of lee-wave trains with non-zero phase speed; the relationship to other systems showing time-scale invariant $1/f$ behaviour, such as $1/f$ noise, is also discussed.

Keywords: Frequency spectra Mountain lee waves MST radar

1. Introduction

Mountain waves have traditionally been considered to be time-independent. A mountain-wave pattern is often assumed to remain unchanged for several hours—over, for example, the duration of measurement campaigns by aircraft (e.g. Shutts and Broad 1993), and the horizontal phase speed is taken to be zero, measured in a ground-based reference frame. However, some recent observations (e.g. Scorer and Verkaik 1989; Bougeault et al. 1993; Adams 1996; Liziola and Balsley 1997) have suggested that mountain waves sometimes propagate horizontally, with non-zero phase speeds as measured from the ground. Attempts to model numerically the time dependence of mountain waves, in response to simple step-like or sinusoidal changes of background wind speed, have begun (e.g. Lott and Teitelbaum 1993a; Hines 1995; Nance and Durrant 1997, 1998); for example, Nance and Durrant (1997, 1998) showed that mountain lee waves propagate upwind or downwind when the background wind speed changes hydrostatically from one value to another. Shutts et al. (1994) and Vosper and Mobbs (1996) treated the horizontal phase speed as a variable term in estimating mountain-wave parameters from the ascent rates of balloons. They found small, non-zero phase speeds, although these were of similar magnitude to the associated standard errors.

A non-zero phase speed for mountain waves will affect both their vertical propagation and momentum flux, and will thereby alter the height distribution of wave drag, which must be simulated in numerical weather prediction and general-circulation models (e.g. Broad 1996). Both wave trapping and critical-layer absorption depend on $c$, the horizontal phase speed observed from the ground. Trapping occurs when the Scorer parameter, given by (e.g. Shutts et al. 1994)

$$\kappa^2(z) \approx \frac{N^2}{(c - U)^2} + \frac{1}{c - U} \frac{d^2U}{dz^2}$$

falls to a smaller value than the horizontal wave number; $N$ and $U$ are, respectively, the Brunt–Väisälä frequency and the speed of the background wind component in the direction

* Corresponding author: Department of Physics, University of Wales, Aberystwyth, Dyfed SY23 3BZ, UK.
of the horizontal wave vector and $z$ is the height. A critical layer occurs when $U$ equals $c$. Lott and Teitelbaum (1993a,b) showed that non-stationary mountain waves can propagate through regions where the background wind passes through zero, and any critical-layer absorption would occur at a higher or lower altitude depending on the sign of $c$. Mountain-wave observations by Worthington and Thomas (1996a) showed vertical propagation that stopped at critical layers where $U$ was approximately zero; however, any estimate of $c$, based on the value of $U$ at the critical layer, would have uncertainties of a few m s$^{-1}$, implying that $c \neq 0$ cannot be ruled out. As suggested by Eq. (1), if $c \neq 0$ then, for a given profile of $\ell^2$, the degree of trapping will be altered from the simple case of stationary waves —affecting the vertical propagation of partially trapped lee waves, the horizontal extent of the wave trains, and the eventual locations of momentum deposition. An estimation of the range of phase speeds associated with mountain waves would, therefore, be useful.

Previous measurements of vertical wind, $W$, during mountain-wave events, using ground-based radars (e.g. Ecklund et al. (1986), and references therein) do not show constant $W$, owing to an unmoving mountain wave above the radar site, nor simple oscillations of $W$ caused by lee waves with $c \neq 0$ propagating above the radar, the basic situation modelled by Nance and Durran (1997). Instead, the frequency spectrum of $W$ shows a power-law behaviour (Ecklund et al. 1986), with a power spectral density $S_W(\omega)$ varying typically as $\omega^{-5/3}$, where $\omega$ is the frequency, during mountain-wave events, compared with a $\omega^0$ dependence during quiet days (Fig. 1). The frequency spectrum of $W$ changes from $\omega^{-5/3}$ to $\omega^0$ over a height interval of just a few km, from below to above a critical layer (Worthington and Thomas 1996a). The $S_W(\omega) \sim \omega^{-5/3}$ spectrum has been interpreted as an interaction between mountain waves and pre-existing horizontal vortical modes (Ecklund et al. 1986; Gage and Nastrom 1990). However, the $W$ variations might,
alternatively, be associated with the time-varying mountain waves themselves (Sato 1990; Worthington and Thomas 1996a).

The present study uses $W$ measurements by MST (mesosphere, stratosphere, troposphere) radar, with fine height and time resolutions, to examine more closely the time dependence of mountain waves. Some previous studies of this time dependence (e.g. Caccia et al. 1997; Nance and Durran 1997, 1998) considered trapped lee waves downwind of mountains, but MST radar data can be used to examine both trapped and untrapped mountain waves. Some possible causes of the $W$ spectrum are examined in sections 3 and 4. In the long term, computer models of mountain-wave events can be run using a forcing background wind where the spectral density $S_{W}(\omega) \sim \omega^{-5/3}$ as is observed (VanZandt 1982), and the resulting time variations of $W$ can be investigated. However, some recent studies in other fields of research have suggested that power-law behaviour of some complex nonlinear systems can be reproduced by very simple models, and this possibility is also investigated in section 5.

2. Results

Data are supplied by the Natural Environmental Research Council MST radar facility at Aberystwyth, Wales. The frequency and peak transmitted power of the radar system are 46.5 MHz and 160 kW, respectively; a more complete description of the operating parameters is given by Slater et al. (1992). Pulses of 8 $\mu$s, coded with a baud length of 2 $\mu$s and sampled at 1 $\mu$s intervals, are used to provide a vertical resolution of 300 m. The data run from 1750 GMT 26 June 1995 to 0910 GMT 27 June 1995. Easterly or northerly winds generate the largest-amplitude mountain waves at Aberystwyth (Prichard et al. 1995), and these data are part of a mountain-wave event with strong easterly winds, which lasted for a week during June 1995. Radar operation was interrupted by mesospheric data gathering in daylight hours, so the data in Fig. 2 last for only 15 hours. However, this data, and the conclusions drawn, are typical of the rest of the event, and of other mountain-wave events; it is used here since the vertical wind field is quiet except for persistent, large-amplitude mountain waves occupying a large height range. The time resolution changes, unfortunately, at 0550 GMT 27 June—from 2.4 min (with $W$ measured every 1.2 min) to 4.0 min, so the later data are excluded from the frequency analyses described in sections 2(b) and 3. Figure 2 shows that the low-level easterly wind of 5–10 m s$^{-1}$ generates mountain waves with $W$ in excess of $\pm 0.5$ m s$^{-1}$, and it is found that $W$ briefly reaches $-3$ m s$^{-1}$ near hour 30. Data from a radiosonde, launched from Aberporth 50 km away, at hour 29, indicate that the tropopause is near 12 km, slightly below the height of 13–14 km at which the mountain waves end their vertical propagation, apparently stopped by critical-layer absorption as the wind speed falls below $\sim 5$ m s$^{-1}$. For much of the data set, clear oscillations of $W$ with height are seen, except between hours 28 and 31 when the mountain waves may be partly trapped.

(a) Consistency with previous studies

The 26–27 June 1995 data are briefly checked for consistency with two types of presentation used in previous studies of mountain waves. Figure 3(a) shows profiles of $W$, averaged over intervals of two hours; the detailed time variations apparent in Fig. 2(b) are partly averaged out, and the resulting profiles clearly show oscillations with height, which are similar to those presented previously by Ralph et al. (1992) and Prichard et al. (1995). In Fig. 3(b), $W$ is time-filtered to retain wave periods of 0.5–3.5 hours, the same processing as used by Bougeault et al. (1993). In their Fig. 13, they bandpass filtered the time series of $W$, identifying the discarded frequency components, with periods longer
Figure 2. Height–time plots of (a) horizontal wind vectors and (b) contours of vertical wind velocity, $W$, measured using the vertical radar beam, for the data set 26–27 June 1995. Positive $W$ indicates upward air movement.

than 3.5 h or shorter than 0.5 h, as ‘slow variations’ and ‘noise’, respectively. The remaining signal, interpreted as a ‘two-hour wave’, with horizontal wavelength $\lambda_h \approx 10$ km estimated from aircraft data, was thereby used to calculate a horizontal phase speed of 1.3 m s$^{-1}$. Figure 3(b) appears very similar to Fig. 13 of Bougeault et al. (1993); however, the $W$ time series from Fig. 2(b) are shown later, in section 2(b), to consist of a power-law frequency spectrum rather than a monochromatic wave. A monochromatic-wave component which visually dominates a time series can appear as an inconspicuous peak in a frequency spectrum which obeys a power law (e.g. Russ 1994); however, an apparent ‘two-hour wave’, as in Fig. 3(b), can be created artificially from just the narrow bandpass filtering of any time series containing harmonics with periods $\sim 2$ hr. In conclusion, Figs. 2 and 3 show no inconsistencies with the radar $W$ measurements attributed to mountain waves in previous studies.

(b) Error sources in $W$ frequency spectra

Before considering an atmospheric origin for the $S_W(\omega) \sim \omega^{-5/3}$ spectra, possible instrumental errors must be discounted. The vertical radar beam could, for example, be
measuring a component of the horizontal wind spectrum, due to tilted aspect-sensitive layers (e.g. Larsen and Röttger 1991; Muschinski 1996). Radar data can be checked for internal consistency using both the vertical beam and symmetric pairs of off-vertical beams to estimate $W$. Using beams at $6^\circ$ from the vertical, the estimates of $W$ from NE6°–SW6° and NW6°–SE6° beam pairs, identified as $W_{6a}$ and $W_{6b}$ respectively, are given by

$$W_{6a} = \frac{v_{\text{NE6}} + v_{\text{SW6}}}{2 \cos 6^\circ}, \quad W_{6b} = \frac{v_{\text{NW6}} + v_{\text{SE6}}}{2 \cos 6^\circ},$$

where $v$ is the line-of-sight velocity in a particular beam. The vertical-beam measurement is identified as $W_0$.

The mean frequency spectra of $W_0$, $W_{6a}$ and $W_{6b}$, each averaged over the height range 2–10 km, are plotted in Fig. 4; note that for clarity the $W_{6a}$ and $W_{6b}$ spectra are displaced vertically. Both the $\omega$ and $S_W(\omega)$ axes use logarithmic scales, so that the spectral index $\beta$, where $S_W(\omega) \sim \omega^\beta$, is given by the gradient of the plotted lines. All three spectra show $\beta$ to be near, or slightly more negative than $-5/3$, over approximately two decades, and down to periods as short as 20 min or less. Above 14 km, the spectra (not shown) are flat, with a peak at the Brunt–Väisälä frequency and decreasing $S_W(\omega)$ at higher frequencies, as already described by Worthington and Thomas (1996a)—similar to the ‘quiet days’ spectra in Fig. 1. The correlation coefficient between pairs of time series ($W_0$ and $W_{6a}$, $W_0$ and $W_{6b}$, $W_{6a}$ and $W_{6b}$), at individual height levels between 2 and 10 km, is better than 0.95 up to heights near 6 km, falling to 0.85 at 10 km. Using cross-spectral analysis in the same height range, the coherence values are better than 0.9 for periods as short as 30 min.
This agreement between the $W_{6a}$ and $W_{6b}$ spectra, and also between the original $W_{6a}$ and $W_{6b}$ time series, suggests that the spatial separation of off-vertical radar beams measuring a mountain-wave pattern, which can show a dependence on azimuth (McAfee et al. 1989), is not a significant factor in the $\omega^{-5/3}$ power law. Also, the agreement between the spectra of $W_0$ and $W_{6a}$ (and also $W_0$ and $W_{6b}$) suggests that aspect-sensitivity effects, arising perhaps from tilted scattering layers (e.g. Worthington and Thomas 1996b), are not an important factor either in measuring the large $W$ due to mountain waves in this data set, since these effects should generally be weaker using beams at 6° zenith angle. McAfee et al. (1994) compare time series of $W$ measured by both VHF and UHF wind profilers at the same site, finding very good agreement at time-scales of 1 min. Since UHF echoes are not affected by aspect sensitivity, the influence of tilted aspect-sensitive layers on the VHF vertical-beam measurements is not significant in their data. McAfee et al. (1995) show further profiles of $W$, averaged for 1 hr, again finding a generally good agreement. VHF aspect sensitivity at a zenith angle of 12° is less significant than at 6° (Hooper and Thomas 1995), but agreement is again found between the $W_0$ spectra and results from a 12° beam pair, using another mountain-wave data set where 12° data are available (Worthington, 1996). This agreement between $W$ frequency spectra of mountain waves in the troposphere, measured by different combinations of radar beams, suggests that the apparent $S_w(\omega) \sim \omega^{-5/3}$ spectrum represents the genuine time dependence of the vertical wind.
3. MOUNTAIN WAVES AND VORTICAL MODES

Although instrumental effects such as aspect sensitivity appear to have been ruled out in section 2(b), at least in the troposphere for this data set, the actual origin of the $W$ frequency spectrum remains unclear. The measured vertical velocities could result from mountain waves alone, or, according to Gage and Nastrom (1990), contain a component of a horizontal vortical-mode (two-dimensional turbulence) spectrum due to tilting of the airflow from the horizontal. Sato (1990) states that deviations of the $W$ spectra from a straight line with slope $-5/3$ imply that the vortical-mode model is not appropriate. However, Sato’s data have since been reinterpreted by Yoe and Rüster (1992), who suggest that several cases may instead represent the vertical air circulation related to the jet stream. The results of Yoe and Rüster (1992) have since been reinterpreted further by Muschinski (1996) in terms of a corrupted vertical-beam measurement caused by Kelvin–Helmholtz instabilities.

If the vortical-mode contribution were to dominate the frequency spectrum of $W$ (Gage and Nastrom 1990) then, on examining any range of frequencies, the corresponding vertical-wave-number spectrum must also be consistent with this model. During mountain-wave activity, the streamlines of the wind flow are being tilted, i.e.

$$W = U \frac{\partial \psi}{\partial x},$$  \hspace{1cm} (3)

where $\partial \psi/\partial x$ is the gradient of the streamlines; the phase of $\partial \psi/\partial x$, and hence of $W$, oscillates with height. The time-varying component of the horizontal wind, $U_{var}$, might also be tilted from the horizontal (Gage and Nastrom 1990), to remain parallel with the streamlines in mountain-wave conditions, and the vertical beam would then measure a component of $U_{var}$, i.e.

$$W = (U_{mean} + U_{var}) \frac{\partial \psi}{\partial x},$$ \hspace{1cm} (4)

where $U_{mean}$ is the mean horizontal wind speed, and $U = U_{mean} + U_{var}$. In the model of Gage and Nastrom (1990), the time dependence of $U_{var}$ is entirely responsible for the $-5/3$ spectral index; the tilt $\partial \psi/\partial x$ is assumed to remain constant with time. Neither vortical motions nor the ‘universal’ spectrum of gravity waves (VanZandt 1982), if they contribute to $U_{var}$, are expected to give clear oscillations of $W$ with height as observed at the longest periods in Figs. 2(b) and 3(a)—such measurements of $W$ are usually interpreted as the vertical component of mountain-wave velocity perturbations (e.g. Ralph et al. 1992; Bougeault et al. 1993; Prichard et al. 1995; Caccia et al. 1997; Ralph et al. 1997). If oscillations of $W$ with height are observed at all frequencies, then the vortical-mode model of Gage and Nastrom (1990), without height structure, is by itself insufficient to explain the radar data—the time-dependent mountain-wave pattern must be implicated in the $-5/3$ spectral index.

Using time filters, the higher-frequency components of $W$ and their associated vertical structure can be investigated. The case-study shown in Fig. 2 is useful for this purpose, since the wind speed is fairly low; the mountain waves therefore have a vertical wavelength short enough to show a whole cycle within the height range of mountain-wave activity. Vertical-wave-number spectra of $W$ can be used to examine the vertical structure; however, with one wavelength in the available height range, the presence of mountain waves tends only to steepen the low-wave-number end of the background vertical-wave-number spectrum without giving a distinct spectral peak. Instead, Fig. 5 shows the vertical structure of $W$ in a height-time plot, with the region above 14 km—where the mountain waves have been removed by a critical layer—acting as a reference for comparison with the lower regions of
Figure 5. Height–time contour plots of W measured using the vertical radar beam: (a) without any filtering, (b) retaining periods longer than 6 hours, (c) retaining periods shorter than 6 hours, (d) retaining periods shorter than 3 hours, (e) retaining periods shorter than 90 min and (f) retaining periods shorter than 45 min. The highest frequencies are smoothed out in the plotted contours.
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mountain-wave activity. Figs. 5(b)–(f) show the height-time variations of \( W \) after applying, in turn, a series of different time filters to the entire height region, both below and above 14 km. Despite the filter cut-offs of progressively shorter period, the measurements of \( W \) still show oscillations with height. These oscillations, featuring a vertical wavelength of several km, are observed for periods as short as 90 min until, finally, with the 45 min filtered data, the regions below and above the critical layer are becoming indistinguishable.

Numerical modelling of two-dimensional turbulence shows that the horizontal-wind structure can consist of multiple horizontal layers, each moving in an approximately two-dimensional way, and independently of other layers above and below (Vallis et al. 1997). However, the long-period component of zonal wind does not change sign in the troposphere in Fig. 2(a), so the oscillations with height of the long-period component of \( W \) (Figs. 2(b), 3(a) and 5(b)) must depend on the changing tilt \( \partial \psi / \partial x \) of the mountain-wave streamlines (Eq. (3)); even after time-filtering as in Fig. 5, the shorter-period components of horizontal wind show no clear oscillatory structure with height.

Although the oscillations of \( W \) with height in Fig. 5 suggest that \( U_{\text{var}} \) does not dominate the vertical-beam measurements, \( U_{\text{var}} \) does introduce a separate time dependence to \( \partial \psi / \partial x \), and hence to \( W \)—the phase of \( W \) changes with time as the position of the mountain-wave pattern varies above the radar (e.g. Caccia et al. 1997; Ralph et al. 1997). Such changes do not even require a variation of wave phase near ground level, or of the horizontal wavelength; any change in background wind speed, for example, which alters the vertical wavelength, will affect the phase of \( W \) at greater heights. It is not clear at present whether this time dependence can give a \( S_w \sim \omega^{-5/3} \) power law.

4. Influences on the Time Dependence of \( W \)

Measurements of the horizontal structure of mountain waves from aircraft (e.g. Cruette 1976; Smith 1976; Brown 1983) often show oscillations of \( W \) with horizontal distance, consistent with mountain-wave theory and quasi-monochromatic. Yet, despite the clear characteristic horizontal wavelength, the ground-based frequency spectrum of \( W \) shows time-scale invariant behaviour, \( S_w \sim \omega^{-5/3} \)—without any characteristic period. The average spectra of atmospheric motions are thought to show a universal power-law behaviour, where the spectral index takes specific values; for the horizontal wind, \( S_U \sim \omega^{-5/3}, \ m^{-3}, \ k^{-5/3} \) over certain scales (e.g. VanZandt 1982; Weinstock 1996), where \( m \) and \( k \) are the vertical and horizontal wave numbers, respectively, and for the vertical wind, \( S_W(\omega) \sim \omega^0 \) in quiet atmospheric conditions as shown in Fig. 1 (Ecklund et al. 1986). The frequency spectrum of \( W \) during mountain-wave events might be included in this list of universal power laws, since mountain-wave events all seem to show \( S_w(\omega) \sim \omega^{-5/3} \) (e.g. Ecklund et al. 1986).

The time dependence of the mountain-wave pattern above the radar results from a combination of several factors—the time dependence of the background-wind speed and direction (Sato 1990), both near ground level and at greater heights; the associated changes in the upwind terrain (Prichard et al. 1995); and the temperature and humidity structures (Ralph et al. 1997; Durran and Klemp 1982). The unsteadiness of mountain waves must be attributed to a combination of all these factors. Sato (1990) attributes time variations of \( W \) to the mountain-wave pattern moving relative to the ground (‘spatial phase modulation’) as the background wind changes. However, the temperature and humidity will also have an effect—on the wave amplitude and wavelength, and not just on the phase. Durran and Klemp (1982) showed that small changes in only the relative humidity can have a major effect on the mountain-wave amplitude, wavelength and phase. Owing to the modification of \( N \) and hence \( \ell^2 \) by humidity, trapped waves can become untracked, or the wave activity
can be suppressed. Difficulties arise, therefore, in relating the $\omega^{-5/3}$ spectrum to specific changes in the background atmosphere which can give the measured time series of $W$ their observed form. The $W$ spectrum does not appear to be linked to a gravity-wave or turbulent cascade process, although Reiter and Foltz (1967) suggested that a cascade process could operate from typical lee-wave wavelengths (10 km) down to turbulence. However, those measurements were not identified uniquely as mountain waves, and more recent aircraft data (e.g. Brown 1983) show mountain waves which are quasi-monochromatic with respect to horizontal wavelength. The spectrum of $W$ does not seem to be derived directly from the $\omega^{-5/3}$ horizontal-wind spectrum; Nance and Durran (1998) report time variations of $W$ in a numerical model, due to nonlinear effects, even when the background wind remains steady, and nonlinear effects may be as important as the background-wind variations. The observed $S_w(\omega) \sim \omega^{-5/3}$ spectrum might depend on the mountain-wave response to the frequency spectrum of horizontal wind, or perhaps on less specific, stochastic, mechanisms which are not unique to mountain waves. This present study does not determine which model is most appropriate, but does explore the potential of one simple stochastic model in section 5.

A possible description of the time series of $W$, used for other geophysical time series, could be coloured noise (e.g. Mandelbrot and Wallis 1969a,b). Such a model does seem to be applicable to the time series of $W$ measured by radar—at least when removed from the context of the vertical structure of the mountain waves. Figs. 6(a) and (b) show two time series of $W$, measured at heights of 17.0 km and 5.9 km during the event shown in Fig. 2; Figs. 6(c) and (d) are two artificial coloured-noise time series with $S(\omega) \sim \omega^0$ and $S(\omega) \sim \omega^{-5/3}$, created from a sum of sine waves with random phases, and amplitudes
dependent on the frequency, such that

\[ f(t) = \sum_{\omega=1}^{512} \omega^{5/2} \sin(c \omega t + \phi(\omega)), \]

(5)

where \( \omega \) is the wave frequency (multiplied by 1024), the time \( t \) represents the interval \( 0 \rightarrow 2\pi \) divided into 1024 sections, and \( \phi \) is a random phase for each harmonic (e.g. Owens 1978; Bacmeister et al. 1996). The apparent similarity between the forms of the atmospheric and artificial time series in Fig. 6 suggests that a coloured-noise model might be considered further.

5. A stochastic model of mountain-wave \( W \) time series

The \( \omega^{-5/3} \) spectra in Fig. 4 appear to be a robust result; additional data indicate that this power law is observed for mountain waves generated by both westerly and easterly low-level winds, implying very different upwind terrain, and also for waves with different vertical structures—e.g. varying and nearly constant phase with height (Worthington and Thomas 1996a, 1997). This might suggest that a ‘first-guess’ model does not require the inclusion of details such as the slope of the mountain-wave phase fronts, or the specific forcing topography. Also, no studies currently exist describing the detailed time variations of the phase of mountain waves, relative to the ground, in response to a background-wind speed which itself shows a power-law behaviour. A stochastic model using the simplest and most general assumptions might therefore be considered, although only to explore the future possibilities of a similar approach, and not as a definitive model of \( W \) time-variability. There are recent precedents, in other fields, for the use of relatively simple models to reproduce the time dependence of systems too complex to be modelled in their entirety (e.g. Hausdorff et al. 1995; Turcotte and Teich 1996). For example, a simple numerical model used by Turcotte and Teich (1996), based on coloured noise, was found to be indistinguishable from genuine data, over all time-scales, and under a wide range of statistical tests.

Since the exact mountain-wave phase variations above the radar site are not known, a ‘random-walk’ model is considered, where a mountain-wave pattern is assumed to drift randomly either upwind or downwind with equal probability, and with a step length which is small (approximately 2%) compared with the horizontal wavelength; a random-walk model is not geophysical, but is nevertheless a very simple and commonly used approach. The resulting time variations of \( W \) and the corresponding frequency spectrum are calculated, the different stages being displayed in Figs. 7(a)–(d). The time series of the displacement of a particle undergoing a random walk is known to correspond to coloured noise with a \( \omega^{-2} \) spectrum (e.g. Turcotte 1992), and this is also found for the modelled time series of \( W \), Fig. 7(d), with little dependence on, for example, the initial position of the wave pattern. Russ (1994) describes without proof that the sine of a coloured-noise signal where \( S(\omega) \sim \omega^\beta \) (e.g. Fig. 7(c)) has a Fourier transform with an unchanged spectral index \( \beta \), and only the intercept is altered. The \(-2\) spectral index is slightly more ‘red’ than the observed \( W \) spectra; however, Eckermann (1990) showed that variability of the wave field can cause a mean spectral index to become less negative, e.g. from \( \omega^{-1.7} \) to \( \omega^{-1.4} \). The sporadic occurrence of mountain-wave events means that the \( W \) time series are highly variable over time-scales of days, and \( W \) may also be variable within mountain-wave events, over time-scales of hours, e.g. Fig. 3(a). The high- and low-frequency limits of the spectrum are also flattened due, respectively, to the background-noise level, and to the overall displacement of the mountain-wave pattern becoming comparable with the horizontal wavelength. These effects also make the mean spectral index less negative.
Figure 7. Outline of a simple stochastic model for the time dependence of vertical wind velocity, $W$, during mountain-wave events. (a) Dependence of $W$ on horizontal distance, assuming a monochromatic mountain wave with horizontal wavelength $\lambda_x = 20$ km; (b) distance moved by a mountain-wave pattern drifting randomly upwind or downwind in a one-dimensional random walk; (c) time series of $W$ that would be measured by a ground-based radar for a mountain-wave pattern displaced, as a function of time, as shown in (b) and (d) frequency spectrum of the $W$ time series in plot (c). The two dotted lines indicate slopes of $\omega^{-2/3}$ and $\omega^{-2}$, where $\omega$ is the frequency.

6. DISCUSSION

A mountain-wave pattern can be considered steady when the forcing background wind varies over time-scales as short as $\sim 1$ h—the time for the wave pattern to propagate over one vertical wavelength (Lott and Teitelbaum 1993a,b). This steady wave pattern can, however, also slowly shift above the radar, so that the phase of $W$ changes with time; Ralph et al. (1992) attribute variations of mountain-wave phase, with periods $\sim 1$ h, to a mountain-wave pattern rotating as the background wind changes direction. At ground-based periods less than 1 h, the waves become ‘non-steady’, and perhaps non-deterministic. It is not clear whether the component of $W$ with ground-based periods of 1 h or less can be the same type of wave as the steady long-period component nor, if the high- and low-frequency limits are to represent different and distinct types of wave motion, why they should together obey a $-5/3$ power law. Similar problems occur in other areas of physics—for example, frequency or wave-number spectra which appear to be scale-independent, although the underlying physics is thought to be scale-dependent. For instance, Sayles and Thomas (1978) suggested that the wave-number spectrum of surface roughness obeys a $S(k) \sim k^{-2}$ power law over eight orders of magnitude, even though the processes creating these different scales of surface roughness were presumably very different. Bak et al. (1987) note that “the common feature for all these systems is that the power-law temporal or spatial correlations extend over several decades where ... one might suspect that the physics would vary dramatically”. Power laws are common throughout physics, and a power law between $\omega^{-1}$ and $\omega^{-2}$ (nominally $\omega^{-5/3}$) is not unique to e.g. a gravity-wave or turbulent cascade.
Figure 8. Frequency power spectra of vertical wind velocity, $W$, during the period 0848 GMT 2 June 1997 to 0907 GMT 4 June 1997, measured using the vertical radar beam (lower line), the NW6°-SE6° beam pair (middle line), and the NE6°-SW6° beam pair (upper line), each averaged over the height range 2–4 km, as in Fig. 4. For clarity the middle and upper lines are displaced by $10^4$ and $10^5$, respectively. The dotted lines indicate a slope of $\omega^{-5/3}$. Individual frequency spectra at each height gate were averaged together, and typical standard errors are plotted. Two arrows show the range of Brunt–Väisälä frequencies measured by eight radiosondes during this event.

For example, one ubiquitous type of power law is ‘$1/f$ noise’, where $S(\omega) \sim \omega^{-1}$, which has been widely studied (e.g. Voss 1989; West and Shlesinger 1989). Actual power-law spectra, which are commonly classed as $1/f$ noise, range from $\omega^{-0.5}$ to $\omega^{-1.5}$, or even $\omega^0$ to $\omega^{-2}$ (Halley 1996). Since power laws are so common in natural processes their occurrence in the time dependence of mountain waves need not be too surprising; however, even in an entirely artificial system, e.g. pitch variations in audio signals (Voss 1989), the observed $S(\omega) \sim \omega^{-1}$ power spectrum has not been explained in a quantitative way.

In studies such as Ecklund et al. (1986) and Gage and Nastrom (1990), the mountain-wave spectrum was examined over approximately one order of magnitude of frequency (e.g. Fig. 1), and Fig. 4 covers approximately two orders of magnitude. Further investigation of the extent of the spectrum is usually limited by the shortness of mountain-wave events at Aberystwyth. However, a spectrum from a two-day data set (0848 GMT 2 June 1997 to 0907 GMT 4 June 1997) is plotted in Fig. 8. The mountain waves were often trapped below 5 km, so this data set could not have been used to investigate vertical structure. While in Fig. 4 $S(\omega)$ falls below the background-noise level at the shortest periods of $10–20$ min, in Fig. 8 the entire spectrum has higher power, and the $-5/3$ power law can be seen to continue down to periods as short as $5$ min—below the local Brunt-Väisälä period of $8–10$ min, and showing a power law over approximately three orders of magnitude.

At the longest ground-based periods, the MST radar measurements of mountain waves approach the limiting case of steady, time-independent waves—as commonly assumed in computer simulations (e.g. Shufts and Broad 1993). This slowly changing component of
the vertical wind field is shown in Fig. 5(b). At shorter periods, where the waves might be expected to become less deterministic, time-dependent simulations such as Lott and Teitelbaum (1993a,b) could be approaching the limit at which mountain waves can be modelled. The $W$ spectrum of mountain waves, relative to a fixed site on the ground, would therefore represent a continuous, gradual change—from static, deterministic mountain waves at long periods of several hours, to an unsteady, non-deterministic component at periods of an hour or less, while retaining the same frequency power spectrum. The traditional assumption of steady mountain waves is only a partial description of a wave field which, relative to the ground, is constantly drifting and changing with time—the most generalized case of the effects modelled by Nance and Durran (1997, 1998). The state of the mountain-wave pattern, on the brink of propagating upwind or downwind in response to the changing background wind, but also tending to remain ‘phase-locked’ relative to the topography, may also be relatable to theories such as self-organized criticality (Bak et al. 1987; Cronise et al. 1996), which model the power-law behaviours of marginally stable systems.

7. CONCLUSIONS

The time dependence of vertical wind velocity, $W$, during mountain-wave events is investigated using MST radar. $W$ shows a power-law frequency spectrum, with $S_W(\omega) \sim \omega^{-5/3}$, rather than the monochromatic signal which would be expected if lee waves with a constant, non-zero phase speed propagated horizontally above the radar. The $\omega^{-5/3}$ frequency spectrum is not explainable in terms of tilted scattering layers, and the vortical-mode model of Gage and Nastrom (1990) also appears inconsistent with the $W$ measurements. Changes in background-wind speed, wind direction, temperature, humidity, and the upwind profile of terrain will all cause time variations of $W$. However, even a very simple model, in which the mountain-wave pattern moves upwind or downwind in a one-dimensional random walk, shows a similar time dependence to genuine radar data. The $W$ spectra appear to represent a continuous, gradual change from steady, deterministic mountain waves at the lowest frequencies, to an unsteady, non-deterministic component at the highest frequencies, while retaining the same power spectrum down to periods as short as 10 min or less.

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REFERENCES


McAfee, J. R., Balsley, B. B. and Gage, K. S.  
Momentum flux measurements over mountains: Problems associated with the symmetrical two-beam radar technique. *J. Atmos. Oceanic Technol.*, 6, 500–508  
1989

McAfee, J. R., Gage, K. S. and Strauch, R. G.  
Examples of vertical velocity comparison from colocated VHF and UHF profilers. *Radio Sci.*, 29, 879–880  
1994

Muschinski, A.  
1995

Nance, L. B. and Durran, D. R.  
1996

Owens, A. J.  
1997

Prichard, I.T., Thomas, L. and Worthington, R. M.  
1998

Ralph, F. M., Crochet, M. and Venkateswaran, S. V.  
The characteristics of mountain waves observed by radar near the west coast of Wales. *Ann. Geophys.*, 13, 757–767  
1995

Ralph, F. M., Neiman, P. J., Keller, T. L., Levinson, D. and Fedor, L.  
1997

Reiter, E. R. and Foltz, H. P.  
The prediction of clear air turbulence over mountainous terrain. *J. Appl. Meteorol.*, 6, 549–556  
1967

Russ, J. C.  
1994

Sato, K.  
Vertical wind disturbances in the troposphere and lower stratoosphere observed by the MU radar. *J. Atmos. Sci.*, 47, 2803–2817  
1990

Sayles, R. S. and Thomas, T. R.  
Surface topography as a nonstationary random process. *Nature*, 271, 431–434 (see also 273, 573)  
1978

Schorer, R. and Verkai, A.  
1993

Shutts, G. J., Healey, P. and Mobbs, S. D.  
1992

‘Overview of the MST radar system at Aberystwyth’. Pp. 479–482 in Proceedings of 5th workshop on technical and scientific aspects of MST radar, Aberystwyth, UK  
1992

Smith, R. B.  
The generation of lee waves by the Blue Ridge. *J. Atmos. Sci.*, 33, 507–519  
1976

Turcott, R. G. and Teitch, M. C.  
1996

Turcotte, D. L.  
Fractals and chaos in geology and geophysics. Cambridge University Press  
1992

Vallis, G. K., Shutts, G. J. and Gray, M. E. B.  
1997

VanZandt, T. E.  
1982

Vosper, S. B. and Mobbs, S. D.  
1996

Voss, R. F.  
1989

Weinstock, J.  
1996

West, B. J. and Shlesinger, M. F.  
1989

Worthington, R. M.  
‘Radar studies of mountain waves, inertia–gravity waves and wave momentum flux in the lower atmosphere’. Ph.D. Thesis, University of Wales, UK  
1996

Worthington, R. M. and Thomas, L.  
1996a

1996b
Yoe, J. G. and Rüster, R.
