The role of mass transfer in describing the dynamics of mesoscale convective systems

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SUMMARY

When deep convection occurs in a rotating stratified fluid, potential vorticity (PV) anomalies form in response to the change in the mass field. This geostrophic adjustment process, and its importance in shaping the dynamics of long-lived convective systems, is studied using a high-resolution numerical model to simulate idealized precipitating and non-precipitating convection.

The first series of experiments considers the partitioning of energy between the balanced flow associated with the adjustment \( E_B \), inertia–gravity waves and turbulent dissipation and how this varies with convected mass \( M_C \). It is found, for two-dimensional non-precipitating convective plumes with a Coriolis parameter equal to \( 10^{-4} \text{ s}^{-1} \), that \( E_B \propto M_C^2 \), and, for the plumes which transport the largest quantities of mass, up to 60% of the energy is retained in the balanced flows — a much higher figure than previous estimates. For three-dimensional (3-D) non-precipitating plumes it is shown that \( E_B \propto M_C^{5/3} \). It is suggested that these results indicate that the geostrophic adjustment to mass transfer is an important process in larger convective systems, such as Mesoscale Convective Systems, and that PV anomalies associated with such a system would be much stronger than the net anomaly associated with an ensemble of thunderstorms giving the same upward mass transfer. It is found, for the 3-D runs, that inertia–gravity waves comprise only 10% of the total energy released.

A further series of experiments are carried out in which the effects of precipitation driven downdraughts are included. The downdraughts do not have the effect of simply reversing the mass transfer of the updraught, and the combination of the updraught and downdraught results in a complex PV anomaly structure with a mid-level cyclonic vortex. Similar conclusions to those for the non-precipitating convection case appear to follow for more realistic precipitating systems.

KEYWORDS: Deep convection Downdraughts Geostrophic adjustment Inertia–gravity waves Turbulence

1. INTRODUCTION

When mass is transferred in a rotating stratified fluid the environment adjusts in response to the new mass field. This problem has been investigated both analytically (e.g. Gill 1981) and experimentally (e.g. Gill et al. 1979; Helfrich 1994). The region to which the mass has been transferred becomes characterized by a bowing outwards of the isentropes and angular momentum surfaces. The resulting low stratification and anticyclonic vorticity means that this mass source region is associated with low potential vorticity (PV). The shape of the isentropes often leads to the mass source region being termed a 'lens'. At the region from where the mass originated (the mass sink) the isentropes and angular momentum surfaces collapse inwards, resulting in high-stratification cyclonic vorticity and, hence, high PV.

This process of geostrophic adjustment is commonly observed in ocean dynamics where water of different origins can be identified by particulate matter and mineral tracers. Mid-ocean vent eruptions can give rise to large ‘megaplumes’ of mineral enriched fluid (Baker et al. 1989) which feel the effect of the earth’s rotation and become trapped within an anticyclonic vortex (Speer 1989). Observations of the benthic ocean by Armi and D’Asaro (1980) showed the existence of lenticular regions of lower stratification in the region above the mixed layer. These lenses possessed characteristics of mixed-layer water and had been formed by bottom water detaching itself from the sea floor. Higher-saline Mediterranean
water has been observed in the North Atlantic in the form of ‘meddies’: anticyclonic lenses of low PV which can exist as coherent structures for as long as two years (Armi et al. 1989).

In the atmosphere, the large cirrus anvils of Mesoscale Convective Systems (MCSs) can be thought of as intrusions of fairly undilute boundary-layer air into the upper troposphere. Bosart and Nielsen (1993) report of a radiosonde ascent which accidently passed through the anvil of an MCS off the Louisiana coast in 1991. Within the anvil a fairly constant value of the wet-bulb potential temperature, $\theta_w$, was recorded, which was 13 K greater than the environment and close to that measured in the boundary layer. The tropopause was raised over the MCS indicating lower stratification within the anvil. Lens-like regions of lower stratification beneath the tropopause coincident with the anvil can also be seen in the studies of long-lived MCSs over the US by Wetzel et al. (1983). The MCS anvil is associated with a region of anticyclonic outflow in both the midlatitudes (Fritsch and Maddox 1981; Fritsch and Brown 1982) and the tropics (Leary 1979; Gamache and Houze 1982), and has also been observed as a region of low PV (Fritsch et al. 1994).

Shutts (1987) extended the analytic solutions of Gill (1981) to the atmosphere by considering penetrative convection as simply a transfer of mass with minimal environmental mixing but with full internal mixing so that the lens was homogeneous in potential temperature, $\theta$, and angular momentum. Using a combination of a mass source and sink he represented the flow state expected in the atmosphere following such convection. These solutions were extended by Shutts and Gray (1994) and Shutts (1995) for lenses with stably stratified interiors. A different analytic approach was used by Fulton et al. (1995) to study the gradient adjustment that followed the injection of mass in the upper troposphere and lower stratosphere. They concentrated on the sensitivity of the resulting anticyclone to latitude (rotation rate) and altitude (stratification). In all these cases an anticyclonic region is observed at the mass source/anvil region and a smaller-scale cyclone at the surface. In Shutts (1987) and related slab-symmetric work this had the appearance of a wind-shear front.

This analytic work was compared with the results of numerical simulations using a primitive-equation model in Shutts and Gray (1994, hereinafter referred to as SG94). They simulated similar convective plumes in rotating and non-rotating environments and showed that, in the rotating cases, the ‘cloudy’ air was associated with an area of near-zero PV, whereas a high PV spike was observed at the surface. These were absent in the non-rotating case. In addition, in the rotating cases, roughly a third of the total energy released by the convection was held in the balanced flows associated with the PV anomalies resulting from the adjustment. Further, they hypothesized, using simple scaling arguments, that the balanced energy ($E_B$) and total energy released ($E_T$) could be related to the mass convected ($M_C$) by:

$$E_T = C_0 M_C - \frac{2}{3} \eta M_C^{3/2} \quad (1)$$

$$E_B \propto M_C^2 \quad (2)$$

where $C_0$ is the initial convective available potential energy (CAPE) and $\eta$ an unknown constant. (Equation (2) was later proved analytically by Shutts (1995) for a two-dimensional (2-D) system.) These relations implied that as more mass is convected the proportion of energy released by convection retained in the balanced flows would increase. This proportion would give an indication of the relative importance that adjustment to mass transfer would have in the final energetics and dynamics of the convective system.

Unfortunately, although agreeing well with their analytic model, SG94’s simulations were highly idealized: they were at rotation rates ten times that observed at mid-latitudes,
only two-dimensional and neglected precipitation-driven downdraughts. The latter transport mass, and their absence makes it difficult to determine the impact of the geostrophic adjustment process in real convective systems. The purpose of this paper is to address these issues and, through idealized experiments, gain a greater insight into how geostrophic adjustment shapes the dynamics of convective systems.

In section 3, Eqs. (1) and (2) are validated for a series of 2-D non-precipitating convective plumes of different mass using a mid-latitude rotation rate. Equation (2) is scaled up to three dimensions and tested for a similar series of 3-D plumes. The effect of precipitation is discussed in section 4 by considering, at first, a single downdraught, then, as an intermediate experiment, an artificial combination of an updraught and precipitation-driven downdraught, and, finally, an idealized 2-D simulation of a long-lived precipitating convective system using low-level shear. The following section contains a description of the numerical model and the modifications applied to enable the above experiments.

2. MODEL AND DIAGNOSTICS

The numerical model used in the experiments presented here is the same as that used in SG94 except with a few modifications. It is based on an anelastic, quasi-Boussinesq equation set:

\[
\frac{\partial \mathbf{V}}{\partial t} + f \mathbf{k} \times \mathbf{V} + \nabla (p' / \rho_s) = (g \theta'_s / \theta_s) \mathbf{k} + \frac{1}{\rho_s} \nabla \tau_{ij}
\]  

(3)

\[
\frac{dT_L}{Dt} = \frac{L_{vap} S_p}{c_p} + \frac{1}{\rho_s} \nabla (\rho_s F_T)
\]  

(4)

\[
\frac{Dr_T}{Dt} = -S_p + \frac{1}{\rho_s} \nabla (\rho_s F_T)
\]  

(5)

\[
\nabla \cdot (\rho_s \mathbf{V}) = 0
\]  

(6)

\[
p = \rho RT_v = \rho RT (1 + 0.608r)
\]  

(7)

where \( \mathbf{V} = (u, v, w) \) is the vector wind; \( p, T, T_L, \theta_s, r \) and \( \rho \) are the pressure, temperature, liquid-water temperature, virtual potential temperature, water-vapour mixing ratio and density respectively. Perturbation values are denoted by a prime and basic-state values denoted by an ‘s’ subscript. \( \mathbf{k} \) is a unit vector pointing vertically upwards and \( f \) is the Coriolis parameter. The turbulent shear stress is \( \tau_{ij} \), \( g \) is the acceleration due to gravity, \( R \) is the gas constant for dry air, \( c_p \) is the specific heat at constant pressure, \( L_{vap} \) is the latent heat of vaporization, \( r_T \) is the total water mixing ratio and \( S_p \) is the source of total water precipitation. \( F_T \) and \( F_r \) are subgrid turbulent fluxes. The liquid-water static energy variable, \( T_L = T + (g z - L_{vap} r_L)/c_p \), is solved in a perturbation form such that \( T'_L = T' - (L_{vap}/c_p) r_L \) and the prognostic equation (4) becomes

\[
\frac{dT'_L}{Dt} + w \frac{d(T_L)_s}{dz} = \frac{L_{vap} S_p}{c_p} + \frac{1}{\rho_s} \nabla (\rho_s F_T).
\]  

(8)

\( \mathbf{V}, r_T \) and \( p \) are the other prognostic variables in the model.

The subgrid turbulence parametrization is based on that of Mason (1989) and is detailed in SG94 and Brown et al. (1994). The total variance diminishing advection scheme used in SG94 (van Leer 1974) is superceded here by the ULTIMATE scheme of Leonard (1991). This similarly preserves monotonicity and can handle sharp gradients but possesses less implicit diffusion. In addition to the ‘instant rain’ microphysics used in SG94, where
liquid water is removed upon formation with no evaporation, the model used here can also operate a warm-rain microphysics parametrization based on Kessler (1974).

The most important modification to the model is the introduction of a stretched horizontal grid. The model has periodic boundary conditions in the horizontal and it was shown in SG94 that this made the partitioning of the energy budget sensitive to domain size, which had to be some ten Rossby radii wide, based upon the height of convection \( 2 N H_C/3 \pi f \), where \( H_C \) is the height of the convection and \( N \) is the Brunt–Väisälä frequency for this sensitivity to be lost. In addition the grid spacing had to be small (\( \approx 200 \) m) to be able to simulate the convective plume consistently with the turbulence parametrization. This combination of large domains and high resolution created a computational paradox which precluded certain simulations, for example, using mid-latitude rotation rates or three dimensions. However, high resolution is only required in the central part of the domain where the convective plume occurs; in the outer regions only large wavelength inertia–gravity waves are present. An ideal solution to this problem, therefore, is to use a stretched horizontal grid. Such a technique has been successfully used in a number of meteorological problems, for example, Wood and Mason (1993) and Fovell and Ogura (1988). However, fast Fourier transforms can no longer be used in solving the Poisson equation for pressure, and an iterative method, derived by Farnell (1980), is used instead.

The solution of Farnell (1980) does require that the grid stretches symmetrically about the centre of the domain and, in 3-D, that it stretches along Cartesian axes rather than radially. Another constraint is that the grid must not stretch excessively. A direct Taylor approximation for the first and second derivatives will produce higher-order truncation terms in \( \Delta x \) for the stretched grid than those derived for an uniform grid. However, Kalnay de Rivas (1972) shows that if the non-uniform grid is transformed to a regular coordinate, assuming that the original coordinate is a smooth function, then the extra truncation errors are of the same order as those of the uniform grid. In order to maintain the smooth function required the grid is constructed by defining an initial grid spacing and a stretch parameter, \( \xi \), such that:

\[
\Delta_{i+1} = \xi \Delta_i
\]

where \( \Delta \) is the grid spacing and \( i \) is the number of grid points from the domain centre. In order to prevent an exponential rise in grid length, \( \xi \) is slowly reduced as \( \Delta_i \) increases. To ensure consistency with the model’s Arakawa C-grid, the grid is calculated such that it is smooth in the half spacings between scalar and velocity points. This restriction in stretching means that the domain cannot be made large enough in 3-D to permit the use of mid-latitude rotation rates.

For 2-D simulations 800 grid points are used in the horizontal, with an initial full grid-spacing of 200 m for the first \( \pm 8 \) km which then stretches up to 8 km at the domain edge at \( x = \pm 1000 \) km. In 3-D the horizontal grid is only 80 points in the x and y directions, with an initial grid-spacing of 200 m increasing up to 9 km at the domain boundary of \( x = \pm 90 \) km. The maximum stretch between adjacent full grid spacings is 1.2% in the 2-D grid and 13% for the 3-D grid. Tests, based on the experiments in SG94 with a uniform horizontal grid spacing, were used to determine the smallest horizontal wavelengths of gravity waves that occurred across the domain, and this information was used to ensure that the grid spacing at any point did not exceed the maximum required to resolve adequately the gravity waves expected in that area.

In both cases the domain extends vertically to \( z = 18 \) km, with a Newtonian damping layer, as described in SG94, acting on all prognostic variables above \( z = 10 \) km. The aim of this is to dissipate the inertia–gravity waves which result as part of the adjustment and would otherwise be trapped in the domain. In the 2-D simulations, 180 grid points are used
in the vertical with a grid spacing of 77 m below $z = 10 \, \text{km}$, smoothly stretched to 160 m in the damping region. For the 3-D set-up 150 grid points are used with a spacing of 90 m below $z = 10 \, \text{km}$ increasing up to 200 m at domain top.

In general, the initial conditions for the simulations are similar to those described in SG94 and are shown schematically in Fig. 1. A cosine-squared warm, moist bubble, of width $X_B = 2 \, \text{km}$, height $H_B = 1.25 \, \text{km}$, and perturbation amplitude of 0.5 K is initiated at the centre of the domain, surrounded by a region of 99% relative humidity of diameter $X_Q$ up to height $H_Q$. Outside of a central region of diameter $X_B$ and height $H_Q = 5 \, \text{km}$ the relative humidity is set to a background value, RH$_B$, normally 60%. The relative humidity is interpolated linearly between the two regions. By varying the parameters $X_Q$ and $X_B$ the convective plume can be made to convect different quantities of mass. The initial thermodynamic profile is idealized by having $N$ constant with height. By changing the
values of $N$ and of the surface potential temperature, $\theta_0$, profiles with different values of CAPE can be formed. $N^2$ is usually set to $1.4 \times 10^{-4}$ s$^{-2}$ and $\theta_0$ to 291 K.

The stretched grid makes the model rather expensive to run, and the initial conditions make it difficult to predict exactly where in parameter space each experiment will fail. A minor modification can be made to the model so that mass can be directly transferred in the atmosphere by adding a source term to the divergence equation, (6):

$$\nabla \cdot (\rho_s \mathbf{V}) = Q_m$$

where $Q_m$ is a mass forcing function. Negative $Q_m$ implies mass sinks and positive $Q_m$ mass sources, with the constraint that $\int Q_m \, dx \, dy \, dz = 0$. This enables controlled mass-transfer experiments to be performed for a range of different situations. Because convection is no longer being explicitly simulated in this type of experiment, larger grid spacings and wider domains can be used. In addition, any effects of the convective plume can be eliminated for sensitivity tests.

The same approach to the energy budget as SG94 is used here, with available potential energy (APE) defined in the Lorenz form:

$$\text{APE} = \frac{1}{2} \rho_s \left( \frac{g \theta'}{N \theta_s} \right)^2.$$

Inertia–gravity wave energy (DMP) is calculated from the energy dissipated in the damping layer. Two further dissipation components are measured: that due to the subgrid turbulence parametrization (DIS), and the implicit diffusion from the ULTIMATE advection scheme (TVD). The total energy released (TOT) is calculated from the latent–heat production and work against the buoyancy force. The balanced energy is determined as the sum of kinetic energy (KE) and APE from the final-state fields after approximately 50 Coriolis time-scales (1/$f$), at which time nearly all of the inertia–gravity waves have been dissipated in the damping layer, and hence the residual flows can be considered to be quasi-balanced. When the Kessler microphysics are used additional source terms are present from water loading and advection of liquid–water substance.

A measurement of the quantity of mass convected is required to validate the expressions (1) and (2). The simplest conceptual method is to measure the upward mass flux integrated over the run and then divide by some length-scale, typically the height of the convection ($H_C$), i.e.

$$M_C \sim \frac{1}{H_C} \int \rho_s w \, dt \, dx \, dy \, dz$$

for $w > 0$. Although this will capture fairly accurately the mass convected in the updraughts it will also contain contributions from any other vertical motion, in particular, inertia–gravity waves, especially later on in the integration. In order to minimize this spurious contribution, time series of $\frac{1}{2} \rho w^2$ are compared with time series of DMP to determine an appropriate time at which to measure $M_C$ before the gravity-wave energy becomes a significant component of the energy of vertical motion.

3. Experiments varying the convected mass

Two series of numerical experiments are conducted that examine the response of the geostrophic adjustment process to convecting different quantities of mass. The first series of experiments is in 2-D, with $f = 10^{-4}$ s$^{-1}$ and run up to 500 000 s. The initial conditions are shown in Table 1. The second series of experiments is in 3-D, with $f = 10^{-3}$ s$^{-1}$ and
TABLE 1. Initial moisture profiles for the two-dimensional 'instant' precipitation runs

<table>
<thead>
<tr>
<th>Run name</th>
<th>(X_B) (km)</th>
<th>(X_Q) (km)</th>
<th>(X_R) (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0303</td>
<td>2.0</td>
<td>2.5</td>
<td>25.0</td>
</tr>
<tr>
<td>L0510</td>
<td>2.0</td>
<td>5.0</td>
<td>100.0</td>
</tr>
<tr>
<td>L1015</td>
<td>2.0</td>
<td>10.0</td>
<td>150.0</td>
</tr>
<tr>
<td>L1525</td>
<td>2.0</td>
<td>15.0</td>
<td>250.0</td>
</tr>
<tr>
<td>L2040</td>
<td>2.0</td>
<td>20.0</td>
<td>400.0</td>
</tr>
<tr>
<td>L3080</td>
<td>2.0</td>
<td>30.0</td>
<td>800.0</td>
</tr>
</tbody>
</table>

See Fig. 1 for definitions of \(X_B\), \(X_Q\) and \(X_R\).

TABLE 2. Initial moisture profiles for the three-dimensional 'instant' precipitation runs

<table>
<thead>
<tr>
<th>Run name</th>
<th>(X_Q) (km)</th>
<th>(X_B) (km)</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E0202</td>
<td>2.0</td>
<td>20.0</td>
<td>1</td>
</tr>
<tr>
<td>E0206</td>
<td>2.0</td>
<td>60.0</td>
<td>1</td>
</tr>
<tr>
<td>E0510</td>
<td>4.0</td>
<td>100.0</td>
<td>5</td>
</tr>
<tr>
<td>E0810</td>
<td>8.0</td>
<td>100.0</td>
<td>5</td>
</tr>
<tr>
<td>E1216</td>
<td>12.0</td>
<td>160.0</td>
<td>9</td>
</tr>
<tr>
<td>E1416</td>
<td>14.0</td>
<td>160.0</td>
<td>13</td>
</tr>
<tr>
<td>E1420</td>
<td>14.0</td>
<td>200.0</td>
<td>13</td>
</tr>
</tbody>
</table>

\(n\) is the number of warm bubbles used to initiate the plume. See Fig. 1 for definitions of \(X_Q\) and \(X_B\).

run to 50 000 s. The initial conditions for the 3-D runs are shown in Table 2. Secondary plumes can be initiated by gravity waves radiating from the plume where the relative humidity gradient changes, at \(x = \pm X_Q/2\). These will give a partial ensemble nature to the convection and cannot be strictly compared with the single-plume experiments. In 2-D these plumes are suppressed by local subsidence warming from the spreading lens for \(X_Q \leq 30\) km, but for 3-D the higher rotation rate constrains the width of the lens, and secondary plumes occur for even small values of \(X_Q\). In order to use sufficiently large \(X_Q\) to attain a wide range of \(M_C\) parameter space this problem is overcome by introducing several warm bubbles into the 99% relative humidity region in the initial state. This allows a large lens to be formed quickly, thus preventing the development of the secondary plumes by gravity waves. Each warm bubble is identical to the original and each centre separated by 2.1 km. This is not an unrealistic mechanism as in convective systems the mass is transported through many updraught cores. Both sets of runs use the "instant" precipitation scheme and so contain no downdraughts.

The \(v\)-fields (winds perpendicular to the cross-section) after 50 Coriolis time periods for 2-D runs of different \(M_C\) are shown in Fig. 2. A similar plot for 3-D runs is shown in Fig. 3. In both sets of runs the vast majority of the inertia–gravity waves have dissipated in the damping layer and the velocity and \(\theta\) fields display the new balanced state anticipated from the studies of Gill (1981) and seen in SG94; in particular the mass-source anticyclonic vortex at the neutral buoyancy level and a mass-sink cyclonic vortex at the surface. Both the amplitude and horizontal scale of the \(v\)-fields increase as more mass is convected. For example, in run L0510 the maximum \(v\) is 3.75 m s\(^{-1}\) whereas in run L3080 it is over 10 m s\(^{-1}\). In the 3-D runs the cyclone is significantly stronger, with velocities about 50%
greater than those around the lens. It is also apparent that instabilities have developed in the lens which results in the speckled appearance of the velocity contours.

The energy budget details for the 2-D runs are shown in Table 3. The values of $E_T$ and $E_B$ represent a complete domain average of energy and are plotted against $M_C$ and $M_C^2$, respectively, in Fig. 4. The best-fit line, calculated using a least-squares algorithm demonstrates that $E_T \propto M_C$ and $E_B \propto M_C^2$ as is anticipated from Eqs. (1) and (2). It would
be intuitively expected for both lines to pass through the origin; though this is almost achieved for the \( E_B \) graph, it is not for \( E_T \). This may be indicative of a residual component of gravity waves in the mass-flux diagnostic which is used to calculate \( M_C \).

The energy and mass diagnostics for the 3-D runs are presented in Table 4. Here the calculation of \( M_C \) is adjusted slightly to account for further weak convection that occurs around 20 000 s in the higher-mass runs. Owing to the presence of gravity waves, a mea-
TABLE 3. THE CONVEXED MASS AND ENERGY BUDGETS, AVERAGED OVER THE LAST 5/f TIME PERIOD OF THE RUN, FOR THE 2-D EXPERIMENTS

<table>
<thead>
<tr>
<th>Run</th>
<th>$M_C$ (10^8 kg)</th>
<th>$E_B$ (J m^-2)</th>
<th>KE (%)</th>
<th>APE (%)</th>
<th>DMP (%)</th>
<th>TVD + DIS (%)</th>
<th>$E_T$ (J m^-2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0303</td>
<td>0.98</td>
<td>0.0051</td>
<td>3.4</td>
<td>4.1</td>
<td>65.1</td>
<td>27.3</td>
<td>0.0683</td>
</tr>
<tr>
<td>L0510</td>
<td>1.71</td>
<td>0.0434</td>
<td>10.2</td>
<td>11.0</td>
<td>44.9</td>
<td>33.8</td>
<td>0.2063</td>
</tr>
<tr>
<td>L1015</td>
<td>2.57</td>
<td>0.0897</td>
<td>13.9</td>
<td>14.5</td>
<td>44.1</td>
<td>27.5</td>
<td>0.3190</td>
</tr>
<tr>
<td>L1525</td>
<td>3.24</td>
<td>0.1732</td>
<td>18.3</td>
<td>18.5</td>
<td>36.2</td>
<td>27.0</td>
<td>0.4748</td>
</tr>
<tr>
<td>L2040</td>
<td>4.49</td>
<td>0.3272</td>
<td>24.6</td>
<td>25.3</td>
<td>32.3</td>
<td>17.8</td>
<td>0.6612</td>
</tr>
<tr>
<td>L3080</td>
<td>5.37</td>
<td>0.4469</td>
<td>28.6</td>
<td>29.3</td>
<td>25.2</td>
<td>16.9</td>
<td>0.7772</td>
</tr>
</tbody>
</table>

$M_C$ is the convected mass, $E_B$ the balanced energy expressed as an energy density for the entire domain, $E_T$ the total energy released similarly expressed, KE the balanced kinetic energy, APE the balanced available potential energy, DMP the total energy dissipated in the damping layer representing inertia–gravity waves and TVD + DIS is the dissipation resulting from both the advection scheme and turbulence parametrization. These final four diagnostics are expressed as a percentage of $E_T$.

![Graph of $E_T$ against $M_C$ and $E_B$ against $M_C^2$](image)

Figure 4. Graphs of $E_T$ against $M_C$ and $E_B$ against $M_C^2$ (see text) for the 2-D simulations. Data points are shown as crosses for which the solid line represents the best-fit straight line.

measurement of the mass flux when this additional convection has ceased would overestimate the value of $M_C$. The extra mass is approximated by calculating the increase in $E_T$ due to this convection and then amending $M_C$ in accordance with Eq. (1). This relationship holds for the 3-D runs, as shown in Fig. 5.

The $E_B$–$M_C$ relationship in Eq. (2) is not universal, and different expressions apply in 3-D. No attempt will be made to derive the exact analytic form here but simple scalings
TABLE 4.  THE CONVECTED MASS AND ENERGY BUDGETS, AVERAGED OVER THE LAST 5/6 TIME PERIOD OF THE RUN, FOR THE 3-D EXPERIMENTS

<table>
<thead>
<tr>
<th>Run</th>
<th>$M_C$ ($10^{11}$ kg)</th>
<th>$E_B$ ($10^{-3}$ J m$^{-3}$)</th>
<th>KE (%)</th>
<th>APE (%)</th>
<th>DMP (%)</th>
<th>TVD + DIS (%)</th>
<th>$E_T$ ($10^{-3}$ J m$^{-3}$)</th>
</tr>
</thead>
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<tr>
<td>E0202</td>
<td>3.8</td>
<td>0.92</td>
<td>6.7</td>
<td>2.6</td>
<td>10.3</td>
<td>80.4</td>
<td>9.9</td>
</tr>
<tr>
<td>E0206</td>
<td>6.3</td>
<td>3.22</td>
<td>11.2</td>
<td>3.2</td>
<td>8.1</td>
<td>77.4</td>
<td>22.3</td>
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<tr>
<td>E0510</td>
<td>13.5</td>
<td>8.35</td>
<td>17.8</td>
<td>3.0</td>
<td>10.8</td>
<td>68.4</td>
<td>40.2</td>
</tr>
<tr>
<td>E0810</td>
<td>15.6</td>
<td>10.6</td>
<td>18.7</td>
<td>3.0</td>
<td>10.4</td>
<td>67.8</td>
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</tr>
<tr>
<td>E1216</td>
<td>21.5</td>
<td>20.7</td>
<td>27.7</td>
<td>5.1</td>
<td>9.2</td>
<td>58.0</td>
<td>63.2</td>
</tr>
<tr>
<td>E1416</td>
<td>22.9</td>
<td>25.3</td>
<td>31.6</td>
<td>6.2</td>
<td>9.4</td>
<td>52.8</td>
<td>66.9</td>
</tr>
<tr>
<td>E1420</td>
<td>21.7</td>
<td>22.2</td>
<td>31.3</td>
<td>6.4</td>
<td>10.1</td>
<td>52.2</td>
<td>58.8</td>
</tr>
</tbody>
</table>

Definitions are as for Table 3.

Figure 5.  Plot of $M_C$ against $E_T$ (see text) for the 3-D runs. The best-fit straight line shows that the linear relationship from Eq. (1) is satisfied for the 3-D runs.

may be inferred for the balanced energy in terms of the lens’s radii and hence mass. Let us assume that the lens has a radius of $a$, uniform density of $\rho$, rotating in the horizontal plane with a constant angular velocity of $\omega$ in a cylindrical polar coordinate system ($r$, $\phi$ and $z$, where $z$ is scaled by $N/f$). It can thus be shown that the total kinetic energy of the lens, $E_L$, can be given as:

$$E_L = \int \frac{1}{2} \rho \omega^2 r^2 \, dV.$$  \hspace{1cm} (13)

In 2-D the kinetic energy is integrated over $r$ and $z$: $z$ over the range $-a$ to $+a$ and $r$ over 0 to $R(z)$, where $R$ is the horizontal distance of the lens edge from the $z$ axis and is given as

$$R(z) = \sqrt{a^2 - z^2}.$$  \hspace{1cm} (14)

Thus, the complete integral for 2-D is

$$E_L = \int_{-a}^{+a} \int_0^{\sqrt{a^2 - z^2}} \frac{1}{2} \rho \omega^2 r^2 \, dr \, dz$$  \hspace{1cm} (15)

which evaluates to:

$$E_L = \frac{\pi}{16} \rho \omega^2 a^4.$$  \hspace{1cm} (16)

The mass of the lens is given as $\rho \pi a^2$, so for two dimensions Eq. (2) is reproduced, i.e. $E_B \propto M_C^2$. 
Figure 6. Plot of $E_B$ against $M_C^{5/3}$ (see text) for the 3-D runs. The best-fit line shows that Eq. (19), the 5/3rds relationship derived from a 3-D integral of an axisymmetric extrapolation of Gill's 2-D solution, gives reasonable agreement.

For three dimensions the integration is over $r$ and $z$ as before, and also $\phi$ from 0 to $2\pi$. The velocity field is assumed to be axisymmetric so the only addition to the integral is $r \, d\phi$. Thus, on evaluating the energy integral

$$E = \int_{-a}^{+a} \int_0^{\sqrt{a^2-z^2}} \int_0^{2\pi} \frac{1}{2} \rho \omega^2 r^3 \, d\phi \, dr \, dz$$

we find

$$E = \frac{4\pi \rho \omega^2 a^5}{15}.$$  \hspace{1cm} (17)

This time the mass is given by $4\pi \rho a^3/3$ and so the balanced energy–mass relationship for the 3-D runs is easily shown to satisfy

$$E_B \propto M_C^{5/3}.$$  \hspace{1cm} (18)

This relation has been derived for only the kinetic energy within the lens and, hence, does not necessarily describe the dependence of the APE and 'environmental' kinetic energy outside the lens. However, for lenses with radii comparable with the Rossby radius of deformation, the APE scales in the same way as kinetic energy, and so the above dependencies of $E_B$ on $M_C$ hold for the total energy. It should also be noted that the total energy exterior to the 2-D lens is unbounded, and a physically admissible solution can only be realized if the complementary mass-sink solution is included (Shutts 1987). Nevertheless, the scaling relationship for the total energy in the 2-D case is not altered by the addition of this frontal solution.

A plot of $E_B$ against $M_C^{5/3}$ is shown in Fig. 6. The best-fit line shows that the relationship between convected mass and balanced energy for 3-D fits Eq. (19) quite well.

For the values of mass convected in these experiments the proportion of energy that is trapped in the balanced flow ranges up to 60% in the 2-D runs and 40% in the 3-D runs. These are certainly appreciable amounts and relate to lenses which have diameters much less than an MCS anvil. For example, L3080 has a diameter of 100 km whereas a Mesoscale Convective Complex (MCC) as defined by Maddox (1980) would require an anvil with a diameter in excess of 360 km. This difference can be attributed to a difference in CAPE: 400 J kg$^{-1}$ for L3080 and, typically, 1500–2500 J kg$^{-1}$ before the formation of an MCC over the US Midwest (Bluestien and Jain 1985).
This sensitivity to CAPE is illustrated by an additional experiment, C750, in which the CAPE is increased to 750 J kg\(^{-1}\) by setting \(N^2 = 1.1 \times 10^{-4}\) s\(^{-2}\) and \(\theta_G = 288\) K. Although the same initial moisture profile as L0510 is used C750 convects 50% more mass. This is because the more energetic updraughts are able to transfer more mass before the rotational stiffness of the surface cyclonic vortex cuts off the moisture supply. In addition, C750 convects roughly the same amount of mass as L1015, and, consequently, both runs have similar levels of \(E_B\). However, \(E_T\) is much higher in C750 and hence \(E_B\) as a proportion of \(E_T\) is lower: 19.2% compared with 28.3%.

Dividing Eq. (2) by Eq. (1) gives the following expression for the proportion of balanced energy:

\[
\frac{E_B}{E_T} = \frac{M_C}{C_0 - \eta M_C^{1/2}}.
\]  

(20)

For \(E_B:E_T\) to remain high any increase in CAPE must be compensated for by an increase in \(M_C\). Comparing L3080 with the scales of US Midwest MCCs it can be seen that the \(6^1\) times increase in CAPE would be compensated for by a 13-fold increase in the mass of the anvil. These figures imply that \(E_B \approx E_T\) for an idealized lens on the MCC scale. This shows that, even with higher CAPEs, it can be anticipated to still have high proportions of balanced flow trapped in larger-scale systems.

The unattainable limit where \(E_T = E_B\) can be used to argue a scaling for the maximum size of a convective system. If \(H\) is some arbitrary height scale then \(N\) and \(N/f\) can be used to non-dimensionalize the velocity and horizontal length, respectively. If this is followed through in the derivation of Eq. (19) it can be shown that

\[
E_B = c f^2 a^5 \left(\frac{f}{N}\right).
\]  

(21)

where \(c\) is some constant and \(f/N\) is the aspect ratio of the lens. From Eq. (1)

\[
E_T = \text{CAPE} \times \frac{4}{3\pi a^3} \left(\frac{f}{N}\right).
\]  

(22)

Equating (21) and (22) gives a scaling for the maximum size of a convectively generated lens, \(r_{\text{max}}\), as

\[
r_{\text{max}} = \beta \frac{(\text{CAPE})^{1/2}}{f}
\]  

(23)

where \(\beta = (4\pi/3c)^{1/3}\). There is limited practical value to such an expression as important processes, such as precipitation and dissipation, have been neglected, although it does provide a useful reference point in this balanced view of penetrative convection.

Tables 3 and 4 show that the partitioning of the energy budget is distinctly different between the 2-D and 3-D runs. Firstly, the levels of numerical dissipation in the 3-D runs are much greater than in the 2-D runs. It varies from about 80% of \(E_T\) in E0202 to just over 45% in E1420, whereas in 2-D runs it does not get much higher than a third, and is about a sixth of \(E_T\) in L3080. High levels of dissipation are not unexpected. For example, a typical value for the fraction of latent heat released by moist convection which can be converted to kinetic energy is 10% (Renno and Ingersoll 1996). Although there are differences between the two series of runs, apart from the number of dimensions, sensitivity runs show that these (coarser grid, 3-D ULTIMATE, higher rotation rate) have a small or negligible impact on the proportion of energy dissipated. It is, therefore, reasonable to conclude that the higher dissipation is a reflection of the added realism of the 3-D experiments. The main consequence of this is in the gravity-wave component of the energy budget.
The second difference is in the ratio of KE to APE in the final states. In the 2-D runs there is almost equipartition; in the 3-D runs the KE is much greater than the APE by a factor of 4 or so. The 3-D runs are characterized by strong, tall positive PV anomalies at the surface extending into the near-zero PV lens. Hoskins et al. (1985) state that, in a tall PV anomaly, vorticity dominates the static stability part, and this is reflected in the high KE:APE ratio observed. The amplification of the positive PV anomaly may be caused by enhanced entrainment into the updraught which would be consistent with the higher proportion of energy dissipation. Vortex stretching by the updraught (absent in 2-D) will also help strengthen the vorticity and extend it into the lens. This extension of the cyclonic core into the anticyclonic lens forms a region of inertial instability which can be seen manifesting itself in Fig. 3.

This effect can be eliminated from the 3-D runs by using the mass-forcing model described in section 2. The idealized mass function, $Q_m$, consists of an elliptic mass source centred at $z = 6.5$ km of radius 4 km and half depth 2 km and a half-ellipse mass sink at the ground of height 3 km and radius 2 km. The amplitude of the mass sink is adjusted so that the total integral of $Q_m$ is zero. In these experiments the KE:APE ratio is approximately 1 and Eq. (19) is fully satisfied for $E_B$ against $Q_m$ (equivalent to $M_C$).

4. INCLUSION OF PRECIPITATION-DRIVEN DOWNDRAUGHTS

(a) Isolated downdraught

In the same way that the updraught can be thought of as a mechanism to transfer mass from the boundary layer to the upper troposphere, so a downdraught can be thought of as a way of transferring mass from the mid-troposphere back to the surface. As the two processes occur near simultaneously on the inertial time-scale in a precipitating convective system it will be the net effect of the two sets of mass sources and sinks which will shape the final geostrophically adjusted fields. It is thus instructive initially to study the effect of a downdraught in isolation.

A precipitation-driven downdraught is simulated in experiment DD by initializing the model with a cloud of a similar scale to the lenses produced by the convective plume and then switching on the microphysics. The initial cloud is an ellipse of horizontal diameter 60 km and depth of 3 km, centred at $z = 6.5$ km. The liquid-water mixing ratio decreases as a cosine-squared function towards the cloud boundary, with a maximum value of $5 \times 10^{-3}$ g g$^{-1}$ in the centre. The model is run in 2-D for 500 000 s using a rotation rate of $10^{-4}$ s$^{-1}$.

The $v$-field at 500 000 s is shown in Fig. 7. Between $z = 5$ to 8 km, where the initial cloud was located, there is a cyclonic vortex, indicating a mass sink, and at the surface an anticyclonic vortex, indicating a mass source, is visible. Analysis of the potential-vorticity field, shown in Fig. 8, enhances the structure of the mass sources and sinks. The concave low PV at the surface is the mass source resulting from the downdraught. It may have been expected that this 'cold pool' would spread out thinly into a convex lens, the final width constrained by rotation. However, the initial water mixing ratio distribution in the cloud concentrates the precipitation in the centre of the downdraught which, through higher fall velocities, spreads out more at the surface than the areas of lighter precipitation on the fringes of the downdraught. This results in a build up of mass in the outer regions of the cold pool. A repeat of the simulation using a uniform initial cloud water mixing ratio eliminates this effect and results in the surface PV anomaly spreading further horizontally and having a convex distribution.

The mass-sink region, indicated by the high PV, covers the whole region that was initially cloudy. This is in contrast to the high PV 'spikes' which are observed as the mass-
Figure 7. The $v$-field for experiment DD (see text) after 500 000 s. Comparing with the same field formed from the convective plume updraughts in Fig. 2 it can be seen that there is now a *cycloonic* vortex at mid-levels, corresponding to a mass sink, and an *anticyclonic* vortex at the surface which is indicative of a mass source. The contour interval is 0.25 m s$^{-1}$ and dashed contours represent negative values.

Figure 8. The potential-vorticity field for experiment DD (see text) after 500 000 s for the central 200 km of the domain. Thick contours are at every 0.5 potential-vorticity units (PVU) whilst thin contours are at every 0.1 PVU between $-0.4$ and $+0.4$ PVU. Dashed contours indicate negative values. Note the concave region of low PV at the surface, the high PV in the cloud and below, and the two $+ve/-ve$ PV doublets either side of the cloud at $z = 5$ km.

sink areas in the convective plume experiments (see SG94's Fig. 13). There is also a distinct high PV mass sink beneath the cloud. This implies that mass has been entrained from the environment into the downdraught which extends the cloud mass sink downwards. The drawing of environmental mass into the downdraught may also explain the $+ve/-ve$ PV doublets observed either side of the cloud at $z = 4.5$ km. Entraining environmental parcels with high absolute momentum are displaced with zero absolute momentum parcels in the downdraught which are detrained out into the environment. The net effect is a reversal of the gradient of $M$ and hence negative PV.

The energy budget for DD is of some interest. After 500 000 s 16.7% of the total energy released remains as KE and APE. Most of the remainder has been dissipated as gravity waves with a very small proportion in the form of numerical dissipation. This low dissipation is almost certainly due to slacker velocity gradients in the downdraught compared with the updraughts in the plume experiments. When errors in the measurement
of $M_C$ are taken into account, the downdraught experiment and the convective plume L0303 can be shown to have transferred roughly similar amounts of mass. However, in DD the KE is 0.0058 J m$^{-2}$ as opposed to 0.0022 J m$^{-2}$ in L0303, and the APE is 0.0098 J m$^{-2}$ compared with 0.0030 J m$^{-2}$ in L0303.

Cooling due to the evaporation of rainwater into unsaturated air entrained into the downdraught can explain some of the higher APE. This is roughly constant at $-0.2$ K over a column 50 km wide and 7 km high which provides an APE contribution of 0.0038 J m$^{-2}$. This should not be considered to be part of the balanced energy which is due to the mass adjustment and, being constant with height, does not have a significant impact on the PV field. When this is accounted for the KE and APE values are roughly equal but they are still two to three times larger than in L0303. This can be attributed to the different shapes of the mass source and sink functions generated by the downdraught compared with those from an updraught, which is shown in the PV field.

Consider conceptual mass source regions for the convective plume (lens) and the downdraught (cold pool) as shown in Fig. 9. The lens is a complete ellipse of mass $M_C$ and so $E_B \propto M_C^2$. However, an equivalent cold pool is a semi-ellipse of mass $M_C$. The balanced energy associated with this is half of that of a complete ellipse of mass $2M_C$, i.e.

$$E_B \propto \frac{(2M_C)^2}{2} = 2M_C^2.$$  \hspace{1cm} (24)

So, in this simple example, twice as much energy is associated with a cold pool as an equivalently sized lens. Additional calculations using the analytic model of Shutts (1995) show that the energy in the balanced flow associated with the conformal transformations for an updraught mass source/sink pair is about 2.25 times less that associated with the flow resulting from a downdraught mass source/sink pair for a downdraught of equivalent magnitude.

The analysis of DD shows that the downdraught is not the simple inverse of the convective plume. It is likely that the combined effect of an updraught and downdraught
will not be a simple reduction in the strength of the updraught mass sources and sinks. In particular, the cold-pool mass source of the downdraught is a completely different size and shape to the line mass sink of the updraught, and the downdraught mass sink extends below the updraught lens/cloud mass source.

(b) Combining updraughts and downdraughts

(i) Without shear. Devising experiments to illustrate the combination of updraughts and downdraughts presents a minor problem. Updraughts and downdraughts only coexist for long periods of time in convective systems in the presence of shear. Shear itself may have some effect on the final balanced flow fields resulting from the mass transfer. Hence, an intermediate experiment is performed in which the effects of both updraughts and downdraughts are considered without shear. To do this, without placing prohibitive restrictions on the amount of mass that can be convected, the activation of the downdraught is delayed until after the convective updraught has been completed. Because the convective stage lasts only a few Coriolis time periods it is hoped that over an integration of 50 such periods this delay will not be significant.

The experiment UD2D is a 2-D simulation similar to L2040 with some modifications: \( N^2 = 1.1 \times 10^{-4} \text{ s}^{-2} \), RH_{B} is set to 20%; and the instant-precipitation parametrization scheme is replaced by Kessler microphysics. Initially, this is switched off, and the convective plume is allowed to form a cloud at the neutral buoyancy level. After 25 000 s, by which time convection has ceased in L2040, the Kessler microphysics is switched on and the run is continued up to a time of 500 000 s.

The resulting \( v \)-field for UD2D is shown in Fig. 10. The flow is distinct from that of the single updraught runs in Fig. 2 owing to the more involved distribution of mass sources and sinks arising from the precipitation. The presence of some residual gravity-wave activity does not help clarify the picture. However, the flow is still anticyclonic between \( z = 5 \) and 8 km, and cyclonic at the surface. This cyclonic region is much weaker than in the non-precipitating experiments and has a smaller lateral range. Furthermore, at \( z = 2 \) km there is a distinct anticyclone almost separating the surface cyclone from the mid-level cyclone at \( z = 2 \) to 4 km.

The weaker surface cyclone has a rather dramatic effect on the proportion of balanced energy which is retained. In UD2D only 11.6% of the total energy released is present in
the balanced flow compared with nearly 50% in run L2040. The thermodynamics of the energy production are different in the two runs, and UD2D is more energetic; however, three times as much energy remains in L2040 as in UD2D. The distribution of this energy in the vertical can be seen in Fig. 11.

At upper levels, above $z = 7.5$ km, the quantities of balanced energy are similar. However, between the surface and the lens there is very little present in UD2D, but in L2040 the energy is at its maximum. Closer study shows that the bottom part of the anticyclone and most of the cyclone have been lost in the new mass source–sink distribution. It appears that, in this experiment, precipitation prevents the formation of the vast majority of the balanced energy which would otherwise be in the mid to lower levels of the troposphere.

The PV field can be used to help identify the position and origin of mass sources and sinks. The field is rather complex but the signature can be clarified by examining the horizontally averaged PV perturbation, $\bar{PV}$. This is plotted against height for L2040, DD and UD2D in Fig. 12.

Above $z = 6$ km in UD2D there is a strong mass source, between $z = 2$ and 6 km a weak mass sink, a weak mass source just above the boundary layer, and a strong mass sink at the surface. L2040 splits cleanly between strong mass source above $z = 4$ km and a mass sink below. Qualitatively, combining the profiles of L2040 and DD gives an insight into the effects of precipitation. The downdraught mass sink has reduced the intensity of the mass source in the cloud in UD2D when compared with L2040. Where this mass sink extends beneath the cloud from the entrainment of environmental air the original cloud mass source becomes a net mass sink. However, below $z = 4$ km, DD is a mass source, so the mass sink observed in UD2D is, therefore, a reduced version of the updraught line mass sink. Hence, the net mass sink between $z = 2$ and 6 km in UD2D is a residual of two
processes — the entrainment of air into the updraught, and the removal of mass due to the downdraught.

The PV anomaly above the cloud at $z = 9$ km in UD2D, which does not appear in L2040 or DD, is believed to represent a small mass sink generated by air entraining into the plume as it overshoots its neutral buoyancy level. Similar observations were made by SG94 and Fulton et al. (1995). When cloudy air is present this feature is amplified; the evaporation of cloud into the entraining dry air reduces its buoyancy and increases the likelihood of more mass being drawn back by the overshoot.

That a mass source does not occur at the surface in UD2D is likely to be due to further shallow convective plumes transporting mass back up to a height of 1 km initiated by local convergence when the downdraught reaches the surface. In a true long-lived convective system low-level shear would result in the surface downdraught mass source (‘mesohigh’) and updraught mass sink (‘pre-squall low’) being spatially separated. This illustrates the limitations of this rather contrived experiment, which, nevertheless, does produce some insight into the interaction between downdraught and updraught mass sources and sinks.

(ii) With shear. By introducing low-level shear, true organized convection can be simulated. Although the surface features are now completely obliterated it does enable some investigation into the coexistence of updraughts and downdraughts and the higher-mass parameter space associated with a long-lived precipitating system.

Two experiments are conducted, based on those of Rotunno et al. (1988), characterized by different levels of CAPE: SL110, with a tropospheric $N^2$ of $1.1 \times 10^{-4}$ s$^{-2}$, has a CAPE of 1300 J kg$^{-1}$, whilst SL95 has $N^2 = 0.95 \times 10^{-4}$ s$^{-2}$ and a CAPE of nearly 1900 J kg$^{-1}$. Both experiments use the 2-D stretched grid domain; $f = 10^{-4}$ s$^{-1}$; boundary-layer shear of $du/dz = 17.5/2500$ s$^{-1}$ in the lowest 2.5 km; an initial 20 km wide cosine-squared warm bubble of maximum perturbation 2.0 K; $\theta_0 = 293$ K; constant $N^2$ below $z = 10$ km and an isothermal stratosphere above; a horizontally uniform relative humidity profile: 95% below
TABLE 5. Energy budgets for the squall-line simulations averaged over the last 2/3 time period of runs SL110 and SL95 (see text)

<table>
<thead>
<tr>
<th>Energy component</th>
<th>Run SL110</th>
<th>Run SL95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total (J m^{-2})</td>
<td>Proportion (%)</td>
</tr>
<tr>
<td>KE</td>
<td>0.0234</td>
<td>4.7</td>
</tr>
<tr>
<td>APE</td>
<td>0.0419</td>
<td>8.4</td>
</tr>
<tr>
<td>TVD + DIS</td>
<td>0.0704</td>
<td>14.1</td>
</tr>
<tr>
<td>DMP</td>
<td>0.3623</td>
<td>72.8</td>
</tr>
</tbody>
</table>

Definitions are as for Table (3).

3 km, 30% above 5 km and linearly extrapolated in between; and a Newtonian damping layer above z = 12.5 km. The Kessler microphysics is continually in operation.

After 200,000 s of integration time both runs have formed extensive cloud fields between z = 9 to 11 km: in SL110 it is 80 km in diameter, whereas in SL95 it is nearly 300 km, which is on the scale of a largish MCS. The dynamic and thermodynamic fields associated with the cloud for SL95 after 100,000 s are shown in Fig. 13. Strong anticyclonic vorticity dominates the mid to upper troposphere, with \( \theta \) attaining magnitudes of 14 m s^{-1}. There is much weaker, and more localized, cyclonic vorticity just above the shear region between z = 2.5 and 4 km. Because of the large horizontal scale, the \( \theta \) plot in Fig. 13(b) clearly shows the low stratification within the mass source lens region. A closer view of the resulting PV anomaly is shown in Fig. 13(c). The near-zero PV stretches from about z = 6 km pushing up the higher ambient PV in the stratosphere to z = 11 km. This is the dominant PV feature: high PV corresponding to mass sinks occur at the top of the cloud due to entrainment, and in a very localized region from z = 3 to 5 km at around \( x = -150 \) km. These positive PV anomalies are fragmented, show little organization and are individually on the convective scale.

As in UD2D, PV* can clarify the overall PV structure, and a plot for SL95 is shown in Fig. 14. The shape of the PV* distribution with height is not too dissimilar to that observed for UD2D in Fig. 12 with a distinct negative region at the cloud levels and weak positive PV* beneath, showing a peak at about z = 4–5 km. The introduction of a high PV stratosphere is responsible for the peaks in PV* around z = 10 km. The intrusion of the low PV lens into the stratosphere results in a minimum in PV* above 10 km, whereas the compensating lowering of the tropopause in the environment causes a maximum in PV* below this level.

Although the localized positive PV anomalies beneath the cloud do appear in the PV* plot they are not consistent with the scales of positive PV anomalies at these levels deduced from MCS studies (e.g. Fritsch et al. 1994). This is because only convective-scale downdraughts occur in SL95 and SL110. The absence of ice microphysics in the simulation, which is essential for the formation of heavy stratiform rain in an MCS anvil (Smull and Houze 1985), precludes the existence of a mesoscale downdraught, which may extend the scale of the positive PV anomaly.

The energy budgets for the two runs are still informative, however, due to the large amounts of mass which have been convected and are shown in Table 5. Levels of balanced energy are low for SL110, which is consistent with UD2D, and it appears that a large proportion of energy released during the convection is ultimately dissipated as inertia-gravity waves. However, the proportions for SL95 are more comparable with the values observed for the higher-mass non-precipitating plumes, such as L2040, although the actual
Figure 13. Fields for SL95 (see text) after 100,000 s: (a) $v$, contour interval 1.0 m s$^{-1}$, negative values dashed; (b) potential temperature, contour interval 3.0 K; (c) potential vorticity (PV), grey shading — black corresponds to 4.0 PVU, white to 0.0 PVU.
values of energy are much higher. By convecting enough mass, significant amounts of energy can be retained in the balanced flow even in a precipitating cloud.

Whether the $E_b \propto M_C^2$ relation can be extended to precipitating systems can be tested using this data. Almost 64 times as much $E_b$ is measured in SL95 compared with SL110. Thus, if Eq. (2) holds for these runs, it would be expected that eight times as much mass has been convected in SL95 than SL110. Because both simulations convect in similar ways, and the precipitation-induced downdraughts are involved in the initialization of new plumes, the integrated mass-flux diagnostic can be used with some confidence for an estimation of the relative $M_C$ in the runs. For SL110 $M_C$ is $2.56 \times 10^8$ kg whilst for SL95 it is $19.3 \times 10^8$ kg, providing a ratio of 7.55 which is fairly close to that expected. The greater gravity-wave energy in SL110 may have led to a slight overestimation of its $M_C$, however.

5. DISCUSSION

In the experiments described here the majority of diabatic processes, with the exception of evaporative cooling, have been neglected. In the single-downdraught case (section 4(a)) it was argued that the evaporative cooling only had a negligible impact on the PV field as it was fairly constant with height. Hence, it is reasonable to assume that the PV anomalies which arise in these experiments, and their associated balanced flows, are primarily the result of geostrophic adjustment to mass transfer.

Diagnosis of the energy remaining in these quasi-balanced flows gives a strong indication of the importance of these PV anomalies. Experiments with non-precipitating plumes have shown that the amount of energy retained in the balanced flows increases proportionately to $M_C^2$ in 2-D and $M_C^{5/3}$ in 3-D. These results underline the original hypothesis that, for a given initial CAPE, the proportion of total energy released and retained in the balanced flows will increase as the quantity of mass convected increases. Hence,
the larger the convective system the more important the role of mass transfer in shaping the final dynamics and PV anomalies. Therefore, it would be expected that a MCS would be associated with more balanced energy, and stronger PV anomalies, than an ensemble of thunderstorms which collectively had convected the same quantity of mass. This also implies that larger convective systems could be thought of as efficient mechanisms for the transfer of energy from the convective scale to the mesoscale in the atmosphere: as more mass is convected the proportion of energy released that is retained in the mesoscale balanced flow increases.

By introducing precipitation-driven downdraughts into the experiments there is a large reduction in balanced energy due to the reduced net mass transfer. A similar observation was made by Vallis et al. (1997). However, the simulations SL95 and SL110 of long-lived convection showed that if sufficient quantities of mass were convected the proportion of energy released held in the balanced flows would still be high, and that Eq. (2) could hold for a 2-D precipitating system. Experiments using the 2-D analytic model of Shotts (1995), but using functions to represent both the updraught and downdraught mass sources and sinks, reinforce this hypothesis. For a downdraught which is a constant fraction of the strength of the updraught it is found that, for a wide range of mass parameter space, the kinetic energy of the resulting balanced fields is proportional to $M_C^2$.

However, the downdraught is not the exact inverse of the updraught and the two do not exactly cancel each other out. This results in a different net distribution of mass sources and sinks which is reflected in the final vorticity and PV fields. In particular, a distinct PV maxima is observed at mid-levels, as have been observed in real MCSs as a mid-level cyclonic vortex (MCV) (e.g. Fritsch et al. 1994), and in idealized studies including evaporative cooling (Shotts et al. 1988; Hertenstein and Schubert 1991). In UD2D it is the convergence due to the combined mass sinks associated with the updraught and the dry air entraining into the downdraught that generate the positive PV anomaly and the weak MCV. However, in SL95 the absence of a mesoscale downdraught, due to the simple microphysics, left this PV maxima on the convective scale and not like observed MCVs.

The experiments presented here are idealized and do not pretend to simulate realistic convective systems. Whereas this is an advantage, in allowing greater understanding of the role of geostrophic adjustment to mass transfer in shaping the dynamics of idealized convective systems, it does pose some limitations in applying the results directly to MCSs. For example, the mass transfer in the experiments takes place on the convective scale, whilst, in real MCSs, the mesoscale updraught and downdraught transport significant quantities of mass. This study has only considered the amount of mass convected and not, directly, the time and spatial scales of mass transfer. The question as to what influence this may have on the larger-scale effects of MCSs is, as yet, unanswered and is an obvious avenue for further research.

**References**


Wetzel, P. J., Cotton, W. R. and McAnelly, R. L.


Wood, N. and Mason, P. J.