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In a recent paper Lin (1997) proposed that the pressure gradient force in the terrain-following σ coordinate be computed using the finite-volume method. He argued that the problem with the coordinate transformation or the interpolation method was that physical principles were often violated in the discrete space due to the inconsistency arising from the use of the continuous hydrostatic relationship for the coordinate transformation while the discretization of the hydrostatic relationship and the two terms representing the pressure gradient force are only discrete approximations. In addition to that he demonstrated that his method considerably reduced the pressure-gradient-force errors compared with the Arakawa–Suarez (1983) schemes.

On the other hand, Janjić (1977, 1980) demonstrated that most σ coordinate pressure-gradient-force schemes could be interpreted as a result of a three-step procedure in the discrete physical space. The three steps are:

1. The values of geopotential φ at σ levels are obtained by integrating numerically a finite-difference approximation of the hydrostatic equation.
2. Using the values obtained in step (1), the values of φ on a constant pressure level are computed by linear interpolation or extrapolation with respect to a monotonic function of pressure, ζ. This interpolation or extrapolation may not necessarily be done using the same finite-difference approximation to the hydrostatic equation as in step (1).
3. The values of φ on constant pressure levels are then used to calculate the finite-difference approximation to the pressure gradient force.

Apparently, the schemes derived using the described procedure do not belong to the class of schemes obtained by discretizing formulas derived by transformations in the continuous space.

In addition, Janjić (1977, 1980) recommended that the requirement for ‘hydrostatic consistency’ be satisfied, i.e. that the same approximation of the hydrostatic equation be used in the first and the second steps of the three-step procedure. The hydrostatic consistency does not automatically increase the accuracy of the pressure-gradient-force approximation in every particular case, but guarantees that the error will remain bounded. As one of his major points, Janjić (1977, 1980) emphasized that for accuracy of the linear interpolation/extrapolation in the second step, the variation of φ with respect to ζ should be as close to linear as possible, and therefore that the pressure-gradient-force error can be minimized by a suitable choice of ζ.

For the staggered distribution of variables considered by Lin, Janjić’s procedure leads to a formula of the form

\[ -\delta_z \phi = -\delta_z \phi^\sigma + \delta_z \phi^\sigma \phi \delta_z \zeta^\sigma. \]  

(1)

Here, the overbar represents the simplest two-point averaging in the direction indicated by the accompanying superscript, and the symbol \( \delta \) denotes the simplest centred two-point finite-difference approximation of the derivative along the axis indicated by the subscript. For example,

\[ \delta_z \phi = \Delta \phi / \Delta \zeta, \]

\[ \delta_z \phi = -\delta_z \phi^\sigma + (\Delta \phi / \Delta \zeta^\sigma) \delta_z \zeta^\sigma. \]  

(2)

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Figure 1. The pressure-gradient-force errors (m²/s⁻²) as functions of the sigma levels from bottom to top for Janjić (1977) schemes with \( \zeta = \sqrt{p^k} \) (empty squares) and \( \zeta = \ln p^2 \) (empty circles) and the Lin (1997) schemes with \( \zeta = p^k \) (filled triangles), \( \zeta = \ln p \) (empty triangles), \( \zeta = \sqrt{p^k} \) (filled squares) and \( \zeta = \ln p^2 \) (filled circles).

See text for further explanation.

Both formulas (1) and (2) are legitimate approximations of the pressure gradient force in the \( \sigma \) coordinate. The difference between the two schemes does not exceed the truncation error.

It can be easily verified that Lin’s schemes are not hydrostatically consistent in the sense defined above. However, as in the case of Janjić’s schemes, the accuracy of the Lin formula should also be sensitive to the choice of the function \( \zeta \) (or \( \sigma \) in Lin’s notation). In particular, it can be easily verified that Lin’s procedure will give the correct answer if \( \phi \) is a linear function of \( \zeta \). In other words, as in the case of Janjić’s schemes, since we generally cannot choose a convenient stratification of the atmosphere, we can choose the function \( \zeta \) in such a way as to minimize the error for the prevailing stratifications.

The experiment of Phillips (1974) and Janjić (1977, 1980) is reproduced here in order to compare the Janjić (1977, 1980) and Lin (1997) schemes. This experiment consists of examining the pressure-gradient-force error in the case when \( \phi \) is only a function of pressure, \( p \). The geopotential profile approximating that of the standard atmosphere was

\[
\phi = 1054.5 + 80397.3\sigma - 7659.0\sigma^2 + 1110.0\sigma^3, \quad \zeta = -\ln p + 11.51292546.
\]

As in Phillips (1974) and Janjić (1977, 1980), two columns located along the x axis at unit distance from each other were considered. The surface pressure at these two points was 800 mb and 1000 mb, respectively. The top of the model’s atmosphere was situated at 200 mb and the model’s atmosphere was represented using 45 equidistant \( \sigma \) layers.

The results are shown in Fig. 1. The sigma levels from bottom to top are on the ordinate, and the errors of the schemes examined are on the abscissa. As can be seen from the figure, the Janjić schemes with \( \zeta = \sqrt{p^k} \) (empty squares) \( (\kappa = R/C_p \) where \( R \) is the gas constant and \( C_p \) the specific heat at constant pressure) and \( \zeta = \ln p^2 \) (empty circles) had the smallest errors. The errors of the Lin schemes with \( \zeta = p^k \) (filled triangles) and \( \zeta = \ln p \) (empty triangles) are larger by almost an order of magnitude. However, as expected on the basis of the considerations about the functional relationship between \( \phi \) and \( \zeta \), the errors of the Lin scheme have been dramatically reduced with \( \zeta = \sqrt{p^k} \) (filled squares) and \( \zeta = \ln p^2 \) (filled circles). At the same time, the elegance and computational efficiency of the Lin scheme has hardly been affected.

It should be noted that even greater accuracy can be achieved by assuming a quadratic, or higher-order, relationship between \( \phi \) and \( \zeta \). This was actually done for the interpolation schemes by Mihailović and Janjić (1986).
REFERENCES


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