Dynamics of fine-scale variables versus averaged observables in a T21L3 quasi-geostrophic model

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SUMMARY

The dynamics of space- and time-averaged observables of a 3-level quasi-geostrophic model is investigated and compared with that of the fine-scale variables, with emphasis on the role of the initial error field on predictability. The invariance of the largest Lyapunov exponent against averaging of up to 10 days is established. Some improvements of predictability of the averaged variables turn out to be possible, at least for certain classes of initial perturbations. Finally, a map of error distribution in space and a classification of weather regimes are derived on the basis of the average properties of the Lyapunov vectors.

KEYWORDS: Atmospheric dynamics Predictability Weather regimes

1. INTRODUCTION

It is by now widely recognized that, owing to the unstable character of atmospheric dynamics, the evolution of the principal meteorological fields cannot be predicted reliably beyond a certain lapse of time of the order of a few days. On the other hand, it is well known that on time-scales larger than 10 days the atmosphere tends to organize in distinct, persistent and recurring configurations, known as weather regimes (Molteni 1994). This observation suggests that the predictability of such dynamical patterns might be enhanced, as compared with the short-scale meteorological events, even though these patterns are uniquely determined from the evolution on synoptic space- and time-scales. The connection between the variability and predictability of the meteorological fields and the dynamics prevailing on larger space- and time-scales is therefore at the heart of the problem of long-term prediction.

One suggestion that comes naturally to mind is that persistent atmospheric configurations may be viewed as states in which small scales have somehow been smeared out. One is therefore tempted to represent them as resulting from the action of a low-pass time and/or space filter on the full, fine-scale variables. The simplest version of such a filter is a plain averaging (Charney 1960; Saltzman 1983), but more sophisticated, weighted-averaging operators can also be envisaged.

Van den Dool and Saha (1990) analysed, from this point of view, 1 to 10-day outputs from a global medium-range forecast model. The corresponding anomaly time series was subjected to a discrete Fourier transform in time. Filtered time series were then obtained by transforming back after zeroing certain Fourier coefficients as desired. Their main conclusion was that the low frequencies are predicted (at a given skill level) over a longer time than the high frequencies, although the predicted skill of current models seems to be far below the theoretical predictability limit for these scales. Further predictability studies, using a variety of other atmospheric models and different types of filters, have been reported by Shukla (1981), Tribbia and Baumhefner (1988) and Roads (1987, 1989). One of the findings of these investigations has been that time- and space-averaged observables may have rather different predictability properties.

Our principal goal in the present work is to carry out a systematic comparative analysis of the predictability properties of fine-scale and averaged meteorological fields in terms of the intrinsic quantities characterizing the underlying dynamics, such as the Lyapunov

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exponents and the Lyapunov vectors. The main idea is to view averaged observables as linear combinations constructed from the initial, fine-scale observable and a number of other observables obtained from it by successive application of time and space lags. This allows one to apply to the study of averaged observables the concepts of dynamical systems theory. Notice, however, that averaged observables, as a rule, do not conform to a closed set of evolution equations, but are hierarchically coupled to the fine-scale ones.

Recently, the dynamical properties of time-averaged observables have been analysed from this standpoint in a low-order atmospheric model (Nicolis and Nicolis 1995). It was shown that, in principle, time averaging does not modify the value of the largest positive Lyapunov exponent, nor the dimension of the underlying attractor. Still, time-averaged observables may be more predictable in the sense that the initial stage of super-exponential error growth, i.e. an error growth rate larger than the dominant Lyapunov exponent, tends to be suppressed by the averaging. These conclusions were confirmed by the analysis of a simple one-dimensional model of thermal convection in which intricate relations between the predictability of space-averaged observables and the distribution of initial error along the different spatial scales were revealed (Vannitsem and Nicolis 1995).

In the present paper we report a comparative study of the predictability of fine-scale variables and of space- and time-averaged observables on a 3-level quasi-geostrophic atmospheric model defined on the global domain, shown recently to display a high-dimensional chaotic attractor with more than 100 positive Lyapunov exponents (Vannitsem and Nicolis 1997). We focus on the behaviour of the mean error of averaged observables and link its evolution to the intrinsic properties of the underlying dynamics, including the Lyapunov exponents and the Lyapunov vectors. Emphasis is also placed on the role of the averaging period and on the initial error distribution in the predictive skill of the model. Finally, a novel classification of some distinct weather regimes in terms of their predictability properties will be attempted.

Section 2 is devoted to a succinct description of the model. In section 3 we compare the statistical properties of the averaged observables with those of the fine-scale variables. In particular, the autocorrelation time, and the spatial and temporal power spectra are deduced. In sections 4 and 5 the predictability properties of fine-scale and averaged fields are compared and explained in terms of the intrinsic properties of the dynamics. In section 6 the geographic distribution of predictability is obtained for the fine-scale and averaged variables. Section 7 is devoted to the classification of weather regimes on the basis of their stability properties, and the main conclusions are drawn in section 8.

2. THE T21 QUASI-GEOSTROPHIC ATMOSPHERIC MODEL

Our study of the dynamics of averaged observables is performed on a 3-level global quasi-geostrophic spectral model (QG model), describing the evolution of the potential vorticity at 200 hPa, 500 hPa and 800 hPa (Marshall and Molteni 1993). In this model, the horizontal fields \( Z \) are expanded in a series of spherical harmonics \( Y_{m,n} \) truncated triangularly at wave number 21:

\[
Z(\lambda, \phi, t) = \sum_{n=0}^{21} \sum_{m=-n}^{n} Z_{m,n}(t) Y_{m,n}(\lambda, \phi)
\]  

(1)

where \( \lambda, \phi, t \) and \( m \) are the longitude, latitude, time and zonal wave number, respectively. The index \( n \) represents a total (two-dimensional) wave number on the sphere and characterizes the size of the two-dimensional horizontal structures. The prognostic equation at
each level $i$ can then be written in terms of the stream function $\Psi$ and the potential vorticity $q$ as

$$\frac{\partial q_i}{\partial t} = -J(\Psi_i, q_i) - D(\Psi_i) + F_i$$  \hspace{1cm} (2)

where $J$ is the Jacobian of the two-dimensional field. The linear term $D$ accounts for the effects of Newtonian relaxation of temperature, a scale-selective horizontal diffusion of vorticity and temperature and a drag on the 800 hPa wind whose coefficient depends on the properties of the underlying surface. A damping time-scale $\tau_d$ at total wave number 21 associated with the horizontal diffusion was introduced and fixed in Marshall and Molteni to 2 days. Finally, the time-independent spatially varying source term, $F$, constrains the solution of the model to an averaged, statistically stable, observed winter climatology ('perpetual winter' conditions), taking into account the influence of diabatic processes and of the divergent part of the wind field. Note that all the fields are computed in dimensionless units: the length unit is the earth’s radius (6371 km) and the time unit is half the inverse of the angular velocity of the earth ($7.292 \times 10^{-5}$ s$^{-1}$). The model equations are integrated in time using a leap-frog scheme for the Jacobian term and a forward scheme for the linear operator $D$ with a time step of 1 hour.

In a previous study (Vannitsem and Nicolis 1997), the present authors have shown that the dynamics of the fine-scale variables are chaotic, comprising about 100 positive Lyapunov exponents. The largest of them is equal to 0.23 day$^{-1}$, and the standard deviation of its local values is about 0.078 day$^{-1}$. It was also shown that for short times the dominant unstable Lyapunov vectors govern the dynamics of the error at synoptic scales, while stable vectors play an important role at very large and very small scales. After this transient behaviour, the error essentially follows the dynamics of the most unstable Lyapunov vector. One of the aims of the present work is to check whether similar features hold for the averaged observables.

3. THE TIME- AND SPACE-AVERAGED OBSERVABLES AND THEIR STATISTICAL PROPERTIES

Following Saltzman (1983) we define the time average of an observable $\Psi(r, t)$ at position $r$ as the running mean of its instantaneous values during a period $T$,

$$\overline{\Psi(r, t)} = \frac{1}{T} \int_t^{t+T} \text{d}\tau \Psi(r, \tau).$$  \hspace{1cm} (3)

In Eq. (3) we use an asymmetric averaging procedure in order to avoid spurious negative values in the integration limits. When $T \rightarrow \infty$, one gets the ergodic mean of the variable $\Psi(r, t)$.

Similarly, we define the average in the space domain as

$$\overline{\Psi(r, t)} = \int_{S(r)} \text{d}x \Psi(x, t)$$  \hspace{1cm} (4)

where $S(r)$ is a surface on the sphere centred on $r$. These surfaces will be successively centred on points of the longitude/latitude ‘Gaussian’ grid ($\lambda, \phi$) (Machtenhauer 1991). To within a multiplicative constant, Eq. (4) can then be written in a discrete form on this grid as

$$\overline{\Psi(\lambda, \phi, t)} = \sum_{l=\lambda-M\lambda}^{\lambda+M\lambda} \sum_{f=\phi-M\phi}^{\phi+M\phi} w(f) \Psi(l, f, t)$$  \hspace{1cm} (5)
where the weighting factors $u(f)$ are the Gaussian coefficients and $M$ is the number of grid points separating the central box $(\lambda, \phi)$ from the boundaries of the surface $S$, over which the coarse graining is performed. The number of grid points used to compute the averaged observable at $(\lambda, \phi)$ is therefore equal to $(2M + 1)^2$. When $S$ is the whole sphere, Eq. (5) reduces to the 0-mode of the stream function which is an arbitrary quantity, fixed to 0.

Let $\tau_1, \tau_2, \ldots, \tau_k$ be the spectrum of characteristic times involved in the dynamics of the fine-scale variables with $\tau_1 = \tau_{\text{min}}$ and $\tau_k = \tau_{\text{max}}$. Obviously, if the averaging period $T < \tau_{\text{min}}$, $\Psi$ is essentially identified to the fine-scale variable $\Psi$. At the opposite end of $T > \tau_{\text{max}}$, $\Psi$ becomes essentially the ergodic average of $\Psi$, in which case any trace of the dynamics has been eliminated. Clearly, then, the relevant range over which $T$ is to be chosen must lie between $\tau_{\text{min}}$ and $\tau_{\text{max}}$ in such a way that fine-scale properties are smoothed and attention is focused on evolution over a scale which is still shorter than $\tau_{\text{max}}$. In the atmospheric model presented in section 2, $\tau_{\text{max}}$ is very large, while the shortest time-scales are of the order of a few hours. The averaging period can therefore be chosen in such a way as to smooth out the effect of synoptic scales, say from 5 to 30 days (Shukla 1981; Tribbia and Baumhefner 1988).

Figure 1(a) displays the autocorrelation function of the time-averaged observable with $T = 10$ days at grid point $\lambda = 140^\circ$ and $\phi = 47^\circ$ at 500 hPa together with the one obtained for the fine-scale variable at the same grid point using 8192 values sampled every 6 hours. Figure 1(b) shows the autocorrelation function of $\Psi$ at $\lambda = 56^\circ$ and $\phi = -36^\circ$ at 500 hPa and its time-averaged counterpart $\bar{\Psi}$ with $T = 10$ days. As expected, in both cases the averaged field shows an autocorrelation function which decreases more slowly than the one of the fine-scale variables, suggesting that $\bar{\Psi}$ now varies over a longer time-scale. As $T$ is increased (not shown) the persistence of correlations in the coarse-grained observable becomes even more pronounced. A novel feature, however, is revealed by Figs. 1(a) and 1(b), namely that the degree of enhancement of correlations as a result of averaging procedure is highly dependent on the location.

Figure 2(a) depicts the power spectrum of $\bar{\Psi}$ with the parameter values of Fig. 1(a). Clearly, the temporal power spectrum of the averaged observable is now sharper, suggesting a greater temporal regularity as contrasted with the flatter spectrum of the fine-scale variables, Fig. 2(b). In addition, a number of well defined, regularly spaced, peaks of decaying amplitude are present. This reflects the existence of a new time-scale resulting from the averaging procedure.

Of interest is also the way time averaging affects the distribution of the power among the space-scales of the motion. To this end, we have computed the spatial power spectrum as a function of the total wave number $n$, defined as,

$$S(n) = \left\langle \sum_{m=-n}^{n} \Psi_{m,n}(t) \Psi_{m,n}(t) \right\rangle$$

(6)

where the angle brackets denote a mean over successive spatial configurations. Figure 3(a) displays $S(n)$ at 500 hPa for the fine-scale variable, and for an averaging period of $T = 5$ days and $T = 10$ days. Clearly, as the averaging period becomes longer, the spatial power spectrum at smaller scales becomes steeper. This behaviour implies that time averaging acts predominantly by erasing part of the short-scale space variability. At first sight this behaviour seems rather surprising, since a time average as long as 10 days is expected to filter out larger scale features as well, such as baroclinic waves. The difficulty is resolved if one realizes that what is actually plotted in Fig. 3(a) refers to the full stream-function field. In Fig. 3(b) the spatial power spectrum of the fluctuation $\delta \Psi$ (anomaly field) from the
reference 'climate' configuration (see for instance Boer and Shepherd (1983)) is instead plotted for different averaging periods, along with its fine-scale counterpart. As one would intuitively expect, time averaging now acts as a filter for larger scales as well.

We now turn to the properties of the space-averaged observables. The autocorrelation function and the temporal power spectrum obtained with a set of 8192 data points sampled every 6 hours are represented (Figs. 4(a) and 4(b)) for a space averaging $M = 10$. As for time averaging, one notes an enhancement of the correlation in time and a clear organization of the temporal power spectrum around well-defined peaks, although, once again, the degree of enhancement of correlations and, concomittantly, the regularity of the spectrum depends both on $M$ and on the grid point considered. The distribution of the power among space-scales for $M = 0$ and $M = 10$ is shown on Fig. 4(c), indicating a drastic reduction of the power at small and intermediate (synoptic) scales when space averaging is performed.

In summary, there is an enhancement of the correlation time and a more organized spectrum associated with a reduction of the number of relevant scales in the dynamical
variability of averaged observables. Note, however, that although similar tendencies are found for space and time averages, differences in the detailed structures of the corresponding correlation and spectral functions are present: when time averaging is performed high-frequency properties of the dynamics are largely removed, but the effect on the spatial power spectrum is less drastic; conversely, when space averaging is performed, high wave-number modes are dramatically dampened but high-frequency modes keep a large amount of power. As we also see in the sequel, the detailed predictability properties of the two types of averages turn out to be rather different.

4. Error Dynamics of Time-Averaged Versus Fine-Scale Observables

(a) Formulation

We turn now to the dynamics of error growth of averaged observables in the QG model. Let \( \zeta_0(\mathbf{r}, 0) \) be an initial state of our dynamical system (here the stream function) represented by a vector in phase space and \( \zeta_1(\mathbf{r}, 0) \) a perturbed state displaced from the
reference field by a small error $\zeta'(\mathbf{r}, 0)$. The instantaneous error at time $t$ is defined as

$$E_t^2 = |\zeta'(\mathbf{r}, t)|^2 = |\zeta_0(\mathbf{r}, t) - \zeta_1(\mathbf{r}, t)|^2$$  

(7)

where $|.|$ is a norm, chosen here to be the Euclidean norm,

$$E_t^2 = \frac{1}{V} \int_V \zeta' \cdot \zeta' \, d\mathbf{r},$$  

(8)

where $V$ is the total volume of the atmosphere. In the spectral domain this norm leads to the following relation,

$$E_t^2 = \frac{1}{3} \sum_{i=1}^{3} \sum_{n=0}^{21} \sum_{m=-n}^{n} |\zeta'_{m,n,i} - \zeta_{m,n,i}|^2$$  

(9)

where $i$ is the index of the vertical level of the model and $^*$ denotes complex conjugation. This global error can also be represented in the spectral domain (at a given level $i$) as a
Figure 4. (a) Autocorrelation function and (b) temporal power spectrum of the space-averaged observable (Eq. (4)) with $M = 10$ (see text), obtained with the time series of Fig. 1(b). (c) Spatial power spectrum of the fine-scale (full line) and the space-averaged observables with $M = 10$ (dashed line) obtained after an integration period of 2000 days.
function of the total wave number,

\[ S_i(n, i) = \sum_{m=-n}^{n} \xi_{n,m,i}^{*} \xi_{n,m,i}. \]  

(10)

This relation will be used in the following in order to assess the role of scales on the error amplification for averaged observables.

Whatever the observable \( \xi \) might be, the instantaneous error \( E_i \) fluctuates in time on the system’s attractor. In order to capture its intrinsic properties we need therefore to adopt a probabilistic viewpoint whereby a large number of realizations starting from different initial conditions, chosen randomly over the entire attractor, are performed and the mean error evolution over these realizations is subsequently evaluated.

In this work we shall be interested in the role of the spectral distribution of the initial perturbation in the subsequent short-term dynamics of the mean error. To this end, we perform three different predictability experiments: The first deals with a generic initial perturbation of \( \Psi \) whose distribution in the spectral domain (Eq. (10)) is flat with respect to the total wave number \( n \), i.e., initial perturbations whose amplitudes are equally distributed among scales (Experiment 1); in the second and third experiments we consider, successively, initial errors introduced selectively at small (Experiment 2) and large (Experiment 3) scales. Notice that the second experiment accounts for the effect of errors arising from subgrid scales whereas the third one is consistent with the spectral error distribution found to be present after the assimilation procedure in large operational atmospheric models (Palmer 1996).

One might argue that experiments involving small scales are not particularly significant in this low-order model, designed for representing synoptic-scale motions. In the sequel, however, we include the upper part of the spectrum in this analysis, our purpose being primarily to emulate predictability experiments in operational forecasting models rather than to describe quantitatively the dynamics on small scales.

A practical problem arising when dealing with time-averaged observables is that the information on these observables is available only after the averaging period \( T \). This implies that the ‘initial’ error \( E_0 \) of the averaged field is, in reality, the one obtained after a period \( T \) beyond the time at which the perturbation is added to the control trajectory of the original system. Inevitably, during this period the instability acting on the fine-scale system will develop whereas time averaging will, in turn, tend to smooth out the evolution. As a consequence, the spectral distribution of the initial averaged error field is slightly deformed, whatever the initial perturbation introduced in the fine-scale variables. Nevertheless, as we will see in the sequel, since we are interested here in averaging periods of up to 10 days, the initial error field keeps its full significance after the averaging time.

\( (b) \) Experiments

As a first predictability experiment (Experiment 1) we add a uniformly distributed random perturbation with zero mean and variance \( 1.5 \times 10^{-13} \) to each spectral mode of the field \( \Psi(\mathbf{r}, t) \) in such a way that the initial mean error, as measured by Eq. (10), is identical at each total wave number \( n \). In addition, the random character of the perturbation precludes any effect due to systematic phase differences between errors in the different modes as well as in the different realizations. Figure 5 displays the evolution of the mean-error growth rate, defined as

\[ RA(t) = \frac{d \ln \langle E_1 \rangle}{dt} \]  

(11)
averaged over 1000 realizations, for the fine-scale variables, the averaged observables with $T = 2$ days, $T = 5$ days and $T = 10$ days. For the fine-scale variables, the mean error displays initially a decrease (negative values in Eq. (11)) which has been associated with the existence of stable directions transverse to the attractor in the multi-dimensional phase space of the model (see Vannitsean and Nicolis (1997)). After this period the mean error exhibits a variable growth rate in time and attains a maximum amplification rate after about 36 hours, equal to 0.25 day$^{-1}$ (full line of Fig. 5). This latter being larger than the value of the most positive Lyapunov exponent, the corresponding regime may be referred to as 'super-exponential' (Trevisan 1993; Nicolis et al. 1995). As regards the mean error for the averaged observables, one verifies, as expected, that for small values of $T$ (typically less than 2 days) the above qualitative features of the fine-scale variables are recovered. But as the averaging period is increased the initial decrease affecting the mean error evolution of the fine-scale variables tends to disappear. In that respect, therefore, the averaged observables are more unstable quantities than their fine-scale counterparts. Moreover, a closer look at the rate of growth of the mean error (Fig. 5) indicates that, for $T = 2$ days, $RA(t)$ attains a maximum higher than the one associated with the fine-scale variable ($\sim 0.28$ day$^{-1}$). As the averaging period $T$ is increased, this maximum value decreases such that for $T = 10$ days the rate of growth is close from the outset to the largest Lyapunov exponent of the original model, i.e. 0.23 day$^{-1}$ (Vannitsean and Nicolis 1997). These results indicate that for a perturbation introduced uniformly over all the scales of motion, there exists a crossover of the predictability properties of time-averaged and fine-scale observables. Indeed, for averaging periods $T \leq 2$ days, the fine-scale variables seem to be less unstable than the time-averaged ones whereas for $T > 2$ days the tendency is reversed.

The second experiment consists in introducing initial errors randomly sampled from a uniform distribution at each grid point of the original system, resulting in an error distribution in the spectral domain in which most of the power is confined at small scales (Experiment 2). The behaviour found is qualitatively similar as far as time averaging is
Figure 6. As in Fig. 5 but for an initial perturbation confined to large scales.

concerned to Experiment 1. However, as we will see in section 5, significant differences will appear when spatial averaging is performed.

Finally, we introduce an initial random perturbation to each spectral mode of $\Psi(r, t)$ in such a way that the initial mean error is essentially confined to large scales of the streamfunction field (Experiment 3). Figure 6 shows the growth rate of the mean error averaged over 1000 realizations associated with the fine-scale variables and the averaged ones with $T = 2, 5$ and 10 days. One observes that after a short ($t < 1$ day) initial decrease, the error associated with the original field grows, at least up to $t = 5$ days, with a rate smaller than the one predicted from the value of the largest Lyapunov exponent of the system (sub-exponential growth). This feature is apparent for the averaged observables with $T = 2, 5$ and 10 days as well. It seems, however, that as the averaging period increases, the sub-exponential behaviour becomes less pronounced and tends to the value of the most positive Lyapunov exponent. From these considerations, one is led to conclude that as the averaging period increases, the growth rate of the mean error of the averaged field at a given time $t$ becomes larger. Actually, a longer integration reveals that this conclusion holds for the first stage of the evolution (10 days). For longer times, all curves grow with a rate equal to the largest Lyapunov exponent until the saturation phase is reached. Note also that this saturation phase is advanced for increasingly large values of the averaging time, $T$ (not shown). If one assumes that the beginning of this stage is the upper limit of a good forecast, these results are in full agreement with the findings of Tribbia and Baumhefner (1988). Namely, as the value of $T$ is increased, the mean error of the time-averaged observables reaches the value of the corresponding climatic variance earlier.

(c) Interpretation

Figure 7 displays the spatial power spectrum of the mean error of the fine-scale variables (Fig. 7(a)) and time-averaged observable (Fig. 7(b)) with an averaging period of $T = 2$ days under the conditions of Experiment 1. The overall resemblance of the two patterns indicates that the stable and unstable directions in phase space (given by the Lyapunov vectors) in both systems are very close. Some qualitative differences are, however, apparent for short times ($t < 1$ day). Specifically, for the averaged observable the
Figure 7. Spatial power spectrum of the error (Eq. (10)) at $t = 0, \ldots, 9$ time units (500 hPa) averaged over 1000 realizations for (a) the fine-scale variables, and the time-averaged observables with averaging period $T$ equal to (b) 2 days and (c) 5 days. Initial perturbation as in Fig. 5.
initial decrease is less pronounced at the smallest and largest scales, whereas a more pronounced amplification occurs at the intermediate ones (synoptic scales). These features can be attributed to two factors. First, according to Fig. 3(a), time averaging reduces the power at small spatial scales and, consequently, also the stabilizing tendency associated with these scales (cf. section 2). As a result, the initial transient decrease of the mean error is dampened and beyond a sufficiently long averaging period ($T \sim 2$ days) it disappears altogether. Second, the very definition of the smoothing procedure, Eq. (3), implies that information on averaged observables is available only after a period of $T$ time units. During this period initial errors tend unavoidably to orient themselves along the most unstable directions in phase space, i.e. along the synoptic scales (compare full lines of Figs. 7(a) and 7(b)), thereby erasing the stabilizing effect of the large scales as well. This is further substantiated by the fact that for large $T$ (e.g. $T = 10$ days, Fig. 7(c)) all modes grow with a rate close to the largest Lyapunov exponent as if the initial perturbations were precisely introduced along the most unstable Lyapunov vector. The qualitative interpretation advanced for Experiment 1 on the averaged observables is valid for Experiment 2 as well (not shown).

Finally, Fig. 8 illustrates the spatial power spectrum of the mean error under the conditions of Experiment 3 (initial perturbations introduced selectively at large scales). Contrary to Fig. 7, we observe a very slow transfer of spectral power between large and small scales. The error dynamics for short times is therefore essentially driven by what happens in large scales. As these are under the direct effect of the stabilizing Lyapunov vectors (cf. section 2) the error growth rate will slow down, resulting in a subexponential stage, which is precisely what is found in this experiment. When time averaging is performed, the mechanism already invoked in connection with Experiment 1 is again operating (Figs. 8(b) and 8(c)); during the averaging time $T$, errors tend to orient themselves along the intermediate scales (the most unstable directions in phase space), thereby partly erasing the stabilizing effect of the large scales. The error behaviour becomes consequently less subexponential. Once again, as $T$ is increased, the subexponential character is progressively erased and the increase at synoptic scales where the principal instabilities are located dominates.

In summary, these results indicate the strong dependence of the predictability properties of the time-averaged observables on the spectral structure of the initial perturbation. Furthermore, they suggest that for time-averaging periods up to 10 days, the largest Lyapunov exponent of the original system keeps its full significance after a transient period. Finally, the spectral properties of the error suggest that the dominant instabilities of the averaged fields are centred around the synoptic scales as in the case for their fine-scale counterparts.

$$(d) \quad \text{The effect of the norm}$$

Up to now, we have used a Euclidean norm (Eq. (8)) to measure the error evolution, usually referred to as the $\Psi$-norm. Here we comment on the influence of the type of norm used on the error behaviour by considering a kinetic energy (K-E) norm,

$$E^2 = \frac{1}{2V} \int_V \nabla \zeta' \cdot \nabla \zeta' \, dr. \quad (12)$$

We first consider a predictability experiment in which the perturbation is introduced at small scales in the above norm. The results turn out to be qualitatively similar to those of Experiment 1 of section 4(b). This is related to the fact that this type of perturbation would correspond to a flat spectrum in the $\Psi$-norm. The interpretation presented in section 4(c) therefore remains valid.
Figure 8. As in Fig. 7 but for an initial perturbation confined to large scales.
Consider next, initial perturbations possessing either a flat spectrum or a spectrum centred predominantly on large scales in the K-E norm. At first sight one would expect to find an error behaviour, as measured by Eq. (12), similar to Experiment 3 of section 4(b) since this type of initial perturbation attributes a small weight to the small scales of the stream-function field. Surprisingly, however, the observed behaviour turns out to be qualitatively similar to Experiment 1. The explanation is to be found in the penalization of large scales in the K-E norm. Indeed, as we saw in Experiment 3, the spectral modes corresponding to these large scales were responsible for the slow increase of the error.

5. Error dynamics of space-averaged versus fine-scale observables

Figures 9(a), 9(b) and 9(c) display, respectively, the mean-error growth rate of the space-averaged observables (Eq. (4)) for Experiments 1 to 3 with \( M = 2 \) and \( M = 10 \). For comparison purposes, we have also plotted the results obtained when no averaging is performed. In Fig. 9(c), it seems that the short-term predictability of space-averaged observables is enhanced since the growth rate of error is substantially smaller than the one corresponding to the fine-scale variable. The situation is completely different when the initial perturbation is selectively introduced at small scales (Fig. 9(b)). Here space averaging seems to favour the unstable character of the dynamics since the growth rate of initial errors is substantially larger than the one corresponding to the fine-scale variable. As the averaging value increases, the maximum value of the growth rate damps and tends to be displaced toward larger times. The case depicted in Fig. 9(a) referring to Experiment 1 is intermediate between the former two. Indeed, it appears that for small \( M \)s (\( M \leq 3 \)) the predictability of averaged fields is weaker than the fine-scale ones, whereas for large \( M \)s the situation is reversed. After about 10 days, the mean error in all cases grows with a similar rate, equal to the largest Lyapunov exponent of the original system (which keeps therefore its full significance for the space-averaged system as well). It should also be noted that the saturation stage (not shown) of the mean error for the fine-scale and averaged observables is very similar, indicating that the long-range skill of both observables is comparable.

To understand these results we first recall that, when spatial averaging is performed, small-scale features are partly smoothed out (see Fig. 4(c)). As a result, the error dynamics of averaged observables is dominated by the behaviour of the larger scales. To confirm this point, we have plotted on Figs. 10(a) and 10(b), respectively, the spectral distribution of the error at 500 hPa for the fine-scale and the averaged observables with \( M = 10 \) in the case of Experiment 1. The initial mean error for the fine-scale variables is here uniformly distributed among all the scales of the motion while the one of the space-averaged observables results in a dominant component at the largest scales (Fig. 10(b)). After a few days, the spectral distribution of the error for the fine-scale field shows an important peak around total wave number \( n = 7 \) (synoptic scales) where the dominant instability is present (Vannitsem and Nicolis 1997). This suggests that the dynamics of the mean error curve of the fine-scale variables (Fig. 10(a)) is essentially driven by synoptic scales. In the case of the averaged observables (Fig. 10(b)), the spectral power of the mean error is centred at \( t = 0 \) around wave number \( n = 1 \) where stable Lyapunov vectors are dominating. The mean error of the averaged observables is thus bound to display initially a slower increase than the fine-scale one while it increases slowly at synoptic scales. After a few days, the spectrum attains a self-similar shape different from that of the fine-scale variables, which is maintained until the saturation level is reached. The absolute maximum of this self-similar spectrum is in this case around \( n = 2 \). The above mechanism explains, a fortiori, the deceleration of error growth arising from averaging in Experiment 3 as compared with the fine-scale behaviour.
Figure 9. Mean-error growth rates, $RA(t)$, (Eq. (11)) of fine-scale (full line) and space-averaged observables with $M = 2$ (dashed line) and $M = 10$ (short-dashed line) (see text) for (a) an initial perturbation introduced uniformly among all the scales of the motion, (b) an initial perturbation confined to the small scales, and (c) an initial perturbation confined to the large space scales. Each curve has been obtained with 500 realizations.
As regards Experiment 2, the crux of the argument is that the perturbations are now introduced at small scales, where the power of the averaged observables is greatly reduced (Fig. 4(c)): the stabilizing effect of the corresponding Lyapunov vectors is thus masked. As a result the system is driven by the transfer of the error toward the scales that lie closest to the small ones, namely the intermediate (synoptic) scales where instability dominates as seen, precisely, in Fig. 10(b). Finally, when the K-E norm is used to quantify the error (cf. section 4(d)) the results obtained are qualitatively similar to the ones presented in Fig. 9.

6. THE ERROR DISTRIBUTION IN SPACE

In recent years, a great deal of effort has been devoted to the analysis of the regional characteristics of the atmospheric predictability (Horel and Roads 1988; Toth and Kalnay 1995). In the present section we analyse and compare the geographic distribution of error of fine-scale and averaged observables of the QG model.
In Fig. 11, the spatial distribution of the local error amplitude field, defined as,

$$
\epsilon(\mathbf{r}, t) = \left\langle |\zeta'(\mathbf{r}, t)| \right\rangle
$$

(13)

associated with the fine-scale variables at time $t = 5$ days is plotted at the three different levels of the model. Three important peaks in the northern hemisphere centered on the Pacific, on the west Asian continent and on the western part of the North Atlantic can be identified. In the southern hemisphere, there is also a maximum in the west Pacific. To understand this result we consider the spatial distribution of the dominant Lyapunov vector of the QG model computed as indicated in Vannitsem and Nicolis (1997). Figure 12 shows the averaged distribution of the amplitude of the Lyapunov vector corresponding to the largest Lyapunov exponent at 500 hPa. The similarity with the spatial distribution of the mean local error amplitude is evident and confirms the crucial role of this vector in the error dynamics. Notice that the particular spatial structure of Fig. 11 is already present after one day, indicating that the three regions concerned are very sensitive to small perturbations during this initial period.

The spatial distributions of the local error amplitude and the dominant Lyapunov vector are remarkably close to the one of the eddy activity concentrated in storm-track regions during the winter season (Hoskins and Valdes 1990; Buizza and Palmer 1995). This suggests that regions of important baroclinic activity are also the regions of major unpredictability.

We now turn to the spatial distribution of the mean error associated with time-averaged observables. Figure 13 displays the local error amplitude field (in arbitrary units) for an averaging period $T$ of 5 days at initial time $t = 0$. Clearly, the spatial structure is similar to that of the fine-scale observables (Fig. 11). This particular structure is maintained during the whole forecast period, suggesting that the different regions show similar persistence in their predictability properties. If $T$ is further increased, a similar distribution of the mean error in space is found. These results confirm further that the most unstable Lyapunov vector associated with the averaged observables displays a similar structure that one of the fine-scale variables.

Finally, we consider the spatial distribution of the error for space-averaged observables. Figure 14 displays the spatial structure of the mean error for $M = 10$ and $t = 3$ days. The maxima of the error are not confined anymore to the storm-tracks but, rather, to high latitudes and to the tropical regions. However, this result should be treated with some caution since the dynamics generated by the QG model corresponds to non-divergent wind fields which play a dominant role only at midlatitudes. Further studies with a more realistic representation of the polar and tropical dynamics should thus be performed in order to capture this particular feature of spatial averaging. Still, our results suggest that the local predictability properties of space-averaged observables are likely to be qualitatively different from both the fine-scale and the time-averaged observables.

7. Weather regimes and their predictability

In the preceding sections emphasis was placed on predictability experiments as a function of the spatial scales. Although the initial perturbation was confined in some of these experiments to a small part of the spectrum, subsequently all modes were becoming excited and as a result one could not associate the evolution of the relevant field with a particular weather regime.

On the other hand, the existence of preferred weather regimes in the atmosphere is now well established (Molteni 1994). Several authors have analysed their persistence properties
Figure 11. Spatial distribution of the mean local error amplitude (in arbitrary units) corresponding to the fine-scale variables after 5 days, averaged over 1000 realizations. The initial perturbation is introduced uniformly at all scales of the motion; (a) 200 hPa, (b) 500 hPa and (c) 800 hPa.
Figure 12. Spatial distribution of the amplitude of the dominant Lyapunov vector at 500 hPa as obtained after an averaging over 300 days.

(e.g. Palmer 1988) and their short-term sensitivity to initial perturbations (Molteni and Palmer 1993; Trevisan 1995; Frederiksen 1997). The present section is devoted to a novel classification of weather regimes of the model on the basis of their instability properties.

We first select weather fields of poor predictability associated with an amplification rate of the dominant Lyapunov vector larger than 0.4 day$^{-1}$. The mean field anomaly of these weather regimes and the mean distribution of the corresponding Lyapunov vectors are depicted in Figs. 15(a) and 16(a), respectively, for the northern hemisphere as obtained after an integration of 20,000 days.

We next select weather configurations of high predictability corresponding to amplification rates of the dominant Lyapunov vector smaller than 0.1 day$^{-1}$. The mean field anomaly and the mean spatial distribution of the corresponding Lyapunov vector are shown in Figs. 15(b) and 16(b).

We notice that the dominant Lyapunov vector associated with local amplifications greater than 0.4 day$^{-1}$ (Fig. 16(a)) possesses a maximum over the western Pacific while for values less than 0.1 day$^{-1}$ (Fig. 16(b)) two maxima appear over the eastern Pacific and over the western Atlantic. More importantly, the corresponding mean anomaly fields (Figs. 15(a) and 15(b)) correspond approximately to opposite anomaly patterns presenting strong similarities with two clusters (C'2 and C'5) found by Molteni et al. (1990) and qualified in Marshall and Molteni (1993) as two characteristic weather regimes of the QG model. This suggests an interesting relation between persistent patterns and predictability.

8. CONCLUSIONS

The dynamics of time- and space-averaged observables have been investigated on a 3-level quasi-geostrophic model and compared with the one of the fine-scale variables. One of the main results of the analysis is the invariance of the largest Lyapunov exponent of the model against averaging periods up to 10 days, indicating that the intrinsic limitations in predictability are identical for averaged and fine-scale variables. Moreover, the saturation phase of the error starts practically at the same time for space-averaged and fine-scale variables, while for time averages it starts earlier. A similar tendency for a poor improvement of the long-range skill in the saturation phase of averaged observables
Figure 13. As in Fig. 11 but for the time-averaged observables with averaging period $T = 5$ days at initial time.
Figure 14. As in Fig. 13 but for the space-averaged observables with $M = 10$ (see text) and after 3 days.
was reported by Tribbia and Baumhefner (1988) for the National Center for Atmospheric Research Community Climate Model.

On a short-term basis some improvement seems to be possible, but it depends crucially on the type of the initial perturbation: for time-averaged observables a decrease of the growth rate of small perturbations was found for large values of the averaging period when perturbations were uniformly introduced at all the scales of the motion; for space-averaged observables and for perturbations uniformly distributed at all the scales of motion or confined to the largest scales, the short-term predictability was largely improved. These behaviours have been interpreted in terms of the evolution of the error in the spectral
domain, driven essentially by the spectral properties of the stable and unstable Lyapunov vectors of the system.

The spatial distribution of the error for averaged and fine-scale observables has also been investigated. The results can be summarized as follows:

- The spatial structures of the mean error of time-averaged observables and fine-scale variables are very similar, suggesting that the mechanisms underlying the intrinsic instability linked to the most unstable Lyapunov vector are similar.
- The most unstable features (the Lyapunov vectors) of the dynamics are confined to the northern hemisphere for time-averaged and fine-scale observables during the winter period.
The spatial distribution of the mean error for space-averaged observables displays a very different pattern than the one of the fine-scale variables. Further studies on more realistic models are needed to confirm that this conclusion holds true in polar and tropical regions.

Finally, a classification of weather regimes on the basis of their predictability properties has been achieved and compared with the results of conventional cluster analysis performed on the QG model. Strong similarities between weather regimes obtained from the two approaches were found. This promising ‘dynamical’ clustering will be investigated further.

The analysis performed in the present work was based on initial perturbations chosen randomly from a uniform distribution. In large operational models, the initial error is the result of a complex data-assimilation procedure, such as the 4-D variational assimilation currently used at the European Centre for Medium-Range Weather Forecasts (Courtier et al. 1994). But, as noted by Pires et al. (1996), such a procedure tends to project the initial error onto the unstable manifold. In order to investigate the possible enhancement of predictability of averaged observables in this context it would be useful to analyse a simple model, such as the 3-level quasi-geostrophic model used here, in which the initial field is generated by a data-assimilation scheme (see, for instance, Houtekamer and Derome (1995)).

Long-range forecasting properties depend also on the slow variations in the boundary conditions and external forcings (Royer 1993; van den Dool 1994; Palmer 1996). These aspects would undoubtedly be worth investigating in the perspective of the present study through an estimation of the sensitivity of the short-term error evolution as a function of these factors.

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