Analysis of the eyes formed in simulated tropical cyclones and polar lows

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SUMMARY

The structure and size of the eyes generated in numerically simulated tropical cyclones and polar lows have been studied. A primitive-equation numerical model simulated systems in which the structures of the eyes formed were consistent with available observations. Whilst the tropical cyclone eyes generated were usually rapidly rotating, it appeared impossible for an eye formed in a system with a polar environment to develop this type of structure. The polar low eyes were found to be unable to warm through the subsidence of air with high values of potential temperature, as the environment was approximately statically neutral. Factors affecting the size of the eye were investigated through a series of controlled experiments. In mature tropical cyclone systems the size of the eye was insensitive to small changes in initial conditions, surface friction and latent and sensible heating from the ocean. In contrast, the eye size was strongly dependent on these parameters in the mature polar lows. Consistent with the findings, a mechanism is proposed in which the size of the eye in simulated polar lows is controlled by the strength of subsidence within it.

KEYWORDS: Eye Eye size Eyewall Numerical model Subsidence

1. INTRODUCTION

The eye formed in a tropical cyclone is one of its most fascinating features. This is a warm central core, often free of clouds, in which the windspeeds are comparatively small. A clear eye is a consequence of subsidence in the eye and is not usually observable until a system becomes relatively intense; as found by Dvorak (1975) from satellite observations of tropical cyclones. In contrast, the eyewall is an outwards sloping ring of intense cumulus convection within which the maximum windspeeds are found.

Similar eye features are also observed in the group of highly convective polar lows that closely resemble tropical cyclones in structure and which have been proposed to intensify through a similar mechanism (Emanuel and Rotunno 1989; Nordeng and Rasmussen 1992). However, the degree of similarity is difficult to determine since there are far less observational data available from polar low cores than from tropical cyclone cores.

Aircraft observations have shown that tropical cyclone intensification is associated with the contraction of the eyewall (Willoughby 1990). Outer convective rings, also local maxima of windspeed, often exist and have been observed to contract round the original eyewall, sometimes replacing it. The factors affecting the size of the eye in a mature tropical cyclone or polar low are still open to debate. Kuo (1959) derived a formula for the radius of maximum winds in a steady state, inviscid, adiabatic, axisymmetric vortex using the constraints of conservation of absolute angular momentum, potential temperature and total energy. However, this radius is dependent on the value of an arbitrary outer radius, the negligible windspeed origin of the converging low-level air in the system.

This paper aims to investigate the topics outlined above through numerical modelling. In the first two parts, the eyes generated in simulated tropical cyclones and polar lows are compared and the dynamical causes of the differences examined. In the final part, the sensitivities of the size of eyes in mature tropical cyclones and polar lows to factors which may vary in nature are studied. It is hoped that the information gained will help determine the minimal physics necessary to incorporate a realistic eye into a simple analytic or numerical balanced model such as that developed and evaluated in Gray and Craig (1998).

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In detail, this paper is divided into five further sections. The first (section 2) describes briefly the numerical model used. Section 3 examines the structures of the eyes generated in a simulated tropical cyclone and polar low and verifies that they are consistent with observational data. The sensitivity of the simulated systems to the magnitude of the parametrized horizontal eddy momentum diffusion used is also discussed in this section. Section 4 investigates the cause of the structural difference between the polar low and tropical cyclone eyes. The factors which control and affect the size of the eye in mature systems are discussed in section 5 which includes comments on the mechanism controlling the eye size in the simulated polar lows. Finally, section 6 contains some conclusions.

2. NUMERICAL MODEL

The experiments are performed using modified versions of the nonhydrostatic, axisymmetric tropical-cyclone model originally written by Rotunno and Emanuel (1987) and used to simulate polar low development by Emanuel and Rotunno (1989). The versions used here include the modifications described by Craig (1995, 1996). Details of the control configurations are given in Table 1. The high horizontal resolution (5 km) used in the simulations provides, at a minimum, tolerable resolution of the region within the radius of maximum windspeed. The axisymmetry of the model is restrictive, however, tropical cyclone eyes (especially those in relatively intense systems) are often observed to be highly axisymmetric in horizontal wind structure (e.g. Marks et al. 1992, Fig. 6).

Different microphysical schemes are used in the versions of this model applicable to tropical cyclone and polar low simulations. The scheme used for the tropical cyclone version is that used by Craig (1996) and carries four water variables: water vapour, cloud droplets, rain and ice particles. In the polar low version of the model the microphysical scheme incorporated into the model by Craig (1995) is used. This parametrization of cloud microphysics was specifically developed for the ice-phase microphysics that occurs in polar lows. This scheme carries just two water variables, water vapour and ice, with all condensed water assumed to exist in the latter state. Both model versions retain the original condensation/evaporation technique employed by Rotunno and Emanuel (1987).

Model simulations are initiated with either a tropical or polar environmental sounding (following the original simulations). Specifically these are mean hurricane season sounding of Jordan (1958) and a modified version of the 12 GMT 13 December 1982 sounding from Bear Island in the Norwegian sea taken just inside the northern limit of the cloud shield associated with a polar low. Imposed low-level vortices are used to initiate convection. These vortices are defined by simple azimuthal windfield functions with maximum

| TABLE 1. MODEL CONFIGURATION FOR THE CONTROL POLAR LOW AND TROPICAL CYCLONE SIMULATIONS |
|------------------------------------------|----------------|----------------|
| Model parameter values                  | Polar low      | Tropical cyclone |
| Horizontal domain size (not including sponge layer) | 2200 km        | 3150 km        |
| Vertical domain size (not including sponge layer) | 14 km          | 22.5 km        |
| Horizontal resolution                   | 5 km           | 5 km           |
| Vertical resolution                     | 1 km           | 1.25 km        |
| Sea surface temperature                 | 279 K          | 300 K          |
| Coriolis parameter                      | $1.36 \times 10^{-4}$ s$^{-1}$ | $6.14 \times 10^{-5}$ s$^{-1}$ |
velocities of 15 m s\(^{-1}\) at 75 km radius and 10 m s\(^{-1}\) at 50 km radius for the control tropical cyclone and polar low simulations respectively. The initial thermodynamic structure of the atmosphere is adjusted to be in thermal wind balance with the azimuthal windfield.

The dependencies of the structure and size of simulated eyes on surface fluxes are examined in this paper (section 5(b)). Fluxes of heat and moisture (sensible and latent heating) and momentum (surface friction) from the ocean surface are parametrized using bulk aerodynamic formulae in this model. These are controlled by nondimensional coefficients and the fluxes are given by

\[
F_\theta = C_T U (\theta_s - \theta) \tag{1}
\]
\[
F_q = C_E U (q_s - q) \tag{2}
\]
\[
F_m = \rho C_D U^2 \tag{3}
\]

respectively, where \(\rho\) is density and \(U\) is the windspeed at the lowest model level. \((\theta_s - \theta)\) is the potential temperature difference between the sea surface and the lowest model level and \((q_s - q)\) is the corresponding difference for the water vapour mixing ratio. The momentum drag coefficient, \(C_D\), is windspeed dependent according to Deacon's formula (following Rotunno and Emanuel 1987) whereas the heat and moisture transfer coefficients, \(C_T\) and \(C_E\), are assumed to be independent of wind speed. The numerical model is described in greater detail in Craig and Gray (1996).

3. **Eye structure**

The structure of the eyes formed in the control tropical cyclone and polar low simulations will be described in this section. The control simulations are those using the parameter values given in the previous section. This analysis will concentrate on the azimuthal and vertical velocity and thermodynamic components of the structure. Observations show that the azimuthal velocity peaks in the region of strong convection defined as the eyewall and that the strongest updraughts are found slightly inside this azimuthal velocity maxima (Jorgensen 1984(a) and (b)). It will be assumed that if the numerical model generates eye structures which are in agreement with available observational evidence then it represents the essential physics required to develop and maintain the eyes produced by these systems in nature.

(a) **Azimuthal velocity structure**

The azimuthal velocity, \(v\), fields of the core regions of the simulated tropical cyclone and polar low are illustrated in Fig. 1 which shows ‘snapshot’ radius-height cross sections. These snapshots are taken during the strongly intensifying stage of the vortices (at 90 and 60 hours for the tropical cyclone and polar low simulations respectively). A region of high azimuthal velocity is apparent in both figures, though at a larger radius in the simulated polar low. The radial peak in windspeed bends slightly outwards with height and the magnitude of \(v\) decays significantly with height, becoming negative (anticyclonic vortex) at high altitudes and radii greater than 300–400 km.

There is a marked difference between the low-level velocity structure within the radius of maximum azimuthal velocity, \(r_{\text{max}}\), in the two snapshots. These differences become clearer on examining the radial distribution of azimuthal velocity at a given height. The strongest azimuthal velocities were often found to exist in the lowest model level (the boundary layer) despite the damping effect of surface friction there. The radial distribution of the boundary-layer azimuthal velocity at ten-hourly intervals for the control tropical
Figure 1. Contour plot of azimuthal velocity from (a) the control tropical cyclone simulation at 90 hours and (b) the control polar low simulation at 60 hours on height against radius axes. Contour interval is 5 m s⁻¹. Raw data are smoothed with a boxcar average of five data point width in the horizontal and vertical.

cyclone and polar low simulations is shown in Fig. 2. As the vortices tend to maturity, the radius and magnitude of the maximum azimuthal velocity approach constant values in both simulations. When reporting the size of mature eyes it will be necessary to first verify that this has occurred during each simulation.

The tropical cyclone eye region is in approximate solid body rotation i.e., \( v \sim (r/r_{\text{max}}) v_{\text{max}} \) with \( r_{\text{max}} \sim 20 \) km for the mature system (Fig. 2(a)). The approximately linear increase in \( v \) with radius outwards from the central axis is tolerably resolved with four grid points although this resolution is insufficient to distinguish between a linear and concave increase in \( v \) with radius. Observations are consistent with rapidly rotating trop-
Figure 2. Radial distribution of the boundary-layer azimuthal velocity at ten hourly intervals for the (a) tropical cyclone and (b) polar low simulations. The last time plotted is in bold.

In contrast, the polar low eye region is not in rapid rotation and the radius of maximum azimuthal velocity is much greater than that observed in the tropical cyclone simulation at \( \sim 60 \) km for the mature system (Fig. 2(b)). The windspeeds inside the eyewall decay from those of the imposed initial vortex over time. The mature eye region is very calm with small \( v \) values outwards from the centre to the eyewall where a sharp transition occurs to high windspeeds. Just inside the eyewall there is a small region where diffusional effects smear out the velocity profile to reduce the abrupt transition from very small to very large
velocities. The width of this transition region increases with height (as can be seen from Fig. 1(b)).

The limited available observational data from polar low cores makes this model prediction for the velocity distribution difficult to verify. However, there is some data which suggest polar lows may have large eyes with very small azimuthal velocities. Weathership data and data from a surface station on Bear Island during the passage of a polar low (Rasmussen et al. 1992, Figs. 12 and 17) indicate a large eye (of radius at least ~50 km) which may contain an area of reasonable calm in its centre. Aircraft data from a polar low over the Northern gulf of Alaska (Douglas et al. 1991, Fig. 21) show a very sharp gradient in azimuthal velocity across the eyewall region, within which the air is very calm. Unfortunately, the aircraft entered the eyewall from the south and exited to the west which makes it difficult to judge the size of the eye from this data.

(b) Temperature structure

A significant, positive temperature anomaly, which peaks at two altitudes, develops in both the tropical cyclone and polar low simulations (Fig. 3). The heights and magnitudes of these peaks in the tropical cyclone simulation agree well with those in observed systems such as Hurricane Inez which, on 28 September 1966, had maximum temperature anomalies of 11 and 16 °C at approximately 4.5 and 11 km altitude respectively (Hawkins and Imbembo 1976). Suitable observational data from polar lows for a similar comparison could not be found.

The development of the thermodynamic structure in the core region of the tropical cyclone and polar low simulations is illustrated in Fig. 4. The radial distribution of the mean midlevel potential temperature is plotted at ten-hourly intervals. Although substantial warming occurs in the eye region in both simulated vortices as the system intensifies, only in the tropical cyclone does the eye region warm more than the eyewall. The potential temperature structure is in approximate thermal wind balance with the azimuthal velocity structure in both systems. The core azimuthal windspeed decays rapidly with height in the tropical cyclone (in balance with the radially decreasing potential temperature) whereas the small low-level windspeeds in the polar low eye are in balance with the approximately radially constant polar low potential temperature.

(c) Sensitivity of the numerical model to the parametrization for turbulence

Malkus (1958) and Kuo (1959) suggested that large eddy fluxes of angular momentum from the eyewall into the eye drive the observed anticlockwise secondary circulation inside the tropical cyclone eye. In the presence of the observed low-level radial outflow, approximately conserving absolute angular momentum, eddy momentum fluxes must exist to form and maintain a rapidly rotating eye. For horizontal momentum diffusion to occur, horizontal gradients in the relative angular momentum must exist. Once the relative angular momentum is well mixed in the eye region, then in order to transport more angular momentum into the eye the azimuthal velocity in the eyewall must increase (or that in the centre decay), disrupting the well mixed state. This implies that the wind profile inside the eye must be (at least slightly) concave in an intensifying system. A concave wind profile is in better agreement with some observations (e.g. Willoughby 1990) than a profile which is in strict solid body rotation.

Eddy momentum fluxes are mainly parametrized in the model so it is important to verify that the structure and development of the eye formed are relatively independent of the amount of interior turbulence occurring. In particular, it must be verified that the different eye structures generated in the simulated tropical cyclone and polar low are not a
consequence of insufficient horizontal diffusion in the latter system, or excessive horizontal diffusion in the former.

The turbulence parametrization used in the model is described thoroughly in Emanuel and Rotunno (1987). In brief, a Richardson number-dependent parametrization (Smagorinsky 1963) is used for the calculation of the eddy viscosity (and the same calculated value is also used for the eddy heat conductivity). To maintain numerical stability in the horizontal, this viscosity value is usually replaced there by one calculated using an assumed mixing length (unless the former value is greater). This leads to typical area averaged horizontal diffusion coefficients of around 600 m² s⁻¹ (peak values exceeding double this) in both the tropical cyclone and polar low eyewall (sample values taken 50 hours into the control simulations, lowest model level, area averaged out to 200 km from the centre of the system.
(tropical cyclone) or from the inner edge of the eyewall (polar low)). It should be noted that there may also be some resolved, axisymmetric eddies in the system.

Experiments were performed to examine the sensitivity of the simulated tropical cyclone and polar low to the amount of parametrized horizontal diffusion (not shown). The horizontal viscosity in the model was constrained to be calculated by the assumed mixing length only and the dissipation was increased and decreased by an order of magnitude. It was found that the simulations were relatively insensitive to the amount of radial diffusion provided it was not extremely small. Although increased diffusion damped the tropical cyclone and polar low growth, increased their eye sizes and, in the simulated polar low, smeared out the transition region between the calm eye and the eyewall, the eye structure was not significantly altered. However, in the tropical cyclone simulation in which the diffusion was decreased by an order of magnitude the vortex intensity was decreased and the eye structure altered such that it resembled that of the control polar low simulation.
(i.e., a relatively calm eye with a sharp azimuthal velocity gradient to the eyewall). This suggests that radial momentum diffusion (exceeding a critical amount) is essential to form a realistic eye. These results are in agreement with similar experiments performed by Emanuel (1989) using a relatively simple, numerical balanced axisymmetric model and the importance of horizontal diffusion to tropical cyclone intensification was recently demonstrated by Emanuel (1997).

4. **Eye Dynamics**

The relatively high temperatures observed within the tropical cyclone eye region, especially in the middle and upper troposphere, are generally agreed to be a result of dry adiabatic warming through subsidence in a statically stable atmosphere (Smith 1980). The temperature of an individual air parcel increases through adiabatic compression as it descends, but, for the *potential* temperature in the eye to increase, high potential temperature air must be advected downwards. Mechanisms for this subsidence have been presented in the literature specifically for tropical cyclone eyes, however, the process which occurs in polar low eyes will be assumed here to be similar.

Subsidence in the eye is part of a forced secondary circulation in an approximately balanced system (in thermal wind balance). Malkus (1958) and Kuo (1959) proposed that the driving force for this circulation is the momentum source of horizontal eddy fluxes of absolute angular momentum from the eyewall. Alternatively, Willoughby (1979) proposed that the heat source generated by cumulus convection in the eyewall drives the circulation. The relative contributions of these two forcing terms is yet to be determined, however, the ratio is constrained such that the heat source cannot act as the sole forcing. Such a forcing would lead to an eye with lower potential temperatures than those in the eyewall. This was illustrated by Schubert and Hack (1982, Fig. 6) for a barotropic vortex with a solid body rotation in the inner region and an outer azimuthal velocity structure decaying more rapidly than (radius)\(^{-1}\). Vertical eddy fluxes of angular momentum caused by cumulus convection may also contribute to the forcing but are unlikely to do so significantly since, if they did so, the eye subsidence would still occur strongly in a mature system. This would contradict observations (Jordan 1961) which indicate stronger subsidence in intensifying tropical cyclones than in steady-state or weakening ones. A decay in the mean vertical velocity in the eye was also observed in the simulated systems as they reached maturity (not shown).

(a) **The difference between polar low and tropical cyclone eyes**

Why does the eye warming exceed the eyewall warming in the tropical cyclone but not the polar low simulation? There are two obvious possible causes: firstly a lack of subsidence in the polar low eye and secondly a lack of warming through this subsidence. The mean vertical velocity between 40 and 50 hours (during the strongly intensifying stage) for the control tropical cyclone and polar low simulations is shown in Fig. 5 on radius-height cross sections. Subsidence occurs at midlevels in both simulations, strongest at heights of ~5 km in the polar low and ~10 km in the tropical cyclone. The magnitudes of the mean vertical velocities are approximately the same in both cases (peaking at ~−0.04 m s\(^{-1}\)). This implies that it is unlikely the polar low eye is failing to warm due to a lack of eye subsidence.

The initial environmental soundings are markedly different in the two simulations. The initial potential temperature, \(\theta\), profiles in the centre of the model vortices are shown in Fig. 6. In the tropical cyclone profile, \(\theta\) increases steadily with height until the tropopause is reached at approximately 15 km, above which \(\theta\) increases rapidly. By contrast, in the
polar low sounding \( \theta \) is virtually constant with height above the boundary layer until the tropopause is reached at approximately 7 km. An almost statically neutral troposphere (\( \theta \) approximately independent of height) prevents the eye region from warming through the downwards advection of high \( \theta \) air, unless air is transported across the tropopause (which does not appear to be happening in this simulation). The sounding maintains this weak stability during intensification as shown by the profile at 60 hours. Hence, in the model at least, the potential temperature in the polar low eye cannot exceed that in the eyewall for any reasonable rate of forced eye subsidence; the eye cannot warm more than the
eyewall. The consequent approximately radially constant potential temperature in the eye implies that the azimuthal windspeed must be independent of height through thermal wind balance. Negligible windspeeds were found in the eyes formed in simulated polar lows which satisfied this condition. The environmental sounding thus prevents the simulated polar lows from forming eyes which are rapidly rotating.

Although the nonexistence of cross-tropopause flow in polar lows formed in nature cannot be proven, some evidence does exist for tropical cyclone systems. Suggestive results were obtained from lidar cross-sections of ozone from Supertyphoon Mireille. These showed no evidence for the downward entrainment of stratospheric air into the eye region (Newell et al. 1996). It should be noted, however, that the high tritium levels observed in the eye of Hurricane Betsy could indicate the addition of subsiding tritium-rich air from the stratosphere (Östlund 1968).

To extend these conclusions to systems in nature the realism of the model soundings must be examined. The tropical cyclone simulations were initiated with a mean tropical sounding (Jordan 1958). This suggests that the initial sounding in the simulated eye is likely to be reasonably realistic when compared to systems in nature. The polar low simulations were initiated with a sounding taken from the highly convective environment of a polar low system which occurred in the Barents Sea rather than a mean polar sounding (see section 2 for further details). To conclusively show that a similar thermodynamic structure exists in the eyes of polar lows in nature, further observational evidence is required.

There is evidence that weak static stability exists near the centre of (at least some) polar low systems in nature which would inhibit the warming of an eye through the downwards advection of air with high potential temperatures. For a polar low which occurred over the northern Gulf of Alaska, Douglas et al. (1991) found a close correspondence between the low-level circulation centre of the system and a minimum in static stability (by analysis of the difference in potential temperature between 950 and 550 mb calculated from omega dropwindsonde and aircraft soundings). A cross-section of potential temperature through the vortex centre showed this minimum to be the result of a midtropospheric dome of cold air between ~700 and 400 mb. Within this cold dome the air at the centre was warmer than the environmental air (at the same pressure level) near the surface but noticeably colder by 500 mb. This meant that weak dry static stability was present between the surface and
600 mb, overlain by highly stable upper tropospheric air. The authors state that this is in agreement with the composite results of Businger (1987) which found that polar low occurrence was closely correlated with a minimum in 1000–500 mb thickness. In a case study of the system from which the initial environmental sounding for these simulations was taken Rasmussen et al. (1992) found that a large, synoptic-scale cold dome also existed across the centre of the system (calculated from omega dropwindsonde soundings and a rawinsonde sounding from a surface station).

5. Eye size

The structure and dynamics of the eyes formed in tropical cyclone and polar low systems were discussed in sections 3 and 4, but, further information is required to explain their size. The eyes formed in tropical cyclones generally contract as the systems intensify (as discussed in the introduction). The size of the eye in mature systems, or the size to which it eventually contracts (in the case of tropical cyclones), will be the focus of this work.

The mechanism which controls the size of the eye was investigated by analysing a series of controlled experiments, described in sections (a) and (b). Simulations were performed to examine the sensitivity of the model eye to the following parameters which could all vary in nature: the absolute angular momentum of the imposed initial vortex used to initiate convection in the model (a function of the Coriolis parameter and the radial distribution of $v$ in the imposed vortex), surface friction, and the intensification rate of the vortex (which is sensitive to the rate of transfer of heat and moisture from the ocean surface).

The model time for which the simulations were run varied with the experiment being performed. Each simulation was run beyond the time at which it reached maximum intensity as measured by the maximum azimuthal velocity. The size of the eyes in the mature systems was estimated from snapshots of boundary layer (first model level) azimuthal velocity against radius at five points in time; three before and two after the time at which the azimuthal velocity peaked. The first model level was chosen because the eyewall was sharpest there in the polar low simulations (see Fig. 1(b)) and thus the size of the eye was easiest to judge. Also, the azimuthal velocity was often greatest there in both the tropical cyclone and polar low systems despite the effects of surface friction.

(a) Sensitivity of the eye size and azimuthal velocity structure to the initial conditions

Tropical cyclone and polar low genesis requires a pre-existent disturbance. For example, for tropical cyclones, possible such disturbances are easterly waves and cyclonic disturbances on fronts which penetrate the tropics (Anthes 1982; Riehl 1954). The structure of a mature system may be sensitive to the initial vortex. It may also be sensitive to the value of the Coriolis parameter at the latitude where it develops. Simulations were performed with both the tropical cyclone and polar low versions of the model in which the value of the Coriolis parameter, $f$, and the structure of the imposed, initial vortex were varied. This vortex is strongest at low levels, decaying with height, and has an initial surface maximum in azimuthal velocity, $v_{\text{max}}$, at a radius $r_{\text{max}}$ (as described in section 2).

In the mature simulated tropical cyclones the size of the eye was found to be approximately insensitive to initial conditions. Simulations were performed in which $r_{\text{max}}$ was reduced to 25 km, $v_{\text{max}}$ was altered to 5, 7.5 and 20 m s$^{-1}$ and $f$ was altered to $3.07 \times 10^{-5}$ and $1.36 \times 10^{-4}$ s$^{-1}$. In all cases the estimated eye sizes were within $\pm 5$ km. These results suggest that if a rapidly rotating eye is formed, then the size of this eye is relatively insensitive to the initial relative and planetary vorticity in the system.
TABLE 2. VARIATION OF THE ESTIMATED MATURE POLAR LOW EYE SIZE WITH CORiolIS PARAMETER, $f$, THE RADIUS OF MAXIMUM WINDS, $r_{\text{max}}$, AND MAXIMUM VELOCITY, $v_{\text{max}}$, OF THE IMPOSED INITIAL VORTEX. ESTIMATED EYE SIZES ARE TO WITHIN APPROXIMATELY ±5 km

<table>
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<th>Experiment</th>
<th>$r_{\text{max}}$ (km)</th>
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<td>Control</td>
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<tr>
<td>$r_{\text{max}} = 25$ km</td>
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<td>$r_{\text{max}} = 35$ km</td>
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<tr>
<td>$r_{\text{max}} = 65$ km</td>
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<tr>
<td>$r_{\text{max}} = 75$ km</td>
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<tr>
<td>$v_{\text{max}} = 5$ m s$^{-1}$</td>
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<tr>
<td>$v_{\text{max}} = 7.5$ m s$^{-1}$</td>
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<tr>
<td>$v_{\text{max}} = 15$ m s$^{-1}$</td>
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<td>$f = 6.14 \times 10^{-3}$ s$^{-1}$</td>
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</tr>
<tr>
<td>$f = 1.43 \times 10^{-4}$ s$^{-1}$</td>
<td>87</td>
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</table>

In contrast, the size of the eye in the mature simulated polar lows is strongly dependent on initial conditions. The estimated eye sizes in the mature simulated polar lows are given in Table 2. The eye size increases with increasing values of $f$ and $r_{\text{max}}$, but decreases with increasing values of $v_{\text{max}}$. It appears contradictory that whilst an increase in planetary vorticity, $f$, increases the eye size, an increase in the amplitude of the imposed initial vortex, which implies an increase in the relative vorticity of the vortex, reduces it (although a reason for this behaviour will be suggested in section (c)). None of the simulated polar lows developed an eye which was rapidly rotating (even in the simulation using the $f$ value used in the control tropical cyclone simulation). This failure to form a rapidly rotating eye in simulations performed with a polar environment is in agreement with the conclusions of section 4.

(b) Sensitivity of the eye size and azimuthal wind structure to the surface friction and the transfer of heat and moisture from the sea surface

The heat and moisture transfer coefficients, $C_T$ and $C_E$, and the frictional drag coefficient, $C_D$, control the transfer of sensible and latent heat energy, and momentum from the ocean to the atmosphere through bulk aerodynamic formulae (described in section 2).

For the simulated tropical cyclones the size of the eye in the mature system is approximately insensitive to these parameters (as to the initial conditions). The experiments performed were identical to those performed with the simulated polar lows which are given in Table 3. The exception to this is the experiment in which the surface friction was reduced to zero. This had a dramatic effect, increasing the eye size markedly (to ~90 km) and changing the eye structure towards that associated with the polar low simulations (the initial windspeeds inside the radius of maximum winds failed to intensify).

The size of the eye in the mature simulated polar lows is sensitive to these parameters and the estimated eye sizes are given in Table 3. The size of the eye in the mature system increases with $C_T$ and $C_E$ (Fig. 7). This is associated with an approximately linear increase in the exponential growth rate of the model vortices (as shown by Gray and Craig 1998). Surface friction has the opposite effect on the eye size with the eye size increasing with decreasing friction (Fig. 7).
TABLE 3. VARIATION OF THE ESTIMATED MATURE POLAR LOW EYE SIZE WITH THE HEAT AND MOISTURE TRANSFER COEFFICIENTS, $C_T$ AND $C_E$, AND THE FRICTIONAL DRAG COEFFICIENT $C_D$. ESTIMATED EYE SIZES ARE TO WITHIN APPROXIMATELY $\pm 5$ km.

<table>
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<th>Experiment</th>
<th>mature $r_{max}$ (km)</th>
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<tr>
<td>Control</td>
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<td>$C_{E,T} - 40%$</td>
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</table>

Figure 7. Radial distribution of the boundary-layer azimuthal velocity in simulated polar lows at ten hourly intervals, where the last two times are after the system reached maximum intensity, for simulations with varying values of the heat and moisture transfer coefficients, $C_T$ and $C_E$, and the frictional drag coefficient, $C_D$. The last time plotted is in bold.
(c) Comments on the mechanism controlling the polar low eye size

The mechanisms which control the size of the eye in mature tropical cyclone and polar low systems are presently not well understood. It is not readily apparent whether the eye size is controlled predominantly by dynamical processes outside or inside the eye or whether a simple balance of forces may exist between the two regions.

The size of the eye in the mature simulated tropical cyclones was found to be approximately constant in all the experiments performed. It is possible that when the structure of the eye becomes rapidly rotating then the eye may act as a boundary layer to the turbulent outer part of the system and that some type of balance between diffusive and inertial forces may control its size. Unfortunately, the experiments performed here are inadequate to justify this or any other theory.

For the simulated polar lows the parameter dependencies of the mature eye sizes are consistent with a mechanism in which the secondary circulation inside the eye affects its size through the low-level outflow of air from the eye with small values of absolute angular momentum which must then be incorporated into the eyewall. Evidence for this is given below.

The mean vertical velocity, from 40 to 50 hours, from the polar low simulation in which the values of the heat and moisture transfer coefficients, $C_T$ and $C_E$, were increased by 40% from their control values is shown in Fig. 8(a). Comparison with the control simulation (Fig. 5(b)) shows that both the strength of the subsidence in the eye (and hence, through continuity, the strength of the low-level outflow) and the size of the eye are greater in the case with increased values of $C_T$ and $C_E$. This increase in eye size is also apparent in the mature system (Table 3).

It is not surprising that the rate of subsidence in the eye increases with $C_T$ and $C_E$ since the forcing mechanism for the secondary circulation (discussed in section 4) predicts a correlation between the rate of intensification of these systems and the rate of subsidence (where the former increases with the values of these coefficients and the correlation between intensification and subsidence rates was observed for tropical cyclones by Jordan 1961). What is not proven, is whether or not there is a correlation between the rate of subsidence and the eye size in the mature simulated polar lows.

Another example of this correlation is shown for the experiment in which the value of the frictional drag coefficient was increased by 100% from the control value (Fig. 8(b)). In this case, a decrease in the rate of subsidence in the eye (on comparison with Fig. 5(b)) is associated with a decrease in eye size both at this time and in the mature system (see Table 3). It is possible that the decrease in the rate of subsidence is associated with a very slight decrease in the rate of intensification of the system (as was found for this model by Craig and Gray 1996). However, it is more probable that it is caused by an increase in the Ekman pumping forced by frictional convergence in the boundary layer which may be sufficient to reduce the rate of subsidence in the eye.

If an association exists between increased subsidence rates in the eye and increased eye size then it is possible that the secondary circulation in the eye affects its size through the low-level outflow of air from the eye with small values of absolute angular momentum which must be incorporated into the eyewall. It is also possible that the effect of the outflow will be greater when $f$ is increased. Conservation (to some degree) of absolute angular momentum implies that increased $f$ will lead to the *a priori* inclusion of air with smaller values of azimuthal velocity into the eyewall. This might be expected to increase the eye size and this prediction is consistent with the finding that the eye size increases with $f$ in the polar low simulations. This mechanism may also explain the sensitivity of the eye size to the relative angular momentum of the vortex used to initiate the simulated polar lows.
The size of the eye in the mature system was found to be smaller if $r_{\text{max}}$ was decreased or $u_{\text{max}}$ increased. Both of these changes lead to increased initial windspeeds at small radii when compared with the control imposed initial vortex. Since the azimuthal windfield in the inner region of the polar low systems decays over time, eventually becoming very calm, this difference in the inner windspeeds at initiation also applies for a given intensity of vortex (measured by the maximum azimuthal windspeed). For increased absolute angular momentum (i.e., increased azimuthal windspeeds) inside the eye the low-level outflow will have less impact on the eyewall windspeeds leading to a smaller eye size in agreement with the simulations (for a given eyewall absolute angular momentum).
6. Conclusions

The factors which control the structure and size of the eye formed in mature tropical cyclones and polar lows have been examined through the analysis of controlled numerical experiments. The structures of the eyes formed in the simulations are consistent with those observed in nature within the restrictions of the available data. It was thus assumed that these eyes were formed by the same processes as those in nature and that the model is a suitable tool for the examination of the eyes formed in these systems.

The eye in the mature, control tropical cyclone simulation was approximately 20 km in radius and close to solid body rotation i.e., the azimuthal velocity inside the eye increased approximately linearly with radius. The thermodynamic structure was in thermal wind balance with the velocity structure such that the eye potential temperature exceeded that in the eyewall. In contrast, rapidly rotating eyes were not formed in the polar low simulations. The polar low eye was typically considerably larger than the tropical cyclone eye (approximately 60 km in radius in the mature control system) and extremely calm outwards to a sharp transition to the high windspeeds found in the eyewall. Again in thermal wind balance with the azimuthal velocity structure, this eye failed to warm more than the eyewall (the potential temperature was approximately constant at all radii from the eyewall inwards).

The difference in structure between the tropical cyclone and polar low eyes was explained by examining the environmental soundings in the eyes. Subsidence in the eye can cause the potential temperature to increase faster there than in the eyewall. However, the environmental sounding must be significantly stable and this is not the case in observed polar lows. In polar low eyes the consequent approximately radially constant potential temperature in the eye implies that the azimuthal windspeed must be independent of height, through thermal wind balance. Negligible windspeeds were found in the eyes formed in simulated polar lows which satisfied this condition. The environmental sounding thus prevents the simulated polar lows from forming rapidly rotating eyes.

The sensitivity of the size of the eye in the mature simulated systems to parameters which vary in nature was investigated through a series of experiments. For the tropical cyclone simulations the eye size was found to be approximately insensitive to small changes in these parameters. The only exception was the case where the surface friction was removed from the system which resulted in a large eye of structure similar to that in the simulated polar lows. In the polar low simulations the size of the eye in the mature system was found to be strongly sensitive to the vortex used to initiate the polar lows, the value of the Coriolis parameter, surface friction and the sensible and latent heating from the ocean surface. The eye size appears to be positively correlated with the strength of the secondary circulation in the eye and the planetary vorticity. It is suggested that the secondary circulation in the eye tends to increase the size of the eye through the low-level outflow of air which must be incorporated into the eyewall and this mechanism was found to be consistent with the dependencies observed. Further work is needed to compare the predictions of these polar low simulations with observations and to determine the mechanism controlling the eye size in tropical cyclones.

References

Kuo, H. L. 1961 Marked changes in the characteristics of the eye in intense typhoons between the deepening and filling stages. *J. Meteorol.*, 18, 779–789


