Conventional and Bayesian approach to climate-change detection and attribution

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(Received 19 December 1997; revised 14 May 1998)
(Symons Memorial Lecture: delivered 21 May 1997)

SUMMARY

The conventional multi-variate, multi-fingerprint theory of climate-change detection and attribution, expressed in terms of existing frequency distributions, is reviewed and generalized to a Bayesian approach based on subjective probabilities. Bayesian statistics enable a quantitative determination of the impact of climate-change detection tests on prior subjective assessments of the probability of an externally forced climate change. The Bayesian method also provides a potentially powerful tool for enhancing statistical detection and attribution tests by combining a number of different climate-change indicators that are not amenable to standard signal-to-noise analyses because of inadequate information on the associated natural-variability statistics. The relation between the conventional and Bayesian approach is illustrated by examples taken from recent conventional analyses of climate-change detection and attribution for three cases of climate-change forcing by increasing greenhouse-gas concentrations, increasing greenhouse-gas and aerosol concentrations, and variations in solar insolation. The enhancement of detection and attribution levels through a joint Bayesian analysis of a number of different climate-change indices is demonstrated in a further example. However, this advantage of the Bayesian approach can be achieved only within the framework of a subjective rather than objective analysis. The conventional and Bayesian approach both exhibit specific advantages and shortcomings, so that a parallel application of both methods is probably the most promising detection and attribution strategy.

KEYWORDS: Attribution Detection Climate change

1. INTRODUCTION

The question whether the predicted global warming due to increasing greenhouse-gas concentrations can be detected today in observed data has long been the subject of debate. In recent years, the consensus opinion has gradually shifted from predominantly negative to cautiously positive. Thus, in contrast to the negative findings of the first scientific assessment of the Intergovernmental Panel on Climate Change (IPCC) (Houghton et al. 1990), the executive summary of the second IPCC scientific assessment, IPCC–95 (Houghton et al. 1996) states that: “The balance of evidence suggests a discernible human influence on climate”. The change in assessment can be attributed mainly to improved model estimates of the anthropogenic signal and natural-variability noise, together with the application of more sophisticated multi-variate fingerprint methods. A contributing factor has also been the increase in signal magnitude in the intervening years.

The IPCC–95 statement has been the subject of considerable (not always scientific) controversy. Since the detection and attribution problem is basically statistical, and the available observational data for estimating both the natural-variability noise and the climate-change signal is limited, it is to be expected that the detection issue should be debated, and will continue to be debated, until the signal has emerged more clearly out of

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the natural-variability noise (Hasselmann 1997b). However, there is another aspect of the controversy that has so far received relatively little attention. This is the different subjective assessments of the evidence for anthropogenic global warming by different individuals.

Most of the work on detection and attribution of climate change has been based on conventional statistical techniques. The methodology is objective and well established, and the uncertainties of the data and model predictions are normally presented in detail in the relevant technical papers (cf. summary in Santer et al. 1996a). Nevertheless, the public and scientific discussion of the detection issue demonstrates that the acceptance of the results of such analyses varies widely from individual to individual. This cannot be explained entirely by the normal scientific process of argument and counter-argument in the evolution of scientific concepts. Rather, it appears that there exists an unavoidable subjective element in the assessment of statistical observational evidence resulting from different individual views on the reliability of models and data. This is not adequately captured by the conventional statistical approach to the climate-change detection and attribution problem.

Traditional statistics requires that probabilities are introduced only for quantities for which there exists an ensemble of realizations that is sufficiently large that it can be regarded as an acceptable proxy for the theoretical construct of an ensemble of realizations defined in some abstract phase space endowed with a positive probability measure. In practice, the only available physical ensemble of realizations approximately satisfying this criterion for the climate-change detection and attribution problem is the natural climate variability. Thus all conventional detection and attribution analyses have focused on the statistics of the climate-variability noise (see, for example, the general multi-variate, multi-pattern theory of climate-change detection and attribution developed by Hasselmann (1979, 1993, 1997a), Bell (1986) and North et al. (1995), the general expositions of Barnett et al. (1991), Pennell et al. (1993), Santer et al. (1995b, 1996a), and other authors, and the applications of these concepts by Bell (1982), Hasselmann et al. (1995), North and Kim (1995), Santer et al. (1995a, 1996b) and Hegerl et al. (1996, 1997)).

In the terminology of conventional statistics, a predicted climate-change signal is said to be detected in the observations at a significance level $s$ if the probability that the signal can be explained by natural climate variability (defined in the following section) is less than $(1 - s)$. A detected signal is then furthermore said to be attributed to a proposed forcing mechanism at some given confidence level if the signal amplitude estimated from the observations is consistent with the predicted amplitude within given confidence bounds, which are estimated again from the natural-variability noise (Hasselmann 1997a).

It can be argued that the focus on the natural-variability noise in these definitions presents only one side of the detection and attribution problem: one should consider not only the probability of the null hypothesis that an observed climate change can be attributed to natural climate variability, but also the probability of the alternative hypothesis that the observed climate change has indeed been brought about by the proposed external-forcing mechanism. However, one faces then the basic difficulty that there exists no adequate data base for the statistical assessment of the alternative hypothesis. In particular, the unknown systematic model errors inherent in predictions of the climate response to external forcing are normally difficult to quantify statistically. If one wishes to balance the outcome of the null hypothesis test against the prior likelihood of the validity of the complementary hypothesis of an external-forced climate change, one must therefore perforce leave the established grounds of classical probability theory founded on existing frequency distributions and venture into the Bayesian theory of confirmation based on the generalized concept of subjective probabilities (cf. Earman 1992).

The relation between the conventional and Bayesian approach can be illustrated by a simple analogue of the climate-change detection and attribution problem. Consider a dice
player A who is approached by a person B who claims that she possesses supernatural powers and offers A a wager at, say, even odds that she can roll two sixes in succession. A calculates the probability of success at \( \frac{1}{36} \), or less than 3%, and accepts the bet. B then does indeed roll two sixes. According to conventional statistical terminology, A should thereafter proclaim that he has detected a person with supernatural powers at a significance level exceeding 97%. In fact, however, A would more likely believe that B was simply lucky or had exchanged the dice. The fact that A accepted the bet at even odds indicates that his prior belief (based on past experience and intuition, but with no supporting statistical data) was that the latter cases were relatively unlikely. After losing the bet, however, A would presumably revise his assessment and would be willing to accept a second bet on the same experiment only at significantly more favourable odds. A third person C, who was perhaps more inclined to suspect that B was an imposter in the first place, would not have accepted the bet at even odds, and her modified assessment of the situation following a positive outcome of the experiment would probably also be different from that of A.

The example illustrates that the subjective assessment of the outcome of a hypothesis test depends not only on the probability of the null hypothesis, but also on one’s prior assessment of the likely validity of the proposed hypothesis, or of possible alternative explanations of the test outcome.

From the viewpoint of conventional statistics, the probability of a non-chance explanation of an experiment for which there exists no statistical data base has no meaning. However, since much of the controversy attending the climate-change detection and attribution issue originates in just this question, namely in the different degrees of confidence attached by different scientists to model predictions of global change, it appears useful to illuminate and quantify the implications of these different levels of confidence within a more general Bayesian statistical framework.

In contrast to conventional ‘objective’ probability theory based on existing frequency distributions, Bayesian statistics employs a ‘subjective’ concept of probability defined with respect to ‘levels of belief’ in the validity of hypotheses. It is assumed that to any hypothesis \( H \) there can be assigned a probability \( p(H) \) that the hypothesis is true. The probability measure can be operationalized by imagining that an individual is asked to bet on the outcome of a test of the hypothesis. In addition to this ‘personalized’ probability one can consider a ‘consensus’ or ‘Delphi’ probability for a group of experts (e.g. IPCC), which may be interpreted as a ‘quasi-objective’ probability measure. However, we shall regard Bayesian probabilities here in the personalized sense. It can be shown that the concept of probability defined in this manner satisfies all the usual properties of a probability, so that one can apply standard methods of statistical analysis (Pearl 1988; Earman 1992).

In addition to providing a framework for quantifying different subjective degrees of belief in a hypothesis, the Bayesian approach has another important advantage. For many of the climate-change variables predicted by models and confirmed qualitatively by observations, such as temperature trends in the stratosphere (Labitzke et al. 1986; Taubenheim et al. 1990) and troposphere (Santer et al. 1997; Ropelewski et al. 1997), modified vertical temperature profiles (Santer et al. 1996b) or changes in sea-ice area (Bjorco et al. 1996), it is difficult to provide reliable estimates of the natural climate variability on the time-scales relevant for climate change. Thus one cannot derive from these data quantitative detection and attribution confidence levels defined in the conventional statistical sense. Within a Bayesian framework, however, all predictions can be combined, irrespective of the objective reliability of the variability estimates, to yield quantitative (albeit subjective) net detection and attribution probability levels.

To clarify the interrelation of the conventional and Bayesian approaches, we first review briefly the conventional multi-pattern fingerprint method of detection and attribution
(section 2), following Hasselmann (1993, 1997a). In section 3, we present as examples some recent applications of the conventional theory (Hegerl et al. 1997) to anthropogenic greenhouse warming, including both greenhouse gases and aerosols, and to natural solar variability. The general Bayesian approach to detection and attribution is developed in sections 4–6. The differences between the conventional and Bayesian approaches are illustrated in section 7, using the examples discussed in section 3, while section 8 considers the application of the Bayesian approach to a set of several different detection variables. The last section 9 summarizes the principal conclusions of the paper.

2. CONVENTIONAL APPROACH

(a) The signal-pattern space

We consider generally the evolution of a climate data vector

$$\phi_a(t) = \phi^s_a(t) + \tilde{\phi}_a(t),$$

consisting of the superposition of a climate-change signal $\phi^s_a$ and a natural-variability component $\tilde{\phi}_a$. Climate variability is defined here, following Hasselmann (1993, 1997a), as climate variations resulting from internal interactions within the climate system, as opposed to climate change generated by external forcing. External forcing can be due either to human activities, such as emissions of greenhouse gases and aerosols, deforestation and land-use changes, or to natural processes, such as the emission of aerosols from volcanic eruptions, or solar variations. Thus natural fluctuations of external origin are regarded for the present purpose of climate-change detection and attribution as climate change rather than climate variability.

The index $a$ of the climate data vector $\phi_a$ designates the different types of climate data, e.g. temperature or precipitation, and the location or averaging region of the data. The data vector can represent either observed data or synthetic data from a model simulation. It will normally consist of only a small subset of the components of the complete climate state vector required for a dynamical representation of the climate system, for example in a model.

For the conventional statistical approach, the climate data vector must be restricted to climate variables for which sufficient data exist to estimate statistical properties, including, in particular, the time-lagged second moments $R_{ab}(t, \tau) = \langle \tilde{\phi}_a(t + \tau) \tilde{\phi}_b(t) \rangle$. If the statistical estimates are based primarily on model simulations, some observational data should also be available to at least check on the plausibility of the model estimates. Here and in the following, cornered parentheses $\langle \ldots \rangle$ denote ensemble means and the climate variability is assumed to have zero mean, $\langle \tilde{\phi}_a(t) \rangle = 0$ (the ensemble mean is assumed to be subtracted in the definition of the climate data vector). The climate variability is regarded as either stationary or cyclostationary, implying that $R_{ab}$ is independent of or a periodic function of time $t$, respectively, for given time lag $\tau$.

Implicit in the decomposition (1) of the climate data vector into the sum of an externally forced climate-change signal and an internally generated climate-variability component is the assumption that the two components can be linearly superimposed without interaction. Numerical experiments with strongly nonlinear general circulation models suggest that this is an acceptable approximation for typical anthropogenic or naturally forced climate-change signals (cf. Haywood et al. 1997; Ramaswamy and Chen 1997). We note in this context that a temperature change of 3 K represents only a 1% change of the global-mean near-surface temperature of 288 K. The superimposition principle is, in fact, the traditional working hypothesis of numerical experiments on climate change, in which
the climate-change signal is determined as the difference between an externally forced model simulation and a control run without external forcing. However, this limitation must be kept in mind in the application of the detection and attribution theory.

For the general space–time dependent theory, it is useful to introduce a compressed notation in which the trajectory of the climate data vector is represented as a vector $\psi = (\psi_i) \equiv (\phi_i(t_i))$, with a composite index $i \equiv (a, b)$ composed of the climate-state index $a$ and the index $b$ of the discretized time variable $(t_b) = (t_1, t_2, \ldots)$. The covariance matrix of the time-lagged second moments $R_{ab}(\tau)$ or $R_{ab}(t, \tau)$ becomes in this notation simply

$$C_{ij} = \langle \psi_i \psi_j \rangle.$$

(2)

However, all relations derived in the following apply formally also to the case that $\psi$ is interpreted as some time-dependent quantity derived from $\phi_a(t)$ by a time-filtering operation applied over a limited time interval (for example, running climate trends defined over time intervals of several decades, cf. Santer et al. (1995a) and Hegerl et al. (1996, 1997)), so that $\psi = \psi(t)$. All following relations apply then for a given time. While the simultaneous inclusion of both time and space variables in a complete space–time dependent detection and attribution analysis is formally more elegant and yields the maximal possible significance levels, the separate treatment of the time dependence is somewhat simpler, both analytically and numerically. Moreover, it yields useful insight into the time evolution of the signal-to-noise level. Most published work on the optimal-fingerprint technique and the applications considered later are based on this second approach. However, some recent interesting work on the full space–time dependent fingerprint method has been carried out by Stott et al. (1997) and North and Stevens (1997).

Essential for a successful detection and attribution strategy is the specification of a strongly reduced subspace in which the signal is to be detected. If the full space of the original-data vector trajectory $\psi_i$ is retained in the statistical tests, it becomes virtually impossible to identify even a large climate-change signal with non-specified signal direction in the presence of noise in the many components of the climate-data phase space (cf. Hasselmann 1979, 1997a). The signal must therefore be postulated to lie in some low-dimensional space spanned by a number of prescribed signal patterns. These can be inferred from model simulations of the response of climate to various candidate forcing mechanisms, or can be simply hypothesized as guess patterns (in the case of greenhouse warming, for example, one could postulate some simple latitudinal temperature distribution modulated by a land–sea mask).

In detection applications, only a single response pattern corresponding to a single forcing mechanism is normally considered (e.g. Hegerl et al. 1996). The detection test in this case is not critically dependent on the correct definition of the guess pattern. An inaccurate guess pattern will normally reduce the inferred significance level of the test, but will not lead to erroneous detection claims. A general multi-pattern analysis is required when the attribution problem for more than one candidate climate-change mechanism is considered (cf. Hegerl et al. 1997). Here again, incorrect signal-pattern predictions will normally only degrade the power of the attribution test, rather than lead to false attribution claims.

In the general multi-pattern case, we consider a climate-change signal

$$\psi_i^* = d^\nu g_{\nu i}$$

(3)

lying in a space of $p$ prescribed signal patterns $g_{\nu i} = (g_{\nu i})$, $\nu = 1, \ldots, p$, with some set of pattern coefficients (amplitudes) $d^\nu$. Here and in the following, pairs of repeated upper and lower indices are summed, both in climate-state space (Latin indices) and pattern space
(Greek indices). Upper and lower indices are identified with co- and contravariant tensors defined with respect to the metric tensors $C_{ij}$ and

$$D_{ij} = g_{vi} C^{ij} g_{uj}$$

(4)

for the climate-state phase space and pattern space, respectively. The associated inverse metric tensors are denoted $C^{ij}, G^{\nu \mu}$, with

$$C_{ij} C_{jk} = \delta^i_k,$$
$$G^{\nu \mu} G_{\mu \lambda} = \delta^\nu_\lambda,$$

(5)

where $\delta^i_k$ is the Kronecker symbol. Tensor indices are raised and lowered in accordance with the rules

$$X_{\cdots}^{\cdots} = C^{ij} X_{\cdots j}^{\cdots};\ X_{\cdots i}^{\cdots} = C_{ij} X_{\cdots j}^{\cdots};$$
$$X_{\cdots}^{\cdots} = G^{\nu \mu} X_{\cdots \mu}^{\nu};\ X_{\cdots}^{\nu} = G_{\nu \mu} X_{\cdots \mu}^{\nu}.$$  

(6)

(7)

The representation (3) permits an arbitrary normalization of the signal patterns. We chose the normalization condition

$$g_{vi} C^{ij} g_{uj} = 1.$$  

(8)

The patterns are not orthogonalized, however, as we wish to retain their physical interpretation as the climate response patterns associated with specific forcing mechanisms, which in general will not be orthogonal.

A tensor notation has been chosen here and in the following rather than a matrix notation as it expresses naturally the invariance of the results with respect to the choice of variables, allowing, for example, the trivial derivation of key relations using orthonormal variables (cf. Hasselmann (1997a)). The transparent index display of the tensor notation is also more convenient when working in more than one space (in the present case, in the climate-state space and the signal-pattern space).

The signal and pattern amplitudes can be inferred either from observations or model predictions. However, we reserve the notation $\psi_i^j$ and $d^\nu$ in (3) for the signal inferred from observations, the model-predicted signal for the $v$th candidate forcing mechanism being denoted

$$\psi^j_v = a_v \cdot \mathbf{g}(v)$$

(9)

(indices in parentheses are exempt from the summation convention).

In the detection problem, the predicted amplitudes $a_v$ are irrelevant; one investigates only whether the climate-signal amplitudes $d^\nu$ inferred from the observations can be distinguished from natural climate variability at some given statistical significance level. Thus one inquires whether the signal inferred from the observations lies outside some significance ellipsoid, 95%, say, of the natural-variability noise centred on the origin. In the attribution problem, on the other hand, one compares the predicted amplitudes $a_v$ for each of the candidate forcing mechanisms with the amplitudes $d^\nu$ inferred from observations. Thus one tests whether the estimated signal lies within the corresponding 95% confidence ellipsoid centred on the predicted signal vector of a given candidate forcing mechanism (Fig. 1). On the basis of the estimated statistical-error ellipsoids for the predicted and observed amplitudes, one can then compute a statistical consistency level for each of the candidate forcing mechanisms.
(b) **Fingerprint patterns**

To estimate the pattern coefficients $d^v$ from the observations $\psi_i$, the observed climate vector $\psi_i$ is represented as the sum

$$\psi_i = d^v g_{vi} + \psi_i^r$$

of the signal, (3), and a residual error $\psi_i^r$, which one seeks to minimize. One needs then to specify the metric defining the magnitude of the error vector $\psi_i^r$. The only available metric consistent with the requirement of invariance with respect to variable transformations is $C_{ij}$. The condition

$$\langle \psi_i^r C^{ij} \psi_j^r \rangle = \text{min}$$

then yields the solution

$$d^v = D^{vi} g_{vi} C^{ij} \psi_j = D^{vi} g_{vi} \psi_i = g^{vi} \psi_i.$$  

(12)

We denote the covariant forms $g^i_v$ of the signal patterns $g_{vi}$ as the *fingerprints*. To distinguish the fingerprints from the signal patterns more clearly, we represent the fingerprints in the following by the separate symbols

$$f^i_v = g^i_v = C^{ij} g_{vi}.$$  

(13)

In this notation (12) becomes (cf. Hasselmann 1997a)

$$d^v = D^{vi} f^i_v \psi_i = f^{vi} \psi_i.$$  

(14)

It can be readily shown (cf. appendix and Hasselmann (1997a)) that the solution (14) that minimizes the invariant least-square residual $\psi^r \psi^r$ is also the solution that maximizes the signal-to-noise ratio. Thus for the purpose of detection, the solution (14) is the optimal estimate of the signal from observations.

We have assumed here that the predicted signal is known through a model climate-change simulation or some ad hoc first-guess assumption. The generalization of the optimal-estimation problem to the case that the predicted signal is defined only statistically has been considered by North *et al.* (1995) and Allen and Tett (1998). In practice,
the estimation of the signal patterns and, still more so, the natural-variability covariance matrix, from model simulations, with some support from limited observational data, is a critical aspect of the problem of climate-change detection and attribution. However, we cannot do justice to this important issue in this general review and refer instead to the detailed discussions in Hegerl et al. (1996, 1997) and Allen and Tett (1998).

The relation between the fingerprints $f^i_v$ and the original signal patterns $g_{vi}$ is best understood by rotating to empirical orthogonal function (EOF) coordinates. In the EOF system, the covariance matrix reduces to the diagonal form $C_{ij} = I_{ij} \sigma^2_{ij}$, where $\sigma^2_{ij}$ is the variance of the $i$th EOF, and $I_{ij}$ is a tensor that has the same values as the Kronecker tensor in the EOF coordinate system (since the covariant tensor $I_{ij}$ is not invariant with respect to coordinate transformations, however, it is not denoted by the Kronecker symbol, which is reserved for the invariant mixed covariant–contravariant unit tensor $\delta^i_j$).

In EOF coordinates, the signal patterns take the form

$$g_{vi} = \sum_k c_{v(k)} e^{(k)}_i,$$

(15)

where $c_{v(k)}$ are the expansion coefficients of the signal pattern $v$ with respect to the EOF patterns $e^{(k)}_i$. For the fingerprint pattern we obtain then, from (13),

$$f^i_v = \sum_k c_{v(k)} \sigma^{-2}_{(k)} I^{ij} e^{(k)}_j.$$

(16)

The factor $\sigma^{-2}_{(k)}$ in (16) attenuates the high-noise EOF components in the fingerprint pattern relative to the low-noise components. Thus the optimal fingerprints are obtained by turning the original signal vectors away from high-noise towards low-noise directions (Fig. 2).

The above geometrical interpretation corresponds to the familiar Euclidean picture, in which the only permitted coordinate transformations are rotations that preserve the Euclidean metric. However, the Euclidean view is not in the spirit of the general invariant structure of the detection and attribution theory presented here. For example, if the rotation to EOF coordinates is followed by a scale transformation yielding a unit covariance matrix $C_{ij} = I_{ij}$, the distinction between signal and fingerprint patterns is lost. In general, one should simply view the fingerprint as the covariant dual of the contravariant signal pattern defined in a space endowed with the non-Euclidean metric $C_{ij}$.

Alternatively, the co- and contravariant tensor notation can be avoided, all relations being expressed in terms of standard linear algebra, if one generalizes the normal definition of scalar products and matrix multiplications to include the metric tensor $C_{ij}$ sandwiched
between the two product factors (cf. Hasselmann 1997a). In this notation, no distinction is made between the signal pattern and the fingerprint, the effect of the non-Euclidean metric being absorbed in the generalized definition of the scalar product.

(c) Detection and attribution tests

For the general multi-variate detection test, we need to relate the signal amplitudes $d_\nu$ inferred from the observations to the $p$-dimensional probability distribution $p_d(\tilde{d})$ of the set of pattern amplitudes $\tilde{d} = (\tilde{d}_\nu)$ that would be obtained if the observations consisted entirely of natural climate-variability noise, without a climate-change signal. We assume that the distribution is Gaussian,

$$p_d(\tilde{d}) = (2\pi)^{-p/2}|D|^{-1/2} \exp(-\tilde{\rho}_d^2/2),$$  \hfill (17)

where, from (4) and (12),

$$D^{\nu\mu} = \langle \tilde{d}_\nu \tilde{d}_\mu \rangle = C^{ij} g_i^{\nu} g_j^{\mu},$$  \hfill (18)

$$\tilde{\rho}_d^2 = D_{\nu\mu} \tilde{d}_\nu \tilde{d}_\mu = D^{\nu\mu} \tilde{d}_\nu \tilde{d}_\mu,$$  \hfill (19)

and

$$|D| = |D^{\nu\mu}|.$$  \hfill (20)

The significance level for the detection of a given estimated signal amplitude vector $\tilde{d}$ is then given by

$$P_{\rho_d} = \int_{\tilde{\rho}_d^2 < \rho_d^2(\tilde{d})} p_d(\tilde{d}) \, d\tilde{d}_1 \ldots d\tilde{d}_p.$$  \hfill (21)

As pointed out above, the detection of a multi-pattern climate-change signal becomes successively more difficult as the number of patterns increases. In practice, the multi-pattern detection approach is feasible only for a relatively small number of candidate patterns of order two or three. For the optimal detection of the climate change for a single forcing mechanism, the detection test is best performed in the one-dimensional space of the single predicted signal pattern. However, for the optimal discrimination between competing forcing mechanisms, the detection test, as well as the attribution test described below, is best carried out in the pattern space of all candidate forcing mechanisms (or in a smaller space corresponding to a subset of patterns that are sufficiently linearly independent).

To test whether the retrieved climate-change signal $\psi^{(m)}$, (3), with pattern amplitudes $d^{(m)}_\nu$ given by (12), is consistent with the predicted climate-change signal $\psi^{(v)}$ for a proposed forcing mechanism $v$, we compare the difference

$$\delta \psi^{(v)} = \psi^{(v)} - \psi^* = (\delta_\nu^{\mu} a_\nu^{(v)} - d^{(v)}_\mu) g_{\mu},$$  \hfill (22)

between the predicted and retrieved climate-change signals with the differences resulting from errors in the model computation of $\psi^{(m)}_\nu$ and in the estimation of $\psi^*$ from observations. Thus we assess the significance of the difference

$$\epsilon_\nu^{(v)} = \delta_\nu^{\mu} a_\nu^{(v)} - d^{(v)}_\mu$$  \hfill (23)

between the predicted and retrieved pattern amplitudes with respect to the $p$-dimensional probability distribution of the net amplitude differences $\tilde{\epsilon}_\nu^{(v)}$ arising from the statistical estimation and model errors.

The errors $\tilde{d}_\nu^{(v)}$ of the retrieved pattern amplitudes due to the natural variability were considered in the previous section. They can be characterized by the covariance matrix $D^{\nu\mu}$, (18).
The errors incurred in computing the predicted climate-change signals from models consist of two parts: sampling errors due to the natural variability of the model, and systematic errors of the model itself. The sampling errors can be estimated from long control simulations (cf. Hegerl et al. 1996, 1997) or Monte Carlo experiments (Cubasch et al. 1994). The errors due to systematic model errors are more difficult to determine, but can be estimated by inter-comparing climate-change simulations of different models. We assume that the net errors from both sources can be characterized by a model amplitude-error covariance matrix $M_{(v)}^{\mu\lambda}$.

Since the modelled and retrieved signal errors are statistically independent, the covariance matrix $E_{(v)}^{\mu\lambda}$ characterizing the statistical differences $\tilde{e}_{(v)}^{\mu\lambda}$ between the modelled and retrieved pattern amplitudes is given by the sum

$$E_{(v)}^{\mu\lambda} = M_{(v)}^{\mu\lambda} + D^{\mu\lambda}$$

(24)

of the covariance matrices of the predicted and retrieved pattern amplitude errors.

The probability distribution of the amplitude differences $\tilde{e}_{(v)}^{\mu\lambda}$ due to statistical sampling and model errors is accordingly given by the Gaussian distribution

$$p_e(\tilde{e}_{(v)}) = (2\pi)^{-p/2} |E_{(v)}|^{-1/2} \exp[-\tilde{e}_{(v)}^2 / 2],$$

(25)

where

$$|E_{(v)}| = |E_{(v)}^{\mu\lambda}|,$$

(26)

$$\tilde{\sigma}_e = \tilde{E}_{\mu\lambda}^{(v)} \tilde{e}_{(v)}^{\mu\lambda},$$

(27)

and $\tilde{E}_{(v)}^{(v)}$ is the inverse of $E_{(v)}^{\mu\lambda}$,

$$E_{(v)}^{\mu\sigma} \tilde{E}_{(v)}^{\sigma\lambda} = \delta_{\lambda\mu}.$$  

(28)

The attribution consistency test can now be defined in an analogous manner to the detection test. The null hypothesis is replaced by the consistency hypothesis, and the retrieved pattern amplitude vector by the difference amplitude vector. Apart from this change in terminology, the concepts are identical to those of the detection test. For any given amplitude difference vector $e_{(v)}$ there exists a surface $\rho_e^2 = \text{const}$ that contains the vector. We consider then the integral

$$\tilde{P}_{\rho_e} = \int_{\rho_e^2 = \rho_e^2} p_e(\tilde{e}_{(v)}) \, \, d\tilde{e}_{(v)}^1 \cdots d\tilde{e}_{(v)}^p$$

(29)

of the $p$-dimensional probability density $p_e$ over the region $\tilde{\rho}_e^2(\tilde{e}_{(v)}) > \rho_e^2(e_{(v)})$ outside the constant probability surface $\tilde{\rho}_e^2(\tilde{e}_{(v)}) = \rho_e^2(e_{(v)}) = \text{const}.$

If $\tilde{P}_{\rho_e}$ is small (5%, say), the hypothesis that the retrieved climate-change signal is consistent with the forcing mechanism $v$ is said to be rejected with a risk of $\tilde{P}_{\rho_e}$, or at a significance level of $P_{\rho_e} = (1 - \tilde{P}_{\rho_e})$ (95%).

A positive outcome of the statistical detection test (i.e. the rejection of the null hypothesis) is formally analogous to a negative outcome of the consistency test (i.e. the rejection of the consistency hypothesis). A positive outcome of the consistency test should therefore be expressed in the double-negative form that the retrieved climate-change signal is not inconsistent with the proposed forcing mechanism at a given significance level $P$. To avoid the double negative in the statement that ‘a retrieved climate-change signal is not inconsistent with a given forcing mechanism at a significance level of $P$ (95%)’, we shall say more simply that ‘a climate-change signal is consistent with the forcing mechanism
within the $P$ (95\%)-confidence region' or 'at a confidence level of $\bar{P}$ (5\%).' Note that the stringency of the consistency test increases with decreasing $P$ or increasing $\bar{P}$. For $P \rightarrow 0$, the confidence region contracts to zero, requiring zero error between the retrieved and predicted pattern amplitudes for a positive outcome of the consistency test, while the confidence level $\bar{P}$ for a consistent signal increases to 100\%.

3. Applications

The general multi-pattern detection and attribution technique, with the fingerprints optimized with respect to space–time, has recently been applied by Hegerl et al. (1997) to a set of three different climate-change mechanisms: forcing by increasing greenhouse-gas concentrations alone, in accordance with the IPCC-90 business-as-usual scenario A, by the same greenhouse-gas concentrations with superimposed aerosol forcing, and by changes in the solar constant. The predicted climate-change signals and estimates of the natural-variability noise were obtained from simulations with coupled ocean–atmosphere general-circulation models. The observations consisted of the global near-surface temperature data of Jones and Briffa (1992). We summarize the results of this investigation briefly as an example to illustrate later the application of the Bayesian approach.

Figure 3 shows the evolution of the observed global-mean temperatures, together with the model simulations for the three forcing cases. The best signal-to-noise ratios for detection were achieved using thirty-year trends. The signal patterns for the first two forcing simulations were computed from the last-thirty-year trends of the simulations (from 2020 to 2050), while the signal pattern for the solar-forcing experiment was inferred from the correlation of the simulated near-surface temperature field with the prescribed solar-forcing variations. The rotated optimal fingerprint patterns were determined using the noise covariance matrix computed from a control run (various other control runs were also considered in a series of sensitivity tests).
Figure 4. Evolution of the detection variable for 30-year trends for the observed data and the three forcing experiments shown in Fig. 3 (from Hegeri et al. (1997)). The horizontal lines represent 95% significance levels estimated from the observed variance and the variance simulated by three models—HADCM2 (Hadley Centre), GFDL (Princeton) and HAM3L (Max-Planck Institute, Hamburg).

The detection tests were first carried out as single pattern tests for each of the three forcing mechanisms separately. The resulting time series of the detection variables for the observations and the three simulations are shown in Fig. 4. The two anthropogenic first-guess signal patterns yield a detected climate-change signal in the latest observed trend at a statistical significance level above 95% (noting that the temperature change is predicted to be positive, the significance level actually exceeds 97.5%). The climate-change component in the direction of the solar signal, however, is not significant.

Single-pattern tests are not good discriminators for attribution. The best discrimination between the three forcing mechanisms was obtained in a two-pattern detection and attribution test for the 50-year trends of the boreal summer-mean temperatures (Fig. 5). A trend interval of fifty years was chosen, as the increase of aerosol concentrations during the last fifty years was relatively linear. The boreal summer season (June, July, August) was selected, since anthropogenic aerosol concentrations are highest in the northern hemisphere, and the impact of the enhanced reflectivity due to aerosols can be expected to be greatest during the summer, when the solar radiation is largest. Although three candidate forcing mechanisms were tested, the detection and attribution test was limited to the two-dimensional space spanned by the signal patterns of the two anthropogenic-forcing experiments, as the component of the solar signal pattern orthogonal to this plane could not be distinguished from noise.

Figure 5 confirms that detection can be achieved at a significance level exceeding 95% for the two anthropogenic-forcing experiments, but not for solar forcing. It demonstrates furthermore that the highest confidence level for attribution is achieved for the combined
greenhouse-gas-plus-aerosol forcing. We shall discuss the interpretation of these results in a Bayesian framework in section 7.

4. Bayes' Theorem

Bayesian statistics, in the form we shall be concerned with here, considers probabilities of hypotheses. A given hypothesis $H$ (e.g., that there exists an anthropogenic greenhouse forcing signal) can be either be true, $H = h = \text{`true'},$ or false, $H = \tilde{h} = \text{`false'}$. The original subjective degree of belief in the hypothesis $H$, expressed by the prior probability, or simply `prior' $p(H)$, can be modified by the outcome of an experiment or some other evidence $E$ (e.g. a measured increase in global-mean temperature above some prescribed value). $E$ again represents a two-valued logical variable, $E = e$ (positive test outcome) or $\tilde{e}$ (negative outcome).

Before the test $E$ is performed, there will normally exist also other information $K$ consisting of a number of different components that can be either true or false, and that affect the prior and posterior probabilities that the hypothesis $H$ is true. We denote by $p(H, E, K)$ the joint probability with respect to the set of all logical variables $H, E, K$.

The prior probability of the hypothesis, for known $K$ but with no information on the test $E$, is given by the marginal-conditional distribution $P(H/K) = \Sigma_E P(H, E/K)$. Performing the test, i.e. determining the evidence $E$, converts the prior probability $P(H/K)$ to the posterior probability $P(H/E, K)$. The relation between the prior and posterior
probabilities is given by Bayes' celebrated theorem:

\[
\frac{p(H|E, K)}{p(H|K)} = \frac{p(E|H, K)}{p(E|K)}. \tag{30}
\]

The relation (30) can be readily verified using the standard definitions for conditional probability distributions: \(p(H|E, K) = p(H, E, K)/p(E, K)\) etc.

Bayes' theorem inverts the conditional probability relation between the hypothesis \(H\) and the evidence \(E\). The probabilities on the right-hand side of (30) referring to the outcome of the test \(E\) are referred to as likelihoods. Thus the theorem states that the ratio of the posterior to prior probabilities of the validity of the hypothesis \(H\) is given by the ratio of the likelihood of a positive (or negative) outcome of the test \(E\) with known \(H\) to the corresponding likelihood without knowledge of \(H\).

In applications in which one is not concerned with changes in the prior information content \(K\), the variables \(K\) can be treated as constant and ignored. Bayes' relation (30) reduces in this case to the simpler form

\[
\frac{p(H|E)}{p(H)} = \frac{p(E|H)}{p(E)}. \tag{31}
\]

In our applications, the prior information \(K\) could be that the hypothesized climate change is based on a computation with a particular coupled ocean–atmosphere general-circulation model, and that the climate exhibits natural fluctuations that are regarded as approximately Gaussian and are characterized by a space–time lagged covariance matrix that has been estimated from data and model simulations. Although these factors will clearly enter into an individual's subjective probability assessments, we shall regard them as constant for a given individual and will therefore consider only the simpler form (31) of Bayes' theorem.

We shall furthermore apply (31) only for the probability that the hypothesis is true and the outcome of the test is positive. In this case (31) becomes

\[
\frac{p(h/e)}{p(h)} = \frac{p(e/h)}{p(e)}. \tag{32}
\]

To simplify the notation we introduce the variables \(c\) (for 'credibility') = \(p(h/e)\) and \(c_0 = p(h)\) for the posterior and prior probabilities that the hypothesis is true, respectively, and \(l = p(e/h)\) for the likelihood of a positive outcome of the test for the case that the hypothesis is true. Furthermore, we represent the net probability \(p(e)\) of a positive test outcome, independent of the hypothesis \(H\), in terms of the conditional likelihood \(l\) and the complementary conditional likelihood \(\tilde{l} = p(e/\tilde{h})\) of a positive test outcome for the case that the hypothesis is false (corresponding to the conventional null hypothesis statistic) using the relation

\[
p(e) = p(e/h)p(h) + p(e/\tilde{h})p(\tilde{h}) = lc_0 + \tilde{l}(1 - c_0). \tag{33}
\]

Bayes' theorem (31) then becomes in this notation

\[
c = c_0l/(c_0l + (1 - c_0)\tilde{l})
\]

\[
= (1 + \beta\tilde{l}/l)^{-1}, \tag{34}
\]

with

\[
\beta = \frac{1 - c_0}{c_0}. \tag{35}
\]
5. APPLICATION TO DETECTION

For the detection problem, the hypothesis $H$ is simply that there exists an anthropogenic climate-change signal in the data. The standard test $E$ of conventional statistical detection analysis is that the amplitude of the climate-change signal inferred from observations, defined with respect to some predicted signal pattern, is greater than a prescribed value, corresponding to a significance level of, say, 95%, above the natural-variability noise. To illustrate the difference between the conventional and Bayesian view, we discuss first this test in a Bayesian frame. However, we consider subsequently a more consistent Bayesian test that combines detection and attribution in a symmetrical treatment of the two possibilities $H = h$ and $H = \bar{h}$.

For a climate modeller, the prior probability $p(h)$ that $H = h$ = ‘true’ will normally lie somewhere in the 90% range, if not 100%. In the latter case, the detection analysis is a purely academic exercise, since the modeller is convinced there is a climate signal in the data, regardless of whether or not the observed signal exceeds the natural climate-variability noise at some given significance level. In this limiting case, Bayes’ theorem yields the same posterior probability for the hypothesis, namely 100%, as the prior probability, independent of the outcome of the test $E$. For a greenhouse-warming sceptic, on the other hand, $p(h)$ is perhaps nearer to 10%. In the limiting case $p(h) = 0$, it follows from Bayes’ theorem that the posterior probability $p(h/E)$ again remains unchanged at the prior probability value $p = 0$, regardless of the outcome of the detection test.

Between these limiting cases, a positive or negative outcome of the detection test will have some finite impact on the level of acceptance of the global-warming hypothesis. The impact depends on the discrimination powers of the test and the individual’s prior level of belief in the global-warming hypothesis.

Figure 6 shows the dependence of the posterior probability $c = c(\hat{l}/l; c_0)$ as a function of $\hat{l}/l$, with $c_0$ as parameter. The figure illustrates several points. In the conventional non-Bayesian detection analysis, the rejection of the null hypothesis at a risk of $\hat{l}$ (e.g. 5%) is
referred to as the detection of a climate-change signal at a significance level of
\[ a = 1 - \hat{l}, \]  
(36)
e.g., 95% (cf. Hasselmann 1979, 1993, 1997a; Hegerl et al. 1996, 1997). However, in the Bayesian approach, the probability that the climate-change hypothesis is true is given by the general expression (34), which can be approximated for small \( \hat{l} \) by
\[ c \approx 1 - \beta \hat{l}/l. \]  
(37)
The relations (36) and (37) differ by the factor \( \beta/l \), which can adopt all values between 0 and \( \infty \), depending on the values of the probabilities \( c_0 \) and \( l \). Thus if \( c_0 \approx 1 \) (high prior confidence in the hypothesis \( H \)), \( \beta \approx 0 \), and we find \( c \approx 1 \), independent of the likelihood \( \hat{l} \) of a positive outcome (rejection) of the null hypothesis test \( E \). In the other limit of an exceptionally low value of \( c_0 \), we obtain a posterior probability \( c \approx 0 \), again independent of the likelihood \( \hat{l} \). Low values of \( \hat{l} \) also yield low values of \( c \) for given \( \hat{l} \). In this case the test \( E \) has only weak confirmation power for the hypothesis. The Bayesian and conventional approach agree for \( \hat{l} \approx 1 \), \( l \approx 1 \) and an non-committal prior \( c_0 = 0.5 \) (which is sometimes proposed as the natural choice if there exists no prior information on the credibility of a hypothesis).

Thus, the impact of a positive outcome of the detection test on the posterior credibility \( c \) is seen to depend strongly not only on the likelihood \( \hat{l} \) of a positive outcome of the test given that the hypothesis is false (null hypothesis test), but also on the assumed prior probability \( c_0 \) of the validity of the hypothesis, as well as on the likelihood \( l \) of a positive outcome of the test \( E \) in the case that the hypothesis is true.

To estimate \( l \) we need to consider the amplitude of the predicted climate-change signal. For a large predicted signal compared with the natural-variability noise, \( l \) will be close to unity, and (37) implies that for a positive outcome of the test, the Bayesian posterior probability that the hypothesis \( H \) is true is greater or smaller than the non-Bayesian significance estimate depending on whether \( c_0 \) is greater or smaller than \( \frac{1}{2} \). In the opposite limit that the predicted mean signal is small compared with the noise level, the two likelihoods are the same, \( l \approx \hat{l} \), so that (34) and (35) yield \( c \approx c_0 \): the outcome of the test \( E \) has no impact on the prior probability \( c_0 \).

The examples underline that it is not possible in the Bayesian approach to separate formally between detection and attribution. In contrast to the conventional approach, in which the signal amplitude is irrelevant for detection and is invoked only when the analysis is extended to the attribution problem (cf. Hasselmann 1997a), for the Bayesian analysis one needs to specify, in addition to the standard test statistic \( \hat{l} \) and the prior \( c_0 \), also the likelihood \( l = p(e, h) \) of a positive outcome of the test for the case that the hypothesis is true, which is strongly dependent on the predicted signal amplitude.

6. Application to Attribution

We turn now to the question of whether a climate-change signal that has been inferred from the observations is consistent with one or several of a set of candidate forcing mechanisms. In the conventional statistical approach, the attribution test is based on a comparison of the difference between the predicted and observed signal amplitudes against the root-mean-square errors of the amplitude difference associated with the natural-variability noise and model errors. The attribution test is termed positive at some confidence level if the amplitude difference lies within a given error confidence interval.
From a Bayesian viewpoint, the conventional attribution test suffers from the same shortcoming as the conventional detection test: it is not symmetrical with respect to the case \( H = h \), that the hypothesis is valid, and the alternative case, \( H = \tilde{h} \) that the null hypothesis (or, in general, an alternative hypothesis that we regard as representing the set of all plausible alternatives to \( H \)) is valid. Since we have already seen that the detection and attribution problem are intrinsically interlinked in the Bayesian approach, we define an alternative Bayesian test that combines detection and attribution and provides a direct intercomparison of the natural-variability and external-forcing hypotheses.

To distinguish between several competing climate-change mechanisms, a general multi-pattern analysis is required. The inferred and predicted climate-change signals are represented as vectors in a low-dimensional space spanned by the predicted signal patterns of the competing forcing mechanisms. As a symmetrical Bayesian test we consider the probability of the signal lying within a small (in the limit, infinitesimal) region of this space containing the retrieved signal, allowing for the different probabilities associated with different hypotheses. The probability of a positive outcome \( E = e \) of this test for any given hypothesis is then infinitesimally small. However, while this would rule out such a test in conventional statistics, the test is meaningful in the Bayesian approach, as we are concerned here only with likelihood ratios, not with absolute likelihoods (cf. (34)). We can test in this manner a number of competing climate-change hypotheses in relation to one another and to the null hypothesis. Apart from the different test definition, the theory and general relations summarized in Fig. 6 apply exactly as in the detection case discussed in the previous section.

7. INTERCOMPARISON OF CONVENTIONAL AND BAYESIAN APPROACHES

As an example illustrating the difference between the Bayesian and conventional approach, we consider the application of the symmetrical Bayesian detection and attribution test to the case of the 50-year trends of northern hemisphere summer-mean near-surface temperatures discussed in Hegerl et al. (1997) and section 3. According to the non-Bayesian analysis, the observed signal was consistent with the predicted signal for the greenhouse-gas-plus-aerosol experiment within 90% confidence bounds, but inconsistent with the other two forcing mechanisms.

For the Bayesian attribution tests, we consider for each of the three climate-change hypotheses and the null hypothesis the probability that the signal inferred from the observations lies within an infinitesimal region containing the retrieved signal (cf. Fig. 5). We then form the ratio \( \tilde{t}/t \) of the conditional likelihood densities for the null hypothesis and the hypothesis being tested, from which the posterior probability \( c \) of the hypothesis can be computed for a given prior \( c_0 \) using (34) and (35).

However, a difficulty for the quantitative evaluation is that the observed climate-change signal lies well outside the 90% confidence ellipses for two of the three forcing hypotheses. The assumption of a Gaussian probability distribution is questionable on the far tails of the distribution, for which little data are available. We shall accordingly characterize probabilities outside the 90% confidence ellipses only by inequality relations, for example, \( l_i < 0.1 \) if the observation lies outside the 90% confidence ellipse for the forcing mechanism \( i \), or \( l_i < \tilde{l}_i \) if the observation lies further outside the confidence ellipse for the forcing hypothesis than for the null hypothesis.

The resulting inequality relations are summarized in Table 1 and Fig. 7. Application of the Gaussian hypothesis on the tails of the distribution yields posterior probabilities close to the lower and upper bounds \( c \approx 0 \) and \( c \approx 1 \), respectively, for the greenhouse-gas-only and
greenhouse-gas-plus-aerosol forcing hypotheses (indicated by the double-hatched regions of Fig. 7). For $\hat{I}/I > 1$, (34) and (35) yield $c > c_0$. In this case the test is more likely to be satisfied for the null hypothesis than for the hypothesis being tested. This applies to the greenhouse-gas-only forcing hypothesis. Thus the prior probability $c_0$ of the validity of the hypothesis is actually reduced through the two-pattern test. (However, if the more realistic hypothesis is made that the climate change consists of a superposition of greenhouse-gas forcing plus natural variability, the confidence ellipse of the hypothesis is expanded, yielding $I > \hat{I}$, and an increase of $c$ relative to $c_0$.)

If both likelihoods are equal, $\hat{I}/I = 1$, we find $c = c_0$: the test is neutral and has no impact on the posterior probability, as pointed out already in the context of the detection test. This is approximately the case for the solar-variability hypothesis, for which the constant probability ellipses for natural variability and the errors of the solar forcing signal lie very close together.

| TABLE 1. BAYESIAN CONDITIONAL LIKELIHOOD RATIOS $\hat{I}/I$ AND POSTERIOR PROBABILITIES $c$ FOR 50-YEAR TRENDS OF NORTHERN HEMISPHERE SUMMER MEAN TEMPERATURES FOR THREE FORCING HYPOTHESES. |
|-----------------|-----------------|-----------------|
| GHG             | GHG + A         | SV              |
| $\hat{I}/I$     | $\approx 1$     | $<0.1$          |
| $c$             | $<c_0$          | $c_0$           |
|                 | $0.14+0.8c_0$   | $\approx c_0$  |

GHG = greenhouse gases only, GHG + A = greenhouse gases plus aerosols, SV = solar variability.
The largest impact of the attribution test is found for the case of the greenhouse-gas-
plus-aerosol forcing, for which the ratio $I/I$ of the likelihood densities for the hypotheses
of the anthropogenic forcing and natural variability is below 0.1.

8. **Bayesian detection tests for multiple climate-change indices**

Many observed climate-change indices, although qualitatively consistent with
climate-change predictions, cannot be subjected to a quantitative signal-to-noise analy-
sis within a conventional statistical framework owing to insufficient information on the
associated natural variability. In a Bayesian analysis, however, the missing information
can be provided by subjective probability assumptions, whose impact can be subsequently
assessed through sensitivity studies. The Bayesian approach thereby provides a means of
combining the information from a number of independent observations, each of which
may carry little weight individually, but which may nevertheless cumulatively provide
significant information on climate change.

As an example, we consider the case of $n$ independent observations $s_i$ of a set of
climate variables $x_i$, $i = 1, \ldots, n$, with associated estimated natural-variability variances
$\sigma_i$. We assume that the natural variabilities of different observations are uncorrelated and
therefore (for Gaussian distributions) statistically independent. Although this will not be the
case generally for an arbitrary set of variables, decorrelated variables can be readily
obtained through a suitable linear transformation. We may apply now the same attribution
test as in the previous section in the $n$-dimensional space of the observed variables $x_i$: we
determine the relative probabilities that all variables are observed in an infinitesimal
region containing the retrieved signal vector $s = (s_i)$ for each of the climate-change forcing
mechanisms and for the null hypothesis. From the ratio of the likelihood densities $I/I$ for
the null hypothesis and a given forcing mechanism, respectively, we can then determine
the posterior probability $c$ of the validity of the forcing mechanism as a function of the
prior probability $c_0$ of the forcing mechanism using (34).

Since the variables are statistically independent, the relevant net probabilities in $n$-
dimensional space are given by the products

$$ I(e) = \prod_i l_i(e_i), \quad (38) $$

$$ \hat{I}(e) = \prod_i \hat{l}_i(e_i), \quad (39) $$

of the one-dimensional probabilities $l_i(e_i), \hat{l}_i(e_i)$ for the individual variables $x_i$. ($E = e$
denotes here the combination of all individual tests $E_i$, where positive $E_i = e$ implies
that the variable $x_i$ lies within an infinitesimal region about the retrieved signal $s_i$. Thus
$E = e$ = 'positive' if and only if all $E_i = e$.) From (34) we obtain then

$$ c = \left( 1 + \beta \prod_i (\hat{l}_i/l_i) \right)^{-1}. \quad (40) $$

Through the product forms (38) and (39), the net attribution probability can become
quite high, even when the corresponding significance levels for the attribution tests of
individual variables are rather low.

Table 2 illustrates the cumulative net effect for a number of climate-change indices for
which published estimates of observed climate trends exist. To avoid biasing the results
through hindsight filtering, it is important that all available data are retained, including
TABLE 2. CLIMATE CHANGE TRENDS IN UNITS PER DECADE (EXCEPT FOR TROPOSPHERIC PATTERN CORRELATIONS, WHICH APPLY TO CHANGES OF ZONAL-MEAN LATITUDE-HEIGHT TEMPERATURE PATTERNS OVER TWO DECADES).

<table>
<thead>
<tr>
<th></th>
<th>$T_{st}$</th>
<th>$T_{st}$</th>
<th>$T_{tr}^\text{pat}$</th>
<th>$T_{tr}^\text{trend}$</th>
<th>$S_{sr}$</th>
<th>$S_{sa}$</th>
<th>net</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0.3°</td>
<td>-2°</td>
<td>0.5</td>
<td>-0.02/0.0°</td>
<td>-2.5%</td>
<td>-0.2%</td>
<td></td>
</tr>
<tr>
<td>class</td>
<td>++</td>
<td>+</td>
<td>[+]</td>
<td>[-1]/[10]</td>
<td>+</td>
<td>[+]</td>
<td></td>
</tr>
<tr>
<td>$s_i/c_i$</td>
<td>2</td>
<td>1</td>
<td>0.5</td>
<td>-0.5/0.0</td>
<td>1</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\bar{\bar{T}}_i/l_i$</td>
<td>0.14</td>
<td>0.6</td>
<td>0.9</td>
<td>1.5/1.0</td>
<td>0.6</td>
<td>0.9</td>
<td>0.06/0.04</td>
</tr>
<tr>
<td>$c_i$</td>
<td>0.9</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4/0.5</td>
<td>0.6</td>
<td>0.5</td>
<td>0.94/0.96</td>
</tr>
</tbody>
</table>

$T_{st}$ = near-surface global-mean temperature (Jones and Briffa 1992; Jones 1994);
$T_{st}$ = stratospheric temperature (Labitzke et al. 1986; Taubenheim et al. 1990);
$T_{tr}^\text{pat}$ = mid-tropospheric temperature, two cases (Ropelewski et al. 1997; Santer et al. 1997);
$T_{tr}^\text{trend}$ = pattern correlation of observed and predicted zonally averaged tropospheric

temperatures (Santer et al. 1996b);
$S_{sr}$, $S_{sa}$ = Arctic and Antarctic sea-ice area, respectively (Bjorgo et al. 1996);
net = resulting net ratio $\bar{\bar{T}}_i/l_i$ and net posterior probability $c$.
A prior climate-change probability of $c_0 = 0.5$ was assumed in computing $c_i$ and $c$. See text for further explanation.

data, such as tropospheric temperatures (Parker et al. 1997; Jones et al. 1997) and Antarctic

sea-ice area (Bjorgo et al. 1996), for which no significant trends have been detected.

Of the variables listed in Table 2, meaningful estimates of natural variability on time-
scales relevant for climate-change detection can be derived from observations only for the
trends in global near-surface temperature. For all other variables, the time series of the
observations are too short to separate recent climate change from climate variability. Thus
with the exception of global near-surface temperature, all likelihood ratios $\bar{\bar{T}}_i/l_i$ listed
in the table are based on subjective estimates. We have therefore simply classed the estimates
into five order-of-magnitude categories, ++, +, [+1], [0] and [−], as indicated.

As hypothesis we have postulated a climate change due to greenhouse-gas-plus-
aerosol forcing. The likelihood ratios $\bar{\bar{T}}_i/l_i$ were computed from the listed signal-to-noise
ratios $s_i/c_i$, assuming a Gaussian relation $l_i/l_i = \exp\{-s_i^2/(2\sigma_i^2)\}$. This may be interpreted
in various ways, cf. Fig. 5. For example, the predicted signal may be assumed to be equal to the observed signal, the ratio $s_i/c_i$ corresponding in this case to the ratio of the observed
signal to the variance of the natural variability. Alternatively, different observed
and predicted signals may be assumed, with corresponding adjustments of the ratio of the
observed signal to the natural-variability variance. The net ratio $\bar{\bar{T}}_i/l_i = \prod (\bar{\bar{T}}_i/l_i)$ is
seen to be considerably smaller, and the resulting posterior probability $c$ (computed for
the prior $c_0 = 0.5$) significantly higher, than the corresponding values for the individual
climate-change indices.

We emphasize that the numbers listed in the table are subjective and can be regarded
only as illustrative (if all estimates could be determined objectively from observed data or
reasonably reliable model simulations, we could have applied conventional multi-variate
statistics). Since the sign of the small trend in global mid-tropospheric temperatures is
still debated (see Hurrell and Trenberth 1996, 1997; Parker et al. 1997; Jones et al. 1997;
Santer et al. 1997; Ropelewski et al. 1997) we have considered two cases: a weak negative
trend [−] and zero trend [0]. We have furthermore ignored correlations between the natural
variability of different climate-change indices (formally, this can be taken into account by
transforming to statistical orthogonal variables). Nevertheless, the marked enhancement
of the detection and attribution levels through a combined analysis of several different
climate-change indices is clearly demonstrated.
Different scientists would presumably enter different numbers into Table 1. Each table should be regarded as an individual expert table. The main purpose of such tables is to provide a rational basis for the analysis of the impacts of different evaluations of climate-change indices on the net assessment of climate-change detection and attribution.

9. Conclusions

In assessing the evidence for climate change due to human activities or other external forcing mechanisms, a comparison of the predicted climate-change signal with the observed climate record is meaningful only in relation to estimates of the natural climate variability and the predicted climate-change signal errors. If both estimates can be quantified through observed data and model simulations, conventional statistics can be applied to derive objective estimates of the significance levels for climate-change detection and attribution. In the general case that several candidate forcing mechanisms are to be considered, the detection and attribution tests need to be carried out in the (low-dimensional) multi-pattern space spanned by the predicted signal patterns of the competing forcing mechanisms.

Application of the single fingerprint method to the detection of a climate-change signal in observed near-surface temperature data yields positive detection above the 95% statistical significance level for forcing due to either anthropogenic greenhouse gases only or to a combination of greenhouse gases and anthropogenic aerosols, but no statistically significant signal for the climate-change pattern predicted for a change in solar insolation. A multi-pattern attribution test, on the other hand, yields good agreement with the combined forcing by greenhouse gases and aerosols, but no agreement at the 90% confidence level for forcing by greenhouse gases alone.

Published estimates of statistical significance levels for conventional detection and attribution tests contain numerous caveats concerning the reliability of the natural variability estimates and the uncertainties of the predicted signals. These derive from the limited length of the observational time series and the unavoidable simplifications of global climate models. Since the uncertainties are difficult to quantify objectively, the results of conventional statistical tests are open to broader subjective interpretation. While it is undoubtedly meaningful to strive, nevertheless, at establishing objective quantitative criteria for the assessment of detection and attribution tests (with careful identification of the uncertainties), it can be illuminating to explore also the inevitable subjective interpretation of the results of such efforts from a Bayesian viewpoint. The Bayesian approach provides a quantitative framework for the investigation of the interplay between statistical detection and attribution tests and prior subjective assessments of climate change and climate variability.

The Bayesian method is furthermore useful for the joint analysis of climate-change data from various sources, such as surface, tropospheric and stratospheric temperatures, sea-ice variations or oceanic data. The predicted climate-change signals for such data are often qualitatively consistent with the observed data, but cannot be used for conventional detection and attribution tests because reliable estimates of the natural climate variability are not available. The Bayesian approach overcomes this difficulty, yielding significantly enhanced detection and attribution power through the combination of independent information from different sources—albeit on a subjective basis.

Bayesian methods will be similarly called for if the present global climate-change detection and attribution methods are extended to the regional scale, or to selected climate-change characteristics, such as the frequencies of El Niños, droughts, storms and other
extreme weather events. Since such variables are not selected for the purpose of maximizing the signal-to-noise ratio for global climate-change detection, but rather for the assessment of climate-change impacts, the estimates of their natural-variability levels and climate-change signals will generally be more uncertain than for climate variables that have been specifically selected for optimal global climate-change detection. Thus conventional detection and attribution techniques will often fail, pointing to the need for a Bayesian approach.

In summary, while the conventional detection and attribution method has the advantage of providing objective quantitative statistical significance estimates, the unavoidable difficulties of reliably estimating all required variability data in the face of limited observations and model uncertainties will inevitably invite subjective assessments of the credibility of a detected climate-change signal. The Bayesian approach provides a useful tool for clarifying and quantifying the impact of such subjective evaluations within the framework of a well-defined theory. The method also opens the door to the joint analysis of a wide set of observational data that are not amenable to conventional detection and attribution analysis. However, both the conventional and Bayesian methods have specific advantages and shortcomings. The most promising approach to the problem of climate-change detection and attribution therefore appears to lie in the further development and simultaneous application of both techniques.

ACKNOWLEDGEMENTS

The author is grateful for stimulating discussions with Gabi Hegerl, Tim Barnett, Ben Santer, Myles Allen, Simon Tett, Gerry North, Phil Jones and many other colleagues. The paper benefitted also from constructive comments on a first version presented as the 1997 Symons Memorial Lecture at the Royal Meteorological Society. The work was supported in part by NOAA's Office of Global Program's Climate Change and Detection Program Element and DOE's Office of Health and Environmental Research.

APPENDIX

In the following we show that the solution (14) minimizing the least-square residual $\psi^v_j \psi^T_j$ maximizes the signal-to-noise ratio, thereby maximizing the statistical significance level of the inferred signal relative to the natural-variability background.

Consider a general linear estimator $d^v$ of the pattern coefficients given by (14), with an as yet unknown set of fingerprints $f^v_i$. In the absence of a signal, application of (14) to the observed natural-variability data $\psi_i$ yields the natural-variability coefficients

$$\tilde{d}^v = f^v_i \tilde{\psi}_i,$$

(A.1)

which are characterized by the covariance matrix (18)

$$\langle \tilde{d}^v \tilde{d}^u \rangle = f^v_i f^u_j C_{ij} = D^{vu}.$$

(A.2)

We assume that the variables $\tilde{\psi}_i$ and thus also $\tilde{d}^v$ are normally distributed.

Consider now an arbitrary signal

$$\psi_i^g = a^v g_{vi},$$

(A.3)

with coefficients $a^v$ in the $p$-dimensional signal-pattern space. We require first that the fingerprint reproduces the signal pattern in the absence of climate-variability noise, i.e. that

$$f^v_i a^v g_{vi} = a^v$$

(A.4)
for arbitrary \( a^\nu \). This yields the condition

\[
f^{\mu \nu} g_{\mu \nu} = \delta_{\nu}^{\nu}.
\]  

(A.5)

We now choose \( f^{\mu \nu} \), under the side condition (A.5), such that the signal-to-noise ratio

\[
\rho_a^2 = a^\nu a^\nu D_{\nu \mu},
\]  

(A.6)

where \( D_{\nu \mu} \) is the inverse of the metric \( D^{\nu \mu} \) defined by (A.2), is maximized. Maximizing \( \rho_a^2 \) is equivalent to minimizing the probability \( P \) that the signal (A.3) can be attributed to the Gaussian distribution of the coefficients \( \tilde{\alpha}^\nu \) due to the natural-variability noise, where

\[
P = \int_{\rho_2}^{\rho_2^2} p(\tilde{\alpha}^1, \ldots, \tilde{\alpha}^p) \, d\tilde{\alpha}^1 \ldots d\tilde{\alpha}^p,
\]  

(A.7)

and

\[
p(\tilde{\alpha}^1, \ldots, \tilde{\alpha}^p) = (2\pi)^{-p/2} |D|^{-1/2} \exp(-\tilde{\rho}^2/2)
\]  

(A.8)

is the Gaussian probability density of \( \tilde{\alpha}^1, \ldots, \tilde{\alpha}^p \), with

\[
\tilde{\rho}^2 = \tilde{\alpha}^\nu \tilde{\alpha}^\mu D_{\nu \mu},
\]  

(A.9)

\[
|D| = |D^{\nu \mu}|.
\]  

(A.10)

Varying \( \rho_a^2 \) with respect to \( f^{\mu \nu} \), noting that \( \delta D_{\nu \mu} = -D_{\nu \lambda} \delta D^{\lambda \eta} D_{\eta \mu} \), and shifting Greek indices, we obtain the variational equations

\[
\delta f^{\nu \mu} \{ a_\nu a_\mu f^{\mu \nu} - \Lambda_{\nu \mu} g^{\mu \nu} \} = 0,
\]  

(A.11)

where \( \Lambda_{\nu \mu} \) is the Lagrange multiplier tensor associated with the side condition (A.5). The solution is

\[
f^{\mu \nu} = g^{\mu \nu},
\]  

(A.12)

as found before for the solution minimizing the statistical least-square error, with Lagrange multipliers \( \Lambda_{\nu \mu} = a_\nu a_\mu \).

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