Inter-hemispheric oceanic exchange

By DORON NOF* and SERGEY BORISOV
The Florida State University, USA

(Received 22 October 1997; revised 6 May 1998)

SUMMARY

The exchange of water between hemispheres is examined theoretically by looking at the behaviour of continuous (double frontal) abyssal currents situated on the bottom of a (parabolic) meridional channel. We used a reduced-gravity (fluid) model with an active lower layer on the bottom of the channel, and a passive infinitely deep upper layer on the top. The topography is the agent responsible for forcing the current across the equator. We first examined analytically the nonlinear, steady, frictionless case and, as a second step, we considered a nonlinear numerical simulation (of the Bleck and Boudra type). We then compared the results of these two (fluid) models to each other and to the solid-balls model of Borisov and Nof which examined the migration of particles in the same cross-equatorial channel.

All three models show that, in general, the current (or a ‘cloud’ of solid particles) advances gradually toward the equator without much change to its structure. Upon reaching the vicinity of the equator, the current width (or the distance between neighbouring balls) decreases dramatically. The current (or cloud) then turns eastward, flows rapidly downslope, and rises on the eastern side. The flow (or cloud) ultimately splits into two parts, one progressing northward and the other recirculating and advancing southward.

Remarkably good agreement is found between the detailed solid-particles analysis and the new numerical fluid simulations, indicating that the inter-hemispheric exchange is primarily an inertial process that depends mainly on the channel geometry. It is shown that the partition of mass flux between the two hemispheres depends on the way that the solid particles, or the current-jet, impinge on the eastern flank of the channel. In contrast to the very good agreement between the solid balls and the fluid simulations, poor agreement was found between the detailed analytical prediction for the cross-equatorial transport and the numerical solution (even when the viscosity was as small as we could use for numerical stability). However, the analytical solution does give the correct prediction for the position of the resulting currents and their general behaviour. It is argued that the disagreement between the analytically calculated fluid mass transports (which, in contrast to the balls, is subject to a potential-vorticity constraint) and the numerical computations is due to the alteration of potential vorticity. Specifically, the friction alters the potential vorticity of the flow and prevents it from acting as a constraint to the crossing. Though crucial to the crossing, the role of friction is, therefore, passive. It merely allows the fluid to alter its potential vorticity in such a manner that the flow can adapt to the crossing pattern imposed by the geometry.

It is suggested that the observed recirculation of Antarctic Bottom Water in the vicinity of the equatorial Atlantic may be a result of our splitting process.

KEYWORDS: Abyssal circulation Frictional processes Potential-vorticity alteration

1. INTRODUCTION

The exchange of deep water between hemispheres has much impact on the earth’s climate and is of considerable interest today. The existence of such an exchange has been confirmed by observations of deep water properties in the two hemispheres, and has been known for quite some time (Emery and Meincke 1986; Mantyla and Reid 1983; DeMadron and Weatherly 1994). However, so far, only limited attention has been given to the mechanisms by which deep oceanic flows cross the equator. This limited attention is at least partly due to the vanishing of the Coriolis force along the equator, which implies that the validity of the quasi-geostrophic approximation near the equator is in doubt.

We shall relax the quasi-geostrophic approximation and look at the nonlinear mechanism of equatorial crossing by double frontal (abyssal) flows on the bottom of meridional underwater channels (Fig. 1). We shall examine two fluid models, one analytical and the other numerical, and then compare results with the recent solid-balls analysis of Borisov and Nof (1998). All three models will show that some water (or balls) can indeed penetrate
from one hemisphere to the other in a cross-equatorial channel. The general scenario is that the flow (or a cloud of balls) approaches the equator along the western side of the channel. In the vicinity of the equator the flow accelerates, flips to the eastern side of the channel and a fraction of the water flows northward. The remaining fluid recirculates and flows southward. The amount of water that actually crosses the equator and ends up in the northern hemisphere depends on the steepness of the channel, the width of the current relative to the width of the channel, the distance between the approaching current and the equator, and the density of the water.
(a) Observational background

(i) Atlantic Ocean. The densest water in the Atlantic is the deep Antarctic Bottom Water (AABW). The fact that it crosses the equator was first documented by Merz and Wüst (1922). The crossing is confined to a channel bounded by the South American continent to the west and the mid-Atlantic ridge to the east (Fig. 1(a)). Wright's (1970) calculations give a northward AABW volume transport of about 6 Sv \((Sv = 10^6 \text{ m}^3\text{s}^{-1})\) at middle latitudes in the South Atlantic, decreasing to about 2 Sv in the equatorial region (see also Hogg et al. 1982). More recent calculations (McCartney and Curry 1993; DeMadron and Weatherly 1994) show slightly higher values: 7 Sv enters the Brazil Basin from the south, and the flux gradually decreases to 6.7 Sv at 23°S and then to 5.5 Sv at 11°S. At the equator, measurements indicate a flow with a transport of 2–3 Sv (Hall et al. 1997). A fraction of the water penetrates into the eastern basin through the Romanche Fracture Zone, but most of the water enters the Guiana Basin. This flow ultimately splits into two approximately equal branches; one of them flows eastward through the Vema Fracture Zone at 11°N to the eastern basin, and the other continues northward along the mid-Atlantic Ridge through the Guiana Basin. See also Whitehead and Worthington (1982) and Whitehead (1989) which address bottom-water flow after it crosses the equator.

(ii) Indian Ocean. The Central Indian Ridge and the Ninetyeast Ridge divide the deep Indian Ocean into three separate parts: the West Australian Basin; the Central Indian
Basin; and a group of basins to the west of the Central Indian Ridge. Cross-equatorial flows, however, have been observed only in the Somali Basin. The source of the bottom water in the Somali Basin is the Lower Circumpolar Water (LCPW) which, in contrast to the Atlantic Bottom Water (consisting of purely AABW) is a mixture of AABW and North Atlantic Deep Water (NADW; Johnson et al. 1991). This water enters the Somali Basin from the Mascarene Basin through the Amirante Passage. About 4 Sv of this water flows northward along the continental rise of Africa. Most of the flow turns east at the equator and impinges on the Carlsberg Ridge. Some water flows northward along this ridge and fills up the northern part of the Somali Basin. As in the Atlantic case, this northern part of the Somali Basin can be viewed as a broad cross-equatorial channel.

(iii) Pacific Ocean. As in the deep Indian Ocean, the densest Pacific water has the properties of LCPW. As mentioned, this water mass consists of fresh and cold AABW, and more saline and slightly warmer NADW (Warren 1981; Wijffels 1993). The bottom water flows northward along the Tonga–Kermadec Ridge in the South Pacific as a western boundary current. It then flows northward through the Samoan Passage at about 10°S (see e.g. Johnson et al. 1994; Rudnick 1997).

According to Stommel and Arons (1960b; see also Kawase 1987 and Kawase et al. 1992), bottom water should also flow in the northern hemisphere as a western boundary current. Recently Johnson and Toole (1993) and Hallock and Teague (1996) verified the existence of such currents in the Pacific. Other recent observations (Sokov 1991, 1992) suggest that at least part of the water flows eastward along the equator until it reaches the Line Islands at about 157°W. At this point some water flows northward (away from the equator) along the western flank of the Line Island ridge. Eastward leakages through the zonal passages created by the Islands are also reported by Sokov (1991, 1992). These observations suggest that some bottom water reaches the East Pacific Rise and penetrates into the eastern part of the Pacific through low-latitude faults. Properties of this water can later be traced at least as far north as 39°N (Mantyla 1975). However, Sokov’s work depends on geostrophic estimates perilously close to the equator to show eastward leakages of bottom water through the Line Islands. Furthermore, Sokov’s results are contradicted by direct measurements (Firing 1989) which show a strong westward bottom current of 6 Sv near the Line Islands. Also, the work of Tsuchiya and Talley (1996) shows the Clarion Passage (~10°N and 165°W), and not the Clipperton Passage (~12°N and 157°W) asserted by Sokov, to be the source of the coldest bottom water.

It is conjectured here that, as in the other two oceans, the cross-equatorial flow can perhaps be viewed as if it is taking place in a cross-equatorial channel.

(b) Theoretical background

The first model of the abyssal circulation in the world ocean was proposed by Stommel (1959). He considered the bottom flows to be driven by sources and sinks (an idea based on Goldsborough’s (1933) calculations for flows driven by evaporation and precipitation). Regions where the bottom waters are formed are the sources; two major areas are the North Atlantic and the Weddell Sea. The ‘sink’ is distributed, i.e. the water rises uniformly everywhere in the ocean and leaves the bottom layer. The model of Stommel and Arons (1960a,b) suggests that the bottom water moves away from the sources as western boundary currents in all oceans. In the interior, the bottom water forms large cyclonic gyres where it is slowly upwelled. This theory has been confirmed by the numerical experiments of Kawase (1987) and Kawase et al. (1992). (For additional information on other attempts to understand related processes, the reader is also referred to Stommel and Arons (1972),
Nof et al. (1991), Speer and McCartney (1992), and Speer et al. (1993) where various aspects of deep flows near the equator are discussed.)

The theory of Stommel and Arons does not explain, however, what happens to the bottom water in the vicinity of the equator. Cross-equatorial flows are subject to a potential-vorticity constraint, resulting from the change in planetary vorticity which accompanies changes in latitude. Since most flows in the ocean interior (i.e. away from western boundaries) are quasi-geostrophic, their relative vorticity is small, so that it cannot compensate for the change in the planetary vorticity required by an inviscid equatorial crossing. This implies that an inviscid cross-equatorial flow that is quasi-geostrophic (i.e. small relative vorticity) requires the depth to change sign which is, of course, impossible.

Consequently, most of the inviscid fluid in the ocean interior cannot easily be displaced from one hemisphere to the other. As a result, it has been believed that cross-equatorial flows occur mainly via boundary currents where either the relative vorticity is high so that it can compensate for the change of planetary vorticity (see e.g. Anderson and Moore 1979; Nof and Olson 1993), or that the dissipation is high so that it can alter the potential vorticity (see also Killworth 1991; Schopp and Colin De Verdiere 1997).

There has been a debate on the issue of whether dissipation is essential for cross-equatorial flows. As mentioned, some of the earlier studies showed that cross-equatorial flows with large Rossby number can occur even without the alteration of potential vorticity. Furthermore, our own preliminary numerical experiments in a narrow (flat bottom) ocean with a source in the South Pole and a sink in the North Pole showed that one can force a high Rossby number flow from one pole to the other without changing the potential vorticity. Other studies (e.g. Killworth 1991; Johnson 1993; Straub et al. 1993; Springer and Kawase 1993) suggest, however, that friction could be important regardless of the value of the Rossby number. It is, therefore, not clear what the specific role of friction is and how large it needs to be; the present article represents an attempt to help resolve this controversy.

(c) Present approach

As mentioned, the fact that deep inviscid currents with large vorticity (i.e. large Rossby number) can cross the equator (due to bottom topography) has been shown analytically by Nof and Olson (1993) for a narrow cross-equatorial channel (i.e. the width of the channel is of the same order as the Rossby radius). By altering its relative vorticity, the inviscid slowly varying double frontal jet flips at the equator from the western to the eastern side of the cross-equatorial channel and then continues to flow poleward. The present article is essentially an extension of the Nof and Olson (1993) inviscid solution to viscous fluid and wide channels where the slowly-varying assumption breaks down at the equator.

It will be demonstrated that the actual picture is considerably more complicated than that described by the inviscid slowly-varying solution of Nof and Olson (1993). Specifically, there are two important differences between the narrow-channel study of Nof and Olson (1993) and the present broad-channel investigation. First, when the channel is broad (rather than narrow), not all the inviscid fluid crosses the equator. The equator acts as a 'splitter', in the sense that some fraction of the fluid recirculates and remains in the same hemisphere from which it originates. Second, friction is crucial to the problem, even when we take the frictional coefficient to be as small as numerical stability allows. This results from the fact that the viscosity causes the formation of equatorial frictional layers which, in turn, force the fluid to alter its potential vorticity. The alteration allows more fluid to cross the equator and end up in the opposite hemisphere.

Our present article is also related to the recent work of Borisov and Nof (1998; hereafter referred to as BN) who examined the behaviour of cross-equatorial eddies. To shed
some light on the eddies in question, BN first examined the behaviour of solid frictionless balls in a meridional equatorial channel. Frictionless balls (or single particles) are, of course, much simpler than real fluid, because they are subject only to inertial forces whereas the fluid is subject to inertial forces, pressure forces and friction. Despite this simplicity, particles sometimes mimic the behaviour of fluids and much can be learned from the particles’ paths. BN showed that the percentage of balls (out of a ‘cloud’ of 10,000) that ultimately settles in the opposite hemisphere depends mainly on the angle at which the balls encounter the eastern flank of the channel. This, in turn, depends on where the balls cross the channel axis. They showed numerically that the penetration of balls into the opposite hemisphere is maximized when the crossing occurs right at the equator (Fig. 2), and that cross-equatorial lens-like eddies display very similar behaviour. As shown in Fig. 2, the symmetrical, maximum penetration situation takes place when the steepness parameter $\alpha = 0.01$ ($\alpha = 2g' H_0 / \beta^2 y_0^2 L^2$, where $g'$ is the reduced gravitational acceleration given by $\Delta \rho / \rho (g)$ where $\rho$ is the density, $H_0$ is the bottom height at the centre of the initial cloud, $L$ and $y_0$ are distances between the centre of the initial cloud and the $x$ and $y$ axes and $\beta$ is the linear variation of the Coriolis parameter with latitude). A reduced penetration occurs wherever $\alpha$ is either larger or smaller than 0.01. The existence of such a critical value for $\alpha$ illustrates that the partition of the balls between the hemispheres is a geometrical property of the field. Note that the upper central panel of Fig. 2 (time = 60; $\alpha = 0.01$) shows a symmetrical situation with a channel axis crossing point situated exactly along the equator. The impingement point (i.e. the easternmost edge of the dark region) is situated in the northern hemisphere and, consequently, the impingement direction is toward the north-east. The lower central panel (time = 30; $\alpha = 0.0246$), on the other hand, shows an impingement point south of the equator and an impingement angle pointing
toward the east-north-east rather than the north-east. A northern (southern) impingement point favours a migration to the northern (southern) hemisphere, because in the northern (southern) hemisphere the sloping bottom forces the balls northward (southward).

The eddies considered by BN are very different from the presently considered continuous currents both topologically and dynamically. First, an eddy cannot adequately represent the continuous flux of deep water observed in all oceans. Second, intense eddies can be sustained anywhere in the ocean, whereas intense currents can usually be sustained only next to western boundaries. Third, the eddies’ drift is usually small compared to their orbital speed, so that the velocity reverses direction across the eddy; no such reversal occurs in the current case.

We shall see that, despite these important differences between eddies and continuous currents, our presently considered continuous cross-equatorial currents show some similarity to the cross-equatorial eddies considered by BN. In particular, as in the eddies’ case, maximum penetration into the opposite hemisphere occurs when the slope of the bottom is ‘critical’ in the sense that it corresponds to the jet crossing the equator along the channel axis. Also, as in the BN eddies’ case, friction causes the alteration of potential vorticity. As we shall see, this similarity results primarily from the filamentation process, which causes both eddies and currents to cross the equator via very narrow features (whose width depends on the frictional coefficient). These equatorial filaments are so narrow compared with both the original off-equatorial eddies and the original off-equatorial currents that they do not ‘remember’ their initial conditions, and consequently behave in an independent manner.

This article consists of two main parts. The first part is an analytical study of an inviscid current splitting in a broad cross-equatorial (parabolic) channel (section 2). We shall use nonlinear reduced-gravity shallow-water equations, with an active bottom layer and a passive infinitely deep upper layer. For this purpose we shall (temporarily) assume that there is no friction in the process and perform the calculations for uniform potential-vorticity currents using the laws of conservation of mass, energy and momentum. This nonlinear analytical calculation is based on integrated properties and, therefore, does not reflect in detail what occurs inside the domain; nevertheless, it does give the desired information (i.e. the ultimate partition of the approaching current). The second part (section 3) is a numerical study of nonlinear currents crossing the equator in similar channels with parabolic bottoms. It shows that, as in the BN balls case, the partition of the current between the hemispheres depends on how the jet impinges on the eastern flank of the channel (section 4). This, in turn, depends on whether the channel axis is crossed north or south of the equator. Maximum penetration into the opposite hemisphere occurs when the crossing takes place exactly at the equator. This shows that the cross-equatorial process is primarily dominated by the geometry of the channel. We shall see that, although friction plays a crucial role in the problem, it is, by and large, a passive role. The friction merely lets the fluid adjust its potential vorticity so that it can follow whatever the geometry of the channel is forcing it to do.

The present numerical simulations demonstrate that the detailed structure of the cross-equatorial current depends on the viscosity coefficient and the mass transport of the inflow. However, it turns out that the structure of the initial inflow has almost no influence on the final outcome. It will be shown that, no matter how small the viscosity is, the current has to pass through narrow frictional layers, where the total energy (potential and kinetic) remains almost unaltered but the potential vorticity changes dramatically.

We shall see that, in contrast to the exceptionally good agreement between the BN balls study and the present numerical simulations, the agreement between the new analytical solution and the new numerical simulations is not so good. The comparison shows that the steady analytical solution gives the correct position of the resulting currents, but does
not give the correct partition of mass flux between the hemispheres. We found this to be
the case even when we used the smallest viscosity that we could possibly use, indicating
that the steady analytical solution is not the limiting case of the numerical solution when
the viscosity decreases to zero. We shall see that this is due to the alteration of potential
vorticity which takes place regardless of the smallness of the eddy viscosity. Despite this
partial disagreement, a comparison between the analytical and numerical solutions is very
useful as it unequivocally points to the importance of potential-vorticity alteration.

The above aspects are addressed in section 4. The results are then summarized and
the application to the AABW in the Atlantic is discussed in section 5.

2. **Analytical solution for a current splitting in a cross-equatorial
parabolic channel**

This is the first part of the investigation, where we calculate analytically the fraction of
water that penetrates into the opposite hemisphere in a broad channel (i.e. we extend here
the inviscid, slowly varying analytical solution of Nof and Olson (1993) to a broad, rapidly
varying, inviscid flow). We shall assume here that the current has reached a steady state
and (temporarily) assume that friction is negligible. Under these conditions, the integrated
momentum and the Bernoulli integral (along streamlines) are conserved. This allows us
to calculate how much water crosses the equator and ends up in the northern hemisphere,
even though the flow is fully nonlinear in the sense that both the amplitude and the Rossby
number are of order unity.

(a) **Upstream depth and velocity profiles of the uniform-potential-vorticity current**

Consider a current flowing along the western side of a meridional (parabolic) channel
and approaching the equator from the south (Fig. 3). Assume that, as long as the current is
away from the equator, it is slowly varying in \( y \) (and hence is in a cross-stream geostrophic
balance) and has a constant potential vorticity. (Such an assumption is very common (e.g.
Røed 1980); it is later verified with our numerical simulations.) Geostrophy implies that:

\[
fv = g'(\partial h / \partial x) + g'(\partial H / \partial x),
\]

where \( f \) is the Coriolis parameter, \( v \) the meridional velocity and \( h \) the current depth. The
channel bottom height, \( H \), is given by:

\[
H = H_0 (x/L)^2.
\]

Here, \( L \) is the distance between the centre of the channel and the upstream centre of the
current, \( H_0 \) is the bottom height at the centre of the current (see Fig. 3).

The potential-vorticity equation is:

\[
f + (\partial v / \partial x) / h = Q,
\]

where \( Q \) is the incoming potential vorticity (negative when the water is native to the
southern hemisphere). Equations (1)–(3) are valid for all \( y \) away from the equator, and the
general solution is, of course, \( y \) dependent. The relevant single equation is:

\[
\frac{\partial^2 h}{\partial x^2} - \frac{f}{f_0} \frac{h}{R_d^2} = -\frac{h_0}{R_d^2} \left\{ \alpha + \left( \frac{f}{f_0} \right)^2 \right\},
\]

where the potential-vorticity depth \( h_0 = f_0 / Q \), where \( f_0 \) is the upstream Coriolis parameter
and \( R_d = (g' h_0)^{1/2} / f_0 \) is the local Rossby radius at \( y = y_0 \). The interested reader is referred
to Nof and Olson (1993) where the general solution to (3a) is discussed. For our integrated momentum technique (which will be employed later) it is sufficient to derive the solution for a given latitude ($y_0$). For this particular upstream section AB (Fig. 3) where $f = f_0 = \beta y_0$ (with $y_0 < 0$), (3a) gives the following single equation for $h$:

$$\frac{\partial^2 h}{\partial x^2} - h/R_0^2 = -h_0(\alpha + 1)/R_0^2.$$  \hspace{1cm} (4)

Equation (4) is a second-order (ordinary) differential equation and requires two boundary conditions. Both are determined by the condition that the thickness vanishes along the
edges of the current:

\[ h(-L + \Delta x) = h(-L - \Delta x) = 0, \]  

(5)

where \( \Delta x \) is half the width of the entering current.

The upstream solution across AB is:

\[ h = h_0(1 + \alpha) \left[ 1 - \frac{\cosh[(x + L)/R_d]}{\cosh(\Delta x/R_d)} \right]. \]  

(6)

By substituting (6) into (1) we get an expression for the upstream meridional velocity:

\[ v = R_d f_0(1 + \alpha) \left[ \frac{\alpha x}{(1 + \alpha) R_d} - \frac{\sinh[(x + L)/R_d]}{\cosh(\Delta x/R_d)} \right]. \]  

(7)

(b) Analytical problem setup

Consider again our slowly varying upstream current that flows along the southwestern side of a meridional cross-equatorial (parabolic) channel (Fig. 3). For simplicity we shall assume (and later verify with our numerical simulations) that the current remains slowly varying until it reaches the equator. At the equator it flows downhill, turns eastward, crosses the centre of the channel and rises up along the eastern side. It then splits into two parts, one turns northward into the northern hemisphere and the other turns back into the southern hemisphere. This behaviour is later verified with our numerical experiments. We also (temporarily) assume that friction can be neglected, implying that the potential vorticity of the current does not change as it crosses the equator. As we have already pointed out, this assumption will not be verified with our numerical simulation; nevertheless, it is instructive to examine this case because it will tell us specifically what the role of friction is.

Clearly, the structure of the southward exiting current at the southern boundary is also given by (6) and (7), except that we substitute \(-L_s\) and \(\Delta x_s\) for \(L\) and \(\Delta x\). (Here, the subscript ‘s’ indicates association with an exiting branch in the southern hemisphere.) However, for the northward exiting current we cannot use the same expressions, because the sign of the associated potential vorticity is opposite to the sign of the local Coriolis parameter. After analogous computations we get:

\[ h_n = h_0(1 + \alpha) \left[ \frac{\cos[(x - L_n)/R_d]}{\cos(\Delta x_n/R_d)} - 1 \right], \]  

(8)

\[ v_n = -R_d f_0(1 + \alpha) \left[ \frac{\alpha x}{(1 + \alpha) R_d} - \frac{\sin[(x - L_n)/R_d]}{\cos(\Delta x_n/R_d)} \right], \]  

(9)

where the subscript ‘n’ indicates association with an exiting branch in the northern hemisphere (i.e. across section EF (Fig. 3) where \(y = |y_0|\)).

It is important to note that the width of the northward branch is bounded. Since \(h_n\) obviously cannot take a negative value, the maximum width of the current is bounded by the first zero of the cosine function, i.e. \(\Delta x_n < \pi R_d/2\). For branches I and II (see Fig. 3) this is not the case, because the function ‘cosh’ is always positive. The solution to the problem will be found by directly connecting the upstream and downstream regions without solving for the region in between. To do so, we shall employ the familiar energy conservation law (Bernoulli) and the not so familiar integrated momentum constraint. The derivation and application of these conservation principles is presented in the next subsections.
(c) **Integrated momentum equation**

Multiplication of the $y$-momentum equation by $h$ and integration over the area bounded by the dashed line shown in Fig. 3 gives:

$$
\iint \left( hu \frac{\partial v}{\partial x} + hv \frac{\partial v}{\partial y} + h\beta y u + \frac{g'}{2} \frac{\partial h^2}{\partial y} \right) \, dx \, dy = 0,
$$

(10)

which, using the continuity equation $\frac{\partial}{\partial x} (hu) + \frac{\partial}{\partial y} (hv) = 0$, can also be written as:

$$
\iint \left\{ \frac{\partial}{\partial x} (huv) + \frac{\partial}{\partial y} (hv^2) + hu\beta y + \frac{g'}{2} \frac{\partial}{\partial y} (h^2) \right\} \, dx \, dy = 0. \quad (10a)

Applying Stokes' theorem to (10a) we get:

$$
\oint huv \, dy - \oint hv^2 \, dx - \frac{g'}{2} \oint h^2 \, dx + \beta \iint yhu \, dx \, dy = 0,
$$

(10b)

where the arrowed circles indicate counter-clockwise integration. The first term in (10b) vanishes because $h$ is zero along the meridional boundaries and $dy = 0$ on the zonal boundaries. Next, we recall our earlier assumption (later verified with our numerical experiment) that, away from the equator, the branches are mainly meridional so that $u$ is small (compared to $v$) and does not contribute much to the fourth term in (10b). Along the equator $u$ is not small but the area is assumed (and later verified) to be small compared to the entire area of integration, so that here again the fourth term is small. Consequently, we are left only with the second and third terms in (10b).

The above procedure implicitly assumes that $uh$ is not very large along the equator, and a somewhat more rigorous way to show that the fourth term in (10b) is small (without assuming how large $uh$ is along the equator) is to introduce a stream function $\Psi$ defined by $\partial \Psi/\partial x = hu; \partial \Psi/\partial y = -hu$. We then write $-\beta y \partial \Psi / \partial y$ as $-\partial (\beta y \Psi)/\partial y + \beta \Psi$, and get, from (10a):

$$
\oint huv \, dy - \oint hv^2 \, dx + \beta \oint \Psi y \, dx - \frac{g'}{2} \oint h^2 \, dx + \beta \iint \Psi \, dx \, dy = 0. \quad (11)
$$

For each branch of the current away from the equator, the stream function does not vary significantly with $y$ and, consequently, $- \oint \Psi y \, dx = \iint \Psi \, dx \, dy$. (Note that a counter-clockwise integration of $y \, dx$ gives minus the area.) Therefore, the third and fifth terms in (11) approximately cancel each other away from the equator. The area near the equator is again taken to be small and, in contrast to the previous procedure where we do not know in advance the value of $u$ along the equator, we now know that $\Psi$ can never exceed its upstream value so that, as before, we are left with only:

$$
\int_B^A \left( hv^2 + \frac{g'h^2}{2} \right) \, dx + \int_C^D \left( hv^2 + \frac{g'h^2}{2} \right) \, dx - \int_B^E \left( hv^2 + \frac{g'h^2}{2} \right) \, dx = 0, \quad (12)
$$

where the limits of integration are shown in Fig. 3.

(d) **Bernoulli integral**

To solve the problem (i.e. to determine how much water flows northward and how much flows southward) we need to know the positions and widths of the currents leaving
the rectangular basin (i.e. the region bounded by the dashed line shown in Fig. 3). This implies that there are four unknowns, \( L_s, L_n, \Delta x_s \), and \( \Delta x_n \). Equation (12) already relates these four unknowns to each other, so we need three more equations. These are provided by an application of the Bernoulli integral to the three bounding streamlines defined by \( h = 0 \).

The general form of the Bernoulli integral for a slowly varying current in a parabolic channel is:

\[
B(\Psi) = g'h + g'H + v^2/2. \tag{13}
\]

Along the bounding streamlines, where \( h = 0 \), the first term drops out and we are left with the second and third terms. The detailed application of (13) to the bounding streamlines is given in the appendix, and the final solution is shown in Fig. 4. Note that, since within each hemisphere the currents are slowly varying with \( y \), our computed transport penetration is applicable for all \( y \) in each hemisphere.

\( e \) Solution and validity regime

The validity of our solution is restricted, of course, to the cases where the thickness is always positive and the velocity does not reverse within the currents; it turns out that the latter is more restrictive than the former. Application of the condition of no flow reversal to the left and right edges of the branches gives:

\[
L \geq -\Delta x + bR_d \tanh(\Delta x/R_d); \quad L \geq \Delta x - bR_d \tanh(\Delta x/R_d) \tag{14a}
\]

\[
L_s \geq -\Delta x_s + bR_d \tanh(\Delta x_s/R_d); \quad L_s \geq \Delta x_s - bR_d \tanh(\Delta x_s/R_d) \tag{14b}
\]

\[
L_n \geq -\Delta x_n + bR_d \tanh(\Delta x_n/R_d); \quad L_n \geq \Delta x_n - bR_d \tanh(\Delta x_n/R_d), \tag{14c}
\]

where \( b = (1 + \alpha)/\alpha \).

Figure 4 shows the solution and the validity regime which is bounded by (14c) and represented by the thick solid curve. The other two curves (corresponding to (14a) and (14b)) lie to the left of this solid curve and, therefore, do not influence the validity regime.

\( f \) Results

Our solution (Fig. 4) shows that when the widths of both the current and the channel are of the order of the deformation radius (i.e. the entire system, rather than merely the upstream and downstream currents, is slowly varying with \( y \)), most of the water flows northward. This is consistent with the slowly varying solution of Nof and Olson (1993). As the widths of the entering current and the channel increase, the fraction of the water that penetrates into the opposite hemisphere decreases. The cross-equatorial amount varies from 10% to 100%, or so, for incoming currents whose width varies from a few to 20 Rossby radii. Note, however, that inviscid double frontal currents (flowing along a sloping bottom) are unstable, and break up in the absence of friction when the width is of the order of the Rossby radius (e.g. Griffiths et al. 1982; Nof 1990); they become stable when the width is much greater than the Rossby radius, because each front does not sense the presence of the other.

We shall see in the next section that, even though some water can cross the equator in the inviscid case, as we have just seen, much more water can cross in the presence of friction. We shall also see that, even though there is not a good agreement between the numerical and analytical prediction for the amount of water that penetrates into the other hemisphere, there is good agreement in the predicted positions of the currents.
Figure 4. (a) Percentage of the northward transport as a function of the incoming current width ($\Delta x/R_d$) and the distance of the entering current from the axis of the channel ($L/R_d$) according to the analytical solution. The steepness $\alpha$ is fixed (0.01) and $R_d$ is the Rossby radius; $\alpha$ involves both the slope and the distance from the equator. Note that for $L/R_d \sim O(1)$ (i.e. the entire crossing is slowly varying) the solution is very close to the origin. It implies that, under these conditions, 100% of the fluid crosses the equator. The Nof and Olson (1993) solution does not exist for $\alpha$ as small as is shown here. (b) The percentage of mass flux penetration into the northern hemisphere ($T_n$) and (half) the width of the entering upstream current ($\Delta x/R_d$) as a function of $\alpha$ (according to the analytical solution). The width of the channel $L/R_d$ and the entering mass flux are fixed. Note that, as expected, the penetration increases and the width decreases as the steepness increases. See text for definitions and further details.

3. **Numerical solution for a continuous current in a meridional cross-equatorial (parabolic) channel**

This is the second part of the investigation, where we solve numerically a system of reduced-gravity equations with an active bottom layer and an inactive infinitely deep upper layer.

(a) **Numerical model**

We use a reduced-gravity version of the Miami Isopycnal Coordinate Ocean Model of Bleck and Boudra (1981, 1986), later improved by Bleck and Smith (1990). The advantage
of this model is the use of the ‘Flux Corrected Transport’ algorithm (Boris and Book 1973; Zalesak 1979) for the solution of the continuity equation. This algorithm employs a higher-order correction to the depth calculations and allows the layers to outcrop and stay positive definite. The resulting scheme is virtually non-diffusive and conserves mass. For these reasons, the model is the most suitable for our problem. The active layer is an enclosed feature, while the rest of the layer has zero depth.

The equations of motion are the two momentum equations:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \beta y v = -g \frac{\partial h}{\partial x} - 2g' H_0 \frac{x}{L^2} + \frac{\nu}{h} \nabla \cdot (h\nabla u) \tag{15}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \beta y u = -g \frac{\partial h}{\partial y} + \frac{\nu}{h} \nabla \cdot (h\nabla v), \tag{16}
\]

where \(\nu\) is the viscosity, and the continuity equation:

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0. \tag{17}
\]

The model uses the Arakawa (1966) C-grid. The \(u\)-velocity points are shifted one-half grid step to the left from the \(h\) points, the \(v\)-velocity points are shifted one-half grid step down from the \(h\) points, and the vorticity points are shifted one-half grid step down from the \(u\)-velocity points. This grid allows for reducing the order of the errors in the numerical scheme. The grid-scale for the current is 2.5 km, 4 km or 10 km depending on the run. With these choices there were at least three grid points across the currents, even in the equatorial regions where the currents were very narrow (compared to the upstream current). The size of the domain is different for different runs, and varies from 151 by 101 to 441 by 441 grid points. The solution is advanced in time using the leap-frog scheme with time steps of 216 s, 864 s, or 1728 s (i.e., from 400 to 50 steps per day). The velocity fields are smoothed in time in order to stabilize the numerical procedure. The velocities for the weightless grid points are set to zero. This allows us to avoid the unphysical acceleration of the infinitely thin layer downslope.

Before discussing the numerical values chosen for our examples, it is appropriate to point out that the actual flow in the deep ocean is, of course, much more complicated than our rather simplified reduced-gravity model. For instance, entrainment, mixing and turbulence are, no doubt, important (see e.g., Polzin et al. 1996) and are not represented in our model. In view of this, the best fit (of the model to the ocean) that one could hope for is that corresponding to oceanic parameters which are of the same order as the model parameters. Furthermore, actual numerical values have to be chosen with care so as to avoid unphysical situations, such as reverse velocities in the upstream current. We performed a total of 25 experiments (Exps.), details of which are given in Table 1.

Our choice of the geometrical dimensions (i.e. width and steepness) as well as the ‘reduced gravity’ \(g'\) was made according to Fig. 1(b) and the limitations involved in the use of a nonlinear reduced-gravity model for deep ocean flows. Namely, we focus on the core of the AABW (rather than the entire current below 3000 m which is usually referred to as the AABW) because nonlinear frontal models are limited to fairly narrow filaments \(\sim O\) (10 km). This is so because in reduced-gravity frontal models the velocities are of the order of the Coriolis parameter multiplied by the Rossby radius, and consequently become unreasonably high, even for filaments that are less than 100 km broad. In view of both of these aspects, our reduced gravity \((g' = 2 \times 10^{-4} \text{ m s}^{-2})\) and the transport of the current \(T\) \(\sim O(10^5 \text{ m}^2\text{s}^{-1})\) are about one or two orders of magnitude smaller than
<table>
<thead>
<tr>
<th>Exp.</th>
<th>$R_d$ (km)</th>
<th>$\alpha$</th>
<th>$v$ $10^6$ cm$^2$s$^{-1}$</th>
<th>$\Delta x$ km</th>
<th>Inflow mass flux $10^4$ m$^3$s$^{-1}$</th>
<th>Category</th>
<th>Inflow potential vorticity $10^{-6}$ m$^3$s$^{-1}$</th>
<th>Percentage of mass transport that ends up in the opposite hemisphere</th>
<th>Grid size km</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.67</td>
<td>0.01</td>
<td>2</td>
<td>20</td>
<td>0.34</td>
<td>broad</td>
<td>6.23</td>
<td>100</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.01</td>
<td>2</td>
<td>80</td>
<td>9.38</td>
<td>broad</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.01</td>
<td>2</td>
<td>40</td>
<td>4.42</td>
<td>narrow</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.01</td>
<td>2</td>
<td>28</td>
<td>2.93</td>
<td>narrow</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.01</td>
<td>2</td>
<td>24</td>
<td>2.44</td>
<td>narrow</td>
<td>1.08</td>
<td>50–100</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>0.01</td>
<td>2</td>
<td>20</td>
<td>1.95</td>
<td>narrow</td>
<td>1.08</td>
<td>30–50</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>0.01</td>
<td>2</td>
<td>100</td>
<td>11.3</td>
<td>broad</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>0.01</td>
<td>1</td>
<td>100</td>
<td>11.3</td>
<td>broad</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>0.01</td>
<td>0.8</td>
<td>100</td>
<td>11.3</td>
<td>broad</td>
<td>1.08</td>
<td>95</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>0.01</td>
<td>5</td>
<td>80</td>
<td>9.38</td>
<td>broad</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>0.01</td>
<td>1.5</td>
<td>80</td>
<td>9.38</td>
<td>broad</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>0.01</td>
<td>1</td>
<td>80</td>
<td>9.38</td>
<td>broad</td>
<td>1.08</td>
<td>95</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>0.01</td>
<td>5</td>
<td>40</td>
<td>4.42</td>
<td>narrow</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>0.01</td>
<td>1.5</td>
<td>40</td>
<td>4.42</td>
<td>narrow</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>0.01</td>
<td>5</td>
<td>24</td>
<td>2.44</td>
<td>narrow</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>0.01</td>
<td>1.5</td>
<td>24</td>
<td>2.44</td>
<td>narrow</td>
<td>1.08</td>
<td>40–60</td>
<td>4</td>
</tr>
<tr>
<td>17</td>
<td>4</td>
<td>0.01</td>
<td>5</td>
<td>20</td>
<td>1.95</td>
<td>narrow</td>
<td>1.08</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>18</td>
<td>4.78</td>
<td>0.01</td>
<td>2</td>
<td>75</td>
<td>9.38</td>
<td>broad</td>
<td>var</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>19</td>
<td>4</td>
<td>0.01</td>
<td>2</td>
<td>60</td>
<td>6.63</td>
<td>broad</td>
<td>var</td>
<td>*</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>9.40</td>
<td>0.005</td>
<td>20</td>
<td>200</td>
<td>56</td>
<td>broad</td>
<td>var</td>
<td>72</td>
<td>10</td>
</tr>
<tr>
<td>21</td>
<td>8.38</td>
<td>0.01</td>
<td>20</td>
<td>126</td>
<td>56</td>
<td>broad</td>
<td>var</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>22</td>
<td>7.84</td>
<td>0.015</td>
<td>20</td>
<td>96</td>
<td>56</td>
<td>broad</td>
<td>var</td>
<td>27</td>
<td>10</td>
</tr>
<tr>
<td>23</td>
<td>7.49</td>
<td>0.02</td>
<td>20</td>
<td>79</td>
<td>56</td>
<td>broad</td>
<td>var</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>24</td>
<td>7.21</td>
<td>0.025</td>
<td>20</td>
<td>68</td>
<td>56</td>
<td>broad</td>
<td>var</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>25</td>
<td>6.98</td>
<td>0.03</td>
<td>20</td>
<td>60</td>
<td>56</td>
<td>broad</td>
<td>var</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

For all experiments we chose $g' = 0.02$ cm s$^{-2}$, $L = 500$ km and $\beta = 2.3 \times 10^{-12}$ cm$^{-1}$s$^{-1}$. The inflow latitude was 4.5°S (except for Exp. 19 where it was 20°S). In Exp. 1 the viscosity was lowered to $0.5 \times 10^6$ cm$^2$s$^{-1}$ at day 300. In Exp. 18 and 20–25, the Rossby radius $R_d$ is based on the central depth of the inflow (rather than $Q/f_0$) because, in these experiments, the potential vorticity was not uniform. It varied from $g'/f_0R_d^2$ at the centre of the inflow to infinity at the edges. In Exp. 19, the fluid (intentionally) did not cross the equator. The 'Category' column distinguishes between 'narrow' and 'broad' currents: narrow currents are defined to be currents with a width less than 10 $R_d$ whereas broad currents have a width larger than 10 $R_d$.

The frequently quoted values, i.e. the ratio of our filament cross-sectional area to the total AABW cross-sectional area reflects the chosen ratio of the transports. It should perhaps be stressed here that these choices are imposed by the physics of the problem, rather than by the details of the numerical model.

For many of the experiments $\alpha = 0.01$ because, as already mentioned, it turns out that this specific value corresponds to a current that crosses the axis of the channel exactly at the equator. We shall see that these are important cases because, as in the BN balls case, the path is as symmetrical as possible (with respect to the intersection of the equator and the channel axis) and the penetration into the opposite hemisphere is maximized (see discussion regarding Fig. 2 in section 2). It will become clear that, as in the BN balls case, when the crossing point is exactly at the equator, the angle at which the current impinges on the right flank of the channel and the position of the impingement point favour a penetration into the opposite hemisphere. That is to say, as the fluid particles encounter the right flank of the channel, the current is oriented as closely as possible toward the north, and the impingement point is situated north of the equator. Again, as in the BN balls case, when
the current crosses the channel axis north or south of the equator (i.e. \( \alpha \) is smaller or larger than 0.01) the impingement angle and position (south or north of the equator) favour at least a partial reflection of the current to the original hemisphere.

Finally, it is appropriate to comment on the possible role of bottom friction, which is excluded from our model. An inclusion of such frictional terms is straightforward and has some use, because in the actual ocean bottom friction is, no doubt, important. However, we wanted to minimize the effects of friction to see whether friction was really essential for cross-equatorial flow and, consequently, we have neglected any bottom drag. In other words, we did not perform any experiments with bottom friction because our goal was to keep the flow as close as possible to the inviscid state. For the same reason, we used the minimal necessary amount of lateral viscosity required for numerical stability.

We now discuss boundary conditions and initialization. The meridional boundaries are situated away from the currents and are, therefore, taken as walls. The zonal boundaries are open: the current enters the basin through the southern boundary along the western side of the channel, and leaves the basin through both the northern and the southern boundaries along the eastern side of the channel. Therefore, we can fix the depth and the velocity profile of the inflow along the western half of the southern boundary of our domain. However, we do not know in advance the position and structure of the resulting outflow and cannot, therefore, specify the details there. It is, of course, very important that the boundaries do not affect the structure of the current. To avoid such an effect we used a ‘sponge’ boundary condition along the eastern side of the southern boundary and along the entire northern boundary—the regions where the outflows will presumably occur.

We structured the ‘sponge’ condition as follows: at each time step the depth at seven grid points next to the wall is reduced by some factor. For most of the runs we used a factor of zero at the boundary, and factors of 0.4, 0.6, 0.8, 0.9, 0.99 and 0.999 as one proceeds into the interior. The velocities at the boundaries were calculated using the already reduced depth. The procedure of ‘clipping’ the depth of the current was placed in the code just after the integration of the continuity equation and before the momentum equations. These boundary conditions have no influence on the interior. We checked this by placing the northern boundary at two different positions. Except within the immediate vicinity of the boundary where the sponge layer is situated (~25 km), the structure of the current was identical. The familiar isopycnal coordinate model allows for both no-slip and free-slip boundary conditions. However, in our case the type of meridional boundary conditions to be used does not matter because the depth around the edges is zero, so that there are no flows along the meridional boundaries.

To initialize the current we use the formulas for the depth and velocity obtained earlier. We define the depth and velocity everywhere in the domain to be initially zero, except in a rectangle next to the western side of the southern boundary where the depth and the meridional velocity are given by (6) and (7). The length of this rectangle in the meridional direction is six grid points for most of the runs. The finite length of the initial current allows for a smooth adjustment during the early stages of the current development.

\((b)\) Results

We performed 25 numerical simulations for currents in a cross-equatorial parabolic channel (Table 1). Experiment 1 (Fig. 5) was performed solely for the purpose of comparing our numerical simulations to the analytical results, and will be discussed later. All of the other experiments are sensitivity experiments.

(i) Sensitivity experiments. A number of parameters could potentially determine the behaviour of the currents, among them the viscosity, the potential vorticity of the incoming
current, its width, and the geometry of the channel. We performed a series of experiments in order to understand the influence of these factors. In this series of experiments the grid-scale was 4 km, the size of the domain was 301 by 251 grid points (i.e. 1200 by 1000 km) and the time step was 846 s (i.e. 100 steps per day). The Rossby radius was 4 km and, as in all of the other experiments that we did, we made sure that there were at least three grid points across the current's narrowest portion. (Note that, to avoid flow reversals, we had to restrict ourselves to currents whose Rossby radius is much smaller than the channel width ($R_d \ll \alpha L$).)

Before describing the results of these sensitivity studies, it is perhaps appropriate to mention again that the percentage of fluid that crossed the equator and ended up in the opposite hemisphere depends on two processes. The first and most important process is the impingement of the jet on the eastern side of the channel. We shall see that, as in the BN balls case, how the fluid is partitioned between the two hemispheres depends on the angle with which the jet encounters the sloping bottom (on the eastern flank of the channel), the height of the impingement point and the location of the impingement point relative to the equator. By 'impingement point' we mean here the point of maximum run-up on the eastern slope. Note that both the angle and the height are related to the steepness of the
Figure 6. Experiments (Exps.) 2–6: (a) Depth distribution corresponding to currents with different inflow widths (2Δx). The initial potential vorticity of the currents, the geometry of the channel (i.e. its steepness α = 0.01) and the viscosity coefficient (ν) are held constant in all the experiments. T is the transport. The narrow current (Exp. 6) shown in the lower right panel exhibits (temporarily) a shape similar to that of the wider currents, but later transforms into a different structure (see Fig. 6(b), lower panels). All other currents are shown at the time when they have nearly reached a steady state. The thickening of the current’s ‘head’ near the exit results from the ‘sponge layer’ that the current encounters. It does not indicate a backward influence of the (exit) boundary condition. The first contour is 0.25 m and the contour interval is 1 m. (b) As Fig. 6(a) except that the currents are narrower; consequently they are less stable, and become time dependent rather than steady. The current shown in the two upper panels is between a steady and a time dependent state. Note that a return flow on the upper left-hand side panel just started to interact with the main current. As before, the thickening of the eddies’ ‘head’ near the exit results from the ‘sponge layer’. This effect is confined to the immediate vicinity of the boundary (<50 km).

See text for further details.

channel, and that the agreement between the balls and the numerical (fluid) simulations will show that the process is inertial in nature (i.e. it is controlled by the channel geometry).

The second process is the alteration of potential vorticity that occurs as the fluid progresses along the equator in the form of frictional layers. The alteration is taking place in such a manner that fluid ending up in the northern hemisphere has positive potential vorticity, whereas fluid left behind in the southern hemisphere has negative potential vorticity. The changes in the fluid potential vorticity are taking place so that the equatorial crossing process imposed by the geometry can primarily occur in an inertial manner. (We shall see later that these results are not very sensitive to the original potential vorticity, because the alteration of potential vorticity along the equator is much greater than the original potential vorticity.) The fluid achieves the changes in potential vorticity (via friction) simply by smoothing out the local velocity gradients.
We shall see that, as in the BN balls case, a maximum penetration into the opposite hemisphere takes place when the crossing of the channel axis occurs exactly along the equator. Under such conditions the problem is as symmetrical as possible, in the sense that the current encounters the eastern slopes of the channel exactly along the equator. Again, as in the BN balls case, it so happens that $\alpha = 0.01$ corresponds to this situation. When the current crosses the equator east or west of the channel axis, the angle at which the flow encounters the east side of the channel and the position of the impingement point (relative to the equator) favour a smaller penetration into the opposite hemisphere.

We began our sensitivity studies (Exps. 2–6) by varying the width of the inflow and holding all the other variables constant (Fig. 6(a) and (b)). For relatively large widths ($\Delta x > 24\text{ km}$) the entire flow crosses the equator and flows northward away from the equator. After some time the amplitude of the transport oscillations decreases, and eventually the currents reach a steady state; for wider currents this happens much sooner than for narrow currents. Note that the frictional forces are more effective in narrower currents and, consequently, there is more of a potential and kinetic energy loss in these particular cases (Exps. 3, 4, and 5). This can easily be seen by noting that, on the eastern side of the channel, the narrow currents do not rise to the same level that they originally had on the western side. (As mentioned, since most of the currents’ energy is potential energy relative to the axis of the channel, and not kinetic energy, this implies a loss of energy.) As a result, the distance of
the outflow from the axis of the channel decreases as the width of the current decreases. Also, the turn that the current makes as it reaches the eastern boundary becomes sharper (Fig. 6(a), lower panels), and eventually, for very narrow currents (Fig. 6(b)) this turn becomes so sharp that the return flow interacts with the main current. At this point in time the current becomes very unstable and strongly time dependent. Therefore, the outflow is a chain of eddies which are shed into both the southern and the northern hemispheres.

The ‘cobra head’ around 2°N results from the current’s response to the impact with the eastern flank of the channel. A very similar structure was observed in the BN solid-balls case (see their Fig. 6) indicating again that the process is nearly inertial. Downstream of the cobra head, the balls’ path oscillates and the analogous current experiences thickening. The thickening is due to the accumulation of water resulting from the relative narrowness of the downstream current and its associated tendency to break up into a chain of eddies. That is to say, downstream of the cobra head the current contains a (clockwise) recirculation region because it is narrower than the upstream current and, therefore, is less stable and has more of a tendency to break up into lenses. The thickening and recirculation process is the way in which the current ‘prepares itself’ to break up into eddies.

The associated time dependent mass fluxes across 4°N and 4°S for these cases are shown in Fig. 7. Experiment 5 represents the transition between steady northward-flowing currents and eddy-generating currents, because at different times it shows different characteristics (Fig. 6(b), two upper panels, and Fig. 7, upper panel). The currents behave in this
Figure 8. Experiments (Exps.) 10–12: Depth distribution corresponding to currents with different viscosity coefficients, \( \nu \). \( \Delta x \) corresponds to half the width of the entering current, and \( T \) is the transport. For comparison, the results of Exp. 2 are also shown. Note that, with the smallest \( \nu \) (lower right panel), some water returns to the southern hemisphere. The contours are as in Fig. 6 except in the lower right panel where the contour interval is now 200 cm.

particular way because as mentioned earlier: first, friction has less impact on the energy of the wider currents and, therefore, the broad downstream currents rise to a higher altitude (on the slope of the channel); and second, narrow currents are less stable than broad currents.

In the next set of experiments (Exps. 10–12) we proceed to examine the influence of viscosity on the behaviour of the current (Fig. 8). For values of \( \nu \) larger than 1.5 \( \times 10^6 \) cm\(^2\)s\(^{-1}\) the structure of the current changes very little as the viscosity increases. However, with the larger coefficient the currents are closer to the steady state; the associated fluxes are shown in Fig. 9. For \( \nu = 1 \times 10^6 \) cm\(^2\)s\(^{-1}\) a fraction of the flow (about 5% of the total flux) recirculates and returns to the southern hemisphere. In contrast to the narrow currents (\( \Delta x = 20 \) and 24 km) which, due to the fact that they are less stable, are strongly time dependent (Fig. 6(b)), this wide low-viscosity current is close to a steady state. With large viscosity coefficient (\( \nu = 5 \times 10^6 \) cm\(^2\)s\(^{-1}\)), however, the narrow currents (\( \Delta x = 20 \) and 24 km) become more stable and, consequently, do not produce eddies (as they do with \( \nu = 2 \times 10^6 \) cm\(^2\)s\(^{-1}\)). Instead, they flow northward much like the wider currents described in the first set of experiments. Experiments 14 and 16 displayed very similar results to those of Exps. 3 and 5 and, therefore, are not shown. Similarly, the narrow current high-viscosity experiments (Exps. 13, 15 and 17) are not very different from the broad current high-viscosity experiment (Exp. 10) and are not shown.
Figure 9. Experiments (Exps.) 10–12: Time series of the mass fluxes across 4°N and 4°S along the eastern side of the channel for currents with constant half-width $\Delta x$, and different viscosity coefficients $\nu$. Again, for comparison, the results of Exp. 2 are also shown. Note that, as expected, for higher $\nu$ the current is closer to a steady state.

After the completion of the above experiments, we examined currents with identical incoming transports but with different depths and velocity profiles (Exps. 2 and 18). We found that the total upstream transport is important, but its distribution has little influence on the final outcome. Figure 10 shows an example of two currents with identical transports but different potential vorticities. The upper panel shows an incoming current with uniform potential-vorticity inflow (Exp. 2), whereas the lower panel shows an incoming current with uniform velocity and parabolic depth profile (Exp. 18). The two outflows look almost identical because the flow experiences significant transformations as it progresses along the equator and "forgets" its initial structure.

Figure 11 summarizes the results of our sensitivity study for constant steepness $\alpha$ (Exps. 2–17). Since we found that the structure of the outflow currents does not depend on the depth and velocity profiles of the inflow, we only show the dependence on the transport and viscosity. Crosses mark the experiments in which the entire flow crosses the equator. The squares represent the cases with some recirculation and southward flow; the open squares correspond to outflows described by a chain of eddies; and the filled squares correspond to nearly-steady states. The square with the cross marks the transitory Exp. 5 discussed above.

(ii) Potential-vorticity changes. As mentioned, during the process of equatorial crossing the current has to pass through a narrow frictional layer. Consequently the flow is altered, and the property that changes the most is the potential vorticity. Specifically, its sign changes from negative to positive as the current crosses the equator (Fig. 12). In mid-latitude the potential vorticity remains constant; most of the changes occur after the current crosses the equator.
The flow in the core of most abyssal currents is relatively weak, and the main contribution to the potential vorticity away from the fronts is $f/h$. This is certainly true in our experiments. Because the core's depth decreases almost linearly as the current approaches the equator, the value of the potential vorticity within the core (i.e. away from the fronts) remains the same. In the opposite hemisphere, the central depth cannot take a negative value, consequently the potential vorticity experiences the changes that we observe in Exp. 2 (Fig. 12(a)). It is noteworthy that the eddies that were shed in the time dependent case (Exp. 5) have positive potential vorticity in the northern hemisphere and negative in the southern hemisphere (Fig. 12(b)).
The changes in the potential vorticity are achieved as follows. As the current encounters the equator it turns rapidly eastward and accelerates downslope. Very high velocities are established, and this causes a dramatic increase in the frictional forces (no matter how small the frictional coefficient is). Specifically, the frictional term becomes so large that the equatorial frictional layer is formed. The scale of the layers, \( \ell \), is \( (vL)^{1/2}/(g'H_0)^{1/4} \) which, for our chosen variables, gives a width of about 12 km, much smaller than the 100 km original current width. As shown by BN, \( \ell \) results from a balance between the frictional forces and advection. To see this, recall that the time that it takes the frictional forces to penetrate (from the sides) into the core of a filament (whose width is \( \ell \)) is \( \ell^2/v \). For frictional effects to be important, this penetration time must be roughly equal to the time that it takes the fluid to slide downhill along the equator and reach the axis of the channel. This sliding time is given by \( L/(g'/H_0)^{1/2} \) where, as before, \( L \) is the distance between the initial position of the current and the axis of the channel, and \( (g'/H_0)^{1/2} \) is the speed at which the fluid slides downhill in the vicinity of the equator. Equating the two times leads to the cross-equatorial length-scale \( \ell \) mentioned above.

The Munk length-scale, \((v\beta)^{1/3}\), which for our variables is about 17 km, is also relevant to the problem because it gives the length corresponding to local dissipation of relative
Figure 12. (a) A snapshot of the potential-vorticity distribution for experiment (Exp.) 2 shown in the upper right panel of Fig. 8 (i.e. an incoming current with inflow half-width $\Delta x = 80$ km and viscosity coefficient $v = 2 \times 10^6$ cm$^2$s$^{-1}$ at time $t = 500$ days; the transport $T = 9.38 \times 10^4$ m$^3$s$^{-1}$). The potential vorticity is nondimensionalized by the absolute value of the inflow potential vorticity. Note the changes that occur as the fluid is crossing the equator. Also note that, in this case, the current is broad and stable and the influence of friction on the current’s energy is minimal. The latter can be easily verified by noting firstly that most of the current’s energy is potential energy, and secondly that on the eastern side of the channel the current rises to approximately the same level that it originally had on the western side. (b) As (a) but for Exp. 5 where $\Delta x = 24$ km, $v = 2 \times 10^6$ cm$^2$s$^{-1}$ at $t = 1500$ days; $T = 2.44 \times 10^4$ m$^3$s$^{-1}$. Note that the eddies have the same sign of potential vorticity as the local Coriolis parameter. Also note that, in contrast to the broad current shown in the upper panel, the initial current is now narrow and, consequently, it is less stable and is subject to a more severe frictional effect. As a result, the downstream branches break into eddies and rise to only a fraction of the initial current altitude (i.e. the eddies are closer to the channel axis than the original current).
vorticity generated by the changes in latitude. The fact that it is somewhat larger than our cross-equatorial-scale indicates that we have a viscosity coefficient somewhat larger than we actually need or, equivalently, that our model can handle latitudinal changes with a \( \beta \) stronger than the one typical for the planet.

As the flow progresses, the frictional forces smooth out the local velocity gradients, forcing the (primary) component of the potential vorticity \( f/h \) to adapt to the local values of \( f \). This implies that the relative vorticity caused by the latitudinal change is destroyed after its creation. As before, since the meridional narrow currents are less stable than the meridional broad currents, they produce eddies more easily. Also, they are more sensitive to the presence of friction and, consequently, in contrast to broad currents which conserve nearly their entire energy, they lose some energy as they cross the equator. Since most of the flow's energy is associated with potential energy, this results in a final downstream eddy altitude that is lower than that of the broad currents (see e.g. Fig. 12(a) and (b)).

To confirm that the changes in the potential vorticity are indeed caused by the presence of the equator rather than the extensive length of the current, we have performed the following numerical run (Exp. 19). We forced the current to advance the same distance as it did in the previous experiments, but we kept it in the same hemisphere. All the parameters of the channel and the current, including the potential vorticity and the width, were the same as in the actual equator-crossing experiment. The only difference is in the position of the bounding latitudes: 20°S and 1°S instead of 4.5°S and 4.5°N. The results of this experiment indicate that the change in the potential vorticity is negligible compared to the change in the cross-equatorial flow. This experiment verifies that, indeed, the equator has a very special influence on oceanic flows. Despite the changes in potential vorticity, the (mechanical) energy is nearly conserved in the broad-currents case (Fig. 13). Narrow currents suffer some energy loss because they are more sensitive than broad currents to frictional losses. Similarly, broad currents subject to high viscosity also show some energy loss; nevertheless, even in these two extreme cases the loss of energy is not severe (Fig. 13).

We also compared the size of the various terms in the momentum equations for Exp. 2 (i.e. the cross-equatorial current, that flows northward without recirculation) at the time when it has nearly reached a steady state (Fig. 14(a)). We found that, indeed, the size of the frictional term becomes comparable to the other terms in the equation when the current flows as a very narrow jet along the equator, and that the main alteration of the potential vorticity occurs there (Fig. 14(b)).

In this context, it is appropriate to point out that, in the presence of our chosen horizontal friction, the modified potential-vorticity equation is:

\[
\frac{D}{Dt} (\partial u/\partial y - \partial v/\partial x + f/h) = \frac{\partial}{\partial y} \left\{ \frac{v}{h} (\nabla \cdot h
\nabla u) \right\} - \frac{\partial}{\partial x} \left\{ \frac{v}{h} (\nabla \cdot h \nabla v) \right\}.
\]  

(18)

Similarly, the modified energy equation (which, with \( v = 0 \), leads to the steady Bernoulli energy) is:

\[
J[(u^2 + v^2)/2 + g'\{h + H_0(x/L)^2\}; \psi] = v \{u \nabla \cdot (h \nabla u) + v \nabla \cdot (h \nabla v)\},
\]  

(19)

where \( J \) is the Jacobian.

As we saw earlier, the viscosity-induced changes in the velocity fields cause \( O(1) \) changes in the potential vorticity. These changes are primarily due to the first term on the right-hand side of (18), because within the equatorial filaments \( u \) is very large and the \( y \)-scale is very small. This situation is analogous to that occurring in other frictional boundary layers. The viscosity-induced changes in the velocity field also cause an \( O(1) \) change in the kinetic energy of the fluid (see the right-hand side of (19)). However, since
the kinetic energy is very small compared to the current's potential energy \((g' H_0 (x/L)^2)\), the influence of the frictional terms on the total energy is relatively small. The current easily adjusts to the kinetic energy loss simply by sliding a short distance downhill, giving the impression that there is almost no energy loss.

(iii) **Sensitivity to the channel steepness** \((\alpha)\). We performed sensitivity studies for currents in channels of different steepness, with all other variables fixed. We found that for any values of \(\alpha\) different from that used earlier (corresponding to a current crossing the channel axis at the equator) we needed to increase the viscosity coefficient to stabilize the solution. Therefore, we undertook several numerical experiments for broad currents with high viscosity \((v = 2 \times 10^7 \text{ cm}^2\text{s}^{-1}; \text{Exps. 20–25})\). Fig. 15 shows the resulting depth distributions for these six experiments. In each case the currents are steady.

As already pointed out, the way that the currents split after encountering the eastern side of the channel depends mostly on the angle and the latitude of the current at the moment of impingement. The maximum northward transport occurs when the equator is crossed exactly in the centre of the channel (Exp. 21). This ensures that the angle at which the jet impinges on the east side of the channel is directed toward the north, and that the impingement region is situated in the immediate vicinity of the equator. When the crossing of the equator is situated to the right (Exps. 22, 23, 24 and 25) or left (Exp. 20) of the channel's axis, the angle at which the jet impinges on the right side of the channel, and its off-equatorial location, force a partitioning of the current (see e.g. Whitehead (1985) for a discussion of a jet impinging on a wall). We shall see later that there is a similarity between the recirculation displayed by these numerical runs (and those shown in Fig. 12(b)) and the recirculation observed by DeMadron and Weatherly (1994).
Figure 14. (a) Experiment (Exp.) 2: alongstream profile of the various terms in the momentum equations (four lower panels) averaged across the sections shown in the upper panel. Left panels show the terms of the x-momentum equation and right panels show the terms of the y-momentum equations for three intervals. The first interval is bounded by the southern boundary and the line marked 'I'. The second interval is bounded by lines 'II' and 'III', and the third is bounded by line 'IV' and the northern boundary. Note that the main balance is between the gravitational force due to the slope of the channel, the Coriolis force and the total acceleration in the x-momentum equation (upper left panel), and between the Coriolis force and the total acceleration in the y-momentum equation (upper right panel). Friction is significant when the current flows along the equator (lower panels) and is responsible for the alteration of potential vorticity. (b) Alongstream profile of nondimensionalized potential vorticity corresponding to the maximum depth of the current shown in (a). Note that most of the changes occur when the current is flowing along the equator.
Figure 15. Experiments (Exps.) 20–25. The depth distribution of broad currents with high viscosity ($v = 2 \times 10^7 \text{ cm}^2\text{s}^{-1}$) and different steepnesses, $\alpha$. Note that the maximum northward transport occurs when the equator is crossed in the centre of the channel (Exp. 21). This assures that the angle at which the jet impinges on the east side of the channel is directed toward the north and that the impingement region is situated along the equator. When the crossing of the equator occurs to the right (Exps. 22, 23, 24 and 25) or left (Exp. 20) of the channel's axis, the angle at which the jet impinges on the right side of the channel and its off-equatorial location force splitting of the current (see e.g. Whitehead (1985) for a discussion of a jet impinging on a wall). Note that, as before, the thickening of the downstream currents results from clockwise circulation associated with accumulation of water. It is a consequence of the currents' downstream narrowing (compared to the upstream state) and their associated tendency to break up into a chain of eddies. The contour interval is 5 m; the transport, $T = 5.6 \times 10^5 \text{ m}^3\text{s}^{-1}$.

Also note that, in some cases, the current turns completely and goes back to the southern hemisphere, so that no water flows to the northern hemisphere (Exps. 23, 24 and 25). Figure 16 shows the dependence of the amount of water that ends up in the northern hemisphere on the steepness of the channel. We should point out that, unfortunately, our attempts to simulate the Nof and Olson (1993) current (i.e. a current whose width is of the order of the Rossby radius in a channel whose width is also of the order of the Rossby radius) were unsuccessful, because such currents are unstable and break up before the equator is reached.

4. DISCUSSION

We shall now compare our three models (i.e. the two new fluid models presented here and the BN solid-balls model) with each other in order to see what can be learned from their similarities and differences. We know by now that all three models display the same general behaviour (i.e. splitting along the eastern flank of the channel) but we shall see that there are important differences in the details of the processes.
(a) Similarity of the present numerical simulations to the BN solid-balls study

All of our simulations show that there is a remarkable similarity between the detailed numerical simulations and the BN balls study. Both show that the impingement angle and position are the most crucial to the problem, and that maximum penetration to the opposite hemisphere occurs when the channel axis is crossed exactly along the equator. This similarity is best illustrated by reference to Fig. 16 which shows that, at least up to $\alpha = 0.015$, the difference between the two relative-penetration curves is small. The discrepancy between the currents clearly increases when $\alpha > 0.015$ and why this is so is not clear to us. One possibility is that it is due to the chaotic behaviour of the balls, discussed by Borisov (1996) and BN.

It is also of interest to note that the path of the current that crosses the equator without any recirculation in the southern hemisphere (e.g. Exp. 2 shown in Fig. 10) is very similar to the trajectories of a cloud of balls (solid particles) in a channel of the same geometry (Fig. 17). The current mimics the particles’ trajectories not only along the western and central part of the channel, but also when they oscillate along the eastern side of the channel. This results from the fact that the inertial forces are dominant whereas the average effect of the pressure gradients is insignificant, so there is no difference between the frictionless balls’ paths and our flow path. Also, as mentioned earlier, the effect of friction on the total energy of the fluid is insignificant, so there is almost no difference between the frictionless balls’ paths and our flow path. Recall that there is actually an O(1) loss in the kinetic energy of the fluid but, since most of the fluid’s energy is potential rather than kinetic, the fluid easily compensates for the loss by sliding a short distance downhill.

(b) Comparison of the numerical results with the analytical solution

As mentioned, to compare the results of our analytical computations in detail with the numerical results, we performed one very high-resolution experiment (Exp. 1). Although this experiment enabled us to perform a detailed comparison between the analytical and the numerical simulation, for reasons of computational economy its parameters were (unavoid-
Figure 17. The path of the current in Experiment (Exp.) 2 (upper panel) and the corresponding trajectories of 20 balls arranged initially along a line (lower panel). The balls' trajectory was adapted from Borisov (1996). The incredible similarity between the two shows that pressure forces play a minor role in the crossing process and that most of the energy is conserved (see text). For other balls' trajectories which do not cross the channel's axis along the equator and show at least partial splitting (i.e. some of the balls turn back and stay in the southern hemisphere) the reader is referred to Borisov (1996) and Borisov and Nof (1998; BN in text).

ably) outside the range corresponding to the real ocean. (The remaining numerical simulations, Exps. 2–25, were carried out with parameters that correspond more directly to the real ocean. However, it turns out that the range of these parameters lies outside the validity regime of the analytical model. Consequently a detailed comparison is impossible for these cases.) For the very high-resolution experiment (Exp. 1) we chose a point on the diagram (Fig. 4(a)) inside the validity regime of the analytical model \((L/R_d, \Delta x/R_d) = (300, 12)\). For this point, the analytical model gives a value of 15\% for the transport flowing northward and 85\% for the transport which is recirculating and flowing southward. For these values of \(L\) and \(\Delta x\) the width of the incoming current is much smaller than its distance from the equator. Therefore, we used a grid-scale of 2.5 km in a basin of 1100 by 1100 km.
As before, we chose this grid size to ensure that even the most narrow filament has at least three grid points across it. The eastern and western boundaries were located 550 km from the axis of the channel. The southern boundary was located 500 km, and the northern boundary 600 km, from the equator. The time step was 216 s (400 steps per day). The viscosity coefficient that we initially used was \( \nu = 2 \times 10^6 \text{ cm}^2\text{s}^{-1} \).

To our surprise, we found that the results of this high-resolution numerical run (shown in Fig. 5) did not agree at all with the analytical computations for the cross-equatorial transport, but agreed (to within 10%) with the analytical computations for the current's position. Specifically, instead of 85% of the water flowing southward and 15% going north, the entire current flowed northward. The current became steady after its head left the basin. This can easily be seen from the first two panels of Fig. 5 which are almost indistinguishable. The analytical solution predicts that the distance between the axes of the outflow branches and the axis of the channel is the same as that between the parent current and the channel axis. This is so because, as before, most of the steady filament's energy is potential energy (relative to the channel axis), so that the analytical position prediction is merely a statement that each branch of the outflow current will rise on the east side of the channel to the same level that the parent current originally had on the south-western side. Figure 5 shows that this is indeed the case.

The large discrepancy between the analytical and the numerically simulated transports is, obviously, caused by the presence of friction in the numerical experiment, as this is the only significant aspect that distinguishes the numerical from the analytical procedure. To examine the sensitivity of the solution to the frictional coefficient and make sure that the above is indeed the case, we reduced the value of \( \nu \) (in Exp. 1) to one quarter of the original value (to \( 5 \times 10^5 \text{ cm}^2\text{s}^{-1} \)) at \( t = 300 \) days. We see from Fig. 5 that at \( t = 305 \) days changes in the behaviour of the current start to occur, and the current becomes thinner in the equatorial region. This is a result of a decrease of the frictional length-scale, \( (\nu/L)^{1/2} (g' H_0)^{1/4} \), which determines the width of the current in the equatorial region. By \( t = 370 \) days some water starts to leak to the south, but no matter how small the viscosity coefficient is, it does not influence the current significantly. Therefore, we conclude that the frictionless analytical prediction for the transport is not the limit of the numerical solution when \( \nu \to 0 \).

It should be pointed out that there are three (potentially) important difficulties with the reasoning leading to the above conclusion. First, the inviscid analytical solution is steady, whereas the numerical simulations used for the comparison are time-dependent. Second, although the horizontal eddy viscosity that we used (\( 0.5 \times 10^6 \text{ cm}^2\text{s}^{-1} \) for Exp. 1, day 300 and onwards) is as small as we could use for numerical stability, it is not that small compared to viscosities used in other numerical simulations. (Usually, a 'diffusion speed' of \( 1 \text{ cm s}^{-1} \) is considered 'average' for general-circulation isopycnic models. For a grid size of 2.5 km it gives an eddy viscosity of \( 0.25 \times 10^6 \text{ cm}^2\text{s}^{-1} \) which is half of our lowest value.) Third, there is some evidence that the field is slightly changing as the viscosity is reduced (see the flow structure of days 320 and 370 of Exp. 1 shown in Fig. 5) suggesting that our simulations do not exactly correspond to the limiting case of \( \nu \to 0 \). Despite these three potential difficulties, however, it is believed that our numerical simulations are not far from the limiting case of \( \nu \to 0 \), because firstly the dissipation due to the viscosity is very small, and secondly the filaments adjust their width to whatever viscosity is imposed on the field.

As pointed out earlier, since our viscous numerical simulations agree with the frictionless balls study, it follows that the friction itself is not the most important issue. The importance of friction is that it lets the fluid alter its potential vorticity so that the geometrically forced exchange process can take place.
Comparison of the present numerical simulation of currents with the BN numerical simulations of eddies

Frontal currents and lens-like eddies are two different oceanic ‘beasts’. First, they are topologically very different: lens-like eddies have a finite amount of water within them whereas currents extend to infinity. Second, they are dynamically very different: lens-like eddies usually have an orbital speed that is much greater than their drift speed, making their absolute speed change sign as one proceeds across them. No such change occurs in continuous currents. (For this reason, a current cannot be thought of as the limit of an elongated eddy.) There are also important differences in the methods that can be employed for their investigations. For instance, as we saw earlier, currents can be studied with the aid of steady analytical models whereas eddies cannot.

In view of these differences, it is to be expected that the two features will behave in a very different way as they cross the equator in a meridional channel. Surprisingly, however, our present numerical simulations for continuous double frontal currents show much similarity to the BN simulations of cross-equatorial eddies. In both cases maximum penetration occurs when the feature crosses the equator along the axis of the channel, and in both cases the partition of mass flux (or mass) between the two hemispheres is very similar to the partition of a cloud of solid balls (see our Fig. 16, and Fig. 17 in BN). Note, however, that the maximum penetration of the currents (100%) agrees exactly with the maximum penetration of a cloud of solid balls, whereas the maximum penetration of eddies is somewhat smaller (about 80%). The above similarities result from the severe equatorial filamentation that both features are subject to. The filamentation, which dramatically reduces the N–S scale to \((vL)^{1/2}/(g' H_0)^{1/4}\), causes so much distortion that the feature does not ‘remember’ whether it was originally a current or an eddy.

5. Conclusions

The behaviour of abyssal oceanic flows in a meridional cross-equatorial channel was examined within the framework of a nonlinear reduced-gravity model. We performed analytical computations for inviscid uniform potential-vorticity currents, based on integrated characteristics of the flow. We then examined the cross-equatorial currents numerically, using an isopycnic coordinate model (with the Flux Corrected Transport procedure). Finally, we compared these new results both with each other and to the BN study of solid balls in a cross-equatorial channel (Fig. 2). All three models show that the current (or ‘cloud’ of balls) advances toward the eastern flank of the channel (along the equator), and that the partition of mass flux occurs along the eastern side of the channel. The details of the three models’ results are not identical, however, and an inter-comparison of the three models leads to the conclusions below:

1. The inviscid analytical study showed that the percentage of water that crosses the equator and settles in the opposite hemisphere depends on the width of the original incoming current and the steepness of the channel \(\alpha\) (Fig. 4(a) and (b)).

2. The numerical (fluid) simulations display a remarkable similarity to the BN solid-balls study. In both cases the penetration into the opposite hemisphere is maximized when the channel axis is crossed exactly at the equator, and this happens when \(\alpha = 0.01\) (Figs. 5, 6(a), 8, 10, 14 and 17). The percentage of mass flux that penetrates into the opposite hemisphere is also remarkably similar to the percentage of balls that cross the equator and settle in the opposite hemisphere (Fig. 16). Since the balls are not subject to a potential-vorticity constraint, the similarity suggests that the geometry of the channel is the most crucial element of the cross-equatorial problem. Potential vorticity is changing as the fluid
crosses the equator, but this takes place merely to accommodate the exchange forced by the geometry.

(3) In contrast to the remarkable similarity between the BN balls study and the present numerical simulations, there is a limited similarity between the analytical solution and the numerical solutions. Specifically, the numerical simulations revealed that the analytical prediction of the percentage of cross-equatorial transport does not hold in the presence of friction because, no matter how small the viscosity is, narrow currents with high speeds are formed along the equator. These high-speed filaments suffer a drastic change in their potential vorticity (Figs. 12 and 13). Due to these changes, more water crosses the equator and less water recirculates. The analytical prediction for the position of the resulting steady currents is accurate, however, because the associated energy losses are very small (Fig. 13).

(4) Sensitivity experiments with the numerical model showed that the viscosity and the transport of the inflow influence the detailed structure of the cross-equatorial flow, but the incoming velocity, depth distribution and potential vorticity of the inflow are irrelevant (Figs. 6–11). The presence of the equator is essential for the potential-vorticity alteration. When the current stays in the same hemisphere, there are no significant changes in the potential vorticity (Fig. 12).

(5) Despite the obvious differences between the continuous currents considered here and the lens-like eddies considered by BN, both show much similarity to the BN balls study. This is due to the dramatic equatorial filamentation that ‘erases’ the fluid memory of its original structure. The scale of the filaments is \((vL)^{1/2}/(g' H_0)^{1/4}\) (where \(L\) is the distance between the feature and the channel axis and \(H_0\) is the feature height relative to the bottom channel).

The results of our comparisons suggest that, even though the flow is influenced by parameters such as viscosity and width of the inflow, the steepness parameter is the most crucial to the problem, because it controls the amount of water that ultimately ends up in the opposite hemisphere. This is so because the steepness of the channel controls the inertial properties of the flow and, most importantly, the angle at which the current impinges on the eastern side of the channel and the latitude where the impingement occurs.

It is suggested that the observed recirculation of the AABW approaching the equatorial Atlantic from the south (Fig. 18) may be a result of our crossing processes. Having said this, however, one should also note that the northern edge of the recirculation shown in Fig. 18 appears to be situated well to the south of the equator, suggesting that it might be caused by a process different from our crossing mechanism. The question whether the discrepancy between the two locations of the recirculation northern edge is real or not is left as a subject for future investigation.

It should also be pointed out that the actual cross-equatorial flow is forced by the local topography to be oriented westward (Hall et al. 1997) rather than eastward. This local discrepancy between the actual east–west geometry and the modelled meridional channel does not invalidate our results because the general inter-hemispheric exchange must take place in a meridional manner.

**ACKNOWLEDGEMENTS**

D.N. acknowledges related conversations with Joe Pedlosky prior to the initiation of this study. Conversations with G. Weatherly regarding the observational aspects were very helpful. Stephen Van Gorder provided helpful comments on the manuscript. The study was supported by the National Science Foundation under grants OCE 9102025 and OCE 9503816, National Aeronautics and Space Administration grants NAG5-4883 and NAG5-4813, and Office of Naval Research grant N00014-96-1-0541.
APPLICATION OF THE BERNOULLI PRINCIPLE TO THE BOUNDING STREAMLINES

Application of (13) to the three bounding streamlines AF, BC and DE (see Fig. 3) gives:

\[
(-L - \Delta x)^2 + 2(-L - \Delta x)R_d \tanh(\Delta x/R_d) + bR_d^2 \tanh^2(\Delta x/R_d)
\]

\[= (-L_n - \Delta x_n)^2 + 2(-L_n - \Delta x_n)R_d \tanh(\Delta x_n/R_d) + bR_d^2 \tanh^2(\Delta x_n/R_d) \tag{A.1}
\]

\[
(-L + \Delta x)^2 - 2(-L + \Delta x)R_d \tanh(\Delta x/R_d) + bR_d^2 \tanh^2(\Delta x/R_d)
\]

\[= (-L_s - \Delta x_s)^2 + 2(-L_s - \Delta x_s)R_d \tanh(\Delta x_s/R_d) + bR_d^2 \tanh^2(\Delta x_s/R_d) \tag{A.2}
\]

\[
(-L_s - \Delta x_s)^2 + 2(-L_s - \Delta x_s)R_d \tanh(\Delta x_s/R_d) + bR_d^2 \tanh^2(\Delta x_s/R_d)
\]

\[= (-L_n - \Delta x_n)^2 - 2(-L_n + \Delta x_n)R_d \tanh(\Delta x_n/R_d) + bR_d^2 \tanh^2(\Delta x_n/R_d), \tag{A.3}
\]

and this closes our system. Note that here, \(b = (1 + \alpha)/\alpha\).
The system of four equations with four unknowns can easily be solved. First, from (A.1) we get an expression for \( L_n \) in terms of \( \Delta x_n \):

\[
L_n = \Delta x_n - R_d \tan(\Delta x_n / R_d) + \left\{ -\frac{1}{\alpha} R_d^2 \tan^2(\Delta x_n / R_d) + B_1 \right\}^{1/2},
\]

where \( B_1 = (-L - \Delta x)^2 + 2(-L - \Delta x)R_d \tanh(\Delta x / R_d) + bR_d^2 \tanh^2(\Delta x / R_d) \). Second, from (A.4) we get an expression for \( L_s \) in terms of \( \Delta x_s \):

\[
L_s = \Delta x_s - R_d \tan(\Delta x_s / R_d) + \left\{ -\frac{1}{\alpha} R_d^2 \tan^2(\Delta x_s / R_d) + B_2 \right\}^{1/2},
\]

where \( B_2 = (-L - \Delta x)^2 - 2(-L - \Delta x)R_d \tanh(\Delta x / R_d) + bR_d^2 \tanh^2(\Delta x / R_d) \). Note that we used only the positive roots in (A.4) and (A.5) to ensure that, as required, \( L_n \) and \( L_s \) are positive.

Here, \( B_1 \) and \( B_2 \) refer to the Bernoulli integral along the left and right sides of the entering current, respectively. We have two remaining equations, the integrated momentum equation (16) and the conservation of the Bernoulli integral along DE (A.3), for the two remaining unknowns, \( \Delta x_s \) and \( \Delta x_r \). Although the resulting equations are algebraic, they cannot easily be solved analytically because of the complexity of the associated exponentials. However, they can easily be solved numerically because their solution is the intersection of two curves on a plane.

References


Firing, E. 1989 Mean zonal currents below 1500 m near the equator, 150°W. *J. Geophys. Res.*, 94, 2023–2028


Sokov, A. 1991 Penetration of Antarctic bottom water into the Northeast Basin of the Pacific Ocean through the deep Clipperton Passage. Oceanology, 31, 412–416


<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Reference</th>
</tr>
</thead>
</table>