The parametrization of entrainment in cloudy boundary layers

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SUMMARY

A parametrization for the entrainment rate in shear-free convective cloud-capped boundary layers is derived. The only information it requires is the external mixed-layer turbulence forcings (the surface buoyancy flux and the net radiative-divergence profile), the inversion jumps of temperature and humidity, the cloud-top liquid water mixing ratio and the cloud and mixed-layer depths. Despite this simplicity it is found to compare well against both a wide range of large-eddy simulations and observations of stratocumulus.

The parametrization is an extension of those derived previously for the idealised cases of smoke clouds, where turbulence is driven by combinations of surface heating and radiative cooling, and liquid water clouds driven solely by buoyancy reversal. The radiative forcing is specified as an indirect forcing, through the buoyant production of turbulence within the boundary layer and a direct forcing, which promotes deepening of the boundary layer when undulations in the cloud top cause part of the cooling to occur within the horizontally averaged inversion. Condensation of water in saturated air reduces the strength of both these terms but otherwise, for radiatively driven liquid water clouds (where evaporative cooling of entrained air does not generate buoyancy reversals), the parametrization is unchanged from that for smoke clouds.

Where evaporative cooling of entrained air is strong enough to generate buoyancy reversal, and therefore drive convective motions, not only does it provide an additional turbulence source, but it is also found to compensate for the reduction in strength of the radiative-forcing terms by the presence of saturated air. Allowing for this enhancement, the entrainment rate is predicted with remarkable accuracy by the sum of previously derived parametrizations for the rates that would have been generated by radiative cooling and buoyancy reversal acting in isolation.

KEYWORDS: Convection; Large-eddy simulation; Stratocumulus

1. INTRODUCTION

The importance of the correct modelling of entrainment in large-scale numerical models, used for global numerical weather prediction (NWP) or climate studies, for example, is increasingly being realised. Beljaars and Betts (1992) found the lack of entrainment in the European Centre for Medium-range Weather Forecasting (ECMWF) NWP model resulted in boundary layers that tended to be too shallow, cold and moist. In a sensitivity study of the ECHAM global-climate research model, Brinkop and Roeckner (1995) found enhanced entrainment of warm dry air into the boundary layer not only led to cloud dilution in some cases, as might be expected, but also that the warmer drier boundary layer could lead to larger surface moisture fluxes which, when coupled with the deeper boundary layer, gave more cloud. In addition, tropical convection was more vigorous, the hydrological cycle was intensified and the whole troposphere in general became warmer and moister.

In section 2, a general entrainment parametrization for both cloud-free and cloudy boundary layers will be presented which is based solely on parameters that would be known in such NWP models, but firstly some existing parametrizations will be discussed.

In shallow non-precipitating systems, the total water mixing ratio, $q_t = q_v + q_l$, and equivalent potential temperature, approximated here by $\theta_e = \theta + (L/c_p)q_v$, are conserved quantities under adiabatic vertical motion and water phase changes, to a reasonable degree of approximation. Here $\theta$ is the potential temperature, $q_v$ and $q_l$ the vapour and liquid water mixing ratios, $L$ is the latent heat of vaporisation of water and $c_p$ the specific heat at constant pressure. The buoyancy, $b$, of the air can similarly be approximated as $g(\theta_e - \theta_0)/\theta_0$, where $g$ is the acceleration due to gravity, $\theta_0$ is a constant reference potential temperature; $\theta_v$ is the virtual potential temperature, given by $\theta_v = \theta + \theta_0(r_mq_v - q_l)$, where $1 + r_m$ ($\approx 1.608$) is the ratio of the molecular weight of dry air to that of water vapour. In the following

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discussion, $\Delta \phi$ is the change in a variable $\phi$ across the inversion capping the boundary layer; $\phi'$ is the deviation from the horizontal average denoted by $\bar{\phi}$, and $w$ is the vertical velocity.

In a cloud-free convective boundary layer it is generally accepted that the magnitude of the buoyancy flux due to entrainment, $\overline{w' b'_e}$, can be approximated as a fixed fraction of that at the surface, $\overline{w' b'_s}$. Estimates from observations suggest this fraction, $A_1$, is approximately 0.2 (see Stull 1976, for example). If the mixed layer is capped by a discontinuous inversion at a height $z = z_i$, with buoyancy jump $\Delta b$ (so that $\overline{-w' b'_e} = w_c \Delta b$, where $w_c$ is, in the absence of subsidence, the rate of rise of the inversion), the above flux relationship for a cloud-free boundary layer can be written in the widely used form

$$w_c = A_1 \frac{w_s^3}{z_i \Delta b}$$  (1)

with

$$w_s^3 = 2.5 \int_0^{z_i} \overline{w' b'} \, dz,$$  (2)

Writing (1) in terms of this generalized $w_s$ expresses the energy available to drive entrainment as a fixed fraction of the vertically integrated buoyant turbulent kinetic energy (TKE) production, as proposed by Deardorff (1976). Equation (1) can be (and often is) applied directly to cloudy boundary layers, but observations of stratocumulus (by Nicholls and Leighton 1986, for example) indicate values of $A_1$ at least an order of magnitude greater than for cloud-free boundary layers.

Deardorff proposed two changes to (1) to allow for the presence of cloud. Firstly, because the air just below the inversion is now saturated, the buoyancy flux there can be approximated in terms of the cloud-top fluxes of $\theta$ and $q$, as $w' b'_e = (g/\theta_0)(\beta w' \theta'_0 |_{z_i} - \theta_0 \overline{w' q' |_{z_i}})$, where the thermodynamic coefficient $\beta$ ($\approx 0.5$) is a weak function of state. Secondly, he allowed a fraction $\alpha$, of the cloud-top radiative divergence $\Delta F$, (which will be measured throughout this paper in temperature flux units, $\text{K m s}^{-1}$) to occur in the discontinuous inversion, with the remainder, $(1 - \alpha)$, within the mixed layer. These assumptions then implied

$$\overline{w' b'_e} = -w_c \delta b + \beta \alpha \frac{g}{\theta_0} \Delta F,$$  (3)

where $\delta b = (g/\theta_0)(\beta \Delta \theta - \theta_0 \Delta q_i)$. Using (3) instead of $\overline{-w' b'_e} = w_c \Delta b$, (1) becomes

$$w_c = \frac{A_1 w_s^3 + \beta \alpha (g/\theta_0) \Delta F}{z_i \delta b}.$$  (4)

With a lack of quantitative information as to its value, Deardorff took $\alpha = 0.5$ and satisfactorily simulated a marine stratocumulus case-study.

Note that by distributing the radiative cooling in this way, it was possible not only to provide a cooling tendency at the top of the mixed layer and so enhance the TKE, but also to promote entrainment directly by reducing the strength of the inversion. Others (Kahn and Businger 1979 and Slingo et al. 1982, for example) have subsequently argued, however, that all but a negligible part of the radiative-flux divergence occurs within the cloud. Therefore, because the cloud is constrained beneath the inversion and so within the turbulent layer, the influence of the radiative cooling must be indirect, serving only to increase the strength of the boundary layer turbulence. Recent results from large-eddy simulations of smoke clouds (Lock and MacVean 1999a and Moeng et al. 1998) have
suggested a new interpretation of this direct radiative term. They assumed the inversion was locally discontinuous, with all of the radiative cooling occurring within cloudy, boundary layer air (as with Kahu and Businger), but that horizontal undulations in the cloud top resulted in part of the radiative cooling appearing within the horizontally averaged inversion (defined to be between the height of the buoyancy flux minimum and where the flux went to zero above, at \( z = h \) say). Integrating the thermodynamic equation over this inversion resulted in the appearance of a direct radiative term (proportional to \( \alpha_i \Delta F \), say) in the equation for \( w_e \), as in (3). In addition, though, because all the radiative cooling was assumed to occur within boundary layer air, the full radiative divergence (i.e. up to \( z = h \)), namely \( \Delta F \), served to increase the strength of the boundary layer turbulence and hence generate entrainment indirectly. This dual rôle for radiative cooling within the turbulent layer (hence the subscript in \( \alpha_i \)) will be returned to in section 2.

Equation (4), however, has a more fundamental shortcoming. Evaporative cooling following mixing of entrained and cloudy air can, under certain circumstances, reduce the buoyancy of the entrained air to less than that of the surrounding unmixed cloudy air. This buoyancy reversal criterion, derived by Randall (1980) and Deardorff (1980), is given by \( \delta b < 0 \). Note, then, that as \( \delta b \) becomes small or negative as is commonly observed in stratuscumulus (see Kuo and Schubert 1988, for example), Deardorff’s parametrization (4) ceases to be valid.

Instead of assuming that a fixed fraction of the integrated buoyancy flux was available to drive entrainment, Stage and Businger (1981) evaluated each physical processes’ contribution to the buoyancy flux integral and separated those that produced TKE from those that consumed it (which, in the case of entrainment, therefore, depended on the sign of \( \delta b \)). If long-wave radiative cooling at cloud top is the only buoyancy source other than evaporative cooling, Stage and Businger’s parametrization can be written

\[
   w_{esB} = \begin{cases} 
   A_1 (g/\theta_0) \Delta F \left[ \zeta^2 + (1 - \zeta^2) \beta \right] / \zeta^2 (g/\theta_0) \Delta \theta_d - A_1 (1 - \zeta^2) |\delta b|, & \text{for } \delta b < 0, \\
   A_1 (g/\theta_0) \Delta F \left[ \zeta^2 + (1 - \zeta^2) \beta \right] / \zeta^2 (g/\theta_0) \Delta \theta_d + (1 - \zeta^2) \delta b, & \text{for } \delta b \geq 0, 
   \end{cases}
\]

(5)

where \( \theta_d = \theta + \theta_0 r_m q_l \) and \( \zeta = (z_{ml} - z_c)/z_{ml} \) is the ratio of the depth of unsaturated air \( z_c \) is the cloud-depth) to the mixed-layer depth, \( z_{ml} \). Here, the liquid water potential temperature, \( \theta_l \), is approximated as \( \theta_l = \theta - (L/(c_p) q_l \). Note that \( z_{ml} \) is not necessarily equal to \( z_i \) as the cloud layer may decouple from the surface. Stage and Businger took \( A_1 = 0.2 \) to recover (1) for idealized cloud-free surface-heated boundary layers.

Note, firstly, that Stage and Businger assumed that all the radiative cooling acted within the cloud, and so served only to generate entrainment indirectly via buoyant production of turbulence. Secondly, the radiative cooling buoyancy source (in the numerators of (5)) has been split, via \( \zeta \), into the contributions from the cloud and sub-cloud layers; the components multiplied by \( \beta \) represents the cloud-layer contribution. Because \( \beta \) is less than one, the radiative source term has been reduced (compared to the same divergence over an unsaturated layer, i.e. \( \zeta = 1 \)) by the presence of saturated air. Physically, temperature perturbations, such as those from radiative cooling, make smaller perturbations to the buoyancy of saturated compared with unsaturated air because of the warming generated by condensing droplets.

It should also be noted, as Stage and Businger themselves pointed out, that (5) is not well-behaved when \( \zeta^2 (g/\theta_0) \Delta \theta_d - A_1 (1 - \zeta^2) |\delta b| \) is small or negative, a regime in which strong buoyancy reversal will tend to decouple the cloud layer from the sub-cloud.
layer. They interpreted this as a stratocumulus to cumulus transition, not relevant to their model. In Siems et al. (1990) and Lock and MacVean (1999b), however, continuous sheets of stratocumulus exhibiting buoyancy reversal showed a very strong tendency to decouple. Mixing was essentially restricted to the cloud layer (implying \( \zeta \to 0 \)) but \( w_c \) remained finite and positive, suggesting that (5) may in fact be ill-defined for most stratocumulus-capped boundary layers where there is buoyancy reversal.

One truly general entrainment-rate parametrization was proposed by Turton and Nicholls (1987). It was based on the cloud-free parametrization, (1), but because their observations from stratocumulus showed values of \( A_1 \) up to an order of magnitude greater than 0.2, they proposed an empirical extension for cloudy boundary layers to include the effect of evaporative cooling. They incorporated the parameter \( \Delta_m \), derived by Nicholls and Turton (1986), which can be written

\[
\frac{g}{\theta_0} \Delta_m = 2 \int_0^1 b(\chi) \mathrm{d}\chi,
\]

where \( b(\chi) \) is the buoyancy excess of an entrained air parcel with mixing fraction of free-atmospheric air \( \chi \). Nicholls and Turton considered that all mixing ratios were equally likely and so they proposed (6) as a physically meaningful measure of the stability of an interface across which mixing would generate evaporative cooling. In the notation used here, \( b(\chi) \) is given by

\[
b(\chi) = \begin{cases} 
\chi \delta b, & \text{for } 0 \leq \chi \leq \chi_s, \\
bs + \frac{\chi - \chi_s}{1 - \chi_s} (\Delta b - \bs), & \text{for } \chi_s < \chi \leq 1,
\end{cases}
\]

where \( b_s \) is the buoyancy of the most negatively buoyant parcel (a known quantity), obtained when the parcel is only just saturated (when \( \chi = \chi_s \)). Substituting (7) in (6) and integrating gives \( (g/\theta_0) \Delta_m = \chi_s \delta b + (1 - \chi_s) \Delta b \). Turton and Nicholls’ parametrization was fitted to the observations of stratocumulus given in Nicholls and Turton to give

\[
w_{\text{CTN}} = \frac{A_1 w^3}{\varepsilon_{\text{ml}} \Delta b} \left[ 1 + a_2 \left( 1 - \frac{(g/\theta_0) \Delta_m}{\Delta b} \right) \right] = \frac{A_1 w^3}{\varepsilon_{\text{ml}} \Delta b} \left[ 1 + a_2 (\chi_s + D) \right],
\]

where \( D (\equiv -b_s/\Delta b) \) is the Siems et al. (1990) cloud-top entrainment instability (CTEI) parameter. \( A_1 \) was given the value 0.2 and the best fit was obtained for \( a_2 = 60 \). Again all radiative divergence was assumed to be within the mixed layer.

Whilst this review is by no means exhaustive, it does demonstrate a common failing with many entrainment-rate parametrizations which effectively renders them unacceptable for NWP. This is that they become invalid in certain commonly encountered regions of parameter space. The alternative presented here (from Turton and Nicholls 1987) does not suffer from this problem but was empirically derived from a very limited set of observations, and so its skill over a wide range of parameter space remains to be tested.

In the next section a new, generally applicable parametrization will be derived and, in section 4, comparison with results from a set of large-eddy simulations will be made.

2. A new general entrainment rate scaling

In Lock and MacVean (1999a) it was found that in dry smoke-cloud boundary layers, with turbulence driven by combinations of cloud-top radiative cooling and surface heating,
both processes appeared to generate entrainment independently, so that a parametrization based on a cubic sum of 'external' velocity scales, characterizing the strength of the forcing, was found to be very successful. Here, it is intended to extend this parametrization to liquid water clouds: firstly to those where evaporative cooling is not sufficiently strong to generate buoyancy reversal; then to those where it is, using the parametrization derived in Lock and MacVean (1999b) for clouds where buoyancy reversal was the sole turbulence generating mechanism.

Take as a starting point the entrainment rates derived in Lock and MacVean (1999a and 1999b), hereafter referred to as LMa and LMb, respectively, for turbulence generated by combinations of surface heating (surf) and radiative cooling (rad) at the top of a smoke-cloud, and solely by buoyancy reversal (br) in a liquid water cloud. These are, respectively,

\[
\dot{w}_e = A_1 \frac{V_{\text{sum}}^3}{z_m \Delta b} + \frac{(g/\theta_0) \alpha_t \Delta F}{\Delta b}, \tag{9}
\]

\[
\dot{w}_e = A_1 \frac{V_{\text{br}}^3}{z_c \Delta b}. \tag{10}
\]

Here \(V_{\text{sum}}^3 = V_{\text{surf}}^3 + V_{\text{rad}}^3\) and

\[
V_{\text{surf}}^3 = z_m \bar{u}'b' \tag{11}
\]

\[
V_{\text{rad}}^3 = \frac{g}{\theta_0} z_m \Delta F, \tag{12}
\]

\[
V_{\text{br}}^3 = A_2 \chi_c D(\Delta b z_c)^{3/2}. \tag{13}
\]

The best agreement was found with \(A_1 \approx 0.23\) and \(A_2 \approx 0.24\). An accurate expression for \(\chi_t\) (and thence \(D\)) was derived in LMb in terms of the cloud-top jumps of \(\theta_e\) and \(q_t\) and the cloud-top liquid water content, \(q_{\text{max}}\), as \(\chi_t = -q_{\text{max}}/(\Delta q_t(1 - C_1 R))\), where the MacVean and Mason (1990) CTEI parameter \(R \equiv \Delta \theta_e/[(L/c_p)\Delta q_t]\), \(C_1 = \gamma_s/(1 + \gamma_s)\) and \(\gamma_s = (L/c_p)\delta q_s/\delta T\), \(T\) is the temperature. Note the dual rôle for the radiative cooling from within the horizontally averaged inversion which appears in both (9), as \(\alpha_t \Delta F\), and implicitly in (12), as discussed in section 1. Note also that, following Betts (1974), the effect of the overlying stability can be included in (9) by replacing \(\Delta b\) with \(\Delta b - N^2 \Delta z\), where \(\Delta z\) is the inversion thickness and the overlying stability \(N^2 = dU/dz\) (see LMa). For the cloud-capped simulations considered here, though, this influence was found to be negligible.

\(\text{(a) Without buoyancy reversal}\)

In LMa, (9) was derived from a parametrization for the entrainment buoyancy flux, \(\bar{u}'b'\), derived in turn by assuming that the entrainment process consisted of an energy conversion from the TKE of the boundary layer air to the potential energy of the entrained air. If it is further assumed here that entrainment is a two-stage process—an initial 'engulfing' process (the forcing down of dry air from above the inversion, at whatever scale) followed by a small-scale mixing process—then it is the first stage to which these energetic arguments will apply. By assumption this stage is a 'dry' process and these energetic arguments should, therefore, be independent of whether the cloud is of smoke or water. Similarly, the change from smoke to liquid water will not affect the interpretation of the rôle of radiative cooling within the horizontally averaged inversion (introduced in LMa and Moeng et al. (1998) and discussed in section 1).

The entrainment-rate parametrization in this regime (i.e. without buoyancy reversal) is, therefore, qualitatively unaltered from that in smoke clouds, given by (9). In section 1,
however, the effect of water phase changes on the buoyancy perturbations resulting from radiative cooling was discussed, and this will still have an effect on the magnitude of both the direct radiative term and the TKE buoyancy source terms here. The necessary changes are straightforward and were introduced in section 1. Following Deardorff (1976), the direct \((\alpha_t \Delta F)\) term must be multiplied by \(\beta\) and, following Stage and Businger (1981), the radiative source term \((12)\) becomes

\[
V^3_{\text{rad}} = \frac{g}{\theta_0} z_{ml} \Delta F \left\{ \xi^2 + (1 - \xi^2) \beta \right\}.
\]

(14)

Similar changes should be made to \(V_{\text{surf}}\) in (11).

To summarize, it is therefore proposed that the entrainment rate in a radiatively cooled liquid water cloud (but without buoyancy reversal) is given by

\[
w_{eAL} = A_1 \frac{V^3_{\text{rad}}}{z_{ml} \Delta b} + \beta \frac{g/\theta_0 \alpha_t \Delta F}{\Delta b} \quad \text{for} \quad \delta b \geq 0,
\]

(15)

with \(V^3_{\text{rad}}\) given by (14).

A further parametrization of \(\alpha_t\) will be derived in section 3, as its magnitude will be strongly dependent on the radiative flux profile.

\(\text{(b)} \quad \text{With buoyancy reversal}\)

If evaporative cooling is strong enough to generate buoyancy reversal of entrained air parcels, this process becomes an additional source of TKE in the boundary layer, capable of indirectly generating entrainment. Given the parametrization structure now in place, the natural way to include this forcing in (15) is to assume (as with the smoke clouds driven by combinations of surface heating and radiative cooling in LMa) that the entrainment rate can be represented by the sum of the entrainment rates that would have been generated by each process acting on its own. For buoyancy reversal, this was found in LMb to be accurately represented by (10), with \(V_{br}\) given by (13). Thus, replacing \(V^3_{\text{rad}}\) in (15) with \(V^3_{\text{sum}} = V^3_{\text{rad}} + V^3_{br}\) might be expected to give reasonable predictions of \(w_e\).

However, initial comparison with the large-eddy simulation data to be presented in this paper prompted two alterations: namely that both \(\xi\) and \(\alpha_t\) should be set to one. While these changes were prompted empirically, the following arguments are proposed by way of physical motivation. For the first of these, setting \(\xi = 1\) is equivalent to ignoring the reduction in the strength of the TKE generation by radiative cooling in saturated air. As has already been stated, this reduction was due to the associated warming generated by the condensation of further liquid water. It is therefore postulated that subsequent mixing with unsaturated entrained air will enhance the buoyancy-reversal generation of TKE by an amount equal to the reduction in the radiative-cooling term. To motivate the second change recall that, in LMa, the energy balance underlying this parametrization was evaluated at the top of the boundary layer, defined as the height where the gain in potential energy of entrained air had its maximum (which was where the entrainment buoyancy flux had its minimum, being negative for smoke clouds). In a liquid water cloud where evaporative cooling can lead to buoyancy reversal of entrained free-atmospheric air, the height at which this energy balance should be evaluated becomes less clear-cut. By analogy with the smoke clouds, it should perhaps be at the height where the buoyancy flux resulting from the entrainment of free atmospheric air has its extremum (its maximum value if there is buoyancy reversal). As described above, radiative cooling will tend to lead to the condensation of further liquid water in the cloud. Mixing with unsaturated entrained air (and subsequent evaporative cooling) will, in this regime, generate negative
buoyancy perturbations thereby contributing to the positive buoyancy flux. The height of the buoyancy flux maximum will, therefore, tend to be at the level where the radiative flux divergence goes to zero. For the purposes of the direct radiative promotion of entrainment, this then suggests that all the radiative divergence could effectively be regarded as occurring within the inversion and so \( \alpha_i \approx 1 \).

For liquid water clouds exhibiting buoyancy reversal, then, it is suggested that

\[
\bar{w}_{w} = A_1 \frac{V_{\text{sum}}^3}{z_{\text{ml}} \Delta b} + \beta \frac{(g/\theta_0) \Delta F}{\Delta b}
\]

for \( \delta b < 0 \), \( \text{(16)} \)

with \( V_{\text{sum}}^3 = V_{\text{rad}}^3 + V_{\text{br}}^3 \) and \( V_{\text{rad}}^3 \) given by (12).

Note that contained in this expression is the assumption that the strength of the buoyancy reversal forcing of entrainment will scale on the cloud depth, \( z_c \), and so \( V_{\text{br}}^3 \) is still given by (13). It is also assumed, however, that there will be a single dominant length-scale for the turbulent eddies which will be the mixed-layer depth, \( z_{\text{ml}} \). Thus, it is this length-scale that appears explicitly in (16).

Finally note that (16) is valid over the entire range of parameter space (with \( V_{\text{br}} \rightarrow 0 \) as \( \delta b \), or equivalently \( D \), tends to zero).

### 3. Numerical Simulations

Several three-dimensional large-eddy simulations (LES) of radiatively cooled cloudy boundary layers have been performed using the UK Meteorological Office model described in LMB. They are listed in Table 1. All simulations used the 'standard' Richardson number definition in the subgrid model, derived by MacVean and Mason (1990), except Bf which used that described in Shutts and Gray (1994), in order to test the sensitivity of the results to subgrid mixing. The latter only allows for adiabatic buoyancy changes during saturated ascent, while the former remains energetically consistent at cloud boundaries by allowing for the possibility of the release of energy by subgrid mixing between cloudy and cloud-free air. Away from cloud boundaries the two definitions are identical. The model's conserved thermodynamic variable is \( T_L = T + (g/c_p)z - (L/c_p)q_L \).

<table>
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<tr>
<th>Simulation designation</th>
<th>Initial ( R )</th>
<th>Initial ( D )</th>
<th>( \Delta \delta_r ) (K)</th>
<th>( \Delta q_r ) (g kg(^{-1}))</th>
<th>( q_{\text{max}} ) (g kg(^{-1}))</th>
<th>( \rho c_p \Delta F ) (W m(^{-2}))</th>
<th>( \kappa ) (m(^2) kg(^{-1}))</th>
<th>( \Delta z_{\text{min}} ) (m)</th>
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<td>60</td>
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<td>5</td>
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<tr>
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<td>0.60</td>
<td>-0.83</td>
<td>0.5</td>
<td>60</td>
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</table>

Simulations with initial letter B had buoyancy reversal throughout, and with E had evaporative cooling but no buoyancy reversal; simulation T was transitional between the two. \( \Delta z_{\text{min}} \) is the minimum vertical grid spacing. All simulations except Bf used the standard subgrid model.
The initial conditions consisted of a well-mixed boundary layer capped by a cloud with cloud top at 650 m and the maximum boundary layer liquid water mixing ratio, \( q_{\text{Lmax}} \), specified. For \( q_{\text{Lmax}} = 0.5 \) g kg\(^{-1}\), the initial state of the boundary layer was \( q_l = 10.89 \) g kg\(^{-1}\) and \( T_L = 291.52 \) K and the cloud was 250 m thick. For \( q_{\text{Lmax}} = 1 \) g kg\(^{-1}\), \( q_l = 11.49 \) g kg\(^{-1}\) and \( T_L = 290.42 \) K and the cloud was 500 m thick. The initial values of \( \Delta \theta_v, \Delta q_t, R \) and \( D \) are included in Table 1. Above the cloud top the profiles of temperature and humidity were set using the same method as in LMb (which, without radiative cooling, resulted in \( R \) remaining constant as the boundary layer grew). The largest value of \( R \) used was 0.5, as it was intended to restrict attention to near-continuous cloud sheets and to leave the area of partial cloudiness (important though it is) for later work. Values of \( R \geq 0.7 \) have been found to lead to rapid cloud break-up, as discussed in MacVean (1993). Jumps across the inversion were calculated as the jump from a level at which conditions could be regarded as ‘free atmospheric’ (taken to be the lowest grid level where \( q_D > 0.95 \)) to the level with the greatest liquid water mixing ratio, \( q_{\text{Lmax}} \) (a height consistently found to coincide with the base of the inversion), and then averaged over the domain. Alternative methods were monitored but, as discussed in LMb, this was found to be the most consistent method for measuring the full strength of the inversion, as would be ‘felt’ by boundary-layer-scale eddies. A passive tracer, \( q_D \), was initially set to zero in the boundary layer and 1.0 above cloud-top and was therefore an indicator of the mixing fraction generated during the simulations.

Turbulence in the model was initialised by a random positive perturbation of \( T_L \) (to avoid artificially generating mixing below cloud base) of at most 0.05 K in the lower half of the cloud. There were no surface fluxes or mean wind. The resolutions used here were chosen as a result of sensitivity studies presented in LMA and LMb. The standard resolution used had a minimum vertical grid-spacing of 10 m around cloud top (between approximately 650 and 800 m). Higher-resolution simulations were performed with approximately 5 m vertical resolution between these heights, except \( T \) (the transitional simulation) for which 5 m resolution was used between 900 and 1000 m as this was the height of the inversion during the transitional period. In the horizontal, all but two simulations had uniform 50 m resolution over a 4 km-square domain (which should be sufficiently large to capture a good statistical sample of the largest eddies—Mason (1989) suggested a domain four times the mixed-layer depth was sufficient for convective simulations). For simulations Bd and Bg mixing was anticipated over smaller depths and so smaller domains were used (2 and 3 km, respectively) to allow greater horizontal resolution to be achieved with the same number of grid-points.

In reality, the radiative flux profiles can be complex even in nocturnal stratocumulus, with a significant warming often occurring at cloud base in addition to strong cloud-top cooling. It was decided, for simplicity, to restrict attention here only to the influence of the cloud-top cooling, the dominant turbulence generating mechanism, although the role of radiative warming deeper in the cloud will be discussed further in section 5 when comparison with observations of stratocumulus is made. In the model, therefore, the long-wave radiative flux was specified, in each column separately, in terms of the radiative flux, \( F_{\text{top}} \), at the top of the domain, \( z_{\text{top}} \), as

\[
F(z) = F_{\text{top}} e^{-x L(z)} \tag{17}
\]

where \( L(z) \) is the liquid water path, given by

\[
L(z) = \int_z^{z_{\text{top}}} \rho q_t dz'.
\]
The absorption coefficient, $\kappa$, and $F_{\text{top}}$ were specified as given in Table 1 (a value of $\kappa = 156 \, \text{m}^2\text{kg}^{-1}$, with $q_{\text{max}} = 0.5 \, \text{g} \, \text{kg}^{-1}$, would give an e-folding decay length-scale, approximated by $(q_{\text{max}} \rho \kappa)^{-1}$, of 12 m, which is typical of subtropical stratocumulus).

As discussed in LMa, if the radiative fluxes are present from the start of a simulation and therefore before overturning circulations have been established, the extended cooling of air near cloud top will initiate an oscillation in the boundary layer TKE. In a water cloud, because radiative divergence generates cooling in the boundary layer without directly affecting $q_{i}$, it tends to make $\Delta \theta_{e}$ less negative without directly changing $\Delta q_{i}$. It therefore has the effect of gradually reducing $R$. In addition, because of its sensitivity to the cloud-top parameters (discussed in LMb), the tendency of radiative cooling will be to reduce $D$ more rapidly. Any initial oscillation in simulations with initially relatively strong buoyancy reversal, but without large-scale advection or surface fluxes which might otherwise act to maintain $D$, would therefore be highly undesirable. In order to alleviate this problem, these simulations were integrated for an hour with the radiative divergence set to zero, to allow boundary layer circulations (driven solely by buoyancy reversal) to develop. Radiative cooling was then introduced gradually, over the following half-hour, so that at least a partial equilibrium with the boundary layer turbulent transport could be maintained. This was achieved by linearly increasing $F_{\text{top}}$ in time until it reached its desired value.

(a) Parametrization of $\alpha_{r}$

The parametrization for $w_{e}$ derived in section 2 and given by (15) depends on the fraction of the radiative divergence that occurs within the horizontally averaged inversion, $\alpha_{r}$. For (15) to be useful as a parametrization of $w_{e}$, a further parametrization for $\alpha_{r}$ must be derived, and it is found here that a very simple one is sufficient (a slightly more complicated parametrization of $\alpha_{r}$ is given by Moeng et al. (1998)). Essentially, $\alpha_{r}$ will depend on two things: the thickness of the inversion and the shape of the radiation profile near cloud top. It has been assumed that the finite thickness of the averaged inversion arose from undulations in an otherwise discontinuous inversion. A length-scale for the vertical scale of these undulations is the penetration depth of upwelling boundary layer circulations, $\Delta z_{p}$, which can be approximated by dimensional arguments as

$$\Delta z_{p} = \frac{V_{\text{sum}}^2}{\Delta b}, \quad (18)$$

where the velocity-scale, $V_{\text{sum}}$, has been used as representative of the turbulence.

In these simulations the radiative flux at a particular height is specified, in (17), from the overlying liquid water path, $\mathcal{F}$, alone. If it is assumed that within the averaged inversion $q_{i}$ decreases linearly from its maximum boundary layer value, $q_{\text{max}}$, at $z = z_{i}$ to zero over a height $\Delta z_{p}$, then $\mathcal{F}(z_{i}) = \rho q_{\text{max}} \Delta z_{p}/2$. If it further assumed that the horizontally averaged radiative flux profile can be approximated by (17) based on this horizontally averaged liquid water path then

$$\alpha_{r} = 1 - e^{-\rho q_{\text{max}} \Delta z_{p}/2}. \quad (19)$$

This parametrization can be tested separately from the entrainment parametrization because $\alpha_{r} \Delta F$ can be diagnosed directly from the simulations, as follows. Given the inversion structure assumed in deriving the entrainment parametrization, integrating the conservation equations for $T_{L}$ and $q_{i}$ over the inversion gives $-w'T_{L} \mid_{z_{i}} = w_{e} \Delta T_{L} - \alpha_{r} \Delta F$ and $-w'q_{i} \mid_{z_{i}} = w_{e} \Delta q_{i}$, respectively. The latter equation has consistently been found to be extremely accurate in these simulations and so it would seem reasonable to assume that the former should hold equally well. Rearranging it therefore implies $\alpha_{r} \Delta F = w_{e} \Delta T_{L}$ +
\( \overline{w'T'_L}_{zi} \). Comparison between this diagnosed value and (19) will be made in section 4. For more realistic radiation schemes than that used here it is anticipated that, given a suitable estimate for \( \kappa \), (19) should still be reasonably accurate.

4. Results

Once turbulent circulations had developed in the boundary layer, entrainment of the drier air above the inversion into the boundary layer caused it to deepen, but the liquid water path fell by no more than 50% in any simulation and the cloud cover was always greater than 98%.

As was discussed in the previous section, with these initial profiles (which were found in the absence of radiative cooling in LMb, to keep \( R \) constant) radiative cooling was expected to reduce \( R \). This can be seen for simulations Bd, Eb and T in Fig. 1(a). Note that for Bd and T, \( R \) only started to change significantly once the radiative cooling had been switched on at the end of the first hour. Similar variations are seen for \( D \), in Fig. 1(b), as well as the rapid decrease in \( D \) from its larger initial value for simulation T. The impossibility of maintaining large values of \( D \), without including large-scale advection or surface fluxes, is discussed in more detail in LMb.

(a) Depth of mixing

It was found in LMb, for simulations driven only by buoyancy reversal, that turbulent mixing was effectively confined to the cloud layer. With the addition of radiative cooling here, mixing extended deeper into the subcloud layer (as measured by the depth over

![Figure 1](image-url). Time series of (a) \( R \), the MacVean and Mason (1990) cloud-top entrainment instability (CTEI) parameter, and (b) \( D \), the Siems et al. (1990) CTEI parameter, from simulations Bd (continuous line), Eb (dashed line) and T (dash-dotted line). One hour has been added to the time for simulation Eb for ease of comparison. See text and Table 1 for details of simulations.
which \( \bar{w}/q_1 \) was greater than 5% of its maximum value) but in none of the simulations with buoyancy reversal did it reach the surface. For those simulations with \( \delta b > 0 \), however, the mixed layer always extended to the surface, unless the simulation had begun with \( \delta b < 0 \) and therefore developed a significant inversion at the base of the mixed layer before buoyancy reversal ceased. The evolution of the mixed-layer boundaries are shown in Fig. 2 for simulations Bd in which \( \delta b < 0 \), and T in which \( \delta b \) changed sign.

For these simulations (where there are no surface fluxes or drizzle), whether downdraughts descend through the initially neutral boundary layer down to the surface is essentially dependent on the balance between the tendency for entrainment to increase, and radiative cooling to decrease, their \( \theta_{el} \) (or their buoyancy beneath cloud-base). If entrainment dominates, then beneath cloud base the deceleration of positively buoyant downdraughts will tend to limit the depth of the mixed layer, and an inversion may form at its base. If radiative cooling dominates, then downdraughts will remain negatively buoyant to the surface. This balance is complicated by the fact that radiative cooling not only cools the cloud layer but also generates entrainment through the generation of turbulence. If the expression (15) is used to quantify the radiative generation of entrainment, it is found that the net effect of radiative cooling in the simulations performed here was always to reduce the buoyancy of the cloud. This is supported by the observation that only if there was the additional entrainment source from buoyancy reversal did entrainment dominate and mixing not extend to the surface. In addition, in simulation T the presence of buoyancy reversal initially was sufficient to decouple the cloud layer (i.e. entrainment increased \( \theta_{el} \) in the cloud faster than radiative cooling decreased it), but after \( \delta b \) had changed sign (after around 2 1/2 hours) the base of the mixed layer grew downwards, as can be seen in Fig. 2(b). Note that the brief period around 1 1/2 hours, when mixing extended to the surface, was generated by the spin-up problem discussed in section 3, where the model’s convective motions were not yet sufficiently developed to transport the radiatively cooled air away from the cloud top.
Figure 3. Total (solid line), resolved (long-dashed) and subgrid (dotted) profiles of the turbulent fluxes, $\overline{w' T_L'}$, where $T_L$ is the model's conserved thermodynamic variable. Also shown are the updraught (dash-dotted) and downdraught (dash-dot-dot-dotted) contributions to the resolved flux for (a) simulation Bd, averaged over the period from $2\frac{1}{2}$ to $2\frac{1}{2}$ hours, and (b) simulation Eb, from $3\frac{3}{4}$ to 4 hours. See text for further details.

(b) Turbulence statistics

The turbulent fluxes of $T_L$ are shown for simulations Bd and Eb in Fig. 3. In the inversion region (above about 750 m for both simulations) first note that the subgrid flux was small for simulation Eb but relatively large and negative for simulation Bd (with buoyancy reversal). This non-trivial subgrid flux reflects the fact that, with buoyancy reversal, negatively buoyant parcels with high $T_L$ (relative to the cloud layer) are formed near cloud top, and so the subgrid model can sustain a non-zero eddy diffusivity (which is dependent on the relative buoyancy of adjacent grid levels through the Richardson number) in regions where $\partial T_L / \partial z$ is positive; this occurs with both the subgrid models employed in this study. For the resolved fluxes in the inversion region, the conditional sampling into updraught and downdraught contributions (based simply on the sign of $w$) was very similar for simulations Bd and Eb, suggesting similar entrainment processes (as was discussed in LMa). Away from the inversion, downdraughts in simulation Eb were found to have lower $T_L$ than updraughts (i.e. radiative cooling dominated over entrainment), while with buoyancy reversal (as in simulation Bd) downdraughts had higher $T_L$ than updraughts implying that entrainment (which increased $T_L$ but generated buoyancy reversal in this case) dominated over radiative cooling. Nevertheless, for both simulations, downdraughts in the cloud layer were narrower and stronger than updraughts, and were responsible for around two-thirds of the total flux.

For simulation T, both when $\delta b$ was negative and when it later became positive (see Figs. 4(a) and (b), respectively), the total resolved $\overline{w' T_L'}$ was always negative (although
As Fig. 3, but for simulation T averaged over the periods (a) from $2 \frac{1}{2}$ to $2 \frac{1}{2}$ hours when $\delta b < 0$ (see text) and cloud base was at approximately 500 m and (b) from $3 \frac{1}{2}$ to 4 hours when $\delta b > 0$ and cloud base was at approximately 600 m.

the magnitude was smaller once $\delta b$ was positive) and downdraughts were always found to have higher $T_L$ than updraughts. This suggests that entrainment of air with high $T_L$ into downdraughts dominated radiative cooling throughout the simulation. In addition, the subgrid flux (dotted in Fig. 4) was always relatively large and negative near cloud top, suggesting continued local buoyancy reversal, even though the horizontally averaged locally calculated $\delta b$ had become positive. Recall that $\delta b$ was calculated in each grid column from the jumps between the height of $q_{1_{\text{max}}}$ and where the air became 'free atmospheric' above, and so essentially over the full depth of the inversion. Calculating $\delta b$ over the two grid points spanning the local cloud top still gave $\delta b > 0$ when horizontally averaged, but the minimum values were significantly less than zero (giving maximum local values of $D \geq 1$). Given the large negative subgrid flux realised at this time, this local generation of negatively buoyant mixtures must have made a significant contribution to the subgrid flux of $T_L$, and the entrainment associated with it a significant contribution to the average downdraught $T_L$. For the simulations which started with $\delta b > 0$, no measure of $\delta b$ gave a negative value, either horizontally averaged or locally.

Interestingly, the budget of resolved turbulence kinetic energy was qualitatively very similar over the cloud layer for all simulations, regardless of the sign of $\delta b$ (including throughout simulation T), as can be seen for simulations Bd and Ea in Fig. 5. Buoyant production in the cloud layer was balanced by local dissipation and transport out of the cloud. Transport into the inversion region, where buoyant production became negligible (although it did not become a net consumer of TKE for either simulation as it had for the
smoke clouds in LMa), was balanced by dissipation and the storage term (as the inversion rose). The lack of a negative 'entrainment' buoyancy flux (either resolved or subgrid) in the simulations without buoyancy reversal is thought to be a diagnostic problem with both horizontal and temporal averaging over a rising thin layer, as discussed in more detail in LMa. The situation is exacerbated in liquid water clouds because buoyancy is not a conserved variable under cloud-top mixing.

There was a difference between the TKE budgets of the two simulations beneath cloud base; with the cloud layer decoupled in simulation Bd, buoyancy became a consumer of TKE indicating entrainment of subcloud air into the turbulent cloud layer, although the mixed-layer depth was found to be almost constant, suggesting there was very little downward growth. In simulation Ea, where the boundary layer was well mixed to the surface, negatively buoyant thermals continued to descend through the subcloud layer and to generate TKE.

It might be thought that some of the attributes of the simulations discussed above could be strongly dependent on the subgrid model, especially since subgrid fluxes were significant in this region (see Figs. 3 and 4, for example). However, simulation Bf used the subgrid model of Shutts and Gray (1994) but was otherwise identical to simulation Bc (which used the standard, see section 3) and, while the precise evolution was slightly different (the entrainment rate increased more rapidly at the start of the simulation, for example), the turbulence statistics for the two simulations were very similar.
ENTRAINMENT PARAMETRIZATION

TABLE 2. LIST OF PARAMETERS AFTER 2 HOURS

<table>
<thead>
<tr>
<th>Simulation designation</th>
<th>$z_{ml}$ (m)</th>
<th>$z_{c}$ (m)</th>
<th>$z_{e}$ (m)</th>
<th>$q_{\text{max}}$ (g kg$^{-1}$)</th>
<th>$\Delta q_{l}$ (g kg$^{-1}$)</th>
<th>$\Delta \theta_{l}$ (K)</th>
<th>$\Delta \theta_{e}$ (K)</th>
<th>$u_{ce}$ (cm s$^{-1}$)</th>
<th>$u_{te}$ (m s$^{-1}$)</th>
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</thead>
<tbody>
<tr>
<td>Ea</td>
<td>798</td>
<td>780</td>
<td>348</td>
<td>0.62</td>
<td>-0.96</td>
<td>0.22</td>
<td>1.12</td>
<td>1.88</td>
<td>0.79</td>
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<tr>
<td>Eb</td>
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<td>755</td>
<td>343</td>
<td>0.58</td>
<td>-0.96</td>
<td>0.14</td>
<td>1.08</td>
<td>1.42</td>
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<tr>
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<td>0.53</td>
<td>1.23</td>
<td>1.28</td>
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<tr>
<td>Ed</td>
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<td>740</td>
<td>538</td>
<td>1.01</td>
<td>-1.66</td>
<td>-0.05</td>
<td>1.66</td>
<td>1.34</td>
<td>0.83</td>
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<tr>
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<td>906</td>
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<td>0.45</td>
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<td>-1.64</td>
<td>2.17</td>
<td>3.17</td>
<td>0.94</td>
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<tr>
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<tr>
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<td>802</td>
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<td>0.28</td>
<td>-1.23</td>
<td>-1.35</td>
<td>0.85</td>
<td>5.70</td>
<td>0.68</td>
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<tr>
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<td>2.90</td>
<td>0.67</td>
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<tr>
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<td>0.80</td>
<td>6.08</td>
<td>0.66</td>
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<tr>
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<td>0.25</td>
<td>-1.21</td>
<td>-1.30</td>
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<td>3.73</td>
<td>0.61</td>
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<td>-4.00</td>
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<td>-0.62</td>
<td>1.34</td>
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<td>0.75</td>
</tr>
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</table>

The second entry for simulation T is for the 15 minutes at the end of the fourth hour, after $\delta b$ had become positive.

(c) Entrainment results

It remains to compare the theoretical models for the entrainment rate discussed in the previous sections with the entrainment rate found in the simulations. Table 2 contains a selection of relevant parameters from the simulations. The data are averages over the fifteen minutes at the end of the second hour, and also the fifteen minutes at the end of the fourth hour of T, after $\delta b$ had become positive. The mixed-layer depth, $z_{mls}$, was diagnosed as the depth over which $\overline{w/q_{l}}$ was greater than 5% of its maximum value. The entrainment rate, $u_{ce}$, was diagnosed from the rate of rise of the horizontal average of the cloud-top height, $z_{c}$, although using other measures of the inversion height gave very similar values.

Figure 6 shows $u_{ce}$ diagnosed from the simulations without buoyancy reversal ('E' in the designations in Tables 1 and 2) plotted against the entrainment rate parametrization derived here (given by (15) or (16), depending on the sign of $\delta b$, referred to as AL), and the parametrizations of Stage and Businger (1981; given by (5); referred to as SB) and Turton and Nicholls (1987; given by (8); referred to as TN). Unless stated otherwise, all constants were given the values they had in the previous studies described in sections 1 and 2, and $u_{ce}$ in (8) was diagnosed from the simulations. In this and subsequent figures, each parametrization was calculated from instantaneous data, and then averaged over 15-minute intervals starting from when the turbulence levels (as measured by $u_{te}$) had, as far as possible, reached a steady state. This tended to be about half-an-hour after the radiative divergence had reached its desired value.

Consider the AL parametrization first. Although the scatter is not negligible, AL is reproducing the actual entrainment rate to within around 20% across the whole range encountered in this regime. Recall that AL incorporated a further parametrization for $\alpha_{t}$, the fraction of the radiative divergence occurring within the horizontally averaged inversion. It can be seen from Fig. 7, that its parametrization, given by (19), agrees well with $\alpha_{t}$ diagnosed from the model's $T_{L}$ flux (see section 3), as does the parametrization of the inversion thickness, $\Delta z_{p}$, given by (18), on which it is based. Note that $\Delta z_{p}$ was diagnosed as the difference between the height where the horizontally-averaged $q_{l}$ fell to less than 5% of its maximum value and the height of the $T_{L}$-flux minimum. Note also that the contribution to AL from this direct term involving $\alpha_{t}$ tends to approach around 50% of the total.
Figure 6. Fifteen-minute averages of entrainment rate $w_e$ diagnosed from the simulations without buoyancy reversal (designation E in tables) compared to the parametrizations by (a) Eqs. (15) and (16) in this study, designated AL; (b) Turton and Nicholls (1987) designated TN (lower-case) and the models of Stage and Businger (1981) designated SB (upper-case). The vertical lines in (a) are the observations of Nicholls and Leighton (1986) ±30% against the parametrization by AL. Letters plotted are the 2nd letters of the E simulations in Tables 1 and 2.

Figure 7. Fifteen-minute averages of (a) $\alpha_t$, the fraction of the radiative divergence occurring within the horizontally averaged inversion, and (b) $\Delta z_p$, the inversion thickness diagnosed from the simulations without buoyancy reversal (designated E as listed in Table 1) compared to the parametrizations in (19) and (18), respectively. Letters plotted are the second letters of the E simulations in Tables 1 and 2.

The results from SB (Fig. 6(b), upper-case letters) follow the pattern found in Nicholls and Turton (1986), which compared various parametrizations with Nicholls and Leighton's (1986) observations, namely that it tended to underestimate $w_e$, here by up to a factor of two. Given the success of AL, which included a direct radiative contribution to $w_e$, it is possible that this error in SB could in part be due to the lack of this term. Note, however, that SB tended always to predict very similar values of $w_e$ and so this is unlikely to be
the only problem. Using their empirically derived constants, TN overpredicted \( w_e \) by up to a factor of 4. The results, shown in Fig. 6(b) (lower-case letters), have \( a_z = 9 \) and the overestimate is still significant, as is the relatively large scatter compared to AL.

Before making these same comparisons for the simulations with buoyancy reversal, it was found in LMB where buoyancy reversal was the only turbulence generating process, that for \( D > 0.3 \) and when \( w_e \) was declining rapidly, there was an apparent time-lead in the entrainment rate predicted by AL of as much as an hour. The cause was traced to the effect of averaging over horizontal inhomogeneities (in particular in the \( q_l \) field) on the necessarily highly sensitive parameters involved in \( V_{be} \). Estimating \( q_{l \text{max}} \) from ‘observed’ local values in updraughts (around which mixing was about to take place) then gave much better temporal agreement. The same time-lead problem was seen in the simulations here, driven by combinations of radiative cooling and buoyancy reversal. For example, Fig. 8(a) shows the time series of the observed entrainment rate (as a thick continuous line) for simulation Bc, which initially had \( D = 1 \), and that predicted by AL (thin continuous line). At the time of maximum entrainment (after about 1 hour 40 minutes) the horizontally averaged \( q_{l \text{max}} \), \( \overline{q}_{l \text{max}} \approx 0.3 \text{ g kg}^{-1} \) but local values as high as the initial 0.5 g kg\(^{-1}\) could still be seen. Consequently, the difference between the maximum \( w_e \) and that predicted by AL was very close to the difference between the \( V_{br} \) contribution to AL calculated from the initial conditions and at the time of the maximum \( w_e \). Alternatively, the result of using an estimated time series of the local \( q_{l \text{max}} \) to calculate AL can be seen as the dotted line in Fig. 8(a). The magnitude of AL is now almost perfect while the time-lead, at about 900 s, is of the order of the eddy-turnover time-scale \( (\sim z_m / \sigma_w \approx 500 \text{ m}/0.5 \text{ m s}^{-1}) \)—a measure of the time taken for a change in the cloud-top buoyancy forcing to translate into a change in the resulting entrainment rate. To test whether the results were dependent on the subgrid model, simulation Bf was identical to simulation Bc except that it was performed with the alternative Richardson number formulation, discussed in section 3. While the time of the
peak $w_c$ was some fifteen minutes earlier and the magnitude of the peak reduced by about 10%, AL continued to predict $w_c$ very well (using a local $q_{\text{max}}$).

For the simulations with smaller initial $D$, horizontal variations within the cloud appeared to be small enough to have little effect on the calculation of AL, as can be seen for simulation Bd, for example, in Fig. 8(b). Calculating AL from $\bar{q}_{\text{max}}$ gives very accurate results, except for a brief period just after the radiative divergence reached its maximum strength, when a time-lead of the order of the eddy-turnover time-scale is visible.

Figure 9 shows the same comparison as Fig. 6 between actual and parametrized entrainment rates, but for the simulations with buoyancy reversal throughout (initial letter 'B' in the designations in Tables 1 and 2). For simulations Ba and Bd, AL was calculated simply from $\bar{q}_{\text{max}}$, while for the others an estimated local $q_{\text{max}}$ was used and the remaining time-lead removed. Note: firstly that adding the potential for buoyancy reversal has significantly increased the entrainment rates that developed in the simulations, demonstrating the
potential potency of the mechanism; secondly the parametrized entrainment rates from AL are sufficiently good (always within 10% of the actual value) to justify the semi-empirical changes made to the radiative forcing terms in section 2(b) (leaving the radiative-forcing terms as they were for the case of no buoyancy reversal, in (15), would have resulted in a significant underestimate of \( w_e \)).

Both TN and SB were found to be far less sensitive to horizontal inhomogeneities, but, probably because of this, they also did not predict \( w_e \) at all well. In addition to the large scatter, SB again tended to underpredict \( w_e \) (as can be seen in Fig. 9), perhaps indicating the lack of a direct radiative contribution. In addition SB also seriously overpredicted \( w_e \) or predicted negative \( w_e \), at various times in three out of the seven simulations, as the denominator in (5) passed through zero. For TN, the best agreement was found with \( a_2 = 15 \) (as was used in Fig. 9), a larger value than was used to compare with the simulations without buoyancy reversal. Note, though, that for the four simulations with \( D > 0.3 \) initially, TN greatly overpredicted \( w_e \) so that some of the data from three of these are off the scale in Fig. 9. This change in \( a_2 \) would appear to be because these simulations are in a somewhat different region of parameter space, in that the term multiplied by \( a_2 \) in (8), namely \( \chi_N + D \), tended to be significantly larger here (at least around 0.4), compared to nearer 0.1 in the observations.

If the standard dry boundary layer parametrization, (1), was used (which is equivalent to setting \( a_2 = 0 \) in TN), the value of \( A_1 \) required to fit the data varied from 0.5 to 5 over all the simulations, with variations of up to a factor of 2 during some simulations. This variation is similar to the ten fold increase in \( A_1 \) for cloudy compared to cloud-free boundary layers observed by Nicholls and Turton (1986).

Finally, Fig. 10 shows the time series of actual (thick continuous line) and parametrized entrainment rates (AL calculated from \( \bar{q}_{\text{max}} \), thin continuous line; SB, dashed line; TN, dotted line) in twenty-minute averages, for simulation T. During this simulation \( \delta b \) changed from negative to positive after about 2 \( k \) hours, suggesting a transition from buoyancy reversal to no buoyancy reversal. As with the simulations with buoyancy reversal throughout, horizontal inhomogeneities led \( q_{\text{max}} \) to be significantly underestimated during the initial, rapidly entraining, phase of the simulation, so that AL underestimated \( w_e \). Not surprisingly, the maximum \( w_e \) is also well predicted here by adding to the prediction of AL the difference between the \( V_{b}^3 \) contribution calculated from the initial conditions and at the time of the maximum \( w_e \). This does not explain, however, why AL consistently underestimated \( w_e \) (by at least 50%) once \( \delta b \) became positive. Recall from subsection (b), however, that there were indications that buoyancy reversal had in fact continued to occur in this simulation (although only locally) right to the end; the subgrid \( T_L \) flux and the downdraught \( T_L \) were significantly different from what had been seen in the simulations without buoyancy reversal throughout. This dependence of the buoyancy reversal forcing on local conditions will be returned to in section 6.

In contrast to AL, SB and TN (with \( a_2 = 9 \) on this occasion) gave better agreement with the simulation (see Fig. 10), although they were both under-predicting \( w_e \) by the end.

5. Comparison with Observations

It must be remembered that the parametrization AL has been determined exclusively from simulation data, albeit with a demonstrated insensitivity to the subgrid model. In this section it will be compared with observations of entrainment in stratuscumulus. Nicholls and Leighton (1986) presented data from 5 flights which were compared with various entrainment parametrizations in Nicholls and Turton (1986) and were used to determine the parametrization TN. Two were nocturnal and all had mean wind speeds, \( \bar{u} \), at or less than
10 m s$^{-1}$ (although for flight 620 the wind speed changed by 7 m s$^{-1}$ over the inversion). In only one of the observed clouds (flight 624) did the mixed layer extend to the surface, but then the surface fluxes were small.

Before a comparison can be made, it will be necessary to generalize AL further to include any additional effects on the entrainment rate resulting from realistic long-wave radiative fluxes and short-wave fluxes. In their recent LES study, Lewellen and Lewellen (1998) concluded that the entrainment rate was ultimately determined by the mixed-layer-scale eddies. It was therefore remarkably insensitive to processes which generated turbulence on smaller length-scales. Thus, the effect of the net radiative divergence on $V_{rad}$ might be expected to depend on its vertical distribution in relation to the mixed-layer boundaries. If the boundary layer is well mixed to a significant distance below cloud base, then Lewellen and Lewellen’s arguments suggest that the net radiative warming (both long- and short-wave) commonly observed around cloud base will not contribute to the energy of boundary-layer-scale eddies and so should not appear in $V_{rad}$. If the cloud layer is decoupled and this warming occurs towards the base of the mixed layer, it might be expected to contribute to $V_{rad}$. On the other hand, short-wave warming which occurs towards cloud top will always act to reduce the magnitude of the net radiative cooling there, and hence reduce $V_{rad}$.

Nicholls and Leighton only presented a computed radiative flux profile for flight 624 (in their Fig. 7). In this nocturnal case the cloud occupied only the top part of the mixed layer, and so only the cloud-top long-wave radiative divergence (given as $\rho c_p \Delta F_{top}$ in Table 3) is included in $V_{rad}$. For the other flights, a radiative-transfer scheme which was more realistic than that used for the simulations in this paper (although still one-
ENTRAINMENT PARAMETRIZATION

TABLE 3. DATA FROM FIVE OF THE FLIGHTS REPORTED IN NICHOLLS AND LEIGHTON (1986) AND FROM THE LAST HOUR OF SIMULATION EA.

<table>
<thead>
<tr>
<th>Flight number</th>
<th>Run</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>511</td>
</tr>
<tr>
<td>$z_{nl}$ (m)</td>
<td>370</td>
</tr>
<tr>
<td>$z_c$ (m)</td>
<td>320</td>
</tr>
<tr>
<td>$R$</td>
<td>-0.12</td>
</tr>
<tr>
<td>$D$</td>
<td>-0.028</td>
</tr>
<tr>
<td>$\bar{u}$ (m s$^{-1}$)</td>
<td>10.5</td>
</tr>
<tr>
<td>$\Delta T_r$ (K)</td>
<td>6.9</td>
</tr>
<tr>
<td>$q_{max}$ (g kg$^{-1}$)</td>
<td>0.45</td>
</tr>
<tr>
<td>$\rho c_p \Delta F_{top}$ (W m$^{-2}$)</td>
<td>79</td>
</tr>
<tr>
<td>$\rho c_p \Delta F_{base}$ (W m$^{-2}$)</td>
<td>20</td>
</tr>
<tr>
<td>$\rho c_p \Delta F_{ca}$ (W m$^{-2}$)</td>
<td>8</td>
</tr>
<tr>
<td>$w_{obs}$ (cms$^{-1}$)</td>
<td>0.50</td>
</tr>
<tr>
<td>$w_{AL}$ (cms$^{-1}$)</td>
<td>0.34</td>
</tr>
</tbody>
</table>

dimensional) was employed to generate flux profiles from the details of the mean boundary layer structure presented by Nicholls and Leighton. From these, estimates of any net radiative divergences at the cloud top and base, $\Delta F_{top}$ and $\Delta F_{base}$, were made. Following Lewellen and Lewellen, the net cloud-base divergence was included in $V_{rad}$ only if the mixing was limited to approximately the cloud layer (i.e. $z_{nl} \approx z_c$, as in flights 511 and 526). The values of $\Delta F_{top}$ and $\Delta F_{base}$ used to evaluate $V_{rad}^3 = (g/\theta_0)z_{nl}(\Delta F_{top} + \Delta F_{base})$ are given in Table 3.

All other quantities needed for AL were presented by Nicholls and Leighton (and are repeated in Table 3 together with the equivalent data from the last hour of simulation Ea, for comparison). In the parametrization of $\alpha_t$, (19), $\kappa$ was given the standard value used in the simulations here, namely 156 m$^2$kg$^{-1}$. Note that only $\Delta F_{top}$ was multiplied by $\alpha_t$. Nicholls and Turton also presented observations of the total long-wave flux divergence over the whole boundary layer, and computations of the divergence over the cloud only. They interpreted the difference between these two as an estimate of the clear-air radiative divergence, $\Delta F_{ca}$, discussed in Nieustadt and Businger (1984). This will act to promote entrainment directly by reducing the strength of the inversion, in the same way as the direct term from within the inversion, the $\alpha_t \Delta F$ term in (15). To allow for this effect, it is simplest to combine $\Delta F_{ca}$ and $\alpha_t \Delta F$ in the direct term of $w_{AL}$, but note that this process will not generate turbulence in the mixed layer and so is not included in $V_{rad}$.

Finally, note that, with $D = 0.003$, flight 528 is an observation of a cloud with buoyancy reversal. Employing the full parametrization (16) to this flight, however, was found to give a large overestimate of $w_c$. In this case, with $D = 0.003$, $V_{br}$ was negligible and so it would seem reasonable to assume that the feedbacks which prompted changes to the radiative-forcing terms should also be negligible. Consequently, in practical use, it is suggested that for small $D$ ($0 < D < 0.05$, say) $\alpha_t$ and $\zeta$ should be linearly interpolated between the values they would have were $D < 0$ and the value of unity they are both given in (16). This is then the method used to calculate $w_{AL}$ for flight 528 in Table 3.

Nicholls and Turton estimated the uncertainty levels of their observations at between 30 and 50%. As can be seen in Fig. 6, all the predictions from AL agree with the observations, fitting within the more conservative error estimate. This agreement would appear to support not only the parametrization AL but also the refinements for realistic radiative profiles suggested by Lewellen and Lewellen's arguments. It should be noted, however,
that the uncertainty estimates are sufficiently large that predictions from AL, which included the observed $\Delta F_{\text{base}}$ in all cases, still fitted within them, although they were then consistently towards the upper limit. For the two flights which have the worst agreement in Fig. 6 (namely 511 and 526), AL tends to underestmate $w_e$. Although they just fit within the uncertainty levels, there are two possible reasons why AL might underestimate $w_e$ in these cases. Firstly, the mean wind speeds were 10.5 and 8.5 m s$^{-1}$, respectively, and turbulence generation by shear in the mean wind is an important process not yet included in AL. Secondly, flights 511 and 526 are the two day-time flights in which mixing was limited to the cloud layer and so cloud-base warming might have been expected to enhance $V_{\text{rad}}$ and hence $w_e$. Despite significant short-wave absorption in the cloud layer, the values of $\Delta F_{\text{base}}$ in Table 3 were exclusively due to long-wave divergence at cloud base. It is possible that a more realistic treatment of the short-wave fluxes (including horizontal inhomogeneities in the cloud, for example) might have enhanced both the cloud-top cooling and cloud-base warming by distributing the short-wave warming deeper into the cloud.

Interestingly, the agreement with flight 620, in which the mean wind speed was low (3.7 m s$^{-1}$) but with significant shear across the inversion, was good. By Lewellen and Lewellen's arguments, shear across the inversion will not enhance the ability of the boundary-layer-scale eddies to transport positively-buoyant air down through the mixed layer, and so should not enhance the entrainment rate. The good agreement between AL and flight 620 would therefore appear to support this argument.

Finally, comparing Table 3 with Table 2 in Nicholls and Turton (1986), which showed the predicted entrainment rates from four different parametrizations, the all-round performance of AL is undoubtedly superior.

6. Conclusions

An extensive range of realistic simulations of nocturnal stratocumulus has been performed (albeit with only a simple radiation scheme designed to produce a realistic cooling near cloud top). Where evaporative cooling of entrained air did not generate buoyancy reversal, the entrainment rate was given accurately by the radiative forcing alone (reduced from that in smoke clouds to allow for the presence of saturated air in the boundary layer). This suggests that in this case the evaporative cooling associated with entrainment played a completely neutral rôle. Where evaporative cooling of entrained air was strong enough to generate buoyancy reversal, and therefore drive entrainment, it appeared that an increase in the turbulence generated by buoyancy reversal compensated for the reduction in the radiative forcing in saturated air, and that the direct radiative term had increased in magnitude (with all the cloud-top radiative divergence now acting within the inversion). The entrainment rate was then found to be given, with remarkable accuracy, by the sum of the rates that would have been generated by radiative cooling and buoyancy reversal acting in isolation, as had been found for combinations of radiative cooling and surface heating in LMA. It is therefore anticipated that the entrainment rate in a boundary layer with turbulence forced by all three processes might be accurately parametrized by setting

$$V_{\text{sum}}^3 = V_{\text{rad}}^3 + V_{\text{surf}}^3 + V_{\text{br}}^3.$$

The parametrization derived here, given by (15) and (16), has also been tested with encouraging success on observations of stratocumulus by Nicholls and Leighton (1986). Although these observations showed no significant buoyancy reversal, those of Weaver and Pearson (1990) from which $D$ can be estimated, show $D$ often as large as 0.2 and even reaching 0.6. Significant generation of entrainment by buoyancy reversal may well therefore be quite widespread (although, unfortunately, Weaver and Pearson did not present measurements of $w_e$).
Even though this parametrization has consistently demonstrated a high level of accuracy over a wide range of boundary layers, the only information it requires is the external buoyancy forcings (the surface buoyancy flux and the net radiative-divergence profile), the inversion jumps of temperature and humidity, the cloud-top liquid water content and the cloud and mixed-layer depths. Consequently, it can easily be implemented in a mixed-layer model entrainment closure and in large-scale NWP or climate models, where the process of entrainment has previously been very poorly represented. A suggested method for incorporating it into a simple first-order turbulence closure, commonly used by such models, is given in the appendix. One restriction on its accuracy, though, is the sensitivity of its term representing the generation of entrainment by buoyancy reversal to the liquid water content. This situation is apparently made worse by the further observation made here that the generation of large horizontal inhomogeneities occurred more readily if there was strong buoyancy reversal. This was the case even comparing simulations with similar turbulence energy and inversion strengths (factors which might otherwise suggest updraughts penetrating a greater distance into the inversion, therefore potentially leading to larger engulfing events and the generation of larger mixing ratios). Therefore, it is proposed that the cause of these inhomogeneities was that, where there was an entrainment event, evaporative cooling generated negative buoyancy locally. The negatively buoyant parcel thus generated then descended away from the interface, and in so doing dragged more air from above the inversion down into the cloud, generating a larger mixing ratio of free-atmospheric air. Thus, there was the potential for a positive feedback, with an initially small entrainment event (generating parcels with only small mixing ratios) leading to further entrainment locally, thus increasing the mixing ratio. This mechanism is similar to that proposed by Siems et al. (1990) for the rapid break-up of stratocumulus by buoyancy reversal or cloud-top entrainment instability, although apparently operating here on a more local scale because in none of the simulations did the clouds show any sign of breaking up.

What this suggests is that a cloud with strong generation of entrainment by buoyancy reversal may be hard to detect from spatially averaged fields—if strong buoyancy reversal also generates significant horizontal inhomogeneities then the liquid water content (and, in particular, that in the penetrating updraughts around which entrainment would be proceeding) will be underestimated in this regime, leading to an underestimate of the buoyancy reversal forcing. It is also very hard to envisage how this problem might be resolved.

The forcing from radiative cooling, on the other hand, was found not to be so sensitive. Even when substantial spatial variations were generated (which occurred only in the buoyancy reversing cases here, as discussed above), they were found not to have any significant impact on the radiative source of turbulence. A possible reason for this (apart from the simplistic radiation scheme which, amongst other approximations, only considered vertical fluxes of radiation) may be that parcels containing entrained air are descending away from cloud top (because of their negative buoyancy in this regime) and so are leaving the radiatively cooled region. This was the conclusion reached by Wang and Albrecht (1994) based on observations of stratocumulus clouds with buoyancy reversal made during FIRE*.

ACKNOWLEDGEMENTS

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* The First Regional Experiment of the International Satellite Cloud Climatology Project.
APPENDIX

Inclusion of an entrainment rate parametrization in a first-order turbulence closure

In order to improve the representation of entrainment in NWP models, whilst keeping within the simple framework of a first-order closure, Beljaars and Betts (1992) proposed imposing a parametrized entrainment flux at the diagnosed top of the cloud-free convective boundary layer ($z_i$). They specified the eddy viscosity for scalar variables at that height as

$$v_h|_{z_i} = \frac{A_1 \overline{w' b'}_S}{(\Delta b / \Delta z)|_{z_i}}.$$  \hfill (A.1)

In a finite-difference model with conserved thermodynamic variables $\theta_i$ and $q_i$, say, a first-order closure gives the flux of $\theta_i$, for example, at model level $k$ as

$$- \overline{w' \theta'_i}|_k = v_h|_k \frac{\Delta \theta_i|_k}{\Delta z_k}.$$  \hfill (A.2)

Using the same eddy viscosity for heat and moisture, and substituting for $v_h|_{z_i}$ from (A.1), this implies that $- \overline{w' b'}|_{z_i} = -(g/\theta_0)\overline{w' \theta'_i}|_{z_i} + \theta_0 r_m \overline{w' q'_i}|_{z_i} = A_1 \overline{w' b'}_S$. This is then the generally accepted scaling for a cloud-free boundary layer, see section 1. Beljaars and Betts showed significant improvement in the boundary layer development for some cloud-free case-studies, but they made no attempt to extend (A.1) to cloudy boundary layers.

Note, however, that in the absence of cloud and assuming a discontinuous inversion, $- \overline{w' b'}|_{z_i} = w_e \Delta b = A_1 \overline{w' b'}_S$. Substituting this into (A.1) gives

$$v_h|_{z_i} = w_e \Delta z|_{z_i}.$$  \hfill (A.3)

Thus, rewriting (A.1) as (A.3) allows a general parametrization for $w_e$ (such as that derived here and given by (15) and (16)) to be used to specify the entrainment fluxes. Note that it is the entrainment fluxes of $\theta_i$ and $q_i$ that are being specified, through the entrainment rate, not the buoyancy flux. Therefore, the question as to whether the entrainment buoyancy flux should take the saturated ($-w_e \delta b$) or unsaturated ($-w_e \Delta b$) form is not an issue.

For cloud topped boundary layers the situation is made more complex by the need to accurately model the direct (as well as the indirect) generation of entrainment. It has been postulated here that the direct term arises from undulations in the inversion generated by impinging turbulent eddies. If, in a one-dimensional finite-difference model, the inversion is spread over two grid levels (which must always occur when the inversion is rising, for example), part of the radiative cooling associated with the cloud top may occur above the base of the inversion and so above the level at which entrainment is specified. This cooling then has the same effect as the ‘real’ direct term but is entirely a numerical artifact. Therefore, care should be taken to ensure that the radiative divergence in the model’s inversion layer is only that associated with the clear air above cloud top. The remainder should be transmitted to the mixed layer. The ‘real’ direct entrainment generation term can then be modelled by including this term in the entrainment rate parametrization.

REFERENCES


