Quasi-Lagrangian energetics of an intense Mediterranean cyclone

By SILAS C. MICHAELIDES\textsuperscript{1*}, NICHOLAS G. PREZERAKOS\textsuperscript{2} and HELENA A. FLOCAS\textsuperscript{3}

\textsuperscript{1}Meteorological Service, Cyprus
\textsuperscript{2}TEI of Piraeus, Greece
\textsuperscript{3}University of Athens, Greece

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SUMMARY

Studies of the energetics of synoptic-scale systems and similar kinds of investigation have traditionally used a Eulerian framework. In this study, the energetics of a synoptic-scale system have been considered using a quasi-Lagrangian method, in order to isolate the disturbance under consideration within a volume which moves together with the system at each stage of its development.

Applying a Lagrangian framework implies that the dimensions of the computational area can be modified on the basis of predetermined criteria. In this study, an area surrounding a depression as shown on the surface analysis, has been selected. This area moves together with the centre of the depression. The energetics results obtained using such a quasi-Lagrangian scheme are compared to those obtained by using a Eulerian framework.

The synoptic-scale system studied here is a wintertime frontal depression, the greatest development of which occurred in the central Mediterranean on 7 December 1991. This depression moved east accompanied by significant temperature changes, heavy precipitation and gale force winds.

**KEYWORDS:** Atmospheric energetics  Available potential energy  Kinetic energy  Mediterranean cyclones

1. INTRODUCTION

Lorenz (1955) introduced the concept of available potential energy (a.p.e.) in atmospheric science as an expansion of the original idea by Margules (1903). In this context, a.p.e. has been defined as the fraction of the total potential energy (i.e., potential energy plus internal energy) that can be liberated by an adiabatic redistribution of the mass of the atmosphere. Kinetic energy (k.e.) is released by an adiabatic transformation process at the expense of a.p.e. Contributions to the generation of a.p.e. originate primarily from diabatic sources such as those resulting in heating by radiation, sensible-heat addition and latent-heat release (see Bullock and Johnson 1971; Fuelberg \textit{et al.} 1985). Friction is identified as the physical process leading to the dissipation of k.e. The diabatic generation of a.p.e., the conversion of a.p.e. into k.e. by reversible adiabatic processes and the dissipation of k.e. by frictional processes, determine a basic energy flow in the atmosphere.

The role of midlatitude synoptic-scale depressions as significant contributors to the global-scale atmospheric energetics has been well documented in the respective literature over the past three decades. Although these studies have shed ample light on various energetics characteristics of these midlatitude systems, they have customarily focused on a single energy form. On the one hand, the kinetic energy budget of midlatitude depressions has been investigated in the majority of such studies (Petterssen and Smebye 1971; Kung and Baker 1975; Kung 1977; Michaelides 1983; Prezerakos and Michaelides 1989, hereafter referred to as PM89); on the other hand, some aspects of the a.p.e. budget have been investigated by a relatively smaller number of researchers (Bullock and Johnson 1971; Lin and Smith 1982). The bulk of these studies reveals the diversity of interests regarding the methods and approaches adopted in the energetics analyses.

Bearing in mind that the required meteorological fields necessary for the computation of the energy flow components are reported on isobaric rather than isentropic surfaces, direct application of Lorenz's isentropic formulations is quite difficult. Therefore, these original isentropic relationships were further approximated by Lorenz (1955, 1967) using

\footnotesize{*} Corresponding author: Meteorological Service, Larnaca Airport, CY-7130, Cyprus.
an isobaric framework. Other approximations to the original isentropic formulations were also developed by Dutton and Johnson (1967).

Lorenz’s approach in formulating the concept of a.p.e. is not unique; Van Mieghem (1973) and Pearce (1978) have also considered the concept of a.p.e. from a different viewpoint. Robertson and Smith (1983) utilized in a cyclone diagnosis an ‘exact’ eddy equation. Nevertheless, Lorenz’s approach yields a unique complete set of relationships involving zonal and eddy energy partitioning.

The present study makes use of Lorenz’s approach for the energetics analysis of a midlatitude baroclinic disturbance, and is one of a series investigating various aspects of the energetics of Mediterranean cyclogenesis (see also Michaelidis and Angouridakis 1980; Michaelides 1983; Michaelides 1987; PM89).

The synoptic-scale circulation studied in this context is a baroclinic depression which acquired its greatest development over the central Mediterranean region on 7 December 1991. The depression formed initially near Sicily in the afternoon of 6 December 1991, deepened rapidly and moved eastwards, producing severe weather conditions marked by torrential rainfall, sharp temperature drops and gale force winds. The dynamic characteristics of this depression were discussed by Prezerakos et al. (1997), hereafter referred to as PFM97. In this companion paper the relative-absolute vorticity and the potential-vorticity spatio-temporal patterns were employed in a detailed diagnostic analysis of the dynamics of the system.

The purpose of this study is to investigate the energetics of an intense Mediterranean depression by using a computational scheme that differs from the traditionally used Eulerian method which is based on fixed volume computations. The quasi-Lagrangian method put forward in this study uses a moving-volume approach in which the computational volume embraces the synoptic-scale system being studied at each time of its development. This means that synoptic-scale circulations other than the cyclone in study are excluded, and the energetics results reflect the cyclone behaviour more genuinely. Eulerian calculations are also performed and the results of the two schemes are compared.

Following this introductory overview on atmospheric energetics, the energetics of open atmospheric systems are discussed in section 2. In section 3 a brief synoptic summary of the developmental stages of the cyclonic system being studied is presented. The data used and the methodology adopted in the investigation are briefly presented in section 4. The results of the energetics analysis using a quasi-Lagrangian framework are discussed in section 5 in terms of the energy contents, transformations and transports; the temporal changes of the respective budget quantities and vertical distributions are also presented. Finally, in section 6, the results of an energetics analysis obtained by using a Eulerian scheme are compared to those obtained by using the quasi-Lagrangian scheme.

2. ENERGETICS OF OPEN ATMOSPHERIC SYSTEMS

Traditionally, midlatitude synoptic-scale baroclinic disturbances have been treated as eddying motions evolving in a zonal current. It is within this context that both k.e. and a.p.e. were further partitioned into zonal and eddy components. Such a partition of energies into zonal and eddy components leads to a reformulation of the energy flow, which is customarily described by a set of differential equations (with respect to time) describing the large-scale atmospheric energy balance (see Lorenz, 1955; 1967). For the study of an open atmospheric system, this set of differential equations must be elaborated further in order to take into account processes associated with the boundaries of the computational area (see Muench 1965; Michaelides 1987; 1992).
In this regard, the various forms of energy, the conversions between them and the boundary processes responsible for the transfer of energy through the boundaries of the computational volume are discussed in the following, using the mathematical formulations presented by Michaelides (1987). An explanation of the mathematical conventions employed is given in appendix A and an explanation of the symbols used is presented in appendix B.

(a) Energy contents

In order to partition a.p.e. into zonal and eddy components, use is made of Lorenz's (1955) approximate formula which defines a.p.e. as a function of the variance of temperature, $T$. Using Eqs. (A.2) and (A.6), the temperature variance can further be resolved as

$$\left( T - [T]_{\lambda \varphi} \right)^2 = \left( [T]_{\lambda} \right)^2 + 2 \left( [T]_{1\lambda} \right)_{\varphi} (T)_{\lambda} + (T)^2_{\lambda}. \quad (1)$$

Integrating over the mass of the atmospheric volume considered and bearing in mind that the term containing only one factor of the department from zonal average, $(T)_{\lambda}$, vanishes when averaged over an area, the zonal a.p.e. ($AZ$) and eddy a.p.e. ($AE$) are defined as

$$AZ = \int_{p_1}^{p_2} \frac{\left( [T]_{\lambda \varphi} \right)^2_{\lambda \varphi}}{2[\sigma]_{\lambda \varphi}} \, dp \quad (2)$$

and

$$AE = \int_{p_1}^{p_2} \frac{\left( [T]_{\lambda \varphi} \right)^2_{\lambda \varphi}}{2[\sigma]_{\lambda \varphi}} \, dp, \quad (3)$$

where the integral refers to the volume defined by the horizontal dimensions of the computational region (confined by meridians $\lambda_1$ and $\lambda_2$ and by latitude circles $\varphi_1$ and $\varphi_2$), and the respective pressure levels ($p_1$ and $p_2$, $p_1 < p_2$), in the vertical. The zonal a.p.e. corresponds to the zonally averaged mass field, and the eddy available potential energy corresponds to the amount of a.p.e. left.

In the above expressions for calculating a.p.e., (and the subsequent formulations for a.p.e. conversions and transfers) and the area (isobaric) average, a static-stability parameter is employed. The approximate mathematical expression for this static-stability measure is taken to be

$$[\sigma]_{\lambda \varphi} = \left[ \frac{g T}{\varepsilon_p} - \frac{pg \, \partial T}{R \, \partial p} \right]_{\lambda \varphi}. \quad (4)$$

This expression for the static stability conforms with the original considerations of Lorenz (1955; 1967). The approximate expression used in this context, makes direct use of the observed meteorological fields.

In the derivation of the above approximate expressions for a.p.e., two basic assumptions have been made implicitly. Firstly, local variations in the static stability on isobaric surfaces are suppressed by using an area (isobaric) average for each level (i.e. $[\sigma]_{\lambda \varphi}$). Secondly, the local rate of change with respect to time of the static-stability term is zero (i.e. $\partial [\sigma]_{\lambda \varphi}/\partial t = 0$, see Muench 1965).

The use of an isobarically averaged static-stability factor in studies of synoptic-scale atmospheric energetics appears to be a practical solution. It has the significant advantage over spatially varying stability parameters in that it can overcome difficulties in determining a.p.e. in regions with a temperature lapse rate approaching its dry adiabatic value.

The issue of the utilization of an area averaged static-stability parameter in diagnostic studies has been criticised by Dutton and Johnson (1967). They argued that the importance
of a variable static stability should not be ignored in diagnostic studies, because contributions to a.p.e. from layers which are nearly neutrally stratified (with respect to the dry environmental lapse rate) can be much higher than the contributions to a.p.e. using an area averaged static-stability parameter. Indeed, local temperature profiles reveal such neutrally stratified layers near the earth’s surface and in medium-level and upper tropospheric layers, usually associated with prolonged subsidence. However, such layers are limited in depth and should be undetected with the presently used vertical resolution of the temperature field. Also, if heating occurs in less-stable air (as it often does in the area of cyclones) the generation of eddy a.p.e. will be enhanced.

Pearce (1978) demonstrated that it is possible to partition a.p.e. into a baroclinicity and a static-stability component. In this way, all the temperature variance information is considered to come from the former component, whereas all the stability information is considered to come from the latter term. As a generalization of Pearce’s (1978) a.p.e. partitioning, Marquet (1991) presented an energy budget in which the available energy is partitioned into a temperature and a pressure component. He refined this partition further by decomposing the temperature component into three terms: a baroclinicity, a complementary and a static stability term. Both Pearce (1978) and Marquet (1991) have shown that, by separating the effect of the static stability from that of the temperature variance, the problem of using an explicit contribution from the static-stability parameter in Lorenz’s (1955) relationships can be overcome. In this way, it is possible to avoid any assumptions and approximations regarding the static stability, and thus study the evolution of a.p.e. taking into account the evolution of the static stability.

In line with the above partitioning of a.p.e., k.e. is also partitioned into zonal and eddy components. This is achieved by using Eq. (A.2) in order to resolve the inner product of the horizontal velocity vector

\[ u^2 + v^2 = [u]_\lambda^2 + [v]_\lambda^2 + 2[[u]_\lambda (u)_{\lambda} + [v]_\lambda (v)_{\lambda}] + (u)_{\lambda}^2 + (v)_{\lambda}^2. \]

(5)

Integrating over the mass of the atmospheric volume considered, and bearing in mind that the terms containing only one factor of departure from zonal average, \((u)_{\lambda}\) or \((v)_{\lambda}\), vanish in an area averaging, the zonal k.e. \((KZ)\) and eddy k.e. \((KE)\) are mathematically defined as

\[ KZ = \int_{p_1}^{p_2} \frac{[u]_\lambda^2 + [v]_\lambda^2}{2g} dp, \]

(6)

\[ KE = \int_{p_1}^{p_2} \frac{(u)_{\lambda}^2 + (v)_{\lambda}^2}{2g} dp. \]

(7)

The zonal k.e. corresponds to the zonally averaged motion, and the eddy k.e. corresponds to the amount of k.e. remaining. Lorenz (1967) clarifies that \(KZ\) as defined above does not refer to the k.e. of the zonal motion or the zonally averaged k.e.

(b) Energy conversions

The transformation between \(AE\) and \(KZ\) is achieved through the thermally driven mean meridional circulations. This process is identified as the coupling of warm air rising at low latitudes with cold air sinking at high latitudes. The transformation between \(AE\) and \(KE\) is achieved through those thermally driven circulations that act in the longitudinal sense. It is accomplished in atmospheric disturbances when warm air rises and cold air sinks at the same latitude. This rate of eddy conversion of a.p.e. into k.e. is considered to be a measure of cyclogenesis.
One issue which is raised in defining energy conversions in limited-area domains, is the applicability of some of the implicit assumptions made in the original definitions on the global-scale. For example, in large computational domains it can readily be assumed that the isobarically averaged vertical wind component in pressure coordinates is zero (i.e. $[\omega]_{\lambda \varphi} = 0$, where $\omega = dp/dt$). This assumption yields expressions for the conversion between a.p.e. and k.e., namely $\langle AZ - KZ \rangle$ and $\langle AE - KE \rangle$, in which no reference is made to the isobarically averaged vertical wind speed. In the above and subsequent expressions, a conversion of energy $X$ into energy $Y$ is denoted by $\langle X - Y \rangle$, and the conversion in the opposite direction is denoted $\langle Y - X \rangle$. This approach was adopted by Muench (1965), Brennan and Vincent (1980) and Michaelides (1987; 1992). However, the a.p.e. to k.e. conversion term can be split into three components based on the following relationship (use is made of Eqs. (A.2) and (A.6))

$$[T \omega]_{\lambda \varphi} = [T]_{\lambda \varphi}[\omega]_{\lambda \varphi} + [(T)_{\lambda}([\omega]_{\lambda})_{\varphi}]_{\lambda \varphi} + [(T)_{\lambda}([\omega]_{\lambda})_{\varphi}]_{\lambda \varphi}. \quad (8)$$

Reiter (1969b) identifies the first and second terms on the right of Eq. (8) as contributing toward the mean meridional circulations, and therefore related to the transformation between $AZ$ and $KZ$. He also distinguishes between two cases regarding these mean meridional circulations: in cases where $[\omega]_{\lambda \varphi} = 0$ the first term vanishes, whereas in cases where $[\omega]_{\lambda \varphi} \neq 0$ the second term would be negligible. The third term on the right of Eq. (8) describes possible correlations between departures from the zonal average in the temperature and vertical motion fields, and is therefore related to the transformation between $AE$ and $KE$. In the present study, the assumption is made that $[\omega]_{\lambda \varphi} = 0$, and therefore only the first and third terms are retained. Implications stemming from the adoption of this assumption in atmospheric energetics calculations over limited domains are discussed by Marquet (1991). Bearing in mind the above, the mathematical expressions for the energy conversions $\langle AZ - KZ \rangle$ and $\langle AE - KE \rangle$ are given by

$$\langle AZ - KZ \rangle = - \int_{p_1}^{p_2} [(T)_{\lambda \varphi}([\omega]_{\lambda \varphi})_{\lambda \varphi}] \frac{R}{g} dp, \quad (9)$$

$$\langle AE - KE \rangle = - \int_{p_1}^{p_2} [(T)_{\lambda}([\omega]_{\lambda})_{\lambda \varphi}] \frac{R}{g} dp. \quad (10)$$

The eddy transport of sensible heat is the process responsible for the transformation between a.p.e. forms: $\langle AZ - AE \rangle$. It is accomplished by the correlation between the meridional wind component and the zonal temperature perturbations acting in a mean meridional temperature gradient. In particular, this occurs when a perturbation is situated within a meridional gradient with warm air being carried poleward and cold air being carried equatorward. The conversion between the two a.p.e. forms is given by

$$\langle AZ - AE \rangle = - \int_{p_1}^{p_2} \left[ \frac{\partial (T)_{\lambda \varphi}}{\partial \varphi} \frac{\partial [\sigma]_{\lambda \varphi}}{\partial \varphi} \right]_{\lambda \varphi} dp + \int_{p_1}^{p_2} \left[ \frac{(T)_{\lambda}([\omega]_{\lambda})_{\lambda \varphi}}{p^{R/\gamma}} \frac{\partial [\sigma]_{\lambda \varphi}}{\partial p} \right]_{\lambda \varphi} dp. \quad (11)$$

Finally, the transformation between k.e. forms is accomplished by the eddy transport of momentum. The transformation $\langle KZ - KE \rangle$ is accomplished by perturbations primarily in the high troposphere near the level of the jet stream. $KZ$ is transferred to or from the eddies when the eddy wind components are positively correlated and situated in a gradient...
of zonal wind speed. The transformation from \( KE \) to \( KZ \) provides for a barotropic maintenance of the zonal westerlies and the jet stream against internal losses due to turbulent dissipation. The reverse transformation, namely from \( KZ \) into \( KE \), is a rather rare phenomenon in midlatitude regions, associated with so-called barotropic instability—when a relatively zonal flow with meridional shear abruptly breaks down into a train of waves with high amplitude. Barotropic instability seems to become important in middle latitudes for the initial growth of waves when the zonal flow is intense and there is a strong latitudinal wind shear. The expression for the barotropic conversion term \( \langle KZ - KE \rangle \) takes the form

\[
\langle KZ - KE \rangle = - \int_{\rho_1}^{\rho_2} \frac{1}{g} \left[ (u)_x \cos \varphi \frac{\partial}{\partial \varphi} \left( \frac{[u]}{\cos \varphi} \right) \right]_{\lambda \varphi} \\
+ \left[ \frac{(u)_x^2}{r} \frac{\partial [v]}{\partial \varphi} \right]_{\lambda \varphi} + \left[ \frac{\tan \varphi}{r} (u)_x^2 [v] \right]_{\lambda \varphi} \\
+ \left[ (u)_x (\omega) \frac{\partial [u]}{\partial p} \right]_{\lambda \varphi} + \left[ (u)_x (\omega) \frac{\partial [u]}{\partial p} \right]_{\lambda \varphi} \, dp. \tag{12}
\]

The above four energy forms with their transformations, the two a.p.e. generation and the two k.e. dissipation terms, define an energy flow sufficient to describe the atmospheric energetics of a closed system provided no lateral boundaries are considered and vertical velocities at the bottom and top of the system are zero. However, in open atmospheric systems several transformation terms are mathematically possible (Lettau 1954). Nevertheless, Smith (1970) states that a sufficient condition for retaining a transformation is that it establishes a direct physical relationship between the two energy forms involved. The four energy transformation terms described above meet this requirement.

(c) **Boundary energy transfers**

As stated above, in open atmospheric systems the specification of lateral boundaries implies that boundary transfers of energy must be taken into account (Muench 1965). These transfer processes for \( AZ \), \( AE \), \( KZ \) and \( KE \) are denoted in the following as \( BAZ \), \( BAE \), \( BKZ \) and \( BKE \), respectively.

The boundary transfer of a.p.e. is defined by the mass integral of the total (horizontal and vertical) divergence of the flux of a.p.e., as explained in appendix A. Bearing in mind that a.p.e. is defined in terms of the variance of the temperature field and that this variance can be resolved further as shown in Eq. (1), the boundary transfers of the zonal and eddy components of a.p.e. can be written as

\[
BAZ = c_1 \int_{\rho_1}^{\rho_2} \int_{\varphi_1}^{\varphi_2} \frac{1}{2[\sigma]_{\lambda \varphi}} \left( u \left[ \langle (T) \rangle_x \right]_{\varphi}^2 + 2 \langle [T]_x \varphi \rangle (T)_x \right)_{\lambda \varphi} \, d\varphi \, dp \\
+ c_2 \int_{\rho_1}^{\rho_2} \int_{\varphi_1}^{\varphi_2} \frac{1}{2[\sigma]_{\lambda \varphi}} \left( \left[ v \cos \varphi \left[ \langle (T) \rangle_x \right]_{\varphi}^2 + 2 \langle [T]_x \varphi \rangle (T)_x \right] \right)_{\lambda \varphi} \, d\varphi \, dp \\
- \frac{1}{2[\sigma]_{\lambda \varphi}} \left[ \langle \omega \left[ \langle (T) \rangle_x \right]_{\varphi}^2 + 2 \langle [T]_x \varphi \rangle (T)_x \right] \right)_{\lambda \varphi} \, dp \tag{13}
\]

\[
BAE = c_1 \int_{\rho_1}^{\rho_2} \int_{\varphi_1}^{\varphi_2} \frac{1}{2[\sigma]_{\lambda \varphi}} u(T)_x^2 \, d\varphi \, dp \\
+ c_2 \int_{\rho_1}^{\rho_2} \int_{\varphi_1}^{\varphi_2} \frac{1}{2[\sigma]_{\lambda \varphi}} \langle v \cos \varphi (T)_x^2 \rangle \, d\varphi \, dp
\]
\[ -\frac{1}{2[\sigma]}(\{\omega (T)^2_{\lambda \phi}\}_{p_1})^p_{\nu_1}, \]  

where \( c_1 = -1/r (\lambda_2 - \lambda_1) (\sin \varphi_2 - \sin \varphi_1), \) \( c_2 = -1/r (\sin \varphi_2 - \sin \varphi_1) \) and \( r \) is the earth's radius.

Although the term containing only one \((T)_{\lambda}\) vanishes in area averaging and the a.p.e. partitioning takes the form of Eqs. (2) and (3), in determining the a.p.e. boundary transfer this term must be retained. The corresponding quantity of energy transfer is hereby ascribed wholly to \( BAZ \), although it does not exclusively represent a boundary transfer of \( AZ \).

Similarly, the boundary transfer of k.e. is defined by the mass integral of the total (horizontal and vertical) divergence of the flux of k.e. Resolving the horizontal velocity as shown in Eq. (5), the boundary transfers of the zonal and eddy components of k.e. can be written as

\[
BKZ = c_1 \int_{p_1}^{p_2} \int_{\varphi_1}^{\varphi_2} \frac{1}{2g} (u ([u]_{\lambda \phi}^2 + [v]_{\lambda \phi}^2 + 2([u]_{\lambda} (u)_{\lambda} + [v]_{\lambda} (v)_{\lambda})))_{\lambda \phi}^p_{\nu_1} d\varphi dp
+ c_2 \int_{p_1}^{p_2} \frac{1}{2g} (v \cos \phi ([u]_{\lambda \phi}^2 + [v]_{\lambda \phi}^2 + 2([u]_{\lambda} (u)_{\lambda} + [v]_{\lambda} (v)_{\lambda})))_{\lambda \phi}^p_{\nu_1} d\varphi dp
- \frac{1}{2g} (\{\omega([u]_{\lambda \phi}^2 + [v]_{\lambda \phi}^2 + 2([u]_{\lambda} (u)_{\lambda} + [v]_{\lambda} (v)_{\lambda})))_{\lambda \phi}^p_{\nu_1},
\]

\[
BK E = c_1 \int_{p_1}^{p_2} \int_{\varphi_1}^{\varphi_2} \frac{1}{2g} (u ([u]_{\lambda}^2 + (v)_{\lambda}^2)))_{\lambda \phi}^p_{\nu_1} d\varphi dp
+ c_2 \int_{p_1}^{p_2} \frac{1}{2g} (v \cos \phi ([u]_{\lambda}^2 + (v)_{\lambda}^2)))_{\lambda \phi}^p_{\nu_1} d\varphi dp
- \frac{1}{2g} (\{\omega([u]_{\lambda}^2 + (v)_{\lambda}^2)))_{\lambda \phi}^p_{\nu_1}.
\]

As explained in the discussion subsequent to Eq. (5), the terms containing only \((u)_{\lambda}\) or \((v)_{\lambda}\) vanish in area averaging and the k.e. partitioning takes the form of Eqs. (6) and (7). However, in determining the k.e. boundary transfer these terms must be retained. The corresponding quantities of energy transfer are hereby ascribed wholly to \( BKZ \), although they do not pertain exclusively to boundary transfer of \( KZ \).

3. SYNOPTIC OVERVIEW

PFM97 examined the synoptic conditions and the dynamics of this case of cyclogenesis over the central Mediterranean. Using isobaric vorticity and potential-vorticity analyses, they suggested that the upper-level dynamics played an important role in the initiation of the event. Furthermore, they demonstrated that the examination of the upper-level circulation pattern along with the isobaric vorticity field over the major European region three days before its initiation is crucial for the forecasting of the surface cyclogenesis. Since the synoptic analysis of the event has already been conducted by PFM97, only a brief summary is presented here.

The chart for 0000 UTC 6 December features a shallow frontal surface extending from the northern coast of the Adriatic Sea to the Black Sea coast through the Balkan peninsula (Fig. 1(a)). At 500 hPa a diffuulent trough is evident to the north-west of the Adriatic coast
Figure 1. Objective analysis of 1000 hPa geopotential height every 3 gpdam (solid thin lines), 500 hPa geopotential height every 6 gpdam (solid thick lines) and 500 hPa temperature every 5 deg C (dashed lines) on (a) 0000 UTC 6 December, (b) 1200 UTC 6 December, (c) 0000 UTC 7 December, (d) 1200 UTC 7 December, (e) 0000 UTC 8 December, (f) 1200 UTC 8 December. The surface fronts are denoted with the conventional symbols and are transferred from the Hellenic National Meteorological Centre’s respective subjective analyses. The locations of the cyclonic and anticyclonic centres are marked with small symbols for 1000 hPa and large symbols for 500 hPa.
Figure 1. Continued.
Figure 1. Continued.
of north-east Italy, with a NE–SW tilt. Horizontal cyclonic wind shear appeared over north-east Italy. The polar-front jet prevails at the western flank of the trough while the cold front is located ahead of its axis.

According to PFM97, the increase of positive relative vorticity over the Tyrrhenian Sea by 1200 UTC 6 December implies deepening of the upper trough, while the frontal zone had moved to the same region being under the maximum of positive vorticity advection (Fig. 1(b)). This is the time that the surface low-pressure centre started developing over the same area, while a closed system had not yet been created at 500 hPa.

By 0000 UTC 7 December, as the orientation of the trough axis at 500 hPa had changed from NE–SW to NW–SE, the low centre at 500 hPa tended to develop. The surface low-pressure system became circularly organized and it deepened significantly near 20°E, exhibiting a significant westward tilt with height (Fig. 1(c)).

As can be seen in Fig. 1(d), the maximum development occurred at 1200 UTC 7 December when the surface system was situated just north-west of Crete. The central pressure had dropped about 20 hPa in twenty-four hours, while the relative vorticity and absolute vorticity advection at 500 hPa had started weakening before the surface pressure reached its lowest value. This stage is marked by a significant increase in the baroclinicity over Greece, while the system became cut-off at all isobaric levels. Its vertical tilt with height implies that the system continued growing almost barotropically until 0000 UTC 8 December when it started gradually dissipating (Figs. 1(e) and 1(f)).

According to Prezerakos and Floca (1996), a remarkable feature at low levels is the anomalously high potential vorticity that formed in the western Mediterranean basin and intensified significantly at the time of the surface development over the Greek area, which might indicate the influence of diabatic heating in the low-level cyclogenesis.

It is evident that both analyses imply the significance of the upper-level dynamics in the initiation of this case of cyclogenesis. Prezerakos (1976; 1991; 1992) and Karein (1979), who dealt with the cyclogenic process in the central and eastern Mediterranean following the classification of cyclogenesis as type A or B by Petterssen and Smebye (1971), both tended to describe cyclogenesis over this region as being of mixed type with a strong leaning towards type B. This is also supported in this case, since the mode of initiation and the role of positive vorticity advection in the middle troposphere could classify it as type B development, while the baroclinic character of the cyclonic system examined could justify its description as a type A development. Moreover, the atmospheric energy conversion during the various steps of the perturbation development could reveal the appropriate type of Petterssen and Smebye (1971) cyclogenesis.

4. DATA AND COMPUTATIONAL METHODOLOGY

Traditionally the Eulerian approach appears to have been utilized in energetics studies of open atmospheric systems. In this approach, a fixed geographical region is selected, bounding the synoptic system under consideration at all stages of its development. This approach means that the area must be large enough to accommodate the synoptic system at all times. Therefore, the computational area must have spatial dimensions considerably larger than the targeted system, in order to take account of the system's temporal changes in position and size. Apparently, such an Eulerian framework simplifies the energetics computations and it has been preferred by almost all the researchers in the field.

The choice of a fixed computational area inevitably embraces significant components from circulations other than the targeted one. Therefore, the energetics results do not exclusively reflect the synoptic vortex studied, but spuriously aggregate the energetics characteristics of adjacent synoptic circulations.
A purely Lagrangian framework would be one in which the synoptic-scale vortex under consideration is 'isolated' throughout its evolution from other non-targeted circulations. This implies that the boundaries of the computational volume can vary in such a way that the vortex is exclusively surrounded by a clearly defined but continuously changing three-dimensional volume, in which a (preferably) conservative atmospheric parameter, peculiar to the system studied, can be traced. However, since such a conservative property characterizes a specific airmass, a Lagrangian scheme based on conservative properties could, at least in principle, be applied to an airmass with uniform characteristics (i.e. non-frontal). In a midlatitude baroclinic cyclonic disturbance, however, the dependence of the conservative properties on the air masses involved (vis-à-vis of Tropical and Polar origin) makes a purely Lagrangian approach practically inapplicable for an energetics analysis of the synoptic-scale vortex as a whole.

Consequently, a quasi-Lagrangian scheme is pursued for the study of the energetics of the synoptic-scale depression. This scheme comprises the atmospheric volume bounded by the 1000 and 100 hPa surfaces in the vertical; the horizontal boundaries of the computational region move in accordance with the position of the centre of the depression.

A purely Lagrangian framework would call for a study of the targeted system in isolation from other spurious circulations. This implies that the boundaries of the computational volume can vary with time, in such a way that the targeted system is exclusively bounded by them. However, it must be appreciated that this exclusive-bounding condition is rather vague. Bearing in mind the complex structure exhibited by baroclinic systems, it is obvious that it is far from straightforward what exactly comprises the system being studied and what does not.

For the computations presented here a uniform grid is used in the meridional and longitudinal directions, with the grid point spacing being $\delta \varphi = \delta \lambda = 2.5^\circ$. Based on the quasi-Lagrangian approach explained above, the computational area is selected as follows. Using the European Centre for Medium-Range Weather Forecasts' (ECMWF's) mean sea level (MSL) pressure analyses (which are contoured at 4 hPa intervals), the centre of the depression is determined to the nearest grid-point. A rectangular area on the sphere is defined, bounded by the meridians and latitude circles determined three grid points away (i.e. 7.5°) from the centre of the depression in the east, west, north and south directions. The computational area defined in this way has horizontal dimensions of $15^\circ \times 15^\circ$ and constitutes the horizontal cross-section of the quasi-Lagrangian framework moving with the depression. Computations in this study refer to an area smaller by 1.25° on each side, because a centred finite-difference scheme has been adopted to approximate space derivatives on the isobaric surfaces. In the vertical, the (isobaric) dimensions of the computational area are determined by the isobaric surfaces of 1000 and 100 hPa at all times.

The horizontal dimensions of the computational framework chosen above define an area of the order of $1650 \times 1650 \text{ km}$; such dimensions are considered by Zverev (1972) to be appropriate for an energetics analysis of a synoptic-scale depression.

The data used in the energetics analysis have been retrieved from ECMWF. They consist of the vertical velocity, temperature and horizontal wind velocity fields (eastward and northward wind components) at the following 10 standard isobaric levels: 1000, 850, 700, 500, 400, 300, 250, 200, 150 and 100 hPa. All of these fields were obtained for both 0000 UTC and 1200 UTC over the period from 6 to 8 December 1991 with values at each grid point of the computational area.

Although the size of the domain containing the surface and upper wave is comparable between various studies, an important point of difference is the grid point spacing employed. They range from around $1.5^\circ$ (Petterssen and Smeyc 1971; Robertson and Smith 1983; Smith and Dare 1986; Michaelides 1983; PM89) to the presently used $2.5^\circ$
Figure 2. The horizontal computational areas embracing the different grids used for the calculations. The area defined by the dotted curve refers to the fixed area used in the Eulerian scheme. The two frames defined by continuous curves refer to the areas used for the initial and final times in the quasi-Lagrangian scheme; for the intermediate times similar frames were used, each centred at the respective position of the cyclone's centre.

(see also Kung and Baker 1975; Kung 1977; Michaelides 1987). The latter grid point spacing is comparable to that commonly used in global energetics studies. The grid point spacing pertains to the resolution of the eddy components, and the grid point spacing employed here is considered to be appropriate in order to place the discussion of cyclone energetics within the framework of the synoptic-scale energetics. The horizontal computational area is shown in Fig. 2 for the initial and final times of the cyclone energetics analysis.

5. QUASI-LAGRANGIAN ENERGETICS RESULTS

In the following, the results of the computations of components of the energy cycle using the quasi-Lagrangian framework described above are presented.

(a) Energy contents

Table 1 presents the amounts of $AZ$, $AE$, $KZ$ and $KE$ for each of the 12-hour periods from 0000 UTC 6 December to 1200 UTC 8 December.

The $AZ$ shows a considerable increase during the first stages of the development of the synoptic vortex, accounted for by initially strong cold air advection from the north; subsequently it displays a decrease. From $5.34 \times 10^8$ J m$^{-2}$ at 0000 UTC 6 December, $AZ$ increases to $9.85 \times 10^8$ J m$^{-2}$ within the next 24 hours; but in the following 24 hours it decreases to $3.43 \times 10^8$ J m$^{-2}$, which is smaller than its initial value.

The $AE$ of the disturbance displays a completely different behaviour: from an initial value of $1.59 \times 10^8$ J m$^{-2}$ it steadily increases and virtually triples in the last stages of the
TABLE 1. ENERGY CONTENTS DURING THE EVOLUTION OF THE DEPRESSION (10^5 J m^{-2})

<table>
<thead>
<tr>
<th>Year.Month.Day.Time</th>
<th>AZ</th>
<th>AE</th>
<th>KZ</th>
<th>KE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991.12.06.00</td>
<td>5.34</td>
<td>1.59</td>
<td>14.87</td>
<td>1.28</td>
</tr>
<tr>
<td>1991.12.07.00</td>
<td>9.85</td>
<td>1.83</td>
<td>21.06</td>
<td>7.07</td>
</tr>
<tr>
<td>1991.12.08.00</td>
<td>5.43</td>
<td>2.46</td>
<td>19.78</td>
<td>3.74</td>
</tr>
<tr>
<td>1991.12.08.12</td>
<td>3.43</td>
<td>4.22</td>
<td>16.51</td>
<td>5.41</td>
</tr>
</tbody>
</table>

Figure 3. The vertical distribution of the energy contents at different times during the evolution of the cyclone. Units are 10^5 J m^{-2} per 100 hPa. Date/time formats are yymmddhh. (a) Zonal available potential energy AZ, (b) eddy available potential energy AE, (c) zonal kinetic energy KZ, and (d) eddy kinetic energy KE.

development, reaching 4.22 \times 10^5 J m^{-2}. With the onset of cyclogenesis the temperature field is deformed, leading to the development of the warm and cold tongues over the area thus intensifying the thermal gradient in the east–west direction.

The KZ is undoubtedly the major form of energy at all stages, exceeding all the other energy amounts put together. KZ follows roughly the same time evolution as AZ, its zonal counterpart. It sharply increases immediately after the formation of the surface
cyclone, when the jet streak to the west of the trough moves into the computational area. It is worth noting that $KZ$ is subject to a continuous decrease afterwards: from a peak of $22.24 \times 10^5 \text{ J m}^{-2}$ at the time of the cyclone formation it drops to $16.51 \times 10^5 \text{ J m}^{-2}$ by the end of the period under study, indicating a weakening of the jet streak.

The onset of surface cyclogenesis is marked by a sharp increase in $KE$. Moreover, $KE$ exhibits the highest rates of energy increase compared to the other three forms of
energy: from $1.28 \times 10^5$ J m$^{-2}$ in the pre-cyclogenetetic period, it reaches $7.07 \times 10^5$ J m$^{-2}$ within the first 24 hours of the cyclone development. Despite the decreases that follow this peak value, the bulk of $KE$ remains at considerably higher levels than the starting pre-cyclogenetetic value.

The time evolution of the vertical distribution of each of the energy forms can be followed in Figs. 3(a)–(d). In these and the following vertical distributions the results have been normalised to read per 100 hPa. Initially $AZ$ is roughly uniformly distributed within the tropospheric layers (see Fig. 3(a)). With the onset of cyclogenesis, however, the $AZ$ of the troposphere increases considerably, with the maximum of this increase occurring within the lowest layers, reaching $2.6 \times 10^5$ J m$^{-2}$ per 100 hPa just above the surface at 1200 UTC 7 December. This is three times as much as the initially calculated energy content at each level. This is the time at which the potential-temperature analysis at 1000 hPa revealed a pronounced distortion, with a thermal ridge over the Aegean Sea and a thermal trough over the Greek mainland, where the thermal gradient was maximized (12 K per 700 km) in the north–south direction (see PFM97). It is worth mentioning that the $AZ$ of the stratospheric layers remains very low at all times, virtually zero.

As can be seen in Fig. 3(b), the amount of $AE$ within various layers is smaller than the respective quantities of its zonal counterpart, $AZ$. The vertical distribution of $AE$ reveals a negligibly small initial contribution of the eddy to the total a.p.e. of the area. However, with the development of the cyclone, such a contribution from the upper tropospheric layers becomes more evident. This behaviour is evidence that the depression is showing a strong leaning towards Petterssen’s type B development (see Petterssen 1956; Petterssen et al. 1955). Also, with the maturity of the system, the lowest layers become the major contributors to the a.p.e., possibly through a diabatic generation of $AE$ accomplished by the release of latent heat to the south of the depression, within the warm sector (see Danard 1966; Bullock and Johnson 1971).

The importance of the upper tropospheric jet stream in the maintenance of the mean zonal flow is well established in Fig. 3(c). Indeed, as mentioned above, the $KZ$ is the major energy form within the area of the cyclone, exceeding all the other energy forms together. A maximum $KZ$ of $6.4 \times 10^5$ J m$^{-2}$ per 100 hPa at around 300 hPa is observed at 1200 UTC 6 December, immediately after the onset of surface cyclogenesis.

As discussed later in this article, this extreme of $KZ$ is largely accomplished by the high rates of import of $KZ$ from the cyclone neighbourhood. Michaelides (1987; 1992) also found that the onset of cyclogenesis is followed by an increase in $KZ$, which is ascribed to a similar import of energy from the area surrounding the cyclone (see also Palmén 1958). In the present study, a secondary maximum in $KZ$ is observed near the earth’s surface at all times. A similar bimodal behaviour in the vertical distribution of k.e. has been noted by Kung and Baker (1975). An increase in the zonal k.e. implies either a strengthening in the zonal flow or a decrease in the upper-wave amplitude.

Compared to $KZ$, the contribution of $KE$ to the k.e. budget of the cyclone area is considerably smaller. Nevertheless, as can be seen in Fig. 3(d), the sharp increase in $KE$ at the level of the jet stream is similar to that of the $KZ$. Also, this sharp increase could equally be accounted for by the import of $KE$ into the area of the cyclone from its surroundings, as discussed below.

(b) Energy conversions

The intensity of each of the energy conversions between the four energy forms, $(KZ - KE)$, $(AZ - AE)$, $(AZ - KZ)$ and $(AE - KE)$, integrated over the entire atmospheric volume which follows the cyclone track, is displayed in Table 2.
TABLE 2. ENERGY CONVERSIONS DURING THE EVOLUTION OF THE DEPRESSION (W m$^{-2}$)

<table>
<thead>
<tr>
<th>Year.Month. Day.Time</th>
<th>$\langle KZ - KE \rangle$</th>
<th>$\langle AZ - AE \rangle$</th>
<th>$\langle AZ - KZ \rangle$</th>
<th>$\langle AE - KE \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991.12.06.00</td>
<td>2.15</td>
<td>-3.90</td>
<td>-1.86</td>
<td>-1.54</td>
</tr>
<tr>
<td>1991.12.06.12</td>
<td>11.30</td>
<td>-7.05</td>
<td>0.53</td>
<td>-6.54</td>
</tr>
<tr>
<td>1991.12.07.00</td>
<td>-7.56</td>
<td>-3.21</td>
<td>6.76</td>
<td>-2.86</td>
</tr>
<tr>
<td>1991.12.08.00</td>
<td>1.40</td>
<td>8.43</td>
<td>1.90</td>
<td>6.99</td>
</tr>
<tr>
<td>1991.12.08.12</td>
<td>3.99</td>
<td>17.63</td>
<td>1.22</td>
<td>5.44</td>
</tr>
</tbody>
</table>

It appears that the initial stages of the cyclone development are accompanied by an intense feeding of the $KE$ at the expense of the $KZ$ through the operation of the barotropic energy conversion term, namely $\langle KZ - KE \rangle$, amounting to a rate of 11.30 W m$^{-2}$ at 1200 UTC 6 December. However, this is followed by a reversal of the energy flow, implying a conversion of $KE$ back to $KZ$ (0000 and 1200 UTC 7 December). A change in the sign of this conversion is noted in the last stages. This behaviour of $\langle KZ - KE \rangle$ indicates an alternate transition of eddy transfer of momentum from areas of excess to areas of deficit and vice versa.

The above results indicate an overall eddy transfer of momentum in the direction of maintaining the k.e. of the disturbance at the expense of the zonal flow prevails during the development of the depression, a result supporting findings of other energetics analyses of Mediterranean baroclinic cyclones using the Eulerian approach (Michaelides 1987; 1992). However, a detailed analysis of the results reveals a barotropic enhancement of the k.e. of the disturbance at the expense of the zonal flow as the system approaches the time of its maximum intensity. The cascading of the k.e. during these stages of the cyclone development implies the maintenance of the cyclonic circulation by utilizing the k.e. of the zonal flow. This is followed by an upgrading of the k.e. as the system starts filling. However, the return to the k.e. cascade regime observed in the final stages, implies a tendency for a revival of the cyclonic circulation by depleting energy from the zonal flow. Obviously, a true revival has not been noticed, because a change in $KE$ depends not only on the transformation $\langle KE - KZ \rangle$ but is the net result of all the respective components of the $KE$ budget.

In the first half of the period studied, the a.p.e. associated with the zonal mass distribution, namely $AZ$, is enhanced at the expense of the a.p.e. of the disturbance, namely $AE$. This conversion reaches 7.05 W m$^{-2}$ at 1200 UTC 6 December. In the second half of the period, $\langle AZ - AE \rangle$ reverses its direction of energy enhancement, thus feeding the $AE$ reservoir at the expense of $AZ$, at increasingly higher rates. Indeed, the highest energy conversion rate observed, 17.63 W m$^{-2}$ at 1200 UTC 8 December, is that which converts $AZ$ to $AE$. The above results suggest that, initially, the eddy flux of heat is against the temperature gradient, thus it enhances the meridional temperature contrast and therefore makes the east–west oriented part of the front more pronounced; in the last stages, the eddy flux of heat is in the direction of the temperature gradient, thus reducing the meridional temperature contrast.

Early global-scale studies have revealed that the sign of $\langle AZ - KZ \rangle$ is uncertain, especially in the wintertime (Wiin-Nielsen 1965). This uncertainty extends to limited-area energetics of Mediterranean cyclones (Michaelides 1992). In the present study, the
conversion between the zonal forms of energy in the neighbourhood of the depression, namely \((AZ - KZ)\), appears to be an effective process for the production of \(KZ\) at the expense of \(AZ\). Indeed, with the single exception of the pre-cyclogenetic period, the sinking at colder and rising at warmer latitudes appears to be the prevailing mechanism over the limited area studied here. These results are in general agreement with the November 1991 case-study (Michaelides 1992) but disagree with the January 1981 case-study (Michaelides 1987).

In the first half of the period under consideration, the feeding of the eddy a.p.e. by the conversion \(\langle AE - KE \rangle\) reaches a maximum of 6.54 W m\(^{-2}\) at 1200 UTC 6 December. In the rest of the period the direction of \(\langle AE - KE \rangle\) changes, converting \(AE\) to \(KE\); the highest rate of this conversion, 6.99 W m\(^{-2}\), is at 0000 UTC 8 December. Palmen (1958) estimates that in the area of a cyclonic system, the conversion from \(AE\) to \(KE\) amounts to 52 W m\(^{-2}\), but a rate of 20 W m\(^{-2}\) is considered by Reiter (1969b) to be a more realistic value (see also Palmen 1966; Danard 1966). However, the estimates presented in the present study, together with the results from the Eulerian approach, support a much smaller rate for this energy conversion in the vicinity of a Mediterranean baroclinic disturbance (see also Michaelides 1987; 1992). Averaging the findings from all the available diagnostic studies of Mediterranean cyclones, it is indeed inferred that the transport of momentum operates in the direction of maintaining the \(KE\) at the expense of its a.p.e. counterpart and that an average value of 5 W m\(^{-2}\) for this rate of transport seems most appropriate.

A positive rate of conversion from \(AE\) to \(KE\) represents a measure of baroclinic cyclogenesis accomplished by the process of warm air rising and cold air sinking at the same latitude. In the baroclinic disturbance under study, this process occurs from 1200 UTC 7 December. However, in the first stages of the development, the feeding of \(AE\) at the expense of \(KE\) is a direct consequence of the rearrangement of the temperature field over the area of interest, where strong vorticity advection in the middle troposphere forces the cold air to ascend or at least to maintain its depth.

The vertical distribution of each of the four energy conversion terms is shown in Figs. 4(a)–(d). In the pre-cyclogenetic phase the barotropic conversion term, \(\langle KZ - KE \rangle\) in Fig. 4(a), is negligible within all layers, but with the onset of the surface cyclogenesis it feeds the kinetic energy of the disturbance utilizing the k.e. of the mean motion, at all levels. However, \(\langle KZ - KE \rangle\) soon changes sign within almost all layers, and the zonal k.e. of the area is enhanced by depletion from the eddy’s k.e. reservoir. In general, as should be expected, the most intense conversion of \(KZ\) into \(KE\) and \textit{vice versa} take place at the level of the tropospheric jet stream.

The energy conversion between the a.p.e. forms, \(\langle AZ - AE \rangle\) shown in Fig. 4(b), is characterized by a conversion of \(AZ\) into \(AE\) taking place within the middle and upper tropospheric layers, at almost all times. However, a striking feature of this conversion term is the extremely intense transformation of eddy a.p.e. into zonal a.p.e., which is of the order of 5 W m\(^{-2}\) per 100 hPa just above the earth's surface at 1200 UTC 8 December.

The vertical distribution of the conversion term \(\langle AZ - KZ \rangle\) is presented in Fig. 4(c). In the first stages, this conversion term appears to enhance the zonal k.e. within the lower tropospheric layers, whereas within the respective middle and upper layers this conversion acts in the direction of \(AZ\) increase. In the final periods, the importance of this conversion becomes negligible within all layers.

The vertical distribution of the conversion between the eddy energy forms, namely \(\langle AE - KE \rangle\) is shown in Fig. 4(d). The behaviour of this conversion term is rather uniform over time and can be generalized as follows: within the upper tropospheric layers this conversion acts in the direction of generating \(AE\) at the expense of \(KE\), whereas in the lower layers the role of the conversion is either reversed or it becomes negligible.
ENERGETICS OF AN INTENSE CYCLONE

TABLE 3. BOUNDARY TRANSFERS OF ENERGY DURING THE EVOLUTION OF THE DEPRESSION (W m⁻²)

<table>
<thead>
<tr>
<th>Year,Month, Day.Time</th>
<th>BAZ</th>
<th>BAE</th>
<th>BKZ</th>
<th>BKE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991.12.06.00</td>
<td>3.88</td>
<td>3.54</td>
<td>−1.50</td>
<td>1.07</td>
</tr>
<tr>
<td>1991.12.06.12</td>
<td>7.28</td>
<td>1.98</td>
<td>28.78</td>
<td>1.06</td>
</tr>
<tr>
<td>1991.12.07.00</td>
<td>3.82</td>
<td>4.45</td>
<td>32.29</td>
<td>47.90</td>
</tr>
<tr>
<td>1991.12.07.12</td>
<td>−0.71</td>
<td>−1.55</td>
<td>9.46</td>
<td>3.76</td>
</tr>
<tr>
<td>1991.12.08.00</td>
<td>4.74</td>
<td>−1.87</td>
<td>62.72</td>
<td>0.43</td>
</tr>
<tr>
<td>1991.12.08.12</td>
<td>2.00</td>
<td>−12.35</td>
<td>30.15</td>
<td>−1.17</td>
</tr>
</tbody>
</table>

(c) Boundary energy transfers

Table 3 shows the surface integrated energy transfers, namely BAZ, BAE, BKZ and BKE, over the computational area at each of the stages in the cyclone’s life-cycle. Bearing in mind the formulation in Eqs. (13)–(16), a positively signed transfer indicates a transfer into the computational area from the surrounding area; a negatively signed quantity indicates a transfer from the computational area to the surrounding space.

In the initial stages of the cyclone development, there exists an import of AZ into the cyclone area reaching a maximum of 6.82 W m⁻² at 0000 UTC 7 December. This import is followed by a much smaller rate of energy export. At 0000 UTC 8 December, the AZ transfer changes sign again, becoming a major energy import at a rate of 4.74 W m⁻². Considering all six times together, the present quasi-Lagrangian analysis reveals an import of AZ into the cyclone area at a mean rate of 3.35 W m⁻².

The course of the other a.p.e. transfer, namely BAE, is quite different from that of BAZ discussed above. In the first half of the period under study AE is imported into the cyclone area. This transfer reverses its direction in the second half of the period, becoming a significant energy export by the end of the period. The overall behaviour of this energy transfer renders the cyclone as an AE exporting eddy at a mean rate of 0.97 W m⁻².

The behaviour of BKZ is rather erratic during the life cycle of the cyclone. A quite large energy import of KZ, amounting to 28.78 W m⁻², is noted with the onset of surface cyclogenesis, followed by a further import of 32.29 W m⁻² at 0000 UTC 7 December. Such a dependence of cyclonic systems on imports of k.e. coinciding with the cyclogenetic process, has been observed by others too (see Michaelides, 1987; Palmen 1958, 1966). In the final stages, the immediate cyclone area appears to be very active in exporting KZ with a calculated peak of 30.15 W m⁻² at 1200 UTC 8 December. Overall, the cyclone import KZ at a mean rate of 5.18 W m⁻².

The rate of KE transfer, namely BKE, maintains a much lower profile than that of its zonal counterpart, BKZ. At most times the cyclone imports KE. Only in the final stages does BKE become exported. It is worth noting that the time of maximum KE import coincides with the time of maximum import of KZ, which is at 0000 UTC 7 December. Overall, the cyclone imports KE at a mean rate of 3.70 W m⁻².

Figures 5(a)–(d) display the vertical distribution of the energy transfers. For uniformity purposes, the results presented in these figures have been normalized to read per 100 hPa. The boundary transfer of zonal a.p.e., namely BAZ, is presented in Fig. 5(a). The results show that initially AZ is transferred into the computational volume at all but the uppermost levels. In the period following, small quantities of AZ are also exported from the region within the lowest tropospheric layers. As the system develops further, the upper layers become more dependent upon imports of AZ. The above rather erratic behaviour
Figure 4. The vertical distribution of the energy conversions at different times during the evolution of the cyclone. Units are in W m\(^{-2}\) per 100 hPa. Date/time formats are yymmddhh. (a) Conversion of zonal kinetic into eddy kinetic (\(KZ - KE\)); (b) zonal available potential into eddy available potential (\(AZ - AE\)); (c) conversion of zonal available potential into zonal kinetic (\(AZ - KZ\)); and (d) conversion of eddy available potential into eddy kinetic (\(AE - KE\)).
of the zonal a.p.e. transfer term within various atmospheric layers has been observed in other boundary transfer calculations (Michaelides 1987; 1992).

The vertical distribution of the transfer $BAE$ is shown in Fig. 5(b). The maximum rates of $AE$ transfer are smaller compared to those of $AZ$, being negligibly small at almost all times.

The most intense rates of energy transfer within various layers are those associated with $KZ$, shown in Fig. 5(c). Indeed, the transfers associated with the tropospheric jet stream, around 300 hPa, appear to be the most prominent feature of $BKZ$ at all times. The import at jet-stream level is quite intense, exceeding 15 W m$^{-2}$ per 100 hPa in the period coinciding with the onset of the surface cyclogenesis. This is followed by a steady reduction in the intensity of energy import, which is eventually reversed to become an energy export in the final stages. It is also worth noting that initially almost all layers deplete $KZ$ from the area surrounding the cyclone; however, as the cyclone develops this behaviour is reversed, with depletion of $KZ$ from the cyclone area and its outwards transfer occurring within almost all layers. This behaviour is clearly reflected in the volume-integrated $BKZ$ results presented in Table 3 and discussed above.

The behaviour of $BKE$ in different layers is shown in Fig. 5(d). Small rates of $KE$ import are noted initially in the middle and upper tropospheric layers, largely offset by comparable export rates in the lower tropospheric and stratospheric layers.

6. Eulerian and Quasi-Lagrangian Schemes

In order to demonstrate that the conclusions regarding the energetics of a synoptic-scale feature are largely dependent upon the adoption of a Eulerian or a quasi-Lagrangian approach, the energy contents, conversions and boundary transfers were also calculated using an appropriate Eulerian scheme. In such a Eulerian scheme, the energetics formulations for an open atmospheric system discussed in section 2 are retained, and are the same as for the quasi-Lagrangian approach. The computational grid spacing of 2.5° in the meridional and longitudinal directions is also maintained. However, the computational area used in the Eulerian approach is fixed, and essentially embraces all of the six computational areas used in the quasi-Lagrangian approach at each of the six times used (see Fig. 2). The Eulerian and quasi-Lagrangian energetics of the depression studied are contrasted in Table 4. The quantities in Table 4 represent overall energy contents, conversions and
boundary transfers as time averages for all of the six times used in the present analysis.

The amounts of zonal energies, $AZ$ and $KZ$, calculated by using either the Eulerian or the quasi-Lagrangian computational methods appear not to differ significantly. However, the two methods yield significantly different amounts as far as the eddy energies are concerned: on the one hand, the quasi-Lagrangian $AE$ amounts to $9.41 \times 10^5$ J m$^{-2}$, whereas its Eulerian counterpart amounts to $3.83 \times 10^5$ J m$^{-2}$; on the other hand, the quasi-Lagrangian $KE$ is less than half the Eulerian $KE$.

The quasi-Lagrangian and the Eulerian computational methods reveal considerably different behaviour regarding the energy transformations. Both methods reveal an overall conversion from $KZ$ to $KE$ and a conversion from $AZ$ to $KZ$. However, these conversions are computed to proceed at higher rates with the Eulerian method that with the quasi-Lagrangian one. The other two transformations $\langle AZ - AE \rangle$ and $\langle AE - KE \rangle$ appear to change sign when the Eulerian approach is adopted.

The average $AZ$ transfer, namely $BAZ$, appears to be the same using either of the two methods. With the quasi-Lagrangian scheme it is calculated that the immediate vicinity of the cyclone exports $AE$ at the rather small overall rate of 0.97 W m$^{-2}$. However, when the larger Eulerian computational region is employed, the broader cyclonic area appears to import $AE$ at a rate of 5.56 W m$^{-2}$. A reversal in sign is also noted for the two
k.e. transfers. The quasi-Lagrangian computational regions appear to absorb $KZ$ and $KE$ at mean rates of 5.18 and 3.70 W m$^{-2}$, respectively; whereas, the larger Eulerian region appears to export $KZ$ and $KE$ at mean rates of 8.59 and 17.61 W m$^{-2}$, respectively.

The above comparisons highlight the fact that the energetics behaviour of the cyclone studied is largely dependent upon the choice of the computational method. The quasi-Lagrangian method, which reflects the immediate vicinity of the cyclone at each time, is
considered to represent the energetics characteristics of the synoptic system under consideration more genuinely, because the cyclone is isolated as much as possible at all times; the calculations are performed for the focused system, excluding as much as possible any other synoptic-scale components of the atmospheric circulation. The Eulerian method allows for circulations other than the cyclonic system under study to infringe into the computational region and spuriously contaminate the cyclone energetics. Bearing in mind the above, the quasi-Lagrangian method elaborated in the present study is considered to offer an appropriate background for the study of the energetics of individual synoptic-scale systems.

7. Concluding Remarks

In this paper some aspects of the energetics of a Mediterranean baroclinic depression are analysed and discussed. The investigation focused on the study of energy contents, transformations and transfers during the development of the system, using a quasi-Lagrangian framework which moves with the position of the centre of the cyclone. For a complete energy budget of the type used in energy budget analyses (see Michaelides 1987, 1992; PM89) the calculation of the local change in energy contents is necessary. However, under the present quasi-Lagrangian scheme, both theoretical and computational difficulties arise regarding the definition of the local (partial) derivative of the energy contents (with respect to time). Numerically, such a derivative is taken to be the difference in the energy content of a fixed atmospheric volume over a given period of time. However, bearing in mind that, in the present study, the computational volume is not fixed, this procedure cannot be adopted.

Other components of a complete energy balance which have not been included in the present study are the frictional dissipation of k.e., the adiabatic generation of a.p.e. and the local production of k.e. by the pressure of work exerted on the boundaries. Bearing in
mind the above, it is useful to clarify that the results presented in Tables 1, 2 and 3 do not exclusively determine a complete energy balance.

Lorenz (1967) defined a.p.e. of the atmosphere considering its entire volume. In this way, the horizontal mass integration involves the entire 'horizontal' extent of the atmosphere, and the respective vertical integration is made from the earth's surface up to the top of the atmosphere, where pressure is zero. Therefore, it is useful at this point to clarify a fundamental issue regarding the present application of the concept of a.p.e. to limited-area energetics. Clearly, the 'horizontal' extent of the atmospheric volume used here is only a portion of the entire 'horizontal' atmospheric extent, and the pressure at the top of the volume used is certainly not zero. Bearing in mind Lorenz's original definition, the amount of a.p.e. computed over a limited atmospheric volume must be considered as the contribution of this volume to the a.p.e. of the global atmosphere. Therefore, the quantities of a.p.e. which are calculated here do not represent the a.p.e. of the synoptic-scale system, but rather represent the fraction of the global a.p.e. contained in the respective computational volume. The above clarification allows for a meaningful extension of the theory of a.p.e. to limited atmospheric volumes (see also Smith 1969; Johnson 1970).

The results obtained in the present analysis are the outcome of the computational manipulation of the specific set of atmospheric data employed here. Therefore it is to be expected that these results are influenced by the various characteristics of the data set upon which the energetics calculations have been based. In an energetics analysis of the general circulation, Kung and Tanaka (1983) have shown that the origin of the data set profoundly influences the computations. In particular, they were able to establish that the energy cycles produced by using data from different sources may present noticeable differences. Such differences are attributed to fundamental differences between the analyses used to prepare the data sets. The extent to which data from global circulation models exercise their influence on energetics calculations must depend on two aspects: firstly, the original observed meteorological data; secondly, the techniques employed in the assimilation of these observed data into their final form utilized in global modelling.

Bengtsson et al. (1982) elaborate on the origin of the meteorological data used in the ECMWF analyses. These data originate from a variety of both synoptic and asymptotic observing systems. Also, these authors give an account of the four-dimensional data-assimilation system used at ECMWF, which consists of a three-dimensional multivariate optimum-interpolation and nonlinear normal-mode initialization. Collected data are assimilated in 6 h time periods assuming geostrophic balance. The application of the nonlinear normal-mode initialization scheme leads to a realistic presentation of initial divergences and pressure tendencies, at least at high latitudes. Retaining the meteorologically significant structures of the fields is a very important aim of the assimilation scheme used.

Bearing in mind the documented dependence of middle latitude cyclonic disturbances upon imports of k.e. in the initial stages of the development and the export of both $KZ$ and $KE$ in the final stages, discussed above, we postulate here that this timing of export/import of k.e. could be related to the concept of downstream development. The exported energy from the decaying cyclone is thought to be used up by a new development or used for the reinforcement of a pre-existing development downstream (see Simmons and Hoskins 1979; Orlanski and Sheldon 1993).

The analysis presented above, together with numerous others, enhances the widely accepted view that cyclonic disturbances of middle latitudes are one of the most active components determining the fate of energy in the atmosphere (see Palmen and Newton 1969; Smith 1980). Finally, with regard to the classification of cyclones put forward by Petterssen and Smebye (1971), the presently analysed case can justifiably be classified as a type B.
One of the main difficulties arising in comparing the results from different energetics studies, is related to the computational scheme employed. Indeed, differences between various energetics studies on individual cyclones can be ascribed, at least partly, to the rather vague reference of the computed Eulerian energetics quantities as representing the characteristics of the studied system. The Eulerian scheme provides a rigid computational tool for general energetics of a fixed atmospheric volume, but it has inherent disadvantages for computing the energetics of individual synoptic-scale systems which are characterized by time evolution and mobility. It is therefore postulated that the quasi-Lagrangian scheme presented in this research provides a more practical approach for the study of mobile systems, such as cyclonic disturbances, because it focuses the computations onto the system itself.

Both the quasi-Lagrangian and the Eulerian approaches underline the active role of baroclinic synoptic systems in the reformulation and redistribution of various large-scale forms of atmospheric energy. However, this role is stressed more clearly by the quasi-Lagrangian approach, which focuses as exclusively as possible on the particular synoptic vortex under study.

Overall, the quasi-Lagrangian scheme has revealed that the cyclonic system studied is quite dependent upon energy transfers from the surrounding region. In the region of this cyclone, both the \( \langle KZ - KE \rangle \) and \( \langle AE - KE \rangle \) conversions enhance the k.e. of the disturbance, and the zonal \( AZ \) appears to feed both \( AE \) and \( KZ \).

Clearly, a typical pattern for the energetics of Mediterranean synoptic-scale cyclones cannot be inferred from a single case-study. Such a pattern, if it actually exists, may be found as a result of energetics analyses of several cyclones. For such an endeavour, uniformity in the areas of the respective synoptic-scale systems covered is essential, and in this respect the quasi-Lagrangian scheme has advantages over the Eulerian approach.

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APPENDIX A

Integral definition of the zonal and isobaric averages and boundary transfers

For the numerical integration of the energy contents, conversions and boundary transfers the following conventions have been used for an atmospheric volume bounded by meridians \( \lambda_1 \) and \( \lambda_2 \), latitude circles \( \varphi_1 \) and \( \varphi_2 \), and isobaric surfaces \( p_1 \) and \( p_2 \), where \( p_1 < p_2 \) (see also Reiter 1969a,b).

(a) The zonal (longitudinal) average of a variable \( X \) is defined as

\[
[X]_\lambda = \frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} X \, d\lambda. \tag{A.1}
\]

The eddy component of \( X \) is defined as the departure of this parameter from its zonal average

\[
(X)_\lambda = X - [X]_\lambda. \tag{A.2}
\]

(b) The meridional average is defined as

\[
[X]_\varphi = \frac{1}{\sin \varphi_2 - \sin \varphi_1} \int_{\varphi_1}^{\varphi_2} X \cos \varphi \, d\varphi. \tag{A.3}
\]
(c) The isobaric (area) average of variable $X$ is defined as

$$[X]_{\lambda, \varphi} = \frac{\int_y \int_x X \, dx \, dy}{\int_y \int_x dx \, dy} = \frac{\int_{\varphi_1}^{\varphi_2} \int_{\lambda_1}^{\lambda_2} X r^2 \cos \varphi \, d\lambda \, d\varphi}{\int_{\varphi_1}^{\varphi_2} \int_{\lambda_1}^{\lambda_2} r^2 \cos \varphi \, d\lambda \, d\varphi} = \frac{1}{(\lambda_2 - \lambda_1)(\sin \varphi_2 - \sin \varphi_1)} \int_{\varphi_1}^{\varphi_2} \int_{\lambda_1}^{\lambda_2} X \cos \varphi \, d\lambda \, d\varphi. \tag{A.4}$$

Obviously, this isobaric average is equivalent to successive applications of the zonal and meridional averaging above. Using the area average of $X$ as defined above and assuming hydrostatic equilibrium, the mass integral of $X$ per unit area is given by

$$\int_{p_1}^{p_2} \frac{1}{g} [X]_{\lambda, \varphi} \, dp. \tag{A.5}$$

The latter is the integral form used for the calculation of the energy contents and energy conversions. The quantity defined in terms of the above relationships

$$(\langle X \rangle_{\lambda})_{\varphi} = [X]_\lambda - [X]_{\lambda, \varphi}, \tag{A.6}$$

is constant along a latitude circle.

(d) The total (horizontal and vertical) divergence of the flux of the variable $X$ is given by

$$\nabla_h \cdot X V_h + \frac{\partial X \omega}{\partial p} = \frac{1}{r \cos \varphi} \frac{\partial u}{\partial \lambda} + \frac{1}{r \cos \varphi} \frac{\partial (v \cos \varphi)}{\partial \varphi} + \frac{\partial X \omega}{\partial p} \tag{A.7}$$

where $\nabla_h$ is the horizontal divergence, $V_h$ the horizontal wind vector ($u$ being its eastward and $v$ its northward components) and $\omega = dp/dt$ the vertical component of motion.

The area average of the total (horizontal and vertical) transfer of variable $X$ is obtained by integrating over the mass of the atmospheric volume

$$B X = \frac{-1}{g(\lambda_2 - \lambda_1)(\sin \varphi_2 - \sin \varphi_1)} \int_{\varphi_1}^{\varphi_2} \int_{\lambda_1}^{\lambda_2} \left( \nabla_h \cdot X V_h + \frac{\partial X \omega}{\partial p} \right) \cos \varphi \, d\lambda \, d\varphi \, dp$$

$$= \frac{c_1}{g} \int_{p_1}^{p_2} \int_{\varphi_1}^{\varphi_2} (Xu)_{\lambda_1}^{\lambda_2} \, dp \, d\varphi + \frac{c_2}{g} \int_{p_1}^{p_2} ([Xv \cos \varphi])_{\varphi_1}^{\varphi_2} \, dp - \frac{1}{g} (X\omega)_{p_1}^{p_2} \tag{A.8}$$

where $c_1 = -1/(r(\lambda_2 - \lambda_1)(\sin \varphi_2 - \sin \varphi_1))$, $c_2 = -1/(r(\sin \varphi_2 - \sin \varphi_1))$, $r$ is the radius of the earth.
APENDIX B

Symbols used

\begin{align*}
AE & \quad \text{Eddy available potential energy} \\
AZ & \quad \text{Zonal available potential energy} \\
BAE & \quad \text{Boundary transfer of eddy available potential energy} \\
BAZ & \quad \text{Boundary transfer of zonal available potential energy} \\
BKE & \quad \text{Boundary transfer of eddy kinetic energy} \\
BKZ & \quad \text{Boundary transfer of zonal kinetic energy} \\
KE & \quad \text{Eddy kinetic energy} \\
KZ & \quad \text{Zonal kinetic energy} \\
R & \quad \text{Gas constant of dry air} \\
T & \quad \text{Temperature (K)} \\
V_h & \quad \text{Horizontal wind vector} \\
u & \quad \text{Eastward wind component} \\
v & \quad \text{Northward wind component} \\
c_1 & \quad \frac{-1}{r} \left( \lambda_2 \lambda_1 \right) \left( \sin \varphi_2 - \sin \varphi_1 \right) \\
c_2 & \quad \frac{-1}{r} \left( \sin \varphi_2 - \sin \varphi_1 \right) \\
c_p & \quad \text{Specific heat of dry air at constant pressure} \\
g & \quad \text{Acceleration due to gravity} \\
p & \quad \text{Pressure} \\
r & \quad \text{Radius of the earth} \\
t & \quad \text{Time} \\
\varphi & \quad \text{Longitude} \\
\lambda & \quad \text{Latitude} \\
\sigma & \quad \text{Static-stability parameter} \\
\omega & \quad \text{The vertical component of motion, } \frac{dp}{dt} \\
(X - Y) & \quad \text{Conversion of energy } X \text{ into energy } Y. \ X \text{ and } Y \text{ stand for } AZ, AE, KZ, \text{ or } KE.
\end{align*}

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