Effects of cloud-droplet spectra on the average surface-temperature of ice accreted on fixed cylindrical collectors

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SUMMARY

An experimental study of the heat balance of a stationary cylindrical collector accreting supercooled water droplets has shown a new dependence on the sizes of the water droplets in the laboratory cloud. In a study in 1967, Macklin and Payne related the steady-state heat-release during accretion to the heat loss by convection and conduction; their equation involved a numerical factor \( \chi \) for which they assumed a value of 0.28. The present study has shown that \( \chi \) is a function of the droplet spectrum with values around 0.5 at a velocity of 4 m s\(^{-1}\) for a mean volume-weighted droplet-diameter of 18 \( \mu \)m, and that \( \chi \) approaches 0.3 for droplets greater than 30 \( \mu \)m diameter. The results have importance to the surface temperatures of riming graupel pellets and hailstones in convective storms which, in the presence of small water-droplets, will be heated to a smaller degree than has been assumed in current models of hailstone growth. Furthermore, the effect of droplet size may influence ice-crystal multiplication and charge transfer in convective clouds, both of which are highly temperature dependent.

KEYWORDS: Accretion Graupel Hailstones Ice multiplication Riming Thunderstorm charging

1. INTRODUCTION

It is well known that hailstones and graupel pellets grow in convective storms as a result of the accretion of supercooled water droplets on ice particles. Numerical models of graupel growth (Musil 1970; Paluch 1978; Heymsfield 1983; Nelson 1983; Xu 1983; Knight and Knupp 1986; Castellano et al. 1994) incorporate parametrizations involving the graupel–droplet collection-efficiency (e.g. Langmuir and Blodgett 1946; Beard and Grover 1974), the heat exchange between graupel and the environment through the Nusselt (\( Nu \)) and Sherwood (\( Sh \)) numbers (Macklin and Payne 1967; Cober and List 1993), the density of accreted ice (Macklin 1962; Pflaum and Pruppacher 1979; Prodi and Levi 1987) and the drag coefficient of the graupel or hailstone (List and Schemenauer 1971; Locatelli and Hobbs 1974; Matson and Huggins 1980). Some of these parametrizations arise from measurements of cloud particles in situ (Zikmund and Vali 1972; Locatelli and Hobbs 1974; Heymsfield 1978), while others come from laboratory studies (Macklin 1962; Pflaum and Pruppacher 1979; Prodi and Levi 1987).

The surface temperature of riming graupel is one of the most important variables involved in the accretion process because it controls the density of the graupel and in consequence affects the rime surface characteristics and the size that the particle achieves. Graupel temperature is also an important variable in several other microphysical processes that occur inside clouds: ice-crystal multiplication (Hallett and Mossop 1974), for instance, takes place in only a narrow band of rime surface temperature; charge separation during collisions between ice particles (Saunders et al. 1991; Avila et al. 1995, 1996) is temperature dependent; so is the occurrence of melting and shedding from the surface of riming graupel and hail (Rasmussen and Heymsfield 1987).

To calculate the surface temperature of riming graupel it is necessary to introduce parametrizations of \( Nu \), \( Sh \), (see appendix for this and other notation) and the graupel–droplet collection-efficiency into the heat balance equation. The Nusselt number \( Nu \) is dimensionless and related to the convective heat-transfer at the surface of the graupel pellet; its value varies across a riming surface, but in general the average over the whole
surface is considered in order to reduce the complexity of the problem. Values of $Nu$ have been extensively determined for smooth-surface bodies, and it is often assumed that $Nu$ is a function of the Reynolds number ($Re$) (Eckert and Drake 1974). However, Bailey and Macklin (1968a) showed that heat transfer from rough hailstones could be several times larger than that from smooth hailstones of similar size, and Achenbach (1974) observed that, for cylinders with $Re > 4000$, the surface roughness increases the ventilation process. Cober and List (1993) determined $Nu$ for riming conical graupel for Reynolds numbers in the range $250 < Re < 1500$. They found values of $Nu$ to be 50% greater than those found by Schemenauer (1972) who measured the convective mass-transfer of smooth conical models; Cober and List (1993) suggested that the difference could be accounted for by the surface roughness of the ice.

There are some indications that the surface roughness of rime may depend on the cloud-droplet spectrum. Bailey and Macklin (1968b), for instance, found that lobe-like growths on artificial hailstones were most pronounced when accreted droplets were small, whereas, for large droplets, the surface irregularities were less marked. They inferred that lobes develop as a consequence of increased collection-efficiency around protuberances. Also, Levi et al. (1991) observed that the shape of accretion depends on the size of the droplets through the Stokes parameter ($K_a$). Since $Nu$ depends on the surface roughness and the roughness may depend on the size of the droplets, it seems reasonable to expect that the heat transfer process will also have some dependence on the size of the droplets, although such a dependence has not previously been reported.

The objective of the present work is to study the influence of the cloud-droplet size spectrum on the surface temperature and heat transfer of rime deposits by means of an experimental study using fixed cylindrical collectors. Many laboratory studies have used this kind of symmetry; cylindrical collectors were used in studies of ice-crystal multiplication and thunderstorm electrification in which the surface of the rime was assumed to be representative of that of natural graupel. It is important to note, however, that, strictly, the present results apply only to ice accretions on fixed cylindrical collectors.

## 2. Theory

The equation that describes the rate of accretion of rime by a cylindrical collector of cross-sectional area $A$ exposed to an air flow of velocity $V$ containing a cloud of supercooled water droplets of liquid-water content $w$ is

$$\Delta m/\Delta t = E w A V,$$

where $E$ is the graupel–droplet collection efficiency. The heat balance equation per unit area of surface is given by

$$E w V \{L_f + c_w(T_a - T_0) + c_l(T_0 - T_s)/\pi = Nu \ K \{(T_s - T_a) + Sh L_s D(\rho_s - \rho_a))/2R,$$

where the left-hand side represents the rate at which heat is released by freezing droplets, and terms on the right-hand side represent exchanges with the environment: the first represents the dissipation of heat to the surroundings by conduction and convection, and the second the exchange of heat through sublimation or deposition.

In principle, $Nu$ and $Sh$ depend on the particular conditions on the surface (Eckert and Drake 1974; Pruppacher and Klett 1978). We, however, are concerned with the average surface-temperature. Consequently, we may make the following approximations

$$Nu = \chi Re^{0.6} Pr^{1/3}$$
\[ Sh = \chi Re^{0.6} Sc^{1/3}, \]

where \( Pr \) and \( Sc \) are the Prandtl and Schmidt numbers and \( \chi \) is the heat transfer coefficient. Thus, following Macklin and Payne (1967), Eq. (2) may be written

\[ Ew V \{ L_r + c_0(T_s - T_0) + c_1(T_0 - T_s)\} / \pi \]
\[ = \chi Re^{0.6}(Pr^{1/3} K(T_s - T_a) + Sc^{1/3} L_s D(\rho_a - \rho_s))/2R. \]

The effective liquid-water content \( Ew \) may be determined by experiment, using Eq. (1).

From Eq. (3), it is possible to obtain the mean temperature of the accreting surface \( T_s \) which depends essentially on \( V, 2R, T_s \) and \( Ew \). There is a dependence of \( T_s \) on the size of the cloud droplets, through the dependence of \( Ew \) on \( E \). If \( Ew \) is kept constant by adjusting \( u \) appropriately, however, there is no dependence of \( T_s \) on the cloud-droplet spectrum. The equations model an ideal situation in which the collector retains its cylindrical shape as it grows by riming, although in nature, and in laboratory experiments, the shape and surface roughness of the rime depend on the size of the cloud droplets. In consequence, the ventilation factor, and hence \( T_s \), could be affected by the size of the droplets.

3. The experiments

The general principle of the experiments was to measure the temperature acquired by a fixed cylindrical rod growing by riming, as a function of \( Ew \) for different values of \( V, T_a \) and cloud-droplet spectra (CDS). Experiments were performed in a cloud chamber, with dimensions 1.5 m \( \times \) 1 m and 2.1 m high, placed inside a cold room capable of being cooled to \(-40^\circ C\). The large volume of the chamber allows the cloud-droplet concentration to be kept fairly constant throughout the period of the experiments. Furthermore, mixing in the cloud keeps temperature gradients to a level which is insignificant for these experiments.

The collector (target) was a commercial platinum resistance thermometer of cylindrical cross-section, 2.8 mm in diameter and 25 mm in length, which also served as the temperature sensor. This element was selected to measure the temperature of the accreted ice because it has a very good thermal response (0.4 s) and great stability (0.05%). The resistance of the sensor was measured with a digital multimeter (Keithley 177) whose analogue output was connected to an X-t chart recorder; consequently, the temperature variation of the target was registered for every measurement. Ambient temperature was determined by a thermistor located 90 mm from the target inside the cloud chamber; the thermistor resistance was measured using another digital multimeter (Fluke 8800A). During the measurements, the variation rate of the ambient temperature was typically only 0.2°C min\(^{-1}\). Collector and ambient temperatures were measured with an accuracy of ±0.1°C.

The cylindrical target was located inside the cloud chamber and was mounted 20 mm inside a horizontal wind-tunnel tube of diameter 36 mm through which the cloud could be drawn. In order to diminish the heat exchanged between the target and tunnel wall, poorly conducting materials (plastic and rubber) were used for the mounting arrangement. The other end of the tunnel was coupled to a suction pump capable of establishing an air flux of controlled velocity in the range 2–20 m s\(^{-1}\) by changing the power supplied to the pump. Velocities of 4 m s\(^{-1}\) and 7 m s\(^{-1}\) were used in the present work, measured by a calibrated Pitot-tube inserted into the tunnel. Taking into account the wind profiles within the tunnel section, the velocity was determined with an accuracy of ±0.5 m s\(^{-1}\).

The water-droplet clouds were generated by water-vapour condensation provided by a boiler whose electrical-power input could be adjusted in order to control the cloud-droplet concentration. The effective liquid water-content was varied in the range 0.5–3 g m\(^{-3}\) and
was determined by weighing (to an accuracy of ±10 μg) the rime deposited on the collector in a measured time. Various CDS were produced by varying the nozzle size at the boiler (Mossop 1984). Nozzles of 20 mm and 155 mm diameter were used to obtain CDS A and B respectively, while CDS C was produced using an ultrasonic humidifier.

The characteristics of the CDS were obtained by taking cloud samples at 5 m s$^{-1}$ on a microscope slide coated with formvar solution, followed by microscopic analysis. The results of Ranz and Wong (1952) give values of the collection efficiencies of slides of width 2.54 cm for droplets of diameters 10 μm, 20 μm and 30 μm at V = 5 m s$^{-1}$ as 8%, 56% and 77% respectively; for a cylindrical collector, the corresponding collection-efficiencies for droplets of 30 μm, 55 μm and 75 μm diameter are 30%, 40% and 55% respectively at V = 4 m s$^{-1}$, and 69%, 75% and 78% at V = 7 m s$^{-1}$. The present study is particularly concerned with the effect of the larger droplets for which it can be seen that the sample is fairly representative of the droplets striking the target.

Figures 1 and 3 show CDS A and B obtained at ambient temperatures of −10°C and −25°C, and Fig. 2 shows three spectra at −1.5°C. Values of mean diameter ($d_m$), mean-volume diameter ($d_v$) and median-volume diameter ($d_{vm}$) are displayed for every spectrum.

4. RESULTS

Figure 4 shows the increase of rime temperature with time after the cloud flow past the target was started, together with the evolution of the ambient temperature ($T_a$) for a run in which the velocity was 7 m s$^{-1}$ and $Ew$ was 0.8 g m$^{-2}$; the arrow shows when the air flux was switched off. Rime temperature increases quickly during the first 20 s, to reach a steady state after which it remains fairly constant during some characteristic time that depends on V and $Ew$; generally, this characteristic time was longer than 60 s. After that time, the rime temperature apparently decreases (not shown in Fig. 4) because the accumulated rime acts as an insulator between the riming surface and the temperature probe beneath. This second, longer, relaxation time also depends on V and $Ew$. For all these reasons, the duration of the runs was typically 60 s. The temperature behaviour shown in Fig. 4 is representative of most of the measurements, but at high rates of accretion in experiments at 7 m s$^{-1}$ and $Ew > 2$ g m$^{-3}$ the runs lasted less than 60 s. On the basis of this behaviour, the steady-state temperature of the collector was determined 30 s after riming started; at this time, both the target and ambient temperatures were noted directly from their multimeter readings in order to have better temperature accuracy than the chart record provided.

Figure 5(a,b) presents the increase in rime temperature above ambient ($\Delta T = (T_s - T_a)$) plotted against the effective liquid-water content $Ew$, for $T_a = -15°C$, $V = 4$ m s$^{-1}$ and 7 m s$^{-1}$ respectively and for the three CDS shown in Fig. 2. Also shown is the theoretical curve calculated from Eq. (3) for the same experimental parameters drawn for the case when $\chi = 0.28$, a value suggested by Macklin and Payne (1967). Figure 5 shows that, in qualitative agreement with theory, the collector steady-state temperature increases with effective liquid-water content and velocity. The new result displayed in Fig. 5 is that, for given values of $Ew$, $V$ and $T_a$, the rime temperature depends on the cloud-droplet spectrum. Furthermore, for both velocities, the warming produced by droplet-spectrum B, containing the largest droplets, is closest to the predictions of Eq. (3) for both velocities. In general, for the same rate of accretion, large droplets raise the mean surface-temperature more than smaller droplets.

Figure 6(a,b) shows how $\Delta T$ varies with $Ew$ at $T_a = -25°C$ for $V = 4$ m s$^{-1}$ and 7 m s$^{-1}$ respectively, for both CDS shown in Fig. 3. Again, the theoretical curve is calculated from Eq. (3). As with the data shown in Fig. 5, the mean surface-temperature depends
Figure 1. Cloud-droplet spectra at $-10^\circ\text{C}$ with values of droplet mean diameter $d_m$ (\(\mu\text{m}\)), mean-volume diameter $d_v$ (\(\mu\text{m}\)) and median-volume diameter $d_{vm}$ (\(\mu\text{m}\)): (a) spectrum A; (b) spectrum B.

on which CDS is used for riming. At $V = 4 \text{ m s}^{-1}$, the warming of the accreted rime is some way below that derived by using Eq. (1) for both droplet spectra; there is better agreement at 7 m s$^{-1}$, but in both cases the warming produced by CDS B (large droplets) was closer to the theoretical values than that produced by CDS A (small droplets).

Figure 7 shows the results obtained for $T_a = -10^\circ\text{C}$, $V = 4 \text{ m s}^{-1}$ and CDS A and B, together with the theoretical curve calculated from Eq. (3). The warming of the accreted rime was broadly similar for both spectra, and only a fraction of a degree below the theoretical prediction.
Figure 2. Cloud-droplet spectra at \(-15^\circ\text{C}\) with values of droplet mean diameter \(d_m\) (\(\mu\text{m}\)), mean-volume diameter \(d_v\) (\(\mu\text{m}\)) and median-volume diameter \(d_{vm}\) (\(\mu\text{m}\)): (a) spectrum A; (b) spectrum B; (c) spectrum C.
5. DISCUSSION

Figures 5, 6 and 7 show that, along with \( Ew \), \( V \) and \( T \), the cloud-droplet spectrum influences the mean surface-temperature of accreting rime: if \( Ew \) and \( V \) are kept constant, so that the rate of accretion of rime by the target is constant and hence the latent heat released to the riming target is constant, the heat exchange between the accreted rime and the surroundings is different for each droplet spectrum. Heat is transferred to the environment by forced convection and sublimation, as described by the right-hand side of Eq. (3). The rate of heat removal depends on \( \Delta T \) and on a ventilation coefficient that in general is a function of the Reynolds number \( (Re) \) and a coefficient \( \chi \), which takes a value of 0.28 for cylinders (Macklin and Payne 1967). As \( Re \) is independent of droplet size, \( \chi \) could depend on the size of the cloud droplets riming on cylindrical collectors. Hence, the values of \( \chi \) were calculated that best fit the experimental points for the conditions appropriate to
Figure 4. Evolution during riming of the ambient temperature $T_a$ and the increase of rime temperature $\Delta T$. The arrow shows when the air flux was switched off.

| TABLE 1. DROPLET MEAN SIZES, AIT TEMPERATURE, DROPLET–TARGET VELOCITY AND $\chi$ FOR VARIOUS DROPLET–SPECTRA |
|---|---|---|---|---|---|
| CDS | $T_a$ (°C) | $V$ (m s$^{-1}$) | $d_m$ (mm) | $d_v$ (mm) | $d_{vm}$ (mm) | $\chi$ |
| A | -15 | 7 | 19 | 22 | 25 | 0.37 |
| B | -15 | 7 | 28 | 31 | 34 | 0.31 |
| C | -15 | 7 | 15 | 18 | 24 | 0.40 |
| A | -15 | 4 | 19 | 22 | 25 | 0.40 |
| B | -15 | 4 | 28 | 31 | 34 | 0.31 |
| C | -15 | 4 | 15 | 18 | 24 | 0.43 |
| A | -25 | 7 | 16 | 18 | 21 | 0.43 |
| B | -25 | 7 | 20 | 23 | 26 | 0.34 |
| A | -25 | 4 | 16 | 18 | 21 | 0.51 |
| B | -25 | 4 | 20 | 23 | 26 | 0.40 |
| A | -10 | 4 | 23 | 25 | 28 | 0.34 |
| B | -10 | 4 | 32 | 35 | 38 | 0.31 |

Figs. 5, 6 and 7. Table 1 lists the values of $\chi$ calculated using Eq. (3) for each spectrum, ambient temperature and droplet–target velocity, together with the characteristic diameters of each spectrum presented in terms of mean diameter ($d_m$), mean-volume diameter ($d_v$) and median-volume diameter ($d_{vm}$). In Fig. 8, the values of the coefficient $\chi$ are shown as a function of $d_v$ for the two sets of data at velocities of 4 m s$^{-1}$ and 7 m s$^{-1}$. It is evident that $\chi$ decreases with increase in velocity, with a clear tendency for it also to decrease when the droplet size increases; for the largest droplets studied here, $\chi$ approaches Macklin and Payne’s (1967) value of 0.28.

Heat exchange between accreted ice and the environment takes place through the boundary layer established around the surface of the rime. The heat-transfer characteristics of the boundary layer depend on $Re$ and the surface roughness: with increasing roughness at lower values of $Re$, first the wake and then the whole boundary-layer become turbulent
Figure 5. Increase in rime temperature $\Delta T$ as a function of effective liquid-water content $E_w$ (g m$^{-3}$) for droplet spectra A, B and C at $T_a = -15^\circ$C: (a) $V = 4$ m s$^{-1}$; (b) $V = 7$ m s$^{-1}$. The lines are theoretical curves calculated from Eq. (3).

which increases heat transfer to the environment. Bailey and Macklin (1968b) found that smaller droplets could build rime accretions with higher surface-roughness than larger ones; because of the form of the growth on stationary objects, they suggested that the roughness was a collection-efficiency effect. Levi et al. (1991) showed that the morphology of ice accreted on fixed cylindrical collectors depends on two parameters: the Stokes number, which is concerned with droplet trajectories, and Macklin's parameter, which is concerned with droplet spreading on the substrate. They pointed out the importance of the droplet spectrum on ice-deposit morphology. Hence, it seems reasonable to expect that different droplet-spectra will develop rime structures which have different roughness and heat-transfer characteristics. Surface roughness would be expected to facilitate cooling by ventilation. This is consistent with the finding that, when two droplet-spectra with the same
effective liquid-water content deposit rime, the spectrum with fewer but larger droplets heats the target more than that with more numerous but smaller droplets.

The results shown in Fig. 6 indicate that the droplet–target velocity also affects the agreement between theory and experimental measurements; at higher velocities there is more heating for the same value of $Ew$, bringing data points and theoretical line into closer agreement. This could be because the Stokes number is proportional to velocity; Levi et al. (1991) observed that the profile of the ice deposit changes from a lobe to a smooth structure when the Stokes number increases, which would decrease the ventilation. Another explanation could be that the spreading of the freezing droplets increases at higher velocity (Macklin 1962), which decreases the surface roughness and reduces the ventilation. Further
Figure 7. Increase in rime temperature as a function of effective liquid-water content for droplet spectra A and B at $T_a = -10^\circ$C and $V = 4$ m s$^{-1}$. The line is a theoretical curve calculated from Eq. (3).

Figure 8. Values of $\chi$, the numerical factor in the heat transfer coefficient, plotted against the droplet mean-volume diameter for all the spectra used, at velocities of 4 m s$^{-1}$ and 7 m s$^{-1}$.
experiments are necessary to determine whether droplet spreading is more important than the shape of the accretion in increasing rime temperature. In general, all the results show that, for the same \( T_a \) and CDS, the experimental points for higher velocities are closest to the theoretical curve.

Figure 8 shows a plot of the values of \( \chi, d_c \), and \( V \) listed in Table 1: for a particular \( d_c \), the value of \( \chi \) is higher at the lower value of \( V \), consistent with the effects of increased surface-roughness and ventilation. Values of \( \chi \) tend to a value around 0.3 at high droplet-target velocities and for large droplets; such conditions can lead to rime surface-temperatures approaching 0°C and, in the limit, wet growth.

Cober and List (1993) determined the heat-transfer parameter for conical graupel growth with a range of droplet sizes and velocities. They stated that, within experimental error, Nusselt number varied with Reynolds number independently of cloud conditions. However, their diagrams show a decrease in Nusselt number with increase in median droplet-diameter from 24 \( \mu \)m to 42 \( \mu \)m and with increase in velocity from 1.5 m s\(^{-1} \) to 3 m s\(^{-1} \). These are consistent with the decreases in \( \chi \) with larger droplets and higher velocities noted in the present work.

Jayaratne (1993) measured the temperature elevation of a rimming cylindrical collector as a function of the flow velocity at an ambient temperature \( T_a \) of -16.5°C. He used a CDS with a mean droplet-diameter \( d \) of 10 \( \mu \)m and mean-volume diameter \( d_c \), of 15 \( \mu \)m. Using Macklin and Payne's (1967) equation, the best fit to Jayaratne's (1993) experimental data is found with \( \chi = 0.40 \) in a velocity range up to about 8 m s\(^{-1} \); increasing the velocity further caused \( \chi \) to decrease to about 0.30. He attributed this result to the relatively smoother rime surface caused by the greater compaction of droplets at higher velocities. Clearly, Jayaratne's results are consistent with the present ones in that \( \chi \) for smoother rime with reduced ventilation approaches the value 0.28. Saunders et al. (1991) determined that \( \chi = 0.48 \) in experiments in which the droplet cloud contained ice crystals which grew at the expense of the water droplets; the smaller droplets led to a higher value of \( \chi \) in agreement with the present work.

Ice-crystal multiplication occurs when supercooled droplets freeze on impact with a riming surface and release ice fragments that grow into ice crystals. Hallett and Mossop (1974) found that the effect takes place at air temperatures between -3°C and -8°C; however, Foster and Hallett (1982) showed that ice multiplication is dependent on the temperature of the surface of the rimming object (the 'rimer'). An increase in rime temperature with accretion may cause splinter production to peak at lower cloud temperatures. For example, increasing the liquid-water content from 1.2 to 1.6 g m\(^{-3} \) shifted the temperature of the peak crystal-production rate from -6°C to -7°C. The conclusion of the present work (that, for the same rate of rime accretion, fewer larger droplets heat the rimer more than do more numerous smaller ones) implies that large supercooled droplets in clouds may lower the active-multiplication temperature-band significantly.

This enhanced heating effect of larger droplets may also play a role in the generation of the negative charge centre in thunderstorms when rimming graupel pellets are charged negatively during collisions with ice crystals. Saunders et al. (1991) and Saunders and Peck (1998), following a suggestion by Baker et al. (1987), based their discussions of charging on the importance of the relative diffusional growth rates of the interacting particles: negative charging of rimming graupel is associated with the natural heating of the rimer by the freezing of accreted droplets, which reduces the diffusional growth at the surface of the rimer below that of the colliding ice-crustals. Avila et al. (1995, 1996) and Jayaratne (1998) heated a rimming surface artificially so that it sublimated; with sufficient heating, the rimer charged negatively during collisions with ice particles. A test of the hypothesis that larger droplets will affect the charging of a rimer during ice-crystal collisions was reported by
Avila et al. (1998) who found that a cloud of droplets extending to 80 µm diameter favoured the negative charging of riming graupel more than did a cloud of smaller droplets. Now, larger droplets lead to increased rime-density (Macklin 1962), but rime density alone does not influence the sign of the charge transfer (Jayaratne 1998). The present work suggests that larger droplets produce a smoother rime-surface that reduces ventilation, leading to increased temperatures for a given rate of rime accretion and consequent enhanced negative charging of the rimer.

In order that the present results may be applied to other situations, and may be included in numerical models, a velocity-dependent function was found that fits the data shown in Fig. 8. Figure 9 shows $\chi$ plotted against the parameter $k = d_v^4 V$, where the equation of the line is

$$\chi = 0.308 + 0.253 \exp(-1.104 \times 10^{-6} k),$$

where $d_v$ is in µm and $V$ in m s$^{-1}$.

6. Conclusion

In the riming cylindrical target heat-balance equations presented by Macklin and Payne (1967), the numerical factor $\chi$ in the heat transfer coefficient was taken to have a value of 0.28. The present work has shown that this value is close to the asymptotic value approached when the rime surface is fairly smooth. Under these conditions, produced by riming with large water-droplets and at high droplet–rimer velocities, the rime surface is smooth enough to lead to reduced ventilation and reduced heat-removal to the environment. With smaller droplets and at lower velocities, the rough nature of the surface leads to enhanced ventilation and heat-transfer so that the rime surface does not heat as much. Values of $\chi$ have been determined here for a range of droplet sizes, temperatures and rimer velocities, and a simplified dependence on velocity and droplet size has been formulated.
which may be used in numerical studies involving cylindrical collectors. The results are relevant to the conditions required for ice multiplication in that larger droplets with more rime-heating will move the temperature band in which ice multiplication operates to lower temperatures. The results may also be important in the area of thunderstorm charging, where the heating by rimming of a graupel pellet is thought to be responsible for negative charging of the graupel during ice-crystal collisions.

Strictly, the current results are applicable to ice deposits accreted on fixed cylindrical collectors. Nevertheless, it is reasonable to expect that rime heating on natural graupel particles will also be affected by the droplet spectrum and rimer velocity. It is therefore important that we increase our knowledge of droplet spectra in convective clouds, particularly in regions of graupel growth. Furthermore, experimenters who simulate conditions inside convective clouds need to ensure that the particular droplet-spectrum used is representative of the cloud regions of interest.

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APPENDIX

List of symbols

$A$ cross-sectional area of the graupel
$c_w$ specific heat of water
$c_i$ specific heat of ice
$d$ droplet diameter
$d_{m}$ mean diameter
$d_{ve}$ mean-volume diameter
$d_{vm}$ median-volume diameter
$D$ coefficient of molecular diffusion of water vapour in air
$E$ collection efficiency coefficient of the graupel for droplets, a function of $R$, $V$ and $d$
$E_w$ effective liquid-water content
$k = d^{4}V$
$K_{st}$ Stokes parameter
$K$ thermal conductivity of air
$L_f$ latent heat of fusion of water
$L_s$ latent heat of sublimation of ice
$w$ liquid-water content of the cloud
$\Delta m$ mass of ice accreted during time $\Delta t$
$Nu$ Nusselt number
$Pr$ Prandtl number
$\rho_s$ density of water vapour at the surface of the ice deposit
$\rho_a$ density of water vapour in the environment
$R$ graupel radius
$Re$ Reynolds number
$Sc$ Schmidt number
$Sh$ Sherwood number
$T_a$ ambient temperature
$T_0$ melting temperature of ice
$T_a$ mean temperature of ice deposit

$V$ relative velocity between droplet and target (or droplet–graupel)

$\chi$ numerical factor in the heat transfer coefficient

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