A numerical study of tropical-cyclone structure: Quasi-stationary spiral bands

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SUMMARY

The evolution of quasi-stationary spiral bands of vorticity, on the periphery of an initially symmetrical tropical-cyclone-like vortex moving on a β-plane, is investigated using a semi-spectral nondivergent nonlinear barotropic numerical model, formulated in a cylindrical coordinate system moving with the vortex. The results obtained with the numerical model are supported by analytic solutions of the linear barotropic vorticity equation in a moving frame of reference.

The analysis of the model data shows the slow evolution and vortex-relative translation of a spectrum of Rossby waves, forming cyclonically curved spiral bands of positive and negative vorticity anomalies in the rear-right quadrant of the moving vortex. The analytic solutions of the linear barotropic vorticity equation and examination of the vorticity tendency in the numerical model indicate that the generation of the spiral bands is mainly a result of the vortex drift and the advection of planetary vorticity by the vortex wind field. Investigation of the kinematical structure of the trailing spiral bands of vorticity and their vicinity suggests that the cyclonic spiral bands in the numerical model may be related, either directly or indirectly, to the existence of quasi-stationary principal spiral bands of convection in real tropical cyclones.

KEYWORDS: Asymmetry Rossby waves Vortex dynamics β-plane

1. INTRODUCTION

Over the last four decades, developments in satellite meteorology have led to great improvements in the observation of meteorological phenomena, especially in regions like the tropical oceans where the regular network of meteorological stations is very sparse. Without satellite imagery, detection of the most dangerous synoptic tropical weather systems, tropical cyclones, would very often not be possible. Besides the detection of tropical cyclones, satellite imagery, together with better radar surveillance, new visualization techniques and enhanced aircraft reconnaissance, has enabled scientists to learn more about the evolution and structure of these storms. One of the more prominent features of tropical cyclones in radar reflectivity maps (see e.g. Willoughby et al. 1984, their Fig. 2; Willoughby 1990, his Fig. 14; or Powell 1990, his Fig. 1) and satellite pictures (for example Jorgensen 1984, his Fig. 1) are quasi-stationary spiral bands of convection. The evolution, maintenance and possible mechanisms for the generation of these bands is the subject of the present study.

Following the definition of Guinn and Schubert (1993), one may distinguish between outer and inner spiral bands of convection. The outer bands normally occur at large radii (typically beyond 500 km), while the inner bands are formed typically between radii of 150 and 500 km. The inner bands are often dominated by a pattern that remains quasi-stationary† relative to the moving vortex. This ‘principal band’ was described by Willoughby et al. (1984) as a “prominent spiral-shaped feature that extended outward toward one side of the vortex”. Commonly, the quasi-stationary band consists of more or less connected convective cells aligned in an elongated, cyclonically curved and relatively narrow spiral. In many cases the principal band is flanked by weaker secondary spiral bands. It exists in tropical cyclones regardless of their intensity. Sometimes the cloud-covered area in the principal band becomes broader with increasing distance from the centre (e.g.

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† The term 'quasi-stationary' is always used in the sense that the translation velocities of the patterns in question are small during the observed time interval; e.g. for the case discussed in the present study they are of the order of, or less than, the drift speed of the vortex.
Barnes and Stossmeister (1986, their Fig. 1). According to Barnes et al. (1983), who investigated the kinematic structure of the principal band of hurricane Floyd of 1981, there was inflow throughout the lowest 3 km of the atmosphere and outflow above. The along-band component of the wind vector (approximately the tangential wind speed relative to the vortex centre) had a local cyclonic maximum at lower levels, which exceeded the surrounding wind speeds by about 3 m s\(^{-1}\) and became weaker at higher altitudes. Qualitatively similar findings were also reported by Barnes and Stossmeister (1986). However, all observational studies reveal considerable differences in the vertical and horizontal kinematic structure of the rain bands, depending on the storm in question.

In many theoretical studies the processes governing the formation and maintenance of convective spiral bands in tropical cyclones have been investigated. For example, Kurihara (1976), Willoughby (1978) and Willoughby et al. (1984) suggested that rain bands are induced by internal gravity waves. The second author stated that perturbations of small amplitude are excited on the periphery of the storm by, for example, frictional forces, the effects of wind shear or topographical influences. As the small-amplitude inertia-buoyancy waves propagate inwards, they may amplify to form spiral bands. Other authors (e.g. Shapiro 1983) held that processes in the boundary layer of a tropical cyclone could be responsible for the formation of asymmetries to the vortex that resemble inner spiral bands (cf. Shapiro op. cit., his Fig. 5). In one recent theory, Guinn and Schubert (1993) argued that inner spiral bands may form as a result of wave breaking of potential vorticity and/or vortex merging. Their experiments with a shallow-water model also suggested that the banded features are not gravity waves. In an observational study, MacDonald (1968) recognized the similarity between the orientation of spiral bands in hurricanes and the orientation of troughs in planetary Rossby waves that surround the polar regions. In the spirit of these observations, Guinn and Schubert op. cit., hypothesized that Rossby-wave breaking is a principal mechanism for the generation of spiral bands. They supported this idea with numerical experiments using an initially asymmetric (elliptic) vortex that develops azimuthally rotating and radially propagating spiral patterns in the vorticity fields (see their Fig. 3). The investigations of Guinn and Schubert were complemented by the work of Montgomery and Kallenbach (1997), who found strong evidence for the existence of Rossby waves that propagate on the radial gradient of vortex vorticity in the absence of a planetary vorticity gradient. Montgomery and Kallenbach also documented the existence of stagnation radii for the outward moving vortex Rossby waves, that confined the wave patterns to regions of two or three times the radius of the maximum tangential wind speed. The excitation of Rossby waves by vortices moving on a β-plane has been investigated in the framework of Gulf Stream rings, e.g. by Firing and Beardsley (1976) in a laboratory experiment using a rotating tank, and by Flierl (1977), Mied and Lindemann (1979), McWilliams and Flierl (1979), Flierl (1984) and McWilliams et al. (1986) in both numerical and analytical studies.

To our knowledge, slowly developing Rossby waves excited by a vortex moving on a planetary vorticity gradient have not been identified with quasi-stationary principal spiral bands observed in real tropical cyclones. Based on analyses of a simulation using a semi-spectral barotropic numerical model in a moving frame of reference, and a simple analytical approach in the spirit of Lighthill (1967), the present study seeks to shed some light on the existence of quasi-stationary spiral bands of vorticity anomaly on the periphery of an initially symmetric vortex moving on a β-plane. Some evidence will be provided that convective spiral bands may be related to Rossby waves induced by a moving vortex. The present considerations complement the findings of the above authors, especially those of Guinn and Schubert (1993) and Montgomery and Kallenbach (1997). However, the results of this study should not detract from the possible existence of other
mechanisms of spiral-band formation and maintenance such as, for example, the interaction of the vortex with its environment, vortex-vortex interaction, or even the formation of spiral bands by internal gravity waves.

2. EXPERIMENTAL DESIGN

The numerical experiment is carried out with a semi-spectral nondivergent nonlinear inviscid barotropic numerical model, formulated in cylindrical coordinates with radius r and azimuthal angle θ in a reference frame moving with the vortex. Details of the model are described in appendix A. The dependent variables, relative vorticity ζ and the radial and tangential components u and v of the wind vector U are defined on a discrete radial grid with variable resolution (Dietachmayer and Droegemeier 1992; Dietachmayer 1992), while in the azimuthal direction they are represented by contributions from wave numbers k of an azimuthal Fourier-decomposition truncated at a given azimuthal wave number N (cf. Ross and Kurihara 1992). The barotropic vorticity equation is integrated forward in time using an explicit third-order Adams–Bashforth scheme, similar to the one described in Durran (1991) but with variable temporal step size. After each time step the azimuthal wave-number contributions of relative vorticity are inverted, following an approach of Adem (1956), to give the corresponding radial distributions of radial and tangential wind speed. The length of each time step is monitored automatically by evaluation of the Courant–Friedrich–Levy criterion. In the moving frame of reference, the barotropic vorticity equation contains terms including the components of the drift speed vector c = (c_r, c_θ), which have to be computed from the wave-number-one contribution of relative vorticity after each time step (Smith and Ulric 1990, their Eq. 2.12).

For the present experiment, a radial integration domain of r_∞ = 2.5 \times 10^6 m was found to be sufficient to minimize boundary effects during the 96-hour period of numerical integration using the relatively narrow initial vortex profile described below. During the time integration the radial grid size varied from 600 m near the vortex centre to 1.5 \times 10^4 m near the outer edge of the radial domain, corresponding to a total number of 301 grid points in radial direction. The Fourier-decomposition was truncated at N = 10. The automatically determined time step ranged from 25 to 35 s, depending on the absolute magnitude of the velocity components and smaller changes of the variable radial grid sizes during the time integration.

The high spatial resolution of the numerical model allows the implementation of a realistic intense symmetric vortex, with initial maximum tangential wind speed v_m = 50 m s^{-1} at a radius of maximum wind r_m = 4 \times 10^4 m. The basic vortex (henceforth identified by the subscript v) is constructed as follows: the tangential wind profile is given as a function of radius by

\[ V_v(r) = ar \exp(1 + br + cr^2), \]

with \( a = (v_a/r_a) \exp\{-1 + \Delta r_m + r_a\}, b = \{A - c(r_m + r_a)(r_m - r_a)\} / (r_m - r_a) \) and \( c = (\Delta r_m + r_m - r_a) / (r_m(r_m - r_a)(r_m - r_m)) \). The constants used in the above expressions are \( v_a = 0.001 \) m s^{-1}, \( r_a = 10^6 \) m and \( A = \ln((v_m r_a)/(v_a r_m)) \). The first constant defines a fixed tangential wind speed at a given large radius r_a. The symmetric tangential wind profile \( V_v(r) \) is modified between the radius r_d = 7 \times 10^5 m and r_a by application of the formula \( V_v(r) = V_v(r) (1 - s^2 \exp(1 - s^2)) \), with \( s = (r - r_d)/(r_a - r_d) \), to ensure that the tangential wind speed tends to zero smoothly between r_d and r_a. At radii greater than r_a the initial tangential wind speed is set to zero*. In the inner radial domain, the corresponding ra-

* Note that the symmetric tangential wind speed, initially set to zero at large radii, becomes non-zero a very short time after the start of the time integration of the numerical model.
Figure 1. Symmetric (a) tangential wind speed \( V_r \) in m s\(^{-1} \) and (b) relative vorticity \( \zeta_r \) in s\(^{-1} \) as functions of the radius \( r \) in km.

dial distribution of relative vorticity \( \zeta_r(r) \) is obtained using a fourth-order finite-difference representation of \( \zeta_r(r) = \partial(r V_r)/r \partial r \). At \( r = r_{\infty} \), \( \zeta_r \) is set to zero, and at \( r = 0 \) the analytic value of \( \zeta_r \) is used. The radial profiles of tangential wind speed and relative vorticity obtained by this procedure are presented in Fig. 1 and show a very intense vortex with wind speed decaying rapidly from 50 m s\(^{-1} \) at 40 km to about 5 m s\(^{-1} \) at 200 km radius. The numerical experiment is carried out with the basic vortex given by Eq. (1) in a run on a \( \beta \)-plane centred at a geographical latitude of \( \varphi_0 = 12.5^\circ \)N. The Coriolis parameter has the form \( f = f_0 + \beta \{ y + y_c(t) \} \), where \( t \) is the time, \( f_0 = 2 \Omega \sin(\varphi_0) \), \( \beta = [2 \Omega \cos(\varphi_0)]/r_E \), \( \Omega \) is the earth's rotation frequency and \( r_E \) is the radius of the earth. The meridional displacement of the vortex centre (the relative vorticity maximum) from the reference latitude, \( \varphi_0 \), is given by \( y_c(t) \), and \( y \) is the meridional distance of a given location from \( y_c \) (cf. also section 3(b)). Analyses of the numerical fields are performed every 8 hours.

It should be noted (cf. Gent and McWilliams 1986; or Weber and Smith 1993) that the above vortex satisfies the necessary conditions for barotropic instability based on the stability criteria obtained by Rayleigh (1880) and Fjørtoft (1950). For this reason a thorough examination of the existence of barotropically unstable eigenmodes was carried out using the algorithm described in Weber and Smith (1993). The linear stability analysis of the current symmetric vortex, performed for azimuthal wave numbers one to six at all analysis times, showed that the vortex used in this study remained barotropically stable to small perturbations over the full period of integration. This result was corroborated by the numerical run, which produced no sign of the well known spatial structures of barotropically unstable eigenmodes (cf. Weber and Smith op. cit.). These would develop during the
integration period due to possibly small temporal changes of the stability characteristic of the symmetric vortex, e.g. associated with the development of the \( \beta \)-gyres.

3. **Quasi-stationary spiral bands**

   (a) General analysis

   As shown in Fig. 2, the vortex moves to the north-west as expected. At early times it accelerates as a result of the development of the \( \beta \)-gyres, but after about 72 hours the drift becomes quasi-steady. During the first 40 hours, the symmetric vorticity increases slightly at radii smaller than 150 km, after which modifications in the inner 150 km remain small. Between 150 and 500 km radius, the symmetric relative vorticity decreases during the first 32 hours, while at later times the changes are positive between 200 and 300 km and negative between 300 and 500 km radius. At larger radii the symmetric vorticity experiences small but positive temporal modifications throughout 96 hours. These findings are in agreement with the temporal (8-hourly) differences of azimuthal mean symmetric wave-kinetic energy \( K_0 \), defined in appendix B, which are positive in the innermost 200 km over a period of 40 hours, and small and both positive and negative at later times. At radii beyond 200 km, temporal changes of \( K_0 \) are small and negative during the first 40 hours, and small and of variable sign later. Note that in an area-integrated sense, the symmetric vortex loses more wave-kinetic energy in the outer part of the radial domain than it gains in the inner part, where local changes are comparably large during early hours of integration. The loss of symmetric wave-kinetic energy at radii greater than 200 km, corresponding to the growth of an outer anticyclonic ring of symmetric tangential wind speed near 400 km radius (reaching \(-0.5 \text{ m s}^{-1}\) after 96 hours), is clearly associated with the continuous development of the \( \beta \)-gyres.

   Figure 3 shows the time-development of the total asymmetric relative-vorticity field (current symmetric vortex removed). During the first 40 hours of integration a quasi-regular

![Figure 2. Vortex track in km of the profile given by Eq. (1) on a \( \beta \)-plane between 0 and 96 hours. Hurricane symbols denote the vortex centre positions (the location of the vorticity maximum) at 8-hourly intervals.]
vorticity dipole pattern* develops as a result of the $\beta$-effect, and its local extrema move radially outwards from 200 to 270 km. At later times the locations of the extrema become stationary relative to the moving vortex. In contrast to the positive anomaly of the dipole, the maximum magnitude of the negative anomaly increases with time from $3 \times 10^{-5}$ to $4 \times 10^{-3}$ s$^{-1}$ between 40 and 96 hours, and a very long and narrow, cyclonically trailing wake-like spiral band of negative vorticity begins to evolve in the rear-right quadrant relative to the moving vortex. Although the whole pattern can be regarded as quasi-stationary relative to the vortex centre, its outermost part seems to lag behind the moving vortex. The highly asymmetric nature of this pattern is documented in Fig. 4, which shows the amplitude development at a point in the region of the negative vorticity spiral in the rear-

* The terms 'dipole', 'dipolar' or 'quasi-regular dipole' etc. are used in the sense that the vorticity field in question is dominated by a wave-number-one contribution. This does not exclude the existence of higher-wave-number contributions.
right quadrant (curve labelled R) and at the opposite point relative to the vortex centre in the front-left quadrant (curve labelled L). After 96 hours the absolute magnitude of asymmetric vorticity in the rear-right quadrant is approximately four times the corresponding value in the front-left quadrant. In concert with the development of the spiral band of negative vorticity, a narrow strip of positive vorticity emerges from the positive part of the vorticity dipole in the front-left quadrant after about 70 hours (cf. Figs. 3(c) and (d)). At 96 hours, this strip forms a long cyclonically curved spiral band at the outer edge of the negative vorticity anomaly in the right-hand quadrants. The different evolution of the asymmetric vorticity patterns in the vicinity of the vortex and at radii beyond about 400 km, especially in the right quadrants, indicates that different mechanisms govern the generation and maintenance of the individual features. To shed some light on these mechanisms, Fig. 5 shows the difference between the total asymmetric vorticity fields at 96 and 88 hours in the moving (Fig. 5(a)) and in a fixed frame of reference (Fig. 5(b)). In the vortex-relative system, the differences are large in the vicinity of the trailing spiral (cf. Fig. 3(d)), while in the region of the quasi-regular vorticity dipole temporal changes are comparably small. In the fixed frame the situation is reversed. These findings suggest that processes on the scale of the vortex are responsible for the development of the quasi-dipolar vorticity pattern, whereas the evolution of the long cyclonic spiral band is a reflection of the vortex drift on the $\beta$-plane. The latter statement is corroborated by a comparison of the drift speed of the vortex (5–6 km h$^{-1}$ north-westward) with an estimate of the relative translation speed of the spiral band (3–5 km h$^{-1}$ south-eastward, depending on the location relative to the vortex). The different development in the individual horizontal regions is indicated also by Fig. 6, showing the decadic logarithm of the absolute value of the radial gradient of absolute vorticity at 96 hours, compiled from the fields of the current symmetric vortex (without the total asymmetric radial vorticity gradient) and the planetary vorticity. Comparisons with the total asymmetric vorticity field of Fig. 3(d) and the difference fields of Fig. 5 show that the vorticity dipole between 200 and 400 km radius is situated right inside the region of the strong radial vorticity gradient, in agreement with the above findings. On the other hand, the negative vorticity wake in the rear-right quadrant of Fig. 3(d) is located in an area where the radial gradient of absolute vorticity is small. The growth of the elongated vorticity asymmetries in the rear-right quadrant is accompanied by wind speed anomalies, shown by the cross-section in Fig. 7 (along the diagonal line in Fig. 3(d)) after 96 hours. The structures of radial and tangential wind components are highly asymmetric, with a
Figure 5. Difference fields resulting from the subtraction of the total asymmetric vorticity, $\zeta$, field in s$^{-1}$ at 88 hours from the corresponding field at 96 hours: (a) in the frame of reference moving with the vortex and (b) in a fixed frame of reference. Contour intervals are $2 \times 10^{-6}$ s$^{-1}$.

Figure 6. Decadic logarithm of the absolute value of the radial gradient of absolute vorticity, compiled from the symmetric vortex at 96 hours and the planetary vorticity. Adjacent contour lines differ by a factor of 10, and regions with magnitudes lower than $5 \times 10^{-11}$ m$^{-1}$ s$^{-1}$ are shaded.

local cyclonic maximum of the asymmetric tangential wind speed near the inner boundary of the area covered by the negative vorticity anomaly. Furthermore, in the inner region of the negative anomaly, the radial wind is directed inward, while in the outer region it is directed outward. In general, in the outer regions the velocity components are larger in magnitude and much more variable in the rear-right quadrant than in all other quadrants.

The wave-number-one contribution to the relative vorticity at 96 hours, displayed in Fig. 8(a), is dominated by the $\beta$-gyres, which grow in magnitude from $7 \times 10^{-6}$ s$^{-1}$ at 8 hours to $2.2 \times 10^{-5}$ s$^{-1}$ at 72 hours. Comparison with Fig. 3(d) shows the relative
Figure 7. Cross-section along the diagonal line of Fig. 3(d), showing: total radial wind speed ($u$, double-dashed); total wind speed ($V$, variable-dashed); and total asymmetric tangential wind speed ($v$, solid). All are in m s$^{-1}$ and relative to the vortex centre at 96 hours. The negative abscissa defines the front-left quadrant and the band of negative vorticity anomaly at 96 hours is marked in the lower-right corner.

dominance of the wave-number-one contribution, i.e. the dipole pattern in the vicinity of the vortex centre. During early hours the extrema translate radially outwards, while at later times they become stationary in the radial direction in accordance with the steady motion of the vortex. At larger radii the wave-number-one pattern has the form of trailing cyclonic spiral arms. Temporal changes of azimuthal mean kinetic energy $K_1$ (suffix $k$ is the wave number) are generally positive in the whole radial domain during the first 48 hours of integration, and negative at later times and beyond 100 km radius, in qualitative agreement with the energy changes of the symmetric vortex. The maximum magnitudes of the 8-hourly differences of $K_1$ are at least one order of magnitude smaller than those of $K_0$, but at radii greater than 200 km the magnitudes of $K_0$ and $K_1$ are approximately of the same order.

The evolution of the wave-number-two vorticity contribution (shown after 96 hours in Fig. 8(b)) is similar to that of the wave-number-one component. However, the development is slower, and after 96 hours the maximum magnitude of the wave-number-two vorticity reaches only half the magnitude of the contribution of wave number one. The extrema of the cyclonic spirals translate slowly to larger radii until they become stationary after about 70 hours. With regard to the evolution of the negative vorticity wake, it is important to note that at radii between 250 and 600 km, the wave-number-two pattern is in phase with the wave-number-one pattern (Fig. 8(a)) only in the rear-right quadrant, while in the other quadrants the corresponding wave-number contributions partly cancel each other. The 8-hourly changes of $K_2$ are generally positive, with a local maximum near 300 km and, until 72 hours, at least one order of magnitude smaller than those of $K_1$, but of equal magnitude at later times.

Figure 8(c) shows the vorticity pattern of wave-number three after 96 hours. The cyclonic trailing spirals develop even more slowly than in the foregoing case, indicating that higher-wave-number contributions are generated by contributions of lower wave numbers through nonlinear interaction. Between 56 and 96 hours, the maximum magnitude of the wave-number-three vorticity asymmetry grows from one third to two thirds of the corresponding value of the wave-number-two contribution. As in the cases discussed above, the extrema of the spirals move slowly away from the vortex centre before they reach their stagnation radius. Again, the vorticity bands are approximately in phase with the bands of
all lower-wave-number contributions only in the rear-right quadrant. The temporal changes of $K_3$ are generally positive, with one local maximum slowly moving outwards from 300 to 500 km radius, and at earlier times at least one order of magnitude smaller than changes of $K_2$, but of comparable magnitude by the end of the numerical integration.

The residual vorticity field at 96 hours, representing all contributions of wave numbers greater than three, is shown in Fig. 8(d). At early hours the magnitude of the residual vorticity is very small compared with the other contributions. However, after 96 hours its maximum absolute magnitude has grown to a value of $1.4 \times 10^{-5}$ s$^{-1}$, greater than the magnitudes of the contributions of wave numbers two and three, and more than half of the magnitude of the contribution wave number one. The highly asymmetric patterns translate radially outwards, and subsequently organize into coherent, very long (up to 1000 km) and narrow, quasi-stationary bands of positive and negative vorticity anomaly in the right quadrants. No comparable structures develop in the other quadrants, and only in the rear-
right quadrant are the bands approximately in phase with the contributions of lower wave numbers.

(b) Analysis of the vorticity tendency

The results in the last section indicate that different mechanisms govern the evolution of the dominant vorticity patterns in the four quadrants and in different radial regimes. To shed further light on the possible processes, it is necessary to examine the vorticity tendency more closely. In a frame moving with the vortex the barotropic vorticity equation takes the form (cf. Smith and Weber 1993)

$$\frac{\partial \zeta}{\partial t} = -\mathbf{U} \cdot \nabla \zeta + \mathbf{v} \cdot \nabla \zeta - \mathbf{U} \cdot \nabla f. \quad (2)$$

Note that in the moving frame, $f$ is a function of time and $\partial f/\partial t = \mathbf{v} \cdot \nabla f$. The terms on the right-hand side of Eq. (2) represent: the advection of relative vorticity by the wind field $U\mathbf{N}Z = -\mathbf{U} \cdot \nabla \zeta$; the advection of relative vorticity by the drift speed of the vortex $CNZ = \mathbf{v} \cdot \nabla \zeta$; and the advection of planetary vorticity by the wind field $UNF = -\mathbf{U} \cdot \nabla f$. Figure 9 shows a series of cross-sections from the front-left (negative abscissa) to the rear-right quadrant (positive abscissa) along the diagonal line in Fig. 3(d), of the three tendency terms at 48 hours (left column) and 96 hours (right column). An analysis of the three tendency terms can be summarized as follows.

- In the rear-right quadrant, $U\mathbf{N}Z$ has large negative values within the innermost 100 km, and large positive values between 100 and 300 km that remain approximately constant with time. Beyond 300 km, a relatively weak wave-like pattern develops after 60 hours that slowly propagates outwards (right-hand parts of Figs. 9(a) and (b)). In the front-right quadrant (not shown), $U\mathbf{N}Z$ has large positive values inside a radius of 100 km, and negative values between 100 and 300 km interrupted by a narrow band of positive or weakly negative tendency centred near 250 km. These patterns slowly translate outwards during the first 60 hours and remain stationary afterwards. In the front-left quadrant (left-hand parts of Figs. 9(a) and (b)), the patterns inside a radius of 250 km are approximately a mirror image of the patterns in the rear-right quadrant. At larger radii, $U\mathbf{N}Z$ develops a wave-like pattern similar to the rear-right quadrant, but the development starts at a later time and the radial wavelength is smaller. The development in the rear-left quadrant (not shown) corresponds qualitatively with the front-left quadrant, but generally with opposite signs of $\mathbf{C}N\mathbf{Z}$.

- In all quadrants and inside a radius of 200 km, $\mathbf{C}N\mathbf{Z}$ (Figs. 9(c) and (d)) is generally of opposite sign to $U\mathbf{N}Z$, which leads to partial cancellation of the two terms. This is also documented in Figs. 10(a) and (b), which show the sum of $U\mathbf{N}Z$ and $\mathbf{C}N\mathbf{Z}$ in the front-left and rear-right quadrant at 48 and 96 hours, respectively. In the rear-right quadrant, $\mathbf{C}N\mathbf{Z}$ develops a local minimum of negative tendency after about 30 hours near 280 km radius, that grows in absolute magnitude and slowly propagates outwards; this is shown on the right-hand sides of Figs. 9(c) and (d). This pattern is not cancelled by any of the other terms in Eq. (2) and, as time passes, the locus of the local minimum moves from the inner edge of the band of negative vorticity anomaly at 96 hours (marked in the upper-right corners of Figs. 9(a) to (f)) to the outer edge at 450 km radius. Hence, $\mathbf{C}N\mathbf{Z}$ can be regarded as a main contributor to the generation and maintenance of the spiral of negative vorticity anomaly in the rear-right quadrant. In all other quadrants, $\mathbf{C}N\mathbf{Z}$ partly or even completely cancels the term $U\mathbf{N}Z$ at radii greater than 250 km. The cancellation is strongest in the front-left quadrant (cf. Figs. 10(a) and (b)), whereas in the rear-left and front-right quadrant the sum of $U\mathbf{N}Z$ and $\mathbf{C}N\mathbf{Z}$ forms wave-like patterns that become weaker with time and slowly
Figure 9. Cross-sections along the diagonal line in Fig. 3(d) of the three vorticity tendency terms on the right-hand side of Eq. (2) in s$^{-2}$ at 48 (left column) and 96 hours (right column). (a) and (b) $UNZ$; (c) and (d) $CNZ$; (e) and (f) $UNF$ (see text for the definition of the acronyms). The negative abscissa corresponds with the front-left quadrant. The region of the negative vorticity anomaly at 96 h, including a schematic diagram of its radial structure, is shown in the upper-right corner of each plot.

translate outwards. After 80 hours, these wave-like tendency patterns remain stationary. The combined effect of $UNZ$ and $CNZ$ represents the main cause for the generation of the so-called ‘bow-waves’ in these quadrants between radii of about 250 and 350 km (cf. Montgomery and Kallenbach 1997).

- The term $UNF$, shown in Figs. 9(e) and (f) after 48 and 96 hours, generally makes the smallest contribution to the development of the vorticity field, and has a very smooth structure that develops early and is not modified much during the 96-hour integration time.
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Figure 10. As Fig. 9, but (a) and (b) show the sum of $UNZ$ and $CNZ$; and (c) and (d) the total vorticity tendency $\partial \xi / \partial t$ in accordance with Eq. (2).

However, in the innermost 200 km, $UNF$ cancels most of the vorticity tendency produced by $UNZ$ and $CNZ$, such that the resulting total tendency remains rather small. The latter is shown in Figs. 10(c) and (d) at 48 and 96 hours; it should be compared with Figs. 9(e) and (f) for $UNF$, and Figs. 10(a) and (b) for the sum of $UNZ$ and $CNZ$. The maximum absolute magnitude of $UNF$ is always smaller by a factor of two in the front-right and rear-left quadrant than in the other two quadrants. Beyond 200 km radius, $UNF$ remains negligible in comparison with the other tendency contributors in all except the rear-right quadrant, where it is negative throughout 96 hours at radii smaller than 370 km, where the centre of the spiral of negative vorticity anomaly is located at 96 hours. Hence, $CNZ$ and $UNF$ form the main contributors to the generation of the negative vorticity anomaly in the rear-right quadrant. At radii greater than 370 km, $UNF$ develops a weak but temporally-growing positive trend that is responsible, together with an equivalent trend of $UNZ$ in the same region, for the evolution of the band of positive vorticity anomaly in the right-hand quadrants near the outer edge of the negative anomaly.

The above results can be summarized as follows.

(i) All terms on the right-hand side of Eq. (2) cancel each other at radii smaller than 200 km, resulting in a rather uniform asymmetric vorticity distribution of small magnitude. A comparison of Figs. 10(c) and (d) with Figs. 10(a) and (b), and Figs. 9(e) and (f), shows that advection of planetary vorticity plays a major role with regard to the evolution of the vorticity fields in the vicinity of the vortex centre.
(ii) The terms UNZ and CNZ are mainly responsible for the generation of the quasi-dipolar vorticity pattern on the periphery of the vortex at radii between 200 and 350 km in Fig. 3(d). Figures 10(a) and (b) demonstrate that the combination of UNZ and CNZ results in outward propagating, temporally weakening, radial wave patterns that stagnate at some radial distance from the centre after a few days. In the sense of the results of Montgomery and Kallenbach (1997), these structures can be identified as vortex Rossby waves or 'bow-waves'. However, the present experiment shows that their general structure is not exclusively the result of the advection of vortex vorticity, it also depends crucially on the drift of the vortex.

(iii) The negative vorticity anomaly in the rear-right quadrant is a direct manifestation of the vortex drift (Figs. 9(c) and (d)) and, to a lesser extent, of the advection of planetary vorticity (Figs. 9(e) and (f)). However, the latter forms an important and necessary prerequisite for the generation of the negative vorticity spiral by the term CNZ. Expressed in a qualitative and simple way, at early hours UNF produces a positive \( \partial \xi / \partial x \) to the east of the vortex by decreasing/increasing \( \xi \) at radii smaller/greater than 370 km. Furthermore, it causes the north-westward drift of the vortex, i.e. a negative value of \( c_y \). With the qualitative assumption that \( \partial \xi / \partial y \) is approximately zero in the same region, the two effects of UNF combine to produce a negative vorticity tendency \( c \cdot \nabla \xi \approx c_x \partial \xi / \partial x \) at later times, leading indirectly to the generation of the elongated negative vorticity spiral by the term CNZ. The qualitative process described above is confirmed by the finding that the location of the outer minimum of CNZ in the rear-right quadrant agrees with the location of the zero of UNF near 370 km radius (cf. Figs. 9(c) and (e)). The influence of UNZ is rather small in this region compared with the two other terms.

(iv) The outermost spiral band of positive vorticity anomaly in Fig. 3(d), developing at later times in the right-hand quadrants, results from a combination of the effects produced by the terms UNZ and UNF, possibly as the result of a process similar to the one suggested in (iii).

(c) Theory

The analysis of the vorticity tendency carried out in the last subsection suggests that most of the banded spiral patterns in the right-hand quadrants are induced by the vortex drift and the advection of planetary vorticity, rather than by the advection of relative vorticity. Therefore, following the statements of e.g. Flierl (1984), it is argued that the spiral or wake pattern represents a spectrum of Rossby waves induced by the vortex as a result of its motion on a \( \beta \)-plane.

The following theoretical calculations are based on the work of Lighthill (1967). In the moving frame, the barotropic vorticity equation is given by Eq. (2). The dependent variables are partitioned as \( U = U_v + U' \) and \( \xi = \xi_v + \xi' \), where primed quantities are perturbations to the basic state. Insertion in Eq. (2) leads to

\[
\frac{\partial \xi'}{\partial t} = -(U_v \cdot \nabla \xi' + U' \cdot \nabla \xi_v + U' \cdot \nabla \xi') + (c \cdot \nabla \xi_v + c \cdot \nabla \xi') \\
- (U_v \cdot \nabla f + U' \cdot \nabla f).
\]  (3)

For the present investigation, following the arguments of Smith et al. (1990) and Smith and Weber (1993), the terms \( \partial \xi_v / \partial t \) and \( U_v \cdot \nabla \xi_v \) have been neglected. Note that here the basic vortex is defined to be independent of time in the moving frame, in accordance with the partitioning approach proposed by Kasahara and Platzmann (1963, their Method III). Moreover, it is assumed that the translation vector \( c \) is constant, reflecting the quasi-steady
vortex drift velocity of approximately $(-0.6, 1.3)$ m s$^{-1}$ in the numerical model at later times. Linearization of Eq. (3) yields

$$\frac{\partial \zeta'}{\partial t} - \mathbf{c} \cdot \nabla \zeta' + \mathbf{U}' \cdot \nabla f = -\mathbf{U}_v \cdot \nabla f. \tag{4}$$

In order to construct a mathematically tractable problem, and to focus on the effect of vortex drift and advection of planetary vorticity in the far field of the vortex, the terms $\mathbf{U}_v \cdot \nabla \zeta'$, $\mathbf{U}' \cdot \nabla \zeta$, and $\mathbf{c} \cdot \nabla \zeta_v$ have been neglected in the derivation of Eq. (4). At the lowest order of approximation these terms can be expected to dominate mainly the field in the vicinity of the vortex centre, where Eq. (4) is generally not valid*. However, it can be argued that the linear equation provides a reasonable approximation to the processes in question at large distances from the vortex centre where the spiral patterns occur and where vortex vorticity and velocity have relatively small values. The right-hand side of Eq. (4) is computed using the basic profile given by Eq. (1).

The procedure to solve Eq. (4) is outlined in appendix C. At any given time, the vorticity and stream function fields are obtained by numerical evaluation of the integrals in appendix C in Fourier- and physical space using Simpson's rule. The analysis of section 3(a) shows that the spiral or wave pattern is quasi-stationary relative to the moving vortex, but that its magnitude grows with time. The linear approach used in the present study allows no growing solutions. Therefore, attention is restricted to time-dependent neutral solutions of Eq. (4). Results are presented for zonal and meridional wave numbers smaller than or equal to 40, in a square domain of 4000 km length in zonal and meridional direction. The vorticity and stream function fields produced by solving Eq. (4) remain largely invariant if the number of wave numbers included in the calculations is doubled or halved.

Figure 11(a) shows the total asymmetric relative-vorticity field after 96 hours. It should be compared with Fig. 3(d). As expected, in the inner part of the field the agreement with the numerical calculation is relatively poor, but beyond radii of 300 km, both fields show similarities and the patterns have approximately the same magnitude. In the theory the vortex produces an elongated wake in form of a vorticity dipole. At radii greater than about 300 km, the negative part of the wake is organized into a cyclonically trailing pattern, located in the same region as the numerical negative vorticity anomaly after 96 hours. The negative pattern is more elongated and stronger than its positive counterpart. The absence of a corresponding positive vorticity pattern in the numerical field of Fig. 3(d) (although there is some evidence of a positive vorticity anomaly in the region of interest) is presumably because of the absence of both linear and nonlinear contributors to the advection of relative vorticity $\mathbf{U} \cdot \nabla \zeta$ in the theoretical calculations. While the negative wake is located near the eastern edge of the strong swirling flow of the moving symmetric vortex, the positive anomaly is always situated to the south of the vortex. This implies that both the western part of the negative anomaly and the complete positive anomaly are homogenized with time by linear and nonlinear advection of relative vorticity in the vicinity of the vortex centre. A homogenization process governed by $\mathbf{U} \cdot \nabla \zeta$ would also explain the greater cyclonic curvature of the negative wake in Fig. 3(d) and the generally smaller magnitudes of the numerical fields in comparison with the theoretically derived fields.

The total fields of relative vorticity and stream function are subjected to an azimuthal Fourier analysis about the vortex centre. The contributions of wave numbers one, two and three and the residual relative vorticity field resulting from this analysis are shown in Figs. 11(b), (c), (d) and (e), respectively, and their outer parts should be compared with

* It should be kept in mind that the structures that develop in the numerical model fields generally result from nonlinear processes that cannot be described fully by the following linear approach.
Figure 11. Vorticity fields in s\(^{-1}\) resulting from the theoretical calculations of section 3(c) after 96 hours. (a) Total asymmetric vorticity. (b) Contribution of wave-number \(k = 1\); (c) \(k = 2\); (d) \(k = 3\); and (e) residual field of all contributions for \(k > 3\). The contour interval is \(5 \times 10^{-6} \text{s}^{-1}\) in (a) and \(2.5 \times 10^{-6} \text{s}^{-1}\) in the other frames. The current vortex centre is marked with a hurricane symbol and in (b) to (e) the core region is excluded for radius \(r \approx 220 \text{ km}\).
the corresponding numerical fields of Fig. 8. With the exception of the residual field of the linear calculation (Fig. 11(e)), which is much larger in magnitude than the corresponding numerical residual field in Fig. 8(d), for the reasons explained in the last paragraph, the agreement in the outer fields of each wave-number contribution is surprisingly good both in magnitude and relative position of the trailing spiral patterns. The comparison of the residual fields at large radii shows qualitative agreement, insofar as in the theoretical calculation the dominant negative anomaly is flanked by two positive anomalies as in the numerical field. Furthermore, most of the locations of the far ends of all vorticity asymmetries in the south-eastern part of both residual fields qualitatively correspond well.

In a supplementary series of theoretical calculations, Eq. (4) is solved for different drift directions of the vortex, using the same constant drift speed as before. The experiments show that the development of a vorticity wake to the rear of the vortex is independent of the direction of motion. However, the structure of the wake depends strongly on the drift direction. In general, vortices moving in a southerly/northerly direction produce anticyclonically/cyclonically trailing spirals of negative vorticity anomaly in their left/right-hand quadrants, and positive anomalies in the complementary quadrants. Moreover, the strength of the anomalies depends on the drift direction: south-westward and north-westward moving storms produce stronger wakes. Further experiments with twice (~10 km h⁻¹) and half (~2.5 km h⁻¹) the original drift speed, using the original drift direction, result in a stronger and more elongated wake in the case of the faster moving vortex, while the slower moving vortex does not produce a discernible wake-like pattern at large radii after 96 hours. Finally, the theoretical calculations do not produce a wake in the far-field of a stationary vortex or in the far-field of a moving vortex on an f-plane, in agreement with the results of section 3(b) that the evolution of a wake is associated both with the vortex drift and the advection of planetary vorticity.

4. Summary and Discussion

In the present study the evolution of asymmetries to a vortex moving on a β-plane has been investigated. In summary the analysis of the numerical fields leads to the following results and conclusions.

(i) After a few days of integration time, elongated cyclonically curved spiral bands of positive and negative vorticity anomaly develop at large radii (typically beyond 6–10 times the radius of maximum tangential wind speed), mainly in the rear-right quadrant of the moving vortex. The spiral bands appear to remain quasi-stationary or translate slowly relative to the vortex centre, such that their far ends successively lag behind the moving vortex. These findings are in agreement with the results of earlier studies on the time-development of Gulf Stream rings, such as those of Mied and Lindemann (1979, their Fig. 5) or McWilliams et al. (1986, their Figs. 3 and 9), who identified the outer asymmetries as slowly moving, trailing Rossby-wave wakes induced by a vortex moving on a planetary vorticity gradient.

(ii) The development of the Rossby-wave wake coincides with the observation in the numerical model that contributions of waves number one, two and three and the residual field of relative vorticity, shown in Fig. 8, are in phase only in the rear-right quadrant, whereas in all other quadrants there is strong cancellation between separate wave-number contributions in the outer fields. This feature is reflected in the temporal growth of the absolute magnitude of the total vorticity anomaly in the rear-right quadrant (cf. Fig. 4) that has no analogue in the other quadrants. Moreover, the residual field cannot be neglected compared with the contributions of lower wave numbers, and shows the highly asymmetric nature of the vorticity structure in the vicinity of the moving vortex.
(iii) Comparison of Fig. 3(d) with Fig. 6 shows that the elongated, cyclonically curved spiral bands develop in regions where the absolute vorticity gradient is weak. This finding suggests that the mechanism of formation and maintenance of the elongated spiral bands differs substantially from the mechanism producing the inner quasi-dipolar vorticity asymmetries in regions between 200 and 350 km radius, where the radial absolute vorticity gradient is large. The vorticity dipole is a result of slowly outward-propagating wave-like patterns of vorticity tendency, that finally stagnate at some radial distance from the vortex centre. They are produced mainly by a combined effect of the advection of vortex vorticity, \(UNZ\), and the effect of the vortex drift, \(CNZ\), and were identified earlier, and in a different context, as vortex Rossby waves by Montgomery and Kallenbach (1997). The identification of the quasi-dipolar vorticity asymmetries as vortex Rossby waves is corroborated also by the finding that these patterns occur in all four quadrants relative to the moving vortex. Their stagnation radius is located right inside the outer edge of the relatively strong radial gradient of absolute vorticity near 300 km radius. In this context it should also be noted that, in the case of a non-moving vortex, the structure of the vortex Rossby waves would differ considerably from the structure shown e.g. in Fig. 3(d).

(iv) The uniform asymmetric vorticity distribution of small magnitude near the vortex centre results from cancellation of all three contributors to the vorticity tendency, hence also of the term \(UNF\) in Eq. (2). This result is rather surprising in view of the small magnitude of the planetary vorticity gradient in comparison with the gradient of vortex vorticity in the vicinity of the vortex core.

(v) The development of the quasi-stationary cyclonically curved spiral band of negative vorticity anomaly in the rear-right quadrant is governed mainly by the effect of vortex propagation, \(CNZ\), and the advection of planetary vorticity, \(UNF\). As Figs. 3 and 9(c) and (d) show, the first term represents the major contributor to the generation and maintenance of the long spiral of negative vorticity anomaly, while the second term (Figs. 9(e) and (f)) is rather small, and both positive and negative in this region. However, \(UNF\) represents an essential pre-requisite for the development of \(CNZ\) in the region of interest. These results are confirmed also by the theoretical calculations summarized below. In contrast to the above findings, the outermost spiral of positive vorticity anomaly that develops at later times is a manifestation of the combined effect of the terms \(UNF\) and \(UNZ\). Apart from this exception, it can be concluded that vortex propagation plays a major role in the formation and maintenance of quasi-stationary spiral bands of vorticity. In the present experiment, the vortex drift is a response to the evolution of the \(\beta\)-gyres, i.e. originally to the advection of planetary vorticity by the wind field of the symmetric vortex, and the drift-dependent Rossby-wave wake forms mainly in the south-east of the centre as the vortex drifts to the north-west. It is reasonable to presume that large-scale environmental vorticity gradients that differ from the planetary vorticity gradient will produce wakes of different orientation, size and strength. Furthermore, the dependence of the formation of outer spiral bands on the vortex drift implies also that stationary vortices do not produce Rossby-wave wakes.

(vi) The trailing spiral pattern discussed in section 3 is not a unique feature of the basic profile used in the present study. Additional experiments show that Rossby-wave wakes can be regarded as general features of vortices moving on a vorticity gradient (cf. also McWilliams and Flierl 1979). However, depending on the particular symmetric vortex, the associated drift speed and the magnitude of the large-scale vorticity gradient, Rossby-wave wakes may have quite variable structures, sizes and magnitudes.

(vii) The negative vorticity anomaly in the rear-right quadrant is accompanied by wind field anomalies. As Fig. 7 shows, total asymmetric tangential wind speed increases near the inner boundary and decreases near the outer boundary of the negative vorticity anomaly.
The radial wind is directed inwards in the inner part of the negative vorticity anomaly and outwards in the outer part. In all other quadrants the total asymmetric radial and tangential wind fields are smaller in magnitude and have a much more uniform structure than in the rear-right quadrant.

The numerical results in respect of the evolution of the vorticity fields are confirmed by the predictions from an analytical solution of the linear barotropic vorticity equation in a moving frame of reference. However, full agreement between the linear theoretical approach and the results of the numerical model cannot be expected, as the latter includes nonlinear processes that become dominant during the numerical integration. In the theory, the wake is produced by the combination of the advection of perturbation vorticity by the vortex drift $c \cdot \nabla \zeta'$ and the advection of planetary vorticity by the total wind field $U \cdot \nabla f$.

The same terms are found to be responsible for the generation of the elongated negative vorticity anomaly by the analysis of the vorticity tendency carried out in section 3(b). The qualitative agreement between the theoretical approach and the numerical analysis is confirmed by the similarity of the vorticity structures of the theoretical and numerical calculations at large radii, where the theory is valid. Hence, the wake of negative vorticity in the far field can be regarded as a spectrum of Rossby waves induced by the moving vortex. The absence of a strong positive vorticity anomaly (as in Fig. 11(a)) in the numerical field of Fig. 3(d) can be explained as a result of linear and nonlinear homogenization processes governed by the advection of relative vorticity, which also strongly distort the western part of the negative anomaly in the numerical fields. This results in a negative vorticity spiral of stronger cyclonic curvature and smaller absolute magnitude in comparison with the pattern produced by the theory. Additional solutions of Eq. (4) have been calculated using the basic vortex given by Eq. (1): (a) moving with the original drift speed in different directions; (b) moving with half and twice the original drift speed in the original direction; (c) with zero drift speed; and (d) moving with the original drift speed on an $f$-plane. In all cases except (c) and (d), a vorticity wake develops in the rear of the moving vortex. However, in the theory the structure of the wake depends strongly on the particular case investigated. For example, slowly translating vortices (with drift speeds of 2 to 3 km h$^{-1}$) do not produce pronounced trailing spiral patterns at large radii as shown in Fig. 3(d) because, after three or four days the theoretically computed wake is still located in the near vicinity of the moving vortex. Finally, in agreement with the statements made previously in this section under (v), the cases (c) and (d) did not produce vorticity wakes.

The similarity, both in structure and location, of the patterns discussed in section 3 with quasi-stationary principal rain bands in real tropical cyclones raises the possibility that the evolution of principal spiral bands may be associated with, or favoured by, dynamical anomalies that occur as a result of the slow development of a Rossby-wave wake in the rear and to the right of a moving storm. Superposition of inertia–gravity waves, as proposed by Willoughby (1978), has been excluded as a main mechanism for the generation of rain bands in the earlier study of Guinn and Schubert (1993). The discovery of similarities between hurricane spiral bands and the orientation of troughs in planetary Rossby waves goes back to the observational study of MacDonald (1968). Later, numerical experiments such as those of McWilliams et al. (1986) on Gulf Stream rings showed that asymmetric vorticity patterns of considerable magnitude can arise in the vicinity of a moving ring as a result of Rossby-wave excitation. To our knowledge, the evolution of a Rossby-wave wake on the periphery of a moving vortex has not been considered as a possible dynamical basis for the formation and maintenance of convective spiral bands in tropical cyclones. The many observational studies of rain bands in tropical cyclones show a rather non-uniform picture of their kinematic structure. At levels up to a few km altitude Ryan et al. (1992), Barnes et al. (1983), Barnes and Stossmeister (1986) and Powell (1990) found inflow
over most of the area covered by the convective band, while at higher levels regions of outflow were found. In many cases the structure of the tangential wind speed shows a detectable cyclonic maximum at lower levels that becomes weaker or even vanishes at higher altitudes. The observations at lower altitudes agree qualitatively with the radial and tangential wind structure in the vicinity of the inner edge of the negative vorticity anomaly in the numerical model* (cf. Fig. 7). For this reason, and provided that there exists a relation between dynamical anomalies and convective spiral bands, it is presumed that in the particular case of this study a preferred location of a quasi-stationary convective rain band would not be the area covered by the negative vorticity anomaly itself, but the region near its inner boundary, where the asymmetric cyclonic tangential wind speed has a local maximum. Furthermore, the vicinity of the outermost positive vorticity anomaly that develops at later times might form another favourable region for the development of a quasi-stationary rain band. The numerical results and the theoretical calculations show that the quasi-stationary spiral bands of vorticity develop relatively late (for example after about 40 hours in the numerical run of the present study). This implies that a spiral band of convection, if associated with the dynamical processes discussed here, would develop rather slowly with time and probably only on the periphery of tropical cyclones that have moved through a relatively quiescent environment for a considerable period of time. The supplementary theoretical calculations of section 3(c) show that quasi-stationary vorticity spirals can be expected to occur in the rear of vortices irrespective of their drift direction. Therefore, the location and orientation of the Rossby-wave wake of Fig. 3 is a reflection of the particular vortex drift to the north-west. Other directions of motion, for example resulting from the advection of large-scale environmental vorticity in addition to planetary vorticity, may lead to different strengths, orientations and structures of wakes, provided that the drift speed of the vortex in question is not too small.

The analysis presented in this study is not intended to preclude the possibility that quasi-stationary spiral bands can be generated by other mechanisms, for example by vortex merger processes or by interaction with elongated vorticity patterns as described by Quinn and Schubert (1993). Moreover, it is evident that a simple barotropic approach cannot fully explain three-dimensional baroclinic phenomena like convective spiral bands that include thermodynamic processes. A clear association of barotropic anomalies with quasi-stationary spiral bands of convection is beyond the scope of the present study and has to be left to future observational and theoretical work. The objective is rather to show that patterns, similar to spiral bands of convection in real tropical cyclones both in structure and relative location, can be generated by a vortex on a β-plane with a simple dynamical model in the absence of any other features with which the observed vortex can interact to produce spiral band patterns.

ACKNOWLEDGEMENTS

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* It should be noted that the absolute magnitude of the Rossby-wave wakes depends inter alia on the structure of the symmetric vortex. Hence, both weaker and stronger wind speed anomalies are possible, depending on the particular vortex profile.
A semi-spectral nondivergent nonlinear barotropic numerical model

The numerical model used here is based on the nonlinear nondivergent inviscid barotropic vorticity equation in a moving frame of reference as given by Eq. (2). A list of the model parameters is given by Table A.1.

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truncation wave-number ( N )</td>
<td>10</td>
</tr>
<tr>
<td>Number of radial grid points</td>
<td>301</td>
</tr>
<tr>
<td>Maximum integration time</td>
<td>96 h</td>
</tr>
<tr>
<td>Radius of earth ( r_E )</td>
<td>6378.161 km</td>
</tr>
<tr>
<td>Centre of ( \beta )-plane ( \phi_0 ) (latitude)</td>
<td>12.5°</td>
</tr>
<tr>
<td>Radial domain size ( r_{\infty} )</td>
<td>2500 km</td>
</tr>
<tr>
<td>Length scale ( L = r_m )</td>
<td>40 km</td>
</tr>
<tr>
<td>Velocity scale ( U = u_0 )</td>
<td>50 m s(^{-1})</td>
</tr>
<tr>
<td>Safety-factor CFL-criterion ( \alpha )</td>
<td>0.1</td>
</tr>
<tr>
<td>Outermost damping radius ( r_D )</td>
<td>20 km</td>
</tr>
<tr>
<td>Damping constant ( q_0 )</td>
<td>0.25</td>
</tr>
</tbody>
</table>

In a cylindrical coordinate system, Eq. (2) takes the form

\[
\frac{\partial \zeta}{\partial t} = -u \frac{\partial \zeta}{\partial r} - v \frac{\partial \zeta}{\partial \theta} + \{c_x \cos(\theta) + c_y \sin(\theta)\} \frac{\partial \zeta}{\partial r} \\
+ \frac{1}{r} \{c_y \cos(\theta) - c_x \sin(\theta)\} \frac{\partial \zeta}{\partial \theta} - \beta \{u \sin(\theta) + v \cos(\theta)\}. \tag{A.1}
\]

The dependent variables are defined as (cf. Ross and Kurihara 1992)

\[
F(r, \theta, t) = \sum_{k=0}^{N} \{F_{kc}(r, t) \cos(k\theta) + F_{ks}(r, t) \sin(k\theta)\}, \tag{A.2}
\]

where \( F \) represents \((u, v, \zeta)\), \( k \) the azimuthal wave number, and the subscripts \((c, s)\) the cosine and sine contribution, respectively. The truncation wave number is denoted by \( N \). Note that the term \( F_{kc}(r, t) \) represents the symmetric vortex and \( F_{ks}(r, t) \) is zero by definition. Furthermore, nondivergence requires that the symmetric radial wind speed \( u_{kc}(r, t) \) is zero. Insertion of Eq. (A.2) in Eq. (A.1) yields, after some algebra,

\[
\sum_{m=0}^{N} \left\{ \frac{\partial \zeta_{mc}}{\partial t} \cos(m\theta) + \frac{\partial \zeta_{ms}}{\partial t} \sin(m\theta) \right\} \]

\[
- \frac{1}{2} \sum_{k=1}^{N} \sum_{n=1}^{N} \left\{ u_{kc} \frac{\partial \zeta_{nc}}{\partial r} - u_{ks} \frac{\partial \zeta_{ns}}{\partial r} + \frac{n}{r} \left( v_{kc} \zeta_{ns} + v_{ks} \zeta_{nc} \right) \right\} \cos((k + n)\theta) \\
- \frac{1}{2} \sum_{k=1}^{N} \sum_{n=1}^{N} \left\{ u_{kc} \frac{\partial \zeta_{sc}}{\partial r} + u_{ks} \frac{\partial \zeta_{ss}}{\partial r} + \frac{n}{r} \left( v_{kc} \zeta_{ss} - v_{ks} \zeta_{sc} \right) \right\} \cos((k - n)\theta) \\
- \frac{1}{2} \sum_{k=1}^{N} \sum_{n=1}^{N} \left\{ u_{kc} \frac{\partial \zeta_{ns}}{\partial r} + u_{ks} \frac{\partial \zeta_{nc}}{\partial r} + \frac{n}{r} \left( v_{ks} \zeta_{ns} - v_{kc} \zeta_{nc} \right) \right\} \sin((k + n)\theta)
\]
\[ \pm \frac{1}{2} \sum_{k=1}^{N} \sum_{n=1}^{N} \left[ u_{kc} \frac{\partial \xi_{ns}}{\partial r} - u_{ks} \frac{\partial \xi_{nc}}{\partial r} - \frac{n}{r} (v_{kc} \xi_{nc} + v_{ks} \xi_{ns}) \right] \sin(|k - n|\theta) \]
\[ + \frac{1}{2} \sum_{k=1}^{N} \left[ c_x \frac{\partial \xi_{kc}}{\partial r} - c_y \frac{\partial \xi_{ks}}{\partial r} + \frac{k}{r} (c_y \xi_{ks} - c_x \xi_{kc}) + \beta (u_{ks} - v_{kc}) \right] \cos((k + 1)\theta) \]
\[ + \frac{1}{2} \sum_{k=1}^{N} \left[ c_x \frac{\partial \xi_{kc}}{\partial r} + c_y \frac{\partial \xi_{ks}}{\partial r} + \frac{k}{r} (c_x \xi_{ks} + c_y \xi_{kc}) - \beta (u_{ks} + v_{kc}) \right] \cos(|k - 1|\theta) \]
\[ + \frac{1}{2} \sum_{k=1}^{N} \left[ c_x \frac{\partial \xi_{ks}}{\partial r} - c_y \frac{\partial \xi_{kc}}{\partial r} - \frac{k}{r} (c_x \xi_{ks} + c_y \xi_{kc}) - \beta (u_{kc} + v_{ks}) \right] \sin((k + 1)\theta) \]
\[ \pm \frac{1}{2} \sum_{k=1}^{N} \left[ c_x \frac{\partial \xi_{ks}}{\partial r} - c_y \frac{\partial \xi_{kc}}{\partial r} + \frac{k}{r} (c_x \xi_{ks} - c_y \xi_{kc}) + \beta (u_{kc} - v_{ks}) \right] \sin(|k - 1|\theta) \]
\[ - \sum_{k=1}^{N} \left( u_{kc} \frac{\partial \xi_{oc}}{\partial r} + \frac{k}{r} v_{oc} \xi_{ks} \right) \cos(k\theta) - \sum_{k=1}^{N} \left( u_{ks} \frac{\partial \xi_{oc}}{\partial r} - \frac{k}{r} v_{oc} \xi_{kc} \right) \sin(k\theta) \]
\[ + \left( c_x \frac{\partial \xi_{oc}}{\partial r} - \beta v_{oc} \right) \cos(\theta) + c_y \frac{\partial \xi_{oc}}{\partial r} \sin(\theta), \]  
(A.3)

where the left-hand side is integrated with time for each separate azimuthal wave number, corresponding with the resultant wave-number contributions on the right-hand side of Eq. (A.3). The ± signs correspond with positive and zero or negative values of the differences in the arguments of the trigonometric functions. Time-integration is carried out using a third-order Adams–Bashforth scheme (cf. Durran 1991) with variable step size. The current time step is determined automatically by evaluating the Courant–Friedrichs–Levy criterion and multiplying this with a safety factor $\alpha = 0.1$. After each time step the velocity components are updated using (cf. Adem 1956)

\[ v_{oc} = \int_0^r \frac{p}{r} \xi_{oc}(p, t) \, dp, \]  
(A.4)

\[ v_{kj} = \frac{1}{2} \int_0^r \left( \frac{p}{r} \right)^{k+1} \xi_{kj}(p, t) \, dp - \frac{1}{2} \int_r^\infty \left( \frac{r}{p} \right)^{k-1} \xi_{kj}(p, t) \, dp, \]  
(A.5)

\[ u_{kc} = \frac{1}{2} \int_0^r \left( \frac{p}{r} \right)^{k+1} \xi_{kc}(p, t) \, dp + \frac{1}{2} \int_r^\infty \left( \frac{r}{p} \right)^{k-1} \xi_{kc}(p, t) \, dp, \]  
(A.6)

and

\[ u_{ks} = -\frac{1}{2} \int_0^r \left( \frac{p}{r} \right)^{k+1} \xi_{ks}(p, t) \, dp - \frac{1}{2} \int_r^\infty \left( \frac{r}{p} \right)^{k-1} \xi_{ks}(p, t) \, dp, \]  
(A.7)

where $p$ is used as a substitute for the radius. The index $j$ in Eq. (A.5) denotes the sine or cosine contribution, respectively, and the infinity sign represents some large radius ($r_\infty$ in the present study). All integrals are evaluated numerically by application of Simpson’s rule and the 3/8 formula of fourth-order accuracy. Closure of the problem is obtained by computation of the drift speed components using

\[ (c_x, c_y) = \left[ \frac{1}{2} \int_0^\infty \xi_{ls}(p, t) \, dp, -\frac{1}{2} \int_0^\infty \xi_{lc}(p, t) \, dp \right] \]  
(A.8)

after each time step. Following Smith and Ulrich (1990), the track error caused by this specific closure assumption is negligible for the time periods considered. The vortex position $\mathbf{x}_c$ is determined using $d\mathbf{x}_c / dt = \mathbf{c}$ in combination with the Adams–Bashforth scheme.
mentioned above. The stream function contributions are computed by radial integration of the corresponding vorticity contributions in the same way as the velocity components. All independent and dependent variables are scaled using the maximum symmetric tangential wind speed and the radius of maximum wind.

The dimensional radial grid size is variable, and depends inversely on the radial structure of the absolute values of the sum of all wave-number contributions of the stream function. As a consequence of the irregular dimensional radial grid, the radial derivatives of the finite-difference form of Eq. (A.3) have to be adapted to the variable radial resolution, requiring the additional computation of terms representing the grid transformation. In the transformed radial coordinate system with fixed (non-dimensional) radial grid size, the radial derivatives are represented by fourth-order finite differences. Besides the gain in computation time as a result of a possible decrease of the number of radial grid points, the use of an irregular grid ensures high radial resolution near the vortex centre and lower resolution at large radii. The irregular radial grid is updated after fixed sequences of time steps (presently 80), using a one-dimensional version of the algorithm described by Dietachmayer and Droegemeier (1992) and Dietachmayer (1992). After each update of the irregular grid, the dependent variables are adjusted to the new grid by linear interpolation. The boundary conditions at the origin of the radial domain are \( \xi_k(r = 0, t) = \xi_{ks}(r = 0, t) = 0 \) for \( k > 0 \) and \( \partial \xi_k / \partial r = \partial \xi_{ks} / \partial r = 0 \) for \( k \geq 0 \). The central value of \( \xi_{bc}(r = 0, t) \) is predicted by Eq. (A.3). At the outer boundary of the radial domain, free boundary conditions are used and the functional values of relative vorticity and its radial derivative are determined using the functional values at inner grid points in combination with fifth-order finite differences.

In order to avoid numerical inconsistencies and to retain computational stability, a numerical filter (Haltiner and Williams 1980; their Eq. 11-87 on p. 393) was applied to the vorticity contributions of the truncation wave-number \( N \). Furthermore, after each time step it was necessary to damp all vorticity contributions at radii smaller than \( r_D = 2 \times 10^4 \text{m} \) by using a dynamical nudging method of the form \( \xi^{n+1} = \xi^n + (\xi^{n+1} - \xi^n) \exp(q \Delta t^n) \exp(q \Delta t^n) \), where \( n \) denotes the time step and with \( q = [q_0(1 - s^2 \exp(1 - s^2))]/\Delta t^n \), \( q_0 = 0.25 \) and \( s = r/r_D \). Tests with and without the dynamical nudging showed that at larger radii, the region of interest here, the vorticity contributions were affected only marginally by the damping. Comparison with the barotropic grid-point model used by Smith et al. (1990) produced qualitatively and quantitatively similar results, but in the new model the evolution of small-scale patterns that were expected to occur because of numerical inconsistencies in the model of Smith et al. op. cit., was largely reduced.

**APPENDIX B**

**Azimuthal mean wave-kinetic energy per unit mass**

In the barotropic numerical model the contributions from all wave numbers lower than or equal to the truncation wave number are computed using Eq. (A.2). In that equation the terms in curly brackets can be re-defined using

\[
F_k(r, t) = F_{kc}(r, t) \cos(\varphi) + F_{ks}(r, t) \sin(\varphi),
\]

with

\[
\varphi = \text{atan}\{F_{ks}(r, t)/F_{kc}(r, t)\},
\]

to give an expression equivalent to Eq. (A.2), i.e.

\[
F(r, \theta, t) = F_{bc}(r, t) + \sum_{k=1}^{N} \{F_k(r, t) \cos(k\theta - \varphi)\}.
\]
In a nondivergent medium the amplitude functions of azimuthal mean wave-kinetic energy $K$ at any time are defined as

$$K_0(r) = \frac{v_0^2(r)}{2} \quad \text{(B.4a)}$$

and

$$K_k(r) = (u_k^2(r) + v_k^2(r))/4, \quad \text{(B.4b)}$$

where $u_k$ and $v_k$ represent radial and tangential velocity components of wave number $k$ computed using Eqs. (B.1) and (B.2). The temporal budget used in the present study is simply the difference between the radial functions of wave-kinetic energy at two successive analysis times, i.e. $\Delta K = K(t + \Delta t) - K(t)$.

**APPENDIX C**

**Solution of Equation (4)**

The right-hand side of Eq. (4) is defined as $F(x', y')$, where $x' = (x', y') = x - ct$ is the Cartesian position vector in the moving frame, $x$ the Cartesian position vector in the fixed frame and $t$ is time. Note that the drift speed vector $c$ is assumed to be constant. For wave numbers $(k, l)$ in the $x'$- and $y'$-direction, respectively, $F(x', y')$ can be expressed in Fourier space as

$$F(x', y') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{F}(k, l) e^{i(kx'+ly')} \, dk \, dl, \quad \text{(C.1)}$$

with the inverse transform

$$\hat{F}(k, l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x', y') e^{-i(kx'+ly')} \, dx' \, dy'. \quad \text{(C.2)}$$

The same procedure is applied to the stream function perturbation $\psi'$ and its Fourier transform $\hat{\psi}'$. Algebraic manipulation of Eq. (4) in Fourier space leads to an equation of the form

$$\frac{\partial \hat{\psi}'}{\partial t} - i \frac{P(k, l)}{k^2 + l^2} \hat{\psi}' = -\frac{\hat{F}(k, l)}{k^2 + l^2}, \quad \text{(C.3)}$$

where the coefficient function $P$ is defined by $P(k, l) = (k^2 + l^2)(ke_x + lke_y) + \beta k$, with $\beta$ being the meridional derivative of the Coriolis parameter. The general solution of Eq. (C.3) can be obtained as a linear combination of an inhomogeneous (stationary) solution and the general solution of the homogeneous equation with the right-hand side of Eq. (C.3) set to zero, using the substitution $\hat{\psi}_h'(k, l, t) = \psi_0(k, l) \exp(-i\sigma t)$. Following the arguments of Lighthill (1967), Eq. (C.3) has no solutions for real $k, l$, and the imaginary part of $\sigma$ positive. Hence, no unstable growing solutions exist. The stationary solution is

$$\hat{\psi}_h'(k, l) = -\frac{i \hat{F}(k, l)}{P(k, l)} \quad \text{(C.4)}$$

Insertion of the expression for the homogeneous solution $\hat{\psi}_h'$ yields a dispersion relation of the form $\sigma(k, l) = -P(k, l)/(k^2 + l^2)$ and the amplitude factor $\psi_0(k, l)$ can be obtained by application of the initial condition that the perturbation is zero for $t = 0$, i.e. $\psi'(x', y', t = 0) = 0$. Evaluation of the integral (C.1) for $\psi'$ instead of $F$ gives

$$\psi_0(k, l) = \frac{i \hat{F}(k, l)}{P(k, l)}, \quad \text{(C.5)}$$
which leads to the general solution of Eq. (4) of the form

\[ \psi'(x', y') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{i \tilde{F}(k, l)}{P(k, l)} (e^{-i\omega t} - 1) e^{i(kx + ly)} \, dk \, dl. \tag{C.6} \]

The integrals in Eq. (C.6) are evaluated numerically using Simpson's rule. The real part of Eq. (C.6) represents the solution in real space.

**References**


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