Frontogenesis and the development of secondary wave cyclones in FASTEX

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(Received 22 April 1998; revised 17 July 1998)

SUMMARY

Two major mechanisms of frontogenesis—deformation and shear—are important in frontal wave cyclone development. Horizontal deformation can suppress the nonlinear wave development. Using an analytic model, Bishop and Thorpe have shown that large strain rates inhibit any wave-slope amplification. For real cases, this ambient strain can be measured using the vorticity–divergence attribution method developed by Bishop. This technique permits us to confirm the crucial role of such strain on the evolution of cases of wave development during the Fronts and Atlantic Storm Track EXperiment (FASTEX).

Horizontal shear in the presence of an along-front thermal gradient is also an important mechanism of frontogenesis. Using an Eady model, Joly and Thorpe have shown that, in cases of large along-front thermal gradient, frontal waves have growth rates smaller than the front itself, and thus would not develop. The domain-independent attribution method developed by Bishop is here extended to a geopotential-field partition. This leads, via a nonlinear balance condition, to the estimation of the ambient along-front potential-temperature gradient. The role of such an along-front potential-temperature gradient is discussed, as well as the relative contributions of the two frontogenesis mechanisms for the FASTEX cases.

KEYWORDS: Deformation Frontogenesis Secondary wave Shear Strain

1. INTRODUCTION

Secondary lows are formed from parent lows (often resulting from baroclinic instabilities), and are commonly found on the trailing cold fronts of mature depressions. These secondary systems occur on the mesoscale, with typical wavelengths of order 1000 km. Sometimes these events are associated with rapid cyclogenesis over the ocean, developing over less than one day, leading to damaging weather on the western sides of continents. Even frontal waves that remain rather weak in terms of pressure anomalies lead to substantial variability in, for example, the distribution and intensity of precipitation events. Failure to predict these weather systems adequately is common even in short-period forecasts (Thorpe and Shapiro 1995).

Some theoretical studies have investigated hypotheses for the development of secondary waves. For instance, instabilities at lower levels can be found if the frontal zone includes a low-level maximum in potential vorticity (Joly and Thorpe 1990). The amplitudes of low-level baroclinicity and potential-vorticity anomalies result from internal heating due to cloud generation during frontogenesis. Vertical coupling in a baroclinic flow between edge waves at the ground and at the tropopause can also lead to baroclinic instability. This interaction between upper and lower-tropospheric features is familiar from primary cyclogenesis. In the case of secondary waves, this vertical coupling often seems to be present, but other mechanisms may dominate the development (see Parker (1998) for a dynamical perspective of current ideas). Among them, the action of the environmental flow is crucial in forming and intensifying fronts through two major mechanisms of frontogenesis—deformation and shear.

Horizontal deformation can suppress the nonlinear wave development. Using an analytic model, Bishop and Thorpe (1994, hereafter BT) have shown that large strain rates inhibit wave-slope amplification, whereas lower strain rates allow large growth of barotropic frontal waves. This role of such a straining environmental flow has been confirmed for several analysed cases by Renfrew et al. (1997), as well in a case study by Rivals et al.

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(1998). Horizontal shear in the presence of an along-front thermal gradient is also an important mechanism of frontogenesis. Using an Eady model, Joly and Thorpe (1991) have shown that, in cases of large along-front potential-temperature gradient (or, equivalently, ambient vertical shear), frontal waves have growth rates smaller than the front itself, and thus do not develop. No observational study has yet confirmed the typical magnitude and role of this vertical shear.

Linkages between model analyses and various theoretical frameworks are explored here by distinguishing the ambient flow from the instability itself through the attribution concept. The way to achieve this partition is to imagine that the flow field obtained after ‘removing’ the instability is attributed to the ambient flow in which the instability is embedded, and thus the difference between the observed and the ambient flow is attributable to the anomaly. The technique used in this study is the vorticity–divergence attribution developed by Bishop (1996a). It provides a wind partition and, thus, the ambient strain rate. The domain-independent Bishop attribution method is extended here to the partition of the geopotential field. This leads, via the nonlinear balance condition, to the estimation of the ambient along-front potential-temperature gradient, and hence the vertical shear.

Frontal-wave examples used here come from the Fronts and Atlantic Storm Track Experiment (FASTEX). This experiment was designed to investigate the structure, evolution and dynamics of frontal cyclones in the North Atlantic region (Thorpe and Shapiro 1995; Joly et al. 1997). It focused on the generation of storms towards the eastern end of the Atlantic storm track. The ultimate objective of FASTEX was to advance the scientific understanding necessary to enable detailed diagnosis and prediction of the life cycles of eastern Atlantic oceanic storms and their associated cloud and precipitation systems. It was the largest mesoscale field study for making multiple observations at different stages of the life-cycles of synoptic systems yet undertaken. The FASTEX campaign took place during January and February 1997, and included 19 Intensive Observing Periods (IOPs). Among them, 8 IOPs have been selected for study in this paper; secondary waves develop in some of these cases but not in others. The data used for the study were obtained from analyses from the UK Meteorological Office Unified Limited Area Model (LAM) data assimilation system. The analyses were processed four times daily at 00 GMT, 06 GMT, 12 GMT and 18 GMT. The LAM was run at a resolution of 0.44° latitude (i.e. approximately 50 km grid resolution) with 19 vertical levels; however, the data used in this study came from a data base in which the LAM fields were routinely stored at a coarser resolution of 100 km.

Section 2 presents the attribution technique and its extension to the geopotential field. The selected IOPs are presented in section 3. The analysis of the frontogenesis function and its partition is carried out in section 4. Section 5 investigates the relation between frontogenesis and wave development. Conclusions are given in section 6.

2. Domain-independent attribution

(a) Horizontal flow

A domain-independent vorticity and divergence attribution technique has been developed by Bishop (1996a). The approach uses free-space Green’s functions to solve Poisson equations to obtain the stream function from the vorticity and the velocity potential from the divergence. These remove the boundary conditions to infinity and so give a certain domain independence to the solutions. It also permits a unique and unambiguous three-component partition of the observed wind, $\mathbf{u}_{\text{obs}}$, on a finite domain.

To summarize the method, each grid point of the domain is considered as a disc source of uniform vorticity/divergence calculated from circulation/flux estimates of the
neighbouring grid points. Each element of vorticity/divergence is therefore associated with a unique wind field obtained via the Poisson equation solutions. The flow obtained by summing the winds induced by all the elements of vorticity in the finite domain is defined as the rotational flow $u_\phi$. Similarly, summing all the elements of divergence in the finite domain gives the divergent flow $u_\chi$. As the domain is finite there is also a remainder flow—the so-called harmonic part of the wind $u_\psi$. It is defined by: $u_\psi = u_{obs} - u_\phi - u_\chi$ and is due to vorticity and divergence outside of the domain. The harmonic wind is non-divergent and irrotational in the finite domain, and so can be represented as either a stream function or a velocity potential. It can be found by solving a Laplace equation for its associated stream function in the domain, subject to Neumann conditions on the boundaries. Note that the reconstruction of the wind field is highly accurate. Root-mean-square and maximum errors are typically 0.1 m s$^{-1}$ and 0.25 m s$^{-1}$, respectively.

Bishop (1996b) partitioned the observed flow into a frontal part and an ambient part by defining a box around a limited part of the flow. The frontal flow, $u_f$, is defined as the wind induced by vorticity, $u_{\psi_f}$, and divergence, $u_{\chi_f}$, inside the box. The ambient flow, $u_a$, is the remainder flow, i.e. the flow induced by vorticity and divergence outside the frontal box, and the so-called harmonic part due to flow outside the finite domain.

The frontal box used here is a rectangular box defined by the vorticity-anomaly strip. In this study it is determined in such a way that the box has to enclose the strip of vorticity, with the edges parallel to the strip. It is this division into frontal and ambient flows that allows a comparison with fronts described in the theoretical models, and their division into perturbation parts and balanced basic states. An example of the motivation for this partition is that, as the ambient flow is defined to be irrotational and non-divergent inside the box, it can be compared locally to an idealized deformation background flow (which is irrotational and non-divergent everywhere). One key measurement of the deformation role of the ambient flow is the ambient along-front stretching, or the strain, defined as $\partial v_\phi/\partial y_b$ ($u$ and $v$ are the across- and along-front components of the wind). The coordinates $x_b$ and $y_b$ refer to the across- and along-front directions respectively. $y_b$ also refers to the long sides of the strip inside the rectangular box. This strain has been compared with its counterpart, the large-scale deformation used in idealized models (such as that of BT). Note that all the variables used hereafter are averaged in the central part of the frontal box, avoiding contributions of frontal vorticity and divergence lying close to the border of the frontal box. A sensitivity study by Renfrew (1995) has shown that the strain rate varies by 10% when the frontal box size and angle were varied by $\pm 25\%$ in width and $\pm 3^\circ$ in angle.

(b) *Extension to the geopotential*

Assuming a balance condition, it is possible to retrieve the geopotential, $\phi$, corresponding to a certain wind field. We use the nonlinear balance equation

$$\nabla^2 \phi = f \left\{ \nabla^2 \psi + \frac{2}{f} (\psi_{xx} \psi_{yy} - \psi_{xy}^2) \right\},$$

where $\psi$ is a stream function, $f$ the Coriolis parameter (taken as constant), and $\phi_{\psi}$ is that part of the geopotential field given by nonlinear balance. This equation is solved in an analogous way to the Poisson equations for vorticity and divergence, as was done by Bishop (1996a), to afford a domain-independent solution. Each grid point of the domain is considered as a disc source of `pseudo-vorticity', $\tilde{\xi}$, defined as

$$\tilde{\xi} = \nabla^2 \psi + \frac{2}{f} (\psi_{xx} \psi_{yy} - \psi_{xy}^2),$$

(1)
Figure 1. (a) Observed geopotential, (b) reconstructed geopotential, and (c) difference between the reconstructed and the observed geopotentials at 900 hPa at 18 GMT 16 February 1997. The units are $10^3$ m$^2$ s$^{-2}$, and the contours are every $2 \times 10^2$ m$^2$ s$^{-2}$ for (a) and (b), and every $10^3$ m$^2$ s$^{-2}$ for (c). The position of the frontal box used for the frontogenesis diagnostics is shown as a grey rectangle.

such that the nonlinear balance equation is transformed into

$$\nabla^2 \phi = f \tilde{\xi}. \quad (2)$$

Each element of pseudo-vorticity in the domain is therefore associated with a unique mass field (as described by the geopotential $\phi_\psi$). There is also a remainder part of the observed geopotential, $\phi_{\text{obs}}$, defined by $\phi_\psi = \phi_{\text{obs}} - \phi_\psi$, which is mainly due to pseudo-vorticity sources outside the domain, and not solely because the nonlinear balance neglects the divergent part of the horizontal velocity. By analogy with Bishop method, the harmonic part is found by solving the Laplace equation for its associated pseudo stream function in the domain. The observed geopotential can then be partitioned into a frontal and an ambient part, in a similar way to the wind partition.

The accuracy of the geopotential-field reconstruction depends on the validity of the nonlinear balance assumption. Over the domain considered here, the root-mean-square and the maximum error are typically $1 \times 10^2$ and $5 \times 10^2$ m$^2$s$^{-2}$, respectively, but there are some large spatial variations (see Fig. 1 showing an example at 900 hPa at 18 GMT 16 February 1997). Indeed, the use of the stream function in the terms on the right-hand side of the balance equation means that only the vorticity-induced winds are considered. It leads to a reconstructed geopotential that is less deep than the observed one in the low-centre regions, where divergent winds are not negligible. The effects of orography (e.g. over Norway, the Alps, etc.) on geopotential are also not caught by the nonlinear
TABLE 1. CLASSIFICATION OF THE LOWS AS DEVELOPING OR NON-DEVELOPING CASES

<table>
<thead>
<tr>
<th>Low</th>
<th>IOP</th>
<th>Date</th>
</tr>
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<tbody>
<tr>
<td>19a</td>
<td>5</td>
<td>20–21 Jan 1997</td>
</tr>
<tr>
<td>22b</td>
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<td>25–26 Jan 1997</td>
</tr>
<tr>
<td>27</td>
<td>9</td>
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</tr>
<tr>
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<td>17</td>
<td>18–19 Feb 1997</td>
</tr>
<tr>
<td>46</td>
<td>19</td>
<td>26–27 Feb 1997</td>
</tr>
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</table>

Developing cases

<table>
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<tr>
<th>Low</th>
<th>IOP</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>19b</td>
<td>5</td>
<td>21–22 Jan 1997</td>
</tr>
<tr>
<td>33</td>
<td>11a</td>
<td>7–8 Feb 1997</td>
</tr>
<tr>
<td>34a</td>
<td>12</td>
<td>9 Feb 1997</td>
</tr>
<tr>
<td>39a</td>
<td>16</td>
<td>16–17 Feb 1997</td>
</tr>
</tbody>
</table>

IOP: Intensive Observing Period

balance. However, Fig. 1 also displays the location of the frontal box used in this study. The maximum error there is around \(1 \times 10^2 \text{ m}^2 \text{s}^{-2}\) corresponding to a 1% error. In this study the frontal boxes are typically located in the central part of the North Atlantic, i.e. far away from any land and from any parent low. The accuracy would be poorer for waves developing near low centres.

From \(\phi\), the potential-temperature field, \(\theta\), can be found by making the Boussinesq hydrostatic assumption:

\[
\theta = \frac{\theta_0}{g} \frac{\partial \phi}{\partial z}.
\]

From Eqs. (1) and (2) it is clear that the potential-temperature field is proportional to minus the vertical gradient of the pseudo-vorticity. For the frontal zone the pseudo-vorticity predominantly decreases with height, and so the frontal potential-temperature anomaly is a maximum in the front. Then, differentiating in the horizontal, the potential-temperature gradients corresponding to certain parts of the flow are obtained:

\[
\frac{\partial \theta}{\partial x_b} = \frac{\theta_0}{g} \frac{\partial}{\partial x_b} \left( \frac{\partial \phi}{\partial z} \right) \quad \text{and} \quad \frac{\partial \theta}{\partial y_b} = \frac{\theta_0}{g} \frac{\partial}{\partial y_b} \left( \frac{\partial \phi}{\partial z} \right).
\]

This allows the ambient along-front potential-temperature gradient, \(\partial \theta_b/\partial y_b\), to be computed. The latter is compared later in this paper with the three different regimes studied by Joly and Thorpe (1991). It is clear that, because the frontal potential-temperature anomaly is a simple warm core, the along-front potential-temperature gradient due to the front will exhibit a dipole pattern.

3. SELECTION AND CLASSIFICATION OF THE LOWS

Among the FASTEX IOPs, nine secondary waves from eight IOPs have been identified and selected. The waves have been classified as either developing or non-developing cases, following the time evolution of a vorticity parameter described in section 5. Table 1 summarizes the FASTEX IOPs used here and their classification as developing or non-developing cases.

As an example, IOP 16 surveyed low 39a, a fast moving frontal wave which developed on the trailing cold front of low 39. At 18 GMT 16 February 1997 this wave had a distinct
Figure 2. Satellite imagery with mean-sea-level pressure (MLSP) contours every 4 hPa and objective frontal analyses, for Intensive Observing Period 16 at 12 GMT 17 February 1997. The grey-scale background is the infrared Meteosat image with brightness temperatures given in °C. The thin black contours are MSLP. Red and blue lines are objective fronts analysed from the model fields (Hewson 1996)—red indicates warm fronts, blue cold fronts, with broken lines representing upper fronts.

Figure 3. As Fig. 2 but for Intensive Observing Period 9 at 00 GMT 3 February 1997.
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900 hPa relative-vorticity signature (with an average relative vorticity of around $0.9 \times 10^{-4}$ s$^{-1}$), had a strong low-level vorticity strip, and was embedded in a strong ambient flow. Two low centres were situated over Iceland (low 38) and Greenland (low 39)—see Fig. 1. The frontal wave was located to the south-west and ran around the flank of the parent system to become, at a later stage, the low 39a. From 00 GMT to 12 GMT 17 February, the wave deepened rapidly in pressure (33 hPa in 12 hours) as it moved into the parent low centre. It developed the typical cloud head of a deepening Atlantic storm depression with associated very heavy precipitation, pronounced dry intrusion, and strong winds (40 m s$^{-1}$ at 900 hPa) over Ireland (see Fig. 2). The mean-sea-level pressure (MSLP) fell to 944 hPa by 00 GMT 18 February with $\xi = 2.4 \times 10^{-4}$ s$^{-1}$ over the Faeroe Islands. The first closed isobars appeared at 18 GMT 17 February off the Hebrides, making low 39a clearly a developing case.

On the other hand, if no cyclone ultimately developed, the case was considered as non-developing. For instance, IOP 9 surveyed low 27, a large well-developed cyclone which moved off the north-east coast of the USA with a cloud head and a dry slot (having a scale of 2500 km). The interesting feature here was its trailing cold front, embedded in a deformation flow, between 18 GMT 2 February and 18 GMT 3 February (see Fig. 3). The front had a well defined potential-vorticity strip over the whole period (around 0.6 PVU* at 900 hPa). At 18 GMT 3 February, the wave was over Ireland with no strong winds or heavy rain reported. The secondary wave associated with low 27 has, therefore, been classified as being a non-developing case.

4. **Partition of the frontogenesis function**

(a) **Partition according to the wind division**

Because of the importance of frontogenesis in the development of frontal waves, we have analysed in detail the various contributions to the frontogenesis in each of the cases being studied. The primary goal was to quantify the relative magnitudes of the various dynamical mechanisms leading to frontogenesis. In addition, the cases were divided into those with and those without the development of frontal waves, to permit an examination to find if there was a systematic difference between the contributions to the frontogenesis. Finally, we have related the observed values to two-dimensional frontogenesis theories. This has provided an insight into the relative importance of the shear and deformation frontogenesis paradigms.

The attribution technique permits the measurement of the frontogenetic effect of different parts of the wind field (Bishop 1996b). As a measure of frontogenetic forcing, the vector frontogenesis function $F$ (Keyser et al. 1988) is used:

$$F_{\text{obs}} = \left( \frac{\partial u_{\text{obs}}}{\partial x_b} \frac{\partial \theta}{\partial x_b} + \frac{\partial v_{\text{obs}}}{\partial x_b} \frac{\partial \theta}{\partial y_b} + \frac{\partial u_{\text{obs}}}{\partial y_b} \frac{\partial \theta}{\partial x_b} + \frac{\partial v_{\text{obs}}}{\partial y_b} \frac{\partial \theta}{\partial y_b} \right).$$

$F_{\text{obs}}$ gives the Lagrangian rate of change of horizontal potential-temperature gradient due to horizontal advection, as a vector field. An associated scalar representation of frontogenesis is given by:

$$F_{\text{obs}} = \frac{\mathbf{F}_{\text{obs}} \cdot \nabla \theta}{|\nabla \theta|} = \frac{D}{Dt} |\nabla_b \theta|,$$

where positive values indicate frontogenesis and negative values indicate frontolysis. (Note that the quasi-geostrophic form of this scalar representation of frontogenesis has been used

* Potential-vorticity unit: 1 PVU = $1 \times 10^{-6}$ m$^2$K kg$^{-1}$ s$^{-1}$
TABLE 2. TIME-AVERAGES OF THE TOTAL FRONTGENESIS FUNCTION, $F_{obs}$, AND OF THE FRONTGENESIS FUNCTION COMPONENTS $F_a$, $F_{\psi}$ AND $F_{\xi}$ (SEE TEXT) EXPRESSED AS PERCENTAGES OF THE TOTAL IN THE 900–700 hPa LAYER, TOGETHER WITH AVERAGES FOR THE NON-DEVELOPING WAVES, FOR THE DEVELOPING WAVES AND FOR ALL CASES

<table>
<thead>
<tr>
<th>Low</th>
<th>$F_{obs}$</th>
<th>$F_a/F_{obs}$</th>
<th>$F_{\psi}/F_{obs}$</th>
<th>$F_{\xi}/F_{obs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19a</td>
<td>0.9</td>
<td>48.2</td>
<td>16.4</td>
<td>35.4</td>
</tr>
<tr>
<td>19b</td>
<td>0.3</td>
<td>-0.1</td>
<td>49.6</td>
<td>50.6</td>
</tr>
<tr>
<td>22b</td>
<td>0.8</td>
<td>77.4</td>
<td>1.1</td>
<td>21.6</td>
</tr>
<tr>
<td>27</td>
<td>0.3</td>
<td>56.6</td>
<td>27.9</td>
<td>15.5</td>
</tr>
<tr>
<td>33</td>
<td>0.2</td>
<td>36.6</td>
<td>19.9</td>
<td>43.4</td>
</tr>
<tr>
<td>34a</td>
<td>0.4</td>
<td>-14.3</td>
<td>56.0</td>
<td>58.2</td>
</tr>
<tr>
<td>39a</td>
<td>0.6</td>
<td>24.2</td>
<td>28.4</td>
<td>47.4</td>
</tr>
<tr>
<td>41 rear</td>
<td>0.7</td>
<td>66.4</td>
<td>9.1</td>
<td>24.5</td>
</tr>
<tr>
<td>46</td>
<td>0.5</td>
<td>53.1</td>
<td>7.1</td>
<td>39.8</td>
</tr>
</tbody>
</table>

Non-developing cases (19a, 22b, 27, 41 rear, 46): $0.7 \pm 0.2$ $60.3 \pm 10.4$ $12.3 \pm 9.2$ $27.4 \pm 9.0$

Developing cases (19b, 33, 34a, 39a): $0.4 \pm 0.1$ $11.6 \pm 20.0$ $38.5 \pm 14.8$ $49.9 \pm 5.4$

All cases: $0.5 \pm 0.2$ $38.7 \pm 28.7$ $23.9 \pm 17.7$ $37.4 \pm 13.5$

The units of the total frontogenesis function, $F_{obs}$, are K (100 km)$^{-1}$(6 h)$^{-1}$. The standard deviations computed for each group of cases are shown after the ± signs.

widely in the past, for example Ruscher and Condo (1996).) Substituting the above wind partition, $F_{obs}$ can be split into three parts such that it is the sum of the frontogenesis-function components $F_{\psi}$ (due to the frontal rotational flow), $F_{\xi}$ (due to the frontal divergent circulation), and $F_a$ (due to the ambient flow). This partitioning, which allows the evaluation of the most important mechanisms, has also been examined by Renfrew (1995) and Rivals et al. (1998).

At each of the 6-hour time steps of the studied cases, the observed frontogenesis functions have been computed at the 900 and 700 hPa levels as well as over the 900–700 hPa layer. As expected for the low-level features studied here, the frontogenesis was always larger at 900 hPa than at 700 hPa, except for low 19a. In this case the frontogenesis functions were of the same order at the two levels, owing to a large deformation flow as the front is stretched out along an upper-level trough. Hence the analysis of the partition of the frontogenesis function has been made as an average over the 900–700 hPa layer. In this layer each component of the frontogenesis function has been calculated and averaged in time for each low, with additional averages being computed for the developing waves, for the non-developing waves, and for all cases (see Table 2).

First of all, the observed frontogenesis function is notably larger (by a factor 1.8) for the non-developing cases than for the developing ones (these figures having relatively small standard deviations). But the non-developed low 27 shows small values of frontogenesis compared with those of lows 34a and 39a, for which a wave has developed. The frontogenesis intensity does not, therefore, invariably permit a prediction of the developing behaviour of a frontal wave.

Nevertheless, the percentage of the total frontogenesis due to the ambient flow distinguishes well the developing cases from the non-developing ones. It is a factor of 5 larger for the latter than for the former (these figures having comparatively larger standard deviations).
For the non-developing cases, the frontogenesis due to the ambient flow typically accounts for 60% of the total. In the context of the BT theory, the non-development of the lows is due to a large gradient of the ambient flow in the along-front direction which squashes the wave along the front. It is consistent, therefore, to conclude that $\mathcal{F}_a$ takes an important part in the frontogenesis for non-developing cases. Moreover, as the wave is not developing, it generates rather small frontal winds with consequently rather weak wind gradients, giving relatively small frontogenesis due to the frontal flow.

On the other hand, for the developing cases the frontogenesis due to the frontal divergent flow is dominant (nearly 50% of the total). Again, from the same theoretical background, the wave development is allowed in a context of small deformation strain. This is consistent with the lesser importance of the ambient part of the frontogenesis, especially compared with the frontogenesis due to an enhanced frontal divergent circulation, as the wave develops into a cyclone. So one might be able to diagnose the development of secondary cyclones by looking at this frontogenesis-function partition. (However note the discussion in section 5(a) which shows that, when considering the time evolution of these parameters, this discrimination is far less clear.)

Finally, the contribution of the frontal rotational wind is the least important term, accounting for around 20% of the total frontogenesis function for all cases (but it dominates the along-front component of $\mathcal{F}$.) The role of the horizontal wind shear associated with the vorticity of the front is negligible compared with the other parts, especially for the non-developing cases. This is true except for lows 19b and 34a, for which the weakness of a deformation flow is notable. So, to simplify the following potential temperature partition, the frontogenesis function is, in what follows, partitioned into $\mathcal{F}_f$, the frontal component, and $\mathcal{F}_a$, the ambient component.

(b) **Partition according to the wind and the potential-temperature division**

The geopotential extension of the domain-independent attribution also leads to an extended partition of the frontogenesis function into a layer (here 900–700 hPa) involving $\theta_a$, the ambient potential temperature, and $\theta_f$, the potential temperature generated within the frontal box (hereafter referred to as the frontal potential temperature). Therefore, the frontogenesis vector is divided into four parts:

- $\mathcal{F}_{a\theta}$, the action of the ambient winds on the ambient potential-temperature gradients,
- $\mathcal{F}_{a\theta_f}$, the action of the ambient winds on the frontal potential-temperature gradients,
- $\mathcal{F}_{f\theta}$, the action of the frontal winds on the ambient potential-temperature gradients,
- and $\mathcal{F}_{f\theta_f}$, the action of the frontal winds on the frontal potential-temperature gradients.

Scalar multiplication of each frontogenesis vector by $\nabla \theta / \| \nabla \theta \|$ then leads to their respective scalar representations $\mathcal{F}_{a\theta}$, $\mathcal{F}_{a\theta_f}$, $\mathcal{F}_{f\theta}$, and $\mathcal{F}_{f\theta_f}$, with their sum equalling the observed frontogenesis.

Table 3 presents the frontogenesis-function partition in the 900–700 hPa layer time-averaged in the same way as for Table 2. For all cases, the frontogenesis partition given by the partition of $\theta$ shows that most of the total $\theta$ gradient comes from its ambient part. The effect of the frontal potential-temperature gradient on the frontogenesis is, therefore, relatively small. However $\mathcal{F}_{a\theta_f}$, which accounts for nearly 10% of the total, is not completely negligible and $\mathcal{F}_{f\theta}$ is of the same order of magnitude, but frontolytic.

The contribution of $\mathcal{F}_{a\theta}$ to the total frontogenesis is significantly larger (by a factor of five) for the non-developing cases than for the developing ones. For the non-developing
TABLE 3. Time-average of frontogenesis function components $F_{adv}$, $F_{vot}$, $F_{10m}$ and $F_{f0}$ (see text) expressed as percentages of the total in the 900–700 hPa layer, together with averages for the non-developing waves, for the developing waves and for all cases

<table>
<thead>
<tr>
<th>Low</th>
<th>$F_{adv}/F_{obs}$</th>
<th>$F_{vot}/F_{obs}$</th>
<th>$F_{10m}/F_{obs}$</th>
<th>$F_{f0}/F_{obs}$</th>
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<tbody>
<tr>
<td>19a</td>
<td>30.3</td>
<td>18.0</td>
<td>33.3</td>
<td>18.5</td>
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<tr>
<td>19b</td>
<td>1.9</td>
<td>-2.0</td>
<td>105.8</td>
<td>-5.7</td>
</tr>
<tr>
<td>22b</td>
<td>68.0</td>
<td>9.4</td>
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<td>-2.9</td>
</tr>
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<td>6.4</td>
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<td>-49.6</td>
</tr>
<tr>
<td>34a</td>
<td>-19.2</td>
<td>4.9</td>
<td>126.9</td>
<td>-12.6</td>
</tr>
<tr>
<td>39a</td>
<td>21.0</td>
<td>3.2</td>
<td>64.1</td>
<td>11.7</td>
</tr>
<tr>
<td>41 rear</td>
<td>64.7</td>
<td>1.6</td>
<td>94.5</td>
<td>-60.8</td>
</tr>
<tr>
<td>46</td>
<td>40.6</td>
<td>12.5</td>
<td>44.9</td>
<td>2.0</td>
</tr>
<tr>
<td>Non-developing cases (19a, 22b, 27, 41 rear, 46)</td>
<td>48.7 ± 14.9</td>
<td>11.7 ± 5.9</td>
<td>49.4 ± 23.6</td>
<td>-9.7 ± 26.9</td>
</tr>
<tr>
<td>Developing cases (19b, 33, 34a, 39a)</td>
<td>8.5 ± 18.9</td>
<td>3.1 ± 3.2</td>
<td>102.4 ± 23.4</td>
<td>-14.1 ± 22.4</td>
</tr>
<tr>
<td>All cases</td>
<td>30.8 ± 26.1</td>
<td>7.9 ± 6.5</td>
<td>72.9 ± 53.5</td>
<td>-11.6 ± 25.1</td>
</tr>
</tbody>
</table>

The standard deviations computed for each group of cases are shown after the ± signs.

cases, the four-term partition shows that $F_{adv}$ and $F_{vot}$ both account for nearly 50% of the total while the 10% due to $F_{10m}$ is roughly counterbalanced by the negative $F_{f0}$.

For the developing cases, $F_{10m}$ is dominant (nearly 100%). So the most important mechanism of frontogenesis in developing cases is clearly the action of the frontal flow on the ambient potential-temperature gradient. Another interesting feature is that the frontogenesis due to the action of the frontal wind on the frontal potential-temperature gradient is frontolytic. As the wave is developing into a cyclone, it acts to remove the front itself. Note also that $F_{adv}$ is frontolytic in the case of low 34a. This has an ill-defined relative-vorticity anomaly at an early stage, which later rapidly amplifies because of the large interaction with an upper-level anomaly. The low-level feature, which is partly frontolytic, appears to be less important in this case.

(c) Purely two-dimensional frontogenesis

The two paradigms mostly used in the theoretical works on frontogenesis are based on two-dimensional models. This is because of the frequently observed elongated aspect of fronts. Consistently for the cases studied here, the across-front component of $F$ accounts for nearly 90% of the total magnitude. This is true except for low 22b for which the acrossfront term is only 80% of the total frontogenesis because of the presence of two parallel fronts. It follows, therefore, that the partition of the across-front term shows the same percentages as shown in the previous subsections.

In the two-dimensional theory, the deformation paradigm is based on the action of a steady horizontal flow on a purely cross-front potential-temperature gradient (see, for example, BT). The flow will tend to advect the potential-temperature field so that the across-front potential-temperature gradient increases. If this paradigm applies, the first part of the across-front component of the frontogenesis vector $F_{adv}$, i.e. the term $D$ defined
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by

\[ D = -\frac{\partial u_a}{\partial x_b} \frac{\partial \theta_a}{\partial x_b}, \]

will be dominant over the second part. The term \( D \) effectively accounts for 90% of the across-front component of \( F_{\theta a} \) in the non-developing cases (i.e. between 80 and 95%) and for 145% in the developing cases (i.e. between 100 and 195%). It shows the validity of the deformation paradigm.

However, as seen in the previous section, the action of the ambient flow on the ambient potential-temperature gradient, (and the deformation frontogenesis as just defined) only accounts for around 50% of the total frontogenesis for the non-developing cases and a smaller percentage for the developing cases. The other important part is \( F_{\theta b} \), the part of the frontogenesis due to the action of the frontal winds on the ambient potential-temperature gradients. Its link with the shear paradigm is now explored.

The shear frontogenesis paradigm assumes the Eady basic state, i.e. an ambient sheared zonal flow in thermal wind balance with a meridional potential-temperature gradient. The simplest form of the paradigm assumes that the zonal part of the frontal wind is always zero. At the initial stage the cross-front potential-temperature gradient is zero, but grows in time. If this paradigm applies then the second part of the across-front component of the frontogenesis vector \( F_{\theta b} \), i.e. the term \( S \), would be dominant, where

\[ S = -\frac{\partial v_f}{\partial y_b} \frac{\partial \theta_a}{\partial x_b}. \]

For all cases, the term \( S \) is of order 60% of the across-front component of \( F_{\theta b} \) (i.e. between 30 and 75% in the non-developing cases, and between 50 and 80% in the developing cases).

It is of interest to estimate the relative importance of the deformation and shear mechanisms. This is shown in Table 4 by the ratio of \( D/S \). The figures show that the term \( D \) is about 2.4 times larger than the term \( S \) for the non-developing cases while it accounts for 40% of the term \( S \) for the developing cases. Whilst the standard deviations are large, the ratio of \( D/S \) is a factor of 6.2 larger for the non-developing cases than for the developing ones. In addition, in Table 4 we show the typical magnitudes of the deformation and along-front thermal gradient that are the input constants used as in two-dimensional frontogenesis theory.

5. FRONTOGENESIS AND WAVE DEVELOPMENT

In the previous section, the waves have been classified as either developing or non-developing cases. The time evolution of the wave development associated with the partition analysis should give a more accurate and complete diagnosis. The measure of the frontal wave growth is characterized by the vorticity waviness, defined by Renfrew et al. (1997) as the peak vorticity minus the maximum value of the along-front average of the vorticity field in the rectangular box. It is a measure of how developed any frontal wave might be. Vorticity waviness is relatively insensitive to small changes in the along-front direction. However, it is sensitive to the size of the frontal region specified, such as the strip length. Care needs to be taken that this strip length is defined consistently through the instability evolution.

(a) Correlation of development with frontogenesis

The growth rate in Fig 4 is the rate of change of the vorticity waviness calculated by differencing the six-hourly sampled values. The corresponding \( F_{\theta a}/F_{\theta b} \) is taken as
TABLE 4. Time-average of the ratio $D/S$, the deformation, $\partial u_1 / \partial x_2$, and the along-front thermal-gradient contribution to frontogenesis, $\partial \theta_a / \partial y_b$ (see text), together with averages for the non-developing waves, for the developing waves and for all cases.

<table>
<thead>
<tr>
<th>Low</th>
<th>$D/S$</th>
<th>$\partial u_1 / \partial x_2$</th>
<th>$\partial \theta_a / \partial y_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(%)</td>
<td>$(10^{-5} \text{s}^{-1})$</td>
<td>$(10^{-5} \text{K m}^{-1})$</td>
</tr>
<tr>
<td>19a</td>
<td>276.4</td>
<td>-1.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>19b</td>
<td>1.5</td>
<td>-0.0</td>
<td>-0.4</td>
</tr>
<tr>
<td>22b</td>
<td>329.3</td>
<td>-1.4</td>
<td>-0.2</td>
</tr>
<tr>
<td>27</td>
<td>77.3</td>
<td>-0.7</td>
<td>-0.3</td>
</tr>
<tr>
<td>33</td>
<td>34.5</td>
<td>-0.2</td>
<td>-0.2</td>
</tr>
<tr>
<td>34a</td>
<td>30.5</td>
<td>-0.6</td>
<td>-1.0</td>
</tr>
<tr>
<td>39a</td>
<td>85.6</td>
<td>-0.5</td>
<td>-0.4</td>
</tr>
<tr>
<td>41 rear</td>
<td>93.1</td>
<td>-1.1</td>
<td>-0.4</td>
</tr>
<tr>
<td>46</td>
<td>214.5</td>
<td>-0.7</td>
<td>-0.2</td>
</tr>
<tr>
<td>Non-developing cases (19a, 22b, 27, 41 rear, 46)</td>
<td>238.1 ± 163.5</td>
<td>-1.0 ± 0.3</td>
<td>-0.3 ± 0.1</td>
</tr>
<tr>
<td>Developing cases (19b, 33, 34a, 39a)</td>
<td>38.0 ± 30.3</td>
<td>-0.3 ± 0.2</td>
<td>-0.5 ± 0.3</td>
</tr>
<tr>
<td>All cases</td>
<td>149.2 ± 158.6</td>
<td>-0.7 ± 0.4</td>
<td>-0.4 ± 0.2</td>
</tr>
</tbody>
</table>

The standard deviations computed for each group of cases are shown after the ± signs.

Figure 4. Growth rate versus $F$th/Fobs, the part of the observed frontogenesis function due to the action of the ambient winds on the ambient potential temperature gradients in percentage terms (see text). For each case, values are given at six-hourly intervals with the initial value being indicated by a large open circle. Dashed lines indicate non-developing cases (open symbols) and dotted lines indicate developing cases (filled symbols). The circles indicate the first point in time for each low.
the mean of these frontogenetic-function values of those times. In the previous section the non-developing cases have shown, on average, frontogenesis to be dominated by the action of the ambient flow on the ambient potential-temperature gradients. Whilst Fig. 4 reflects this statement, it does not permit any precise distinction between developing and non-developing cases. For example the frontogenesis of the developing low 33 is also largely dominated by $\mathcal{F}_{a\partial_b}$.

(b) Ambient strain and shear

The theoretical work by BT has pointed out the role of the ambient strain on the wave development. Here the ambient strain is defined as the quantity $\partial v_a / \partial y_b$. The theory developed by BT suggests that this ambient strain should be the best parameter for discriminating development from decay of incipient frontal waves. If the theory is correct, then stratifying all the cases by the ambient strain will provide a clearer picture of development than the frontogenesis parameter we have been considering thus far in this paper. Assuming that the ambient flow is predominantly non-divergent, then $\partial v_a / \partial y_b \simeq -\partial u_a / \partial x_b$ and the typical magnitude for all cases is given in Table 4. The BT theory indicates that it is the size of the ambient strain relative to a measure of the (vorticity) strength of the front that determines development. In what follows we therefore use a normalization of the strain given by BT.

Following the work of Renfrew et al. (1997), Fig. 5 plots the growth rate against the ambient normalized strain. The ambient along-front stretching rate is taken as the mean of the six-hourly sample values, and has been normalized by the minimum strain rate $\alpha_{\text{min}}$ necessary to suppress all barotropic frontal wave growth according the BT theory. The following equation has been used:

$$\alpha_{\text{min}} = \frac{f}{4} \left( \frac{\zeta_b - \zeta_a}{\zeta_b + \zeta_a} \right) e^{-2\mu},$$
where $\zeta_I$ is the absolute vorticity averaged within the frontal box, $\zeta_a$ is the absolute vorticity of the neighbouring region, and $2\mu$ is the non-dimensional wave number of the disturbance, taken here as zero, i.e. an infinite wavelength. This normalization accounts for the different strengths of the vorticity strips in each case. The normalization implies that the strain threshold required to suppress all barotropic frontal wave growth is a normalized stretching rate of 1. All the non-developing cases show values of strain greater than 1, thus inhibiting the role of the strain, whereas frontal wave development occurs when the normalized strain is less than 1. It is clear that plotting the data in this way collapses all the data along a diagonal from high strain with decay to low strain with growth. So Fig. 5 clearly confirms the BT scenario and the observations of Renfrew et al. (1997). Furthermore, a value of unity for the normalized strain appears to split the cases into the two categories of developing and non-developing waves. However, it should be noted that if the data being used were at a higher horizontal resolution, then the normalized strain which makes this split may have a different value. It is sufficient for this study to note that the normalized strain is indeed the most critical parameter in allowing development of wave cyclones.

In Fig. 6, the normalized vertical shear is defined as the ambient along-front potential-temperature gradient divided by $-1 \times 10^{-5}$ K m$^{-1}$ (the largest value of the vertical-shear regime in the study by Joly and Thorpe (1991)). The theory indicates that, for normalized vertical shear greater than unity, a two-dimensional front amplifies faster than a three-dimensional wave. Hence, values of normalized vertical shear less than unity are necessary to allow the possibility of wave growth. The range of values of the ambient along-front potential-temperature gradient is narrow for these FASTEX cases. Most have values around $-0.2 \times 10^{-5}$ to $-0.4 \times 10^{-5}$ K m$^{-1}$, with low 34a having an exceptionally large value. These show that all non-developing cases with strong strain also have weak ambient vertical shear. This is also true for most of the developing cases in the context of low strain and agrees well the results of Joly and Thorpe (1991). In their model, only frontal waves in a
small-shear regime (i.e. a normalized shear of 0.1) have growth rates significantly greater than the front itself, on scales of 1000 km and 1.5 day.

From their results, Joly and Thorpe have also suggested that larger values of large-scale baroclinicity do not favour frontal waves. Consequently, other interactions are needed for consistency with their results, particularly for lows 19b and 34a. The presence of upper-level interactions is clear for low 34a, and possibly to a lesser degree for low 19b.

6. Conclusions

The role of frontogenesis on frontal wave development has been investigated on several FASTEX cases and compared with theory. This has been done using the attribution concept, which offers a separation, in a given synoptic situation, between the part due to the front and the part due to the frontal wave. The method used here to partition the flow is the vorticity/divergence attribution of Bishop (1996a). It leads to an evaluation of the action of deformation on the wave by calculating the strain due to the ambient flow, as well as of its importance in frontogenesis.

Firstly, the results confirm the role of the ambient strain on the frontal wave development as modelled by BT and observed by Renfrew et al. (1997). In the case of strong deformation no frontal wave development is allowed, and the ambient flow dominates the frontogenesis. When the ambient strain is weak, frontal waves can grow, in particular by the action of the ageostrophic convergent circulation. The frontogenesis has been observed to be usually more intense for the non-developing cases than for the developing ones, and largely dominated by the action of the ambient flow on the potential-temperature gradient. However, the rate of frontogenesis alone does not permit as clear a wave-development diagnostic as is obtained by considering the ambient strain.

This domain-independent attribution method has been extended to a geopotential partition by using nonlinear balance. This extension affords a partition of the frontogenesis function into four components, according to the ambient and frontal parts of the wind and potential temperature. Whilst the action of the ambient winds on the ambient potential-temperature gradient is much more important for the non-developing cases compared with the developing ones, this cannot be generalized to all the cases.

This extension also permits the evaluation of the action of the vertical shear of the ambient flow on the wave, by calculating the ambient along-front potential-temperature gradient. Results show that the ambient along-front potential-temperature gradients (with values around \(-0.2 \times 10^{-5}\) to \(-0.4 \times 10^{-5} \) K m\(^{-1}\)) are rather small compared with those used in theoretical studies (Joly and Thorpe 1991). With such values, the role of the shear is less discriminating than the action of the strain. However, some of the developing cases were associated with large shear, in disagreement with the Joly and Thorpe (1991) theory. This could be attributed to the presence of upper-level interactions, which have not been taken into account by Joly and Thorpe. Further research is needed to explain this behaviour, perhaps using a more complete numerical model.

Acknowledgement

This research has been carried out using support from a research grant from the Natural Environment Research Council (GR3/9036). The authors would like to thank Craig Bishop and Ian Renfrew for the use of computer code, and Morwenna Griffiths and Ian Renfrew for useful discussions concerning aspects of this research.
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