A microphysically based precipitation scheme for the UK Meteorological Office Unified Model

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SUMMARY

Many numerical weather prediction models are moving towards more prognostic schemes for the prediction of ice and liquid water contents within clouds. This paper describes a large-scale cloud and precipitation scheme developed for the UK Meteorological Office's Unified Model. It uses physically based transfer equations to predict ice as a prognostic variable. We review similar schemes and then describe our new scheme, giving examples of its performance in mesoscale forecasts compared with the current operational scheme and with observations. The microphysical processes occurring in a frontal cloud are well modelled. The prediction of supercooled stratocumulus cloud is much improved.

KEYWORDS: Cloud Microphysics Numerical weather prediction Precipitation

1. INTRODUCTION

Cloud and precipitation are very important features of a numerical weather prediction model. The amount, height and thickness of cloud cover play an important part in governing the air temperature through interactions with radiation. This paper assesses the mesoscale forecast performance of two precipitation schemes within the framework of the UK Meteorological Office’s Unified Model. The older scheme uses a function of temperature to partition condensate into liquid and ice; the newer scheme predicts ice and liquid water contents by a more physical method.

In order to successfully model a cloud system, the correct prediction of cloud phase is essential. Firstly, ice cloud contains larger particles than liquid cloud (due to the smaller number of ice condensation nuclei than liquid condensation nuclei in the atmosphere) and hence has a much greater tendency to fall. Bergeron (1935) predicted that, without the presence of ice in a cloud, precipitation could not form since the particles could not grow large enough. Although this is incorrect, it is true that the vast majority of rainfall in the UK occurs as a result of melting ice particles. Secondly, liquid water can, under conditions where activated ice nuclei are low in number, remain in a cloud even when the temperature is well below the thermodynamic freezing point of water. Such supercooled liquid will freeze on contact with any surface at a similarly low temperature, such as an aeroplane wing. This can produce dangerous flying conditions.

Clouds can very often be observed to undergo a life cycle, beginning as liquid and ending their lives as ice. The time-scale over which ice grows is around half an hour, over an order of magnitude slower than that for liquid water (because the number of particles is less, diffusional growth is less efficient). A model which can successfully predict a cloud life cycle should have considerable advantages over one which diagnoses the properties of a cloud based on such factors as temperature and relative humidity.

Many large-scale cloud and precipitation schemes are used for climate prediction and numerical weather prediction, most based on the work of Sundqvist (1978) who modelled precipitation by instantaneously removing some of the condensate produced by a cloud scheme. Although most schemes still remove rain water from the system in a single time step, physical processes are often allowed to act upon the falling rain.

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Lin *et al.* (1983) described an extensive parametrization for a two-dimensional cloud-resolving model. They use six water quantities and twenty-seven transfer terms. Such a complex description is clearly not possible under the time considerations of numerical weather prediction. They conclude that a snow variable is crucial in predicting the correct rainfall characteristics for a deep convective system.

Tiedtke (1993) describes a detailed prognostic scheme used operationally at the European Centre for Medium-Range Weather Forecasting. It has prognostic equations for the grid-box mean cloud fraction and condensate but relies on a diagnostic temperature-dependent split between the ice and liquid. The large-scale precipitation mechanisms are based almost directly upon the Sundqvist formulation.

Lohmann and Roeckner (1996) used a prognostic representation of cloud liquid and cloud ice. Precipitation quantities were treated diagnostically, and cloud fractions were calculated as a function of relative humidity. This scheme produced an increase in the ice-water path predicted by a general circulation model (GCM).

Fowler *et al.* (1996) used five water quantities in a GCM: vapour; cloud water; rain water; cloud ice; and snow. They formulated a large-scale cloud and precipitation scheme similar to that of a cloud-resolving model. Detrainment of ice from cumulus clouds provided a link between the ice in the convection and large-scale parametrization schemes. They showed that satisfactory distributions of ice and liquid water contents could be modelled. Detrainment of ice from tropical convection to the large-scale cloud and precipitation scheme produced improvements in their tropical-radiation budget. They also demonstrated that changes in some of the microphysical parameters, such as the autoconversion threshold, could produce large changes in the radiation budget. However, they did not include any subgrid-scale cloud fraction or account for its effects.

Ghan *et al.* (1997) described GCM results with a microphysics scheme which included prognostic values for cloud ice and an ice-number concentration. They showed that the ice-number concentration was an important factor in determining the climate of their model, but it was computationally expensive to include. They did not include a cloud fraction parametrization.

Rotstayn (1997) described a model based on vapour, cloud water and cloud ice with diagnostic rain and snow quantities. It also produced realistic water contents in a GCM. A major advantage of this scheme is its treatment of cloud fraction. By modifying the Smith (1990) scheme, Rotstayn was able to obtain an estimate of both the liquid and ice cloud fractions. Assumptions could then be made to obtain a consistent description of the vertical overlap of the liquid and ice cloud fractions. However, the treatment assumed that ice clouds form to remove all supersaturation with respect to ice in a grid box. The measurements of Heymsfield and Milosevich (1995) clearly show this to be a poor assumption, with ice taking a considerable time to remove supersaturation. At mesoscale resolution this is likely to be a significant factor, hence the need to nucleate and grow ice as a rate term.

The current cloud scheme employed at the UK Meteorological Office is a diagnostic scheme, described by Smith (1990). It diagnoses only the amount of condensate in a grid box, and the fraction of the grid box covered by the condensate. The scheme does not predict the phase of the condensate. This is diagnosed using a linear temperature function. If condensate is warmer than 0 °C then the condensate is assumed to be liquid. If it is colder than −9 °C then it is assumed to be ice. For intermediate temperatures a linear fit is made for the fraction of condensate which is liquid. Although in a statistical sense, as shown by the aircraft sampling of Moss and Johnson (1994), this is a reasonable assumption to make, it cannot be used when considering the vertical coherence of clouds from model layer to model layer. An individual cloud will very
rarely produce a phase distribution described by the temperature function, yet the
calculation of precipitation is based upon that assumption.

The current precipitation scheme is also described by Smith (1990). The diagnosed
ice is allowed to fall, whereas the diagnosed liquid is not. Precipitation is formed directly
from condensate by an autoconversion term, similar to Sundqvist (1978), and by an extra
accretion term which removes extra condensate as the precipitation rate is increased.
Both precipitation terms are crudely parametrized with tunable constants.

The Smith (1990) precipitation scheme can result in two problems. Firstly, precipi-
tation may develop even if the cloud top is only a little colder than freezing. In reality
such a cloud will be completely liquid since very few ice nuclei can be activated, and
those that are will have fallen out of the cloud. If the entire boundary layer is below
freezing then a modelled stratocumulus cloud will lower because of the falling ice. This
lowering continues until the cloud reaches the ground, producing an incorrect fog fore-
cast and the collapse of the boundary layer structure. The second problem occurs when
the cloud base is near to or above 0°C. Here the falling ice is transported to higher
temperatures where the parametrization diagnoses it as liquid to an increasing degree.
Hence the downwards flux reduces and this allows a considerable build-up of moisture
just above the melting layer.

The drawbacks of the current scheme are considerable. Not only is the diagnosis of
phase inadequate, but the scheme relies heavily on parametrized values which are not
specified from a physical basis. The new scheme described below avoids these problems,
to a large degree, and in many circumstances produces considerably better forecasts.

2. THE MIXED-PHASE PRECIPITATION SCHEME

In this paper we describe a new large-scale cloud and large-scale precipitation
scheme based on that of Rutledge and Hobbs (1983), but adapted for use in the
UK Meteorological Office’s numerical weather prediction model. It incorporates a
prognostic ice water content variable. Convective cloud and convective precipitation are
handled by a different, diagnostic scheme. The Rutledge and Hobbs (1983) formulation
provides a suitable starting point for the scheme because it is designed to model
processes which occur in frontal systems rather than intense convection. The large-scale
precipitation scheme described, uses four quantities to describe water in the atmosphere:
vapour; liquid droplets; raindrops; and frozen water. Only one quantity, which we will
refer to as ‘ice’, is used to describe all frozen water in large-scale clouds, including
aggregated snow, pristine ice crystals and rimed particles. This is required in order to
allow the model to run at a reasonable speed and was used by the UK Meteorological
such as Fowler et al. (1996) split frozen water into two quantities, ‘ice’ and ‘snow’. This
paper shows that a reasonable prediction of ice cloud can be produced without such a
split. The scheme has no need for any temperature-dependent splitting between ice and
water based upon statistical measurements.

Physically based transfer terms link the four water quantities. They are summarized
in Fig. 1. The transfer terms are: condensation/evaporation of cloud liquid water from
vapour; homogenous and heterogenous nucleation of ice crystals; deposition and evapo-
ration of ice; riming of liquid water by ice particles; capture of raindrops by ice particles;
everaporation of melting snow; melting of snow to rain; evaporation of rain; accretion
of liquid water by raindrops; and autoconversion of liquid water to rain. Ice is allowed to
fall from layer to layer and rain is treated as a diagnostic, all predicted rain falling out
within a single time step as suggested by Ghan and Easter (1992). Each of these terms
is explained in detail in the appendix. The principal terms which govern the ice content are the fall of ice, the deposition/evaporation term and the riming term.

The condensation/evaporation of cloud liquid water is parametrized using a diagnostic scheme. Since liquid clouds respond to changes in supersaturation in a matter of several seconds, the diagnostic treatment is justified. It is not justified for ice, though, which can take tens of minutes to adjust to equilibrium. The rest of the transfer terms require information about the particle size distribution of ice and rain. Also required is information about the fall-speed characteristics of particles, their densities, and their shapes. The formulations used are given in the appendix. These parametrizations are all that is required to form the bulk of the transfer terms. Since many of the constants and parametrizations (but not the autoconversion parametrization) can be compared against observational studies, the arbitrarily parametrized values forming the basis of the Meteorological Office’s current precipitation parametrization scheme are largely removed.

3. THE UNIFIED MODEL AND ITS IMPLEMENTATION

The large-scale cloud scheme and the large-scale precipitation scheme are two of the physical process schemes used within the UK Meteorological Office Unified Model, described by Cullen (1993). The new scheme described here requires a small change to the cloud scheme, and a total rewriting of the large-scale precipitation scheme. The structures of the schemes are outlined below. Detailed descriptions of transfer terms are to be found in the appendix.

The main difference in the new scheme is that the ice content is considered to be a prognostic variable. Ice is advected around the model domain using the Unified Model’s positive semi-definite advection scheme designed for tracer variables (Roe 1985). It is also mixed within the boundary layer by a tracer-mixing scheme. Additional changes are
required in the code to convert ice from a diagnostic to a prognostic variable, but these are not discussed in this paper. No direct coupling occurs between the large-scale cloud and precipitation schemes and the convection scheme, unlike the detrainment schemes of Fowler et al. (1996) and Tiedtke (1993).

(a) Cloud scheme

The inputs to this scheme are the vapour plus liquid water content, the ice content and the 'liquid' temperature (the temperature the air would have if the liquid was evaporated). This scheme uses the method of Smith (1990) to produce a cloud liquid mixing ratio and a dry-bulb temperature. The Smith scheme assumes a triangular distribution of total water content minus saturation water content, with a width determined by a critical relative humidity. Condensation occurs in regions where the total water content exceeds saturation. The Smith scheme is modified to use the saturation vapour pressure with respect to liquid water for its calculation, and works with the vapour plus liquid water content (rather than the total water content) to produce a liquid content at any temperature. This also produces a liquid cloud fraction.

The ice cloud fraction is calculated as a function of the ice water content by inverting this relationship in the Smith (1990) cloud fraction scheme (but using the saturation vapour pressure with respect to liquid). This results in the expression:

\[
\begin{align*}
  c_f \text{ice} &= 0 & n_c \leq 0 \\
  c_f \text{ice} &= 0.5(6n_c)^{2/3} & 0 < n_c \leq 1/6 \\
  c_f \text{ice} &= (1 - 4 \cos^2 \phi) & 1/6 < n_c < 1 \\
  \text{where } \phi &= [\cos^{-1} \left(3(1 - n_c)/2^{3/2}\right) + 4\pi]/3 \\
  c_f \text{ice} &= 1 & l \leq n_c \\
  \text{where } n_c &= q_c/(1 - RH_{\text{crit}} q_{\text{sat liq}})
\end{align*}
\]  

where \(c_f \text{ice}\) is the ice cloud fraction, \(q_c\) is the ice content in kg kg\(^{-1}\), \(RH_{\text{crit}}\) is the critical relative humidity required for the formation of liquid clouds, and \(q_{\text{sat liq}}\) is the saturation specific humidity with respect to liquid water. Such a formulation is used in order to be as consistent as possible with the liquid-fraction calculation, although any expression could be used. The total cloud fraction (calculated assuming minimum overlap between ice and liquid cloud), liquid water content and ice water content are used by the radiation scheme. The ice content (stored as a mixing ratio) is itself unaffected by the cloud scheme.

(b) Precipitation scheme

The input values are temperature, liquid water mixing ratio, ice water mixing ratio and water vapour mixing ratio. Starting from the top level, the transfer equations are sequentially applied to convert between the different phases of water and apply any necessary correction to the temperature due to latent heating. A part of the ice content falls to the next layer down and any rain produced is passed straight to the next layer (the rain is a diagnostic quantity in this scheme, so any generated rain falls out in one time step). Then transfer calculations are performed on the layer below, also using the ice and rain that fell from the layer above. After the transfers at the bottom layer have been calculated the water content variables contain new values at each grid point, and the fluxes out of the bottom layer represent surface rain or snow.

The transfer terms are applied sequentially, so each one acts on the values generated by the term before. The terms are applied in the following order:
- Fall of ice from one layer to the next;
- Homogenous nucleation of ice from liquid;
- Heterogenous nucleation of ice from liquid or vapour;
- Depositional growth/evaporation of ice from liquid or vapour;
- Rimming of liquid by ice;
- Capture of raindrops by ice;
- Evaporation of melting ice;
- Melting of ice to rain;
- Evaporation of rain;
- Accretion of liquid water droplets by rain;
- Autoconversion of liquid cloud water to rain.

The appendix lists the formulation of each term. Figure 1 shows the transfers which are used.

(c) Subgrid-scale treatment

It is relatively simple to formulate such a scheme for use where grid boxes in the numerical model are extremely small, such as in a high-resolution cloud-resolving model. However, the assumption that moisture and condensate are evenly spread across a grid box can only be true for grid boxes a few tens of metres in size at the largest (and perhaps not even at these scales). Hence for a numerical weather prediction grid box (and especially for a GCM grid box) some account must be made of the subgrid-scale nature of the processes which occur. This is an extremely difficult problem to quantify. Only a few schemes, such as Rotstayn (1997), make consistent assumptions about the fractional coverage of different condensate and precipitation types, and their overlap in the vertical.

A proper treatment must know about the distribution and overlap of vapour, liquid, ice and temperature. The new scheme uses local water contents in the fall-of-ice calculation and the autoconversion of the liquid-to-rain term. Other precipitation terms assume that ice and liquid are spread over the whole grid box. The terms which involve vapour diffusion, such as deposition and evaporation, are modified. Here, instead of using the grid-box mean supersaturation, a modified calculation produces an effective supersaturation:

\[ (S_i - 1)_{\text{effective}} = (q + q_{cl} - \alpha q_{sat\ ice})/q_{sat\ ice}, \]  

where \((S_i - 1)_{\text{effective}}\) is the effective supersaturation for ice, \(q\) is the vapour mixing ratio, \(q_{cl}\) is the liquid water mixing ratio, \(q_{sat\ ice}\) is the saturation mixing ratio with respect to ice, and \(\alpha\) is a function of ice cloud fraction, \(c_{f\ ice}\), and a critical relative humidity for cloud formation, \(R_{H\ crit}\):  

\[ \alpha = R_{H\ crit}(1 - c_{f\ ice}) + 1c_{f\ ice}. \]  

A similar term applies for liquid processes, where \(q_{sat\ liq}\) and \(c_{f\ liq}\) are used. \((S_i - 1)_{\text{effective}}\) is limited to a maximum of \((q_{sat\ liq} - q_{sat\ ice})/q_{sat\ ice}\) since at larger supersaturations liquid would form and reduce the supersaturation. This term allows effective supersaturation when there is no cloud but the relative humidity, \(R_{H}\), is greater than \(R_{H\ crit}\), and the effective supersaturation equals the grid-box mean supersaturation when cloud is present throughout the grid box. This is an extremely crude method of accounting for subgrid-scale variations, which account for many of the problems in a parametrization scheme, but gives reasonable asymptotic behaviour for low and high relative humidities.
Precipitation quantities falling from one layer to the next are assumed to be uniformly distributed across the horizontal extent of the grid box. This neglect of fractional precipitation cover is undesirable, and is inconsistent with the assumptions by which the total cloud fraction is calculated. Results from Jakob and Klein (personal communication) show that different treatments of vertical overlap in calculating precipitation have a large impact on the predicted surface precipitation rate.

4. RESULTS FROM THE SCHEME

In this section results from two much studied mesoscale forecasts are presented. The control run represents the current Unified Model forecast and it is compared with the new scheme and observations.

(a) Supercooled stratocumulus cloud

It is a common occurrence in winter for a sheet of stratocumulus cloud to cover a significant area of the UK. Even though temperatures within the cloud and throughout the boundary layer are below freezing, the cloud remains in the liquid phase. The reason is that very few ice nuclei can be activated unless the temperature falls significantly below $-10\,^\circ C$. The ice nuclei which are activated often grow quickly and fall out of the cloud, leaving it in a liquid state.

The case of 8 January 1997 shows the problems of supercooled stratocumulus cloud for a model. There was a high-pressure area influencing most of the country and producing slack winds, shown in the synoptic analysis of 0600 UTC (Fig. 2). Such a situation in January is frequently associated with cloudy, overcast conditions as the sun is unable to provide enough heating to remove the cloud sheet. Surface observations clearly show a well-defined sheet of stratocumulus across much of the southern UK. The surface visibilities were good and there was very little fog reported. Figure 3 shows a summary of surface reports at 0600 UTC and 1200 UTC.

However, the current scheme forecast a considerable amount of fog across the country (Fig. 4, bottom) and vastly underestimated the visibilities. Although this case was handled well by human forecasters, the operational model provided an extremely poor cloud forecast. A vertical cross section (at $51^\circ N$), through the low cloud modelled by the current scheme after the six-hour forecast, reveals the way in which it fails to capture the cloud physics (Fig. 5, bottom). The automatic diagnosis of a significant amount of ice at temperatures below $0\,^\circ C$ results in an appreciable downward moisture flux as the scheme models the falling ice. The flux is greater at lower temperatures due to the phase ratio parametrization. Although the boundary layer scheme attempts to maintain a mixed boundary layer, it cannot prevent enough moisture reaching the lowest levels to produce fog in the model. In part, the severity of the problem is increased by the entire boundary layer being below $0\,^\circ C$, but even if the surface was warmer there would be nothing to stop the cloud descending to the $0\,^\circ C$ isotherm. On many days the height of a stratocumulus deck in the current scheme is just above the $0\,^\circ C$ isotherm. It is thought that a cloud deck can slowly fall until it reaches this height, where the cloud is diagnosed as liquid and can fall no further, although autoconversion processes can convert this liquid into rain.

The new scheme deals with this situation reasonably well (Fig. 5, top); no ice can be nucleated by this scheme unless the temperature falls to below $-10\,^\circ C$. Even if ice is present then it can only grow slowly at temperatures just below freezing (the saturation vapour pressure with respect to ice is very close to the saturation vapour pressure with respect to liquid at these temperatures). Hence a layer of supercooled liquid is
maintained above the surface. The small amount of ice which is formed increases towards the base, rather than the top, of the cloud, since it is falling out. Since the downward moisture flux due to precipitation is much smaller than in the current scheme, the amount of fog predicted is much reduced (Fig. 4, top). Observations (Fig. 3) show that snow grains were reported at some stations in southern England. This indicates that there was some production of ice and that riming contributed significantly to the growth of the particles. A cross-section (Fig. 6) of the model growth rates of ice due to deposition (top) and riming (bottom) indicates that the model riming rate can be similar to the deposition rate.

The cloud is trapped below a sharp boundary layer inversion, as observed by a tethered balloon ascent at Cardington, Bedfordshire (52.11°N 0.43°W) and shown in Fig. 7. The observed boundary layer is well mixed, the temperature profile lying close to a dry adiabat. The new scheme produces a temperature profile which maintains the sharp inversion, although the adiabatic mixed layer is not well modelled (Fig. 8). In contrast, the current scheme has completely destroyed the inversion and the temperature profile is almost isothermal, which is a very poor forecast of a simple well-mixed boundary layer. The boundary layer collapse produces a marked effect on the diagnosed cloud cover. Figure 9 shows the cloud cover assuming that clouds in adjacent model layers are maximally overlapped and clouds with a clear model layer between them are randomly overlapped. The new scheme predicts well the almost continuous sheet of cloud across the UK, although it incorrectly predicts a break in central England. The current scheme predicts large regions of broken cloud across the country; the collapsed boundary layer can no longer produce the turbulent moisture fluxes necessary for a stable, continuous cloud sheet.
Figure 3. Surface observations at 0600 UTC and 1200 UTC on 8 January 1997. The station circle shows cloud fraction (filled is total cloud cover, unfilled is clear skies), the temperature in degrees Celsius is given to the top left of the station circle, current significant weather to the left, past significant weather to the bottom right, and the pressure (coded) to the top right. Wind direction is shown by the arrow, with the long 'feather' representing ten knots and the short feather five knots. The triangular present weather symbol represents snow grains.

Figure 10 shows a plot of the phase ratio (the liquid water content divided by the sum of the liquid and ice water contents) of the cloud in the new scheme for points near Cardington. Although a statistical average of many clouds shows that clouds contain more ice than liquid as the temperature falls, for this particular cloud system such a temperature-dependent parametrization is clearly wrong. In this case, the phase ratio increases with decreasing temperature. Employing cloud physics methods to predict the phase results in a significant advantage.

(b) A decaying front

An aircraft of the Meteorological Research Flight flew on 25 October 1996 through a decaying frontal system east of the UK. The synoptic situation at 1200 UTC is shown in Fig. 11 with a complex cold front to the east of the UK. A visible (channel 4) satellite picture for 1206 UTC is shown in Fig. 12. The flight was a spiral descent, extensively described by Field (1999). This allows a group of particles to be followed as they fall through the system. The analysis by Field (1999) of detailed measurements by two-dimensional probes allows an assessment of the assumptions and predictions
Figure 4. Fractional coverage of fog in grid boxes as diagnosed from model fields at 0600 UTC on 8 January 1997 after a six hour forecast: (a) the new scheme; (b) the current scheme.
Figure 5. East–west cross-sections of ice and liquid mixing ratios in kg kg\(^{-1}\) at 51°N. (a) Ice and (b) liquid mixing ratios for the new scheme; (c) ice and (d) liquid mixing ratios for the current scheme. The dashed lines are temperature contours in degrees Celsius. Water content contours represent: 0.01; 0.02; 0.05; 0.1; 0.15 g kg\(^{-1}\); and further increments of 0.05 g kg\(^{-1}\). The vertical eta coordinate is the pressure divided by the surface pressure.
of the new scheme. The current operational scheme contains no assumptions about the microphysical nature of drop size distributions, or particle densities, so cannot be analysed in a similar way.

The modelled cloud fractions are shown in Fig. 13. Both schemes predict the extensive cloud cover associated with the front, although the front is too far advanced in the model. Slightly less cloud cover is predicted by the new scheme for the front, which is probably detrimental, although the modelled cover remains very high.

Cross-sections of model-predicted ice and liquid contents at 55°N latitude are shown in Fig. 14. Very little liquid cloud was measured by Field (1999). The phase ratio for the new scheme cross-section is plotted in Fig. 15. It shows a very different pattern to that from the supercooled stratocumulus cloud, with no liquid water except at relatively
Figure 7. The vertical profile of temperature plotted on a tephigram for tethered balloon profiles from Cardington on 8 January 1997. The solid line represents the 0740 UTC ascent and the dashed line the 0830 UTC descent. The isotherms and isentropes are separated by 10 °C. The thick isotherm represents 0 °C. The pressure contours represent 700 hPa (top), 800 hPa, 900 hPa and 1000 hPa (bottom). The dashed lines are saturation specific-humidity contours representing 0.8 (top left), 1, 1.5, 2, 2.5, 3, 4 and 5 g kg\(^{-1}\) (bottom right).

Figure 8. Vertical temperature and dew-point profiles at 0600 UTC 8 January 1997 for a grid point representing Cardington, plotted as a tephigram. (a) The new scheme; (b) the current scheme. The background lines are the same as for Fig. 7.
Figure 9. The fractional coverage of cloud for: (a) the new scheme; (b) the current scheme at 0600 UTC 8 January 1997 from a six hour forecast.
Figure 10. The phase ratio (fraction of condensate which is liquid) for model points around Cardington for the new scheme.

Figure 11. The analysis chart for 1200 UTC 25 October 1996.
high temperatures. This is consistent with weak updraughts and a decaying front. The development of the ice particles must have been by deposition and aggregation only. The aggregation in the new scheme is implicitly parametrized using a temperature variation in the particle size distribution. By studying the behaviour of the particle size spectra of both the observations and the model, inferences can be made about the relative importance of aggregation and deposition.

Slope, $\Lambda$, and intercept, $N_0$, are parameters in the distribution $N(D) = N_0 \exp(-\Lambda D)$, where $D$ is the equivolumetric diameter of the particle, and $N(D) dD$ is the number of particles with equivolume diameters between $D$ and $D + dD$. $\Lambda$ and $N_0$ can be fitted to observations made using the High Volume Precipitation Spectrometer (Stratton Park Engineering Company Inc.); the instrument measures particles greater than 800$\mu$m in diameter, and the fitted $N_0$ and $\Lambda$ follow a well-defined pattern. Field (1999) shows that $N_0$ increases with decreasing height in the cloud until the temperature reaches $-14$ °C, consistent with depositional growth, but $\Lambda$ remains fairly constant. At higher temperatures $N_0$ and $\Lambda$ fall steadily. Near cloud base particles break up as they evaporate, increasing $\Lambda$. The observations suggest that at temperatures higher than $-14$ °C the relationship $N_0 \propto \Lambda^2$ holds reasonably well. Theoretical and empirical work by Mitchell (1988) suggests that the exponent in the power law should be 2.1 ± 0.2 in an aggregation region. The new scheme in the model produces profiles fitted extremely well by the $N_0 \propto \Lambda^{2.1}$ relationship, as demonstrated by Fig. 16. The modelled profiles are consistent with the observations of Field (1999) as well as with observational data from a variety of other sources (Lo and Passarelli 1982; Gordon and Marwitz 1986; Passarelli 1978). Hence, in this case, the model appears to be parametrizing aggregation reasonably well. It also produces plausible particle size distributions.
Figure 13. The fractional coverage of cloud in grid boxes from model fields at 1200 UTC 25 October 1996 after a six hour forecast: (a) the new scheme; (b) the current scheme.
Figure 14. East–west cross-sections of ice and liquid mixing ratios at 55°N, close to the aircraft spiral. (a) Ice and (b) liquid mixing ratios in the new scheme; (c) ice and (d) liquid mixing ratios in the current scheme. The dashed lines are temperature contours in degrees Celsius. Water content contours represent: 0.01; 0.02; 0.05; 0.1; 0.15 g kg$^{-1}$; and further increments of 0.05 g kg$^{-1}$.
The modelled values of $\Lambda$ against temperature are plotted in Fig. 17 along with the relationship of Ryan (1996):

$$\Lambda = 1600 \times 10^{-0.023T},$$

where $\Lambda$ is in $\text{m}^{-1}$ and $T$ is temperature in degrees Celsius. Some similar modelling studies choose to define $\Lambda$ as a function of temperature, rather than $N_0$. The new scheme modelled $\Lambda$ fits reasonably well with the relationship above.

Further analysis can be used to calculate the relative importance of deposition and aggregation in the new scheme. The aircraft observations show that deposition is more important in the upper regions of the cloud, and aggregation in the lower regions. Although the model also gives an increasing contribution to aggregation at higher temperatures, aggregation is much more significant than deposition, except at the cloud top. This is shown in Fig. 18. The new scheme also does not model particle break-up at the base of a cloud.

The model in this case-study agrees reasonably well with the observations, validating its microphysical nature directly. However, the observations also show quite clearly a bimodal spectrum (Field 1999), which is impossible to obtain in the new scheme since an exponential particle size distribution is used. A two-quantity description of ice particles (e.g. ‘ice’ and ‘snow’) may be a better fit to the observations, especially if gamma functions, rather than exponentials, are chosen to model their distributions as in Mitchell et al. (1996). However, this is thought to be unnecessary to obtain a reasonable evolution of the ice content.
Figure 16. Modelled $N_0$ and $\Lambda$ (see text) from the new scheme for profiles within the cross-section shown in Fig. 14 and observed $N_0$ and $\Lambda$ from the spiral descent by Field (1999). A line of gradient 2 is also shown (dashed).

Figure 17. Modelled $\Lambda$ and $T$ (see text) values from the new scheme for the profiles shown in Fig. 16 and observed $\Lambda$ and $T$ values from Field (1999). The line represents the expression of Ryan (1996).
Figure 18. Cross-sections of the inferred growth rate of ice particle diameter in the new scheme due to: (a) deposition; (b) aggregation. The dashed lines are temperature contours in degrees Celsius. The solid contours represent: 0.1; 0.2; 0.5; 1; 2; 3 mm hr⁻¹.

The current operational parametrization scheme produces a similar cross-section. However, the amount of ice is different, as shown by the cross-section of ice content in Fig. 14. Near the melting layer the new scheme predicts ice contents of less than 0.25 g kg⁻¹ whilst the current scheme predicts contents approaching 1 g kg⁻¹. The difference at temperatures less than −9 °C is mainly due to the differently parametrized fall-speeds of the ice particles, the new scheme having reduced fall velocities at lower temperatures and hence greater ice content. The greater ice contents near cloud base are due to the reduction in flux as ice gets parametrized as liquid. This greatly increases the condensed water content. The aircraft observations suggest that the lower ice contents of the new scheme are more realistic. Using data from Field (1999) and assuming that the larger particles have a density 0.1 times that of solid ice and the smaller particles are
solid ice, gives an ice content of around 0.15 g kg\(^{-1}\) at a temperature of \(-10^\circ\text{C}\). Note also that the lower edge of the system is more sharply defined in the current scheme than in the new scheme. This is because evaporation of ice upon entering a dry layer is instantaneous in the current scheme, whereas in the new scheme the ice can fall a considerable distance before evaporating. This can result in a slightly increased area of rainfall around a front in the new scheme, as more ice reaches the melting layer before it is evaporated.

5. Conclusions

The new scheme is a physically based transfer scheme which uses the four water contents of vapour, cloud liquid water, ice, and rain to describe the moisture in the atmosphere. It has a prognostic ice variable, which gives it several advantages over the current scheme which must diagnose ice without knowledge of the history of the cloud system. We have presented results from two case-studies. There are a number of points which can be made about the relative performance of the new scheme and the current scheme, although some of these are not covered in the results shown. The main similarities and differences in the predictions between each scheme are:

(i) The precipitation rate is very similar. This is primarily determined by the dynamical forcing. The details of the precipitation scheme play only a minor role in the distribution of precipitation across the domain. They act instead to adjust water contents in order that the model is approximately in balance, so that the flux of vapour in updraughts is equal to the flux of precipitation downwards.

(ii) The new scheme produces a larger amount of high-altitude ice cloud than the current scheme. This is primarily due to reduced fall-speeds at lower temperatures. Different rates of ice growth in the two schemes provide secondary differences; the ice is diagnostic in the current scheme, so deposition and evaporation are effectively instantaneous, but in the new scheme ice takes a finite time to grow.

(iii) The lower edges of cloud in deep precipitation systems are less well-defined in the new scheme because of the finite time it takes to evaporate ice falling into a dry layer.

(iv) The cloud fraction in deep precipitation systems is slightly reduced in the new scheme as a result of the parametrization of ice cloud as a function of condensate rather than as a function of vapour plus condensate.

(v) The evaporation of rain is reduced in the new scheme, leading to drier boundary layers, less low cloud and greater visibility. In heavy rain the reduced evaporation leads to increased surface temperatures. The autoconversion of liquid to rain is less likely to occur in the new scheme (not demonstrated in the examples above).

(vi) Supercooled liquid stratocumulus clouds are much better predicted, since they are not diagnosed as falling ice in the new scheme. There is no excessive build-up of water content just above the 0 °C isotherm.

(vii) Freezing rain and drizzle can naturally be predicted by the new scheme. The current scheme melts and freezes precipitation instantly as it crosses the 0 °C surface. Also, regions of supercooled liquid water can be predicted on a more reliable basis.

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APPENDIX

Formulation of transfer terms in the scheme

The formulation of the transfer terms requires parametrization of the mass, fall speed and particle size distributions of ice and rain as a function of temperature. These are described first and then subsequently used to derive each of the transfer terms.

The ice particle size distribution is parametrized as:

\[ N_{\text{ice}}(D) = N_{0_{\text{ice}}} \exp(-0.1222T) \exp(-\Lambda_{\text{ice}}D), \quad (A.1) \]

where \( D \) is the equivalent diameter of the particle in m, \( N_{0_{\text{ice}}} = 2.0 \times 10^6 \text{ m}^{-4} \), and \( T \) is the temperature in degrees Celsius. \( \Lambda_{\text{ice}} \) represents the slope of the exponential distribution. The temperature function \( \exp(-0.1222T) \) represents the fact that ice particles tend to be smaller at lower temperatures, and is an implicit way of parametrizing aggregation. Ice particles are assumed to be spheres for the riming and deposition calculations. The particle size parametrization follows Cox (1988).

The mass of an ice particle is parametrized as a function of \( D \):

\[ m_{\text{ice}}(D) = aD^b, \quad (A.2) \]

where \( a = 0.069 \text{ kg m}^{-2} \) and \( b = 2.0 \). This is similar to the parametrization of Cox (1988). Although such a power law implies that particle densities can get above that of solid ice, this is an easy expression to integrate.

The fall-speed of an ice particle at an air density of 1 kg m\(^{-3}\) is similarly parametrized:

\[ v_{\text{ice}}(D) = cD^d, \quad (A.3) \]

where \( c = 25.2m^{0.473} \text{ s}^{-1} \) and \( d = 0.527 \). The combination of the distribution, mass and velocity relationships yield a fall-speed ice water content relationship similar in form to that of Heymsfield (1977). For a given ice content and temperature, \( \Lambda_{\text{ice}} \) can be calculated by integrating (A.2) across the particle size distribution (A.1). This gives the result that, for a given temperature, \( \Lambda_{\text{ice}} \) is proportional to the inverse cube root of the ice water content.

Raindrops are assumed to be spherical. The mass/diameter relationship is then just that for constant-density spheres. The raindrop size distribution is taken to be that from Marshall and Palmer (1948):

\[ N_{\text{rain}}(D) = N_{0_{\text{rain}}} \exp(-\Lambda_{\text{rain}}D), \quad (A.4) \]

where \( N_{0_{\text{rain}}} = 8.0 \times 10^6 \text{ m}^{-4} \) and \( \Lambda_{\text{rain}} \) represents the slope of the exponential distribution.

The fall speed relationship for rain is also expressed as a power law. This makes the transfer terms simple to integrate over a distribution:

\[ v_{\text{rain}}(D) = eD^f, \quad (A.5) \]

where \( e = 368.8 m^{0.33} \text{ s}^{-1} \), \( f = 0.67 \) and the air density is taken to be 1 kg m\(^{-3}\). This expression is that used by Sachidananda and Zrnić (1986).
There is a correction factor applied to the fall-speeds, due to the density of air. At low air densities a particle will fall faster than at high air densities. The correction factor used is given by Pruppacher and Klett (1978):

$$\nu(\rho) = (\rho_0/\rho)^{0.4}\nu(\rho_0),$$  \hspace{2cm} (A.6)

where $\nu(\rho)$ is the fall-speed of a particle at arbitrary air density $\rho$, and $\rho_0$ is 1 kg m$^{-3}$. $\nu(\rho_0)$ is given by (A.3) and (A.5) above.

We now summarize the transfer terms used in the scheme. They are applied sequentially in the model and are shown below in the order in which they are applied. Temperature increments due to latent heat are also applied where appropriate, but are not discussed here. Comparison with Lin et al. (1983) shows that only a few processes are completely missing in the new scheme. Ice is always assumed to consist of large enough particles to melt to the rain category and not the cloud liquid water category. Spontaneous freezing of raindrops is assumed to be of little importance. The new scheme does not parametrize graupel or hail, since it is concerned with large-scale processes. Although these particles can be said to be part of the ice category, the ice parametrizations are not designed for such dense particles. Hence hail processes, such as wet growth, are not considered. The new scheme does not parametrize the Hallett and Mossop (1974) process, which can result in the rapid conversion of liquid to ice. Brown and Swann (1997) show that, in a cloud-resolving model, this is an important process. With only a single variable representing the ice content of a cloud, specifying the Hallett–Mossop process is difficult.

(a) Condensation of liquid

As described in section 3, this uses the Smith scheme to diagnose vapour and liquid contents from a vapour-plus-liquid input.

(b) Fall of ice

The mass weighted fall-speed of a group of particles in a grid box is obtained by integrating the equations describing the distribution, fall-speed and mass over the particle diameters. This produces an equation giving the fall-speed as a function of temperature and slope of the particle size distribution, $\Lambda_{ice}$, which is easily converted into an expression for the fall speed in terms of temperature, air density and ice mixing ratio. The amount of ice which falls from one layer to the next is given simply by:

$$\Delta q_{cf} = \nu_{ice}q_{cf}\Delta t/\Delta z,$$  \hspace{2cm} (A.7)

(limited to the amount of ice which is in the layer to begin with) where $\nu_{ice}$ is the fall-speed of the ice entering the layer, $\Delta t$ is the time step and $\Delta z$ the thickness (in metres) of the layer. This is stored as a flux (SNOW) given by

$$\Delta q_{cf} = \text{SNOW}\Delta t/(\rho \Delta z).$$  \hspace{2cm} (A.8)

$q_{cf}$ is the ice mixing ratio.

If ice entering a layer falls through the layer in one time step then it is moved straight through the layer (and all the ice originally in the layer is assumed to fall out of the layer), except for a mixing ratio given by:

$$q_{cf} = \text{SNOW}/(\nu_{ice}\rho),$$  \hspace{2cm} (A.9)

which is retained in the layer. This is retained because it is assumed that the flux of ice falling into the layer is constant throughout the time step, which implies this mixing
ratio. This formulation allows ice to fall through more than one layer in a time step. This is probably the part of the scheme which is most sensitive to time step; explicit fall schemes often have large sensitivities to time step. A test for the cold-front case showed that the domain-averaged ice water path increased by two percent when the time step was doubled to ten minutes. An alternative to an explicit fall scheme is to analytically integrate the fall of ice from one layer to the one immediately below, as described by Rotstayn (1997). These schemes are generally less time-step dependent.

The subgrid nature of the problem is handled by using the local ice water content in the calculation of the fall speeds. Note that the local value is only used in this calculation and not other terms.

(c) Homogenous nucleation of ice

This term simply converts all liquid water to ice if the temperature is less than a given threshold of \(-40 ^\circ C\).

(d) Heterogenous nucleation of ice

This uses the Fletcher (1962) equation to nucleate a small amount of ice every time step. The nucleation term itself does not provide a major source of ice in the model, but acts as a seed for the deposition term, and later riming, to grow ice from liquid or vapour. The number of ice nuclei activated per cubic metre at \(T (^\circ C)\) is given by the Fletcher (1962) relation, but limited to a maximum value:

\[
n = \min(0.01 \exp(-0.6T), 1 \times 10^5). \tag{A.10}
\]

Each of these nuclei is given an arbitrary small mass of \(1 \times 10^{-12} \text{ kg}\). Heterogenous nucleation removes liquid cloud water in preference to vapour (Bergeron–Findeisen process). However, heterogenous nucleation is only applied if all the following conditions are met:

(i) The liquid cloud fraction is greater than 0;
(ii) \(q > RH_{\text{crit}}q_{\text{sat, ice}}\);
(iii) \(q > RH_{\text{crit}}q_{\text{sat, liq}} g(T)\) where \(g(T)\) is the minimum of 1 and \((188.92 + 2.81T + 0.013336T^2 - 10)/100\) where \(T\) is in \(^\circ C\);
(iv) \(T < -10 ^\circ C\).

These restrictions act to restrict nucleation to low temperatures and to regions of high vapour content. Condition (i) ensures that ice cannot be nucleated unless liquid water exists. Condition (iii) is from Heymsfield and Milosevich (1995). The removal of condition (iii) results in the additional formation of small amounts of transient cirrus clouds which do not effect the evolution of the major cloud systems. Condition (iv) is necessary to avoid excessive ice being produced from supercooled liquid clouds with relatively high cloud-top temperatures; although if significant ice falls, or is advected, into these clouds then the nucleation term is unimportant compared to the depositional growth of the ice.

(e) Deposition and evaporation of ice

This is the principal way in which ice can grow in the model. A single ice particle will grow by vapour diffusion according to the following equation:

\[
\frac{dm}{dt} = \frac{4\pi C(S_i - 1)F}{[\{L_s/(RT) - 1\}L_s/(k_a T) + RT/(Xe_{sat, ice})]} \tag{A.11}
\]

where \(dm/dt\) is the rate of change of mass of the particle; \((S_i - 1)\) is the supersaturation of the atmosphere with respect to ice; \(R\) is the gas constant for water vapour; \(k_a\) is the
thermal conductivity of air at temperature $T$, $X$ is the diffusivity of water vapour in air at temperature $T$ and a given air pressure; $\varepsilon_{\text{sat \ ice}}$ is the saturated vapour pressure over ice; $L_a$ is the latent heat of sublimation of ice; $C$ is a capacitance term and $F$ is a ventilation coefficient. $C$ is assumed to appropriate to spheres, so is equal to $D/2$. $F$ is given by Pruppacher and Klett (1978) as $F = 0.65 + 0.44 Sc^{1/3} Re^{1/2}$ where $Sc$ is the Schmidt number, equal to 0.6, and $Re$ is the Reynolds number, $v(D) \rho D / \mu$, where $v(D)$ is the fall-speed of the particle and $\mu$ is the dynamic viscosity of air.

This equation is integrated over the particle size distribution parametrization using the fall velocity diameter and mass diameter relationships. The subgrid-scale supersaturation parametrization, $(S_i - 1)_{\text{effective}}$, replaces $(S_i - 1)$ (see (2)), although remaining values are calculated taking grid box mean values for simplicity, and not local contents.

The same equation is used for both deposition and evaporation. The limits which apply for evaporation are: the amount of additional vapour required to bring the atmosphere to saturation; and the amount of ice there is to evaporate. The deposition limit is the amount of supersaturation available. Note that $(S_i - 1)_{\text{effective}}$ in the rate equation is limited to $q_{\text{sat \ liq}}/q_{\text{sat \ ice}} - 1$. The spare or excess moisture capacity of the atmosphere is given by solving for when $(S_i - 1)_{\text{effective}} = 0$. The Bergeron–Findeisen process is considered by having the deposition term remove the liquid water in preference to the vapour. This term is applied only for temperatures below 0 °C.

\[ (f) \quad \textbf{Riming of liquid water by ice} \]

This is the second main way by which ice can grow from liquid. The rate at which a single ice particle collects supercooled liquid water is given by the equation:

\[ \frac{dm}{dt} = (\pi/4) D^2 \nu_{\text{ice}}(D) \rho q_{\text{cl}} \]  
\[ (A.12) \]

where $D$ is the diameter of the ice particle; $\nu_{\text{ice}}(D)$ is the fall velocity; and $q_{\text{cl}}$ is the liquid water mixing ratio. This equation assumes spherical ice particles and an efficiency of collection of unity. This equation is integrated over the particle size distribution. Grid-box mean values are used to calculate the transfer term, with no attempt made to consider subgrid-scale distributions. This term is only applied for temperatures below 0 °C.

\[ (g) \quad \textbf{Capture of raindrops by ice particles} \]

This represents contact freezing of raindrops by collision with ice particles at freezing temperatures. The collision rate between an ice particle of diameter $D_{\text{ice}}$ and a raindrop of diameter $D_{\text{rain}}$ can be written as:

\[ \text{Collision rate} = E(D_{\text{ice}}, D_{\text{rain}}) \pi / 4(D_{\text{ice}} + D_{\text{rain}})^2 |\nu_{\text{ice}}(D) - \nu_{\text{rain}}(D)|. \]  
\[ (A.13) \]

The collision efficiency, $E$, is assumed to be unity. This expression must be integrated over both particle size distributions and weighted by the mass of each raindrop to obtain the expression for the rate of change of ice mass. Because of the modulus term in the velocities, (A.13) is complicated to integrate exactly. Hence an approximation is made, replacing $|\nu_{\text{ice}}(D) - \nu_{\text{rain}}(D)|$ by a function of the grid box mean velocities, $f(\nu_{\text{ice}}, \nu_{\text{rain}})$, which is then not integrated. The function is:

\[ f(\nu_{\text{ice}}, \nu_{\text{rain}}) = |\nu_{\text{ice}} - \nu_{\text{rain}}| \quad \text{if } |\nu_{\text{ice}} - \nu_{\text{rain}}| > (\nu_{\text{ice}} + \nu_{\text{rain}})/8 \]
\[ f(\nu_{\text{ice}}, \nu_{\text{rain}}) = (\nu_{\text{ice}} + \nu_{\text{rain}})/8 \quad \text{if } |\nu_{\text{ice}} - \nu_{\text{rain}}| \leq (\nu_{\text{ice}} + \nu_{\text{rain}})/8. \]  
\[ (A.14) \]

This function is used by Brown and Swann (1997) in the UK Meteorological Office’s cloud-resolving model.
(h) \textit{Evaporation of melting snow}

This term allows ice which is at temperatures higher than 0 \degree C to be subject to an evaporation term before it melts. The formulation is very similar to that of the deposition and evaporation of ice. The only changes are that the supersaturation with respect to liquid water and the latent heat of vaporization are used in the rate calculation.

\[ \frac{dm}{dt} = -4\pi CF\left\{ k_a / L_m(T_w - T_0) \right\}, \]  
(A.15)

where \( L_m \) is the latent heat of melting of ice, \( T_w \) is the wet-bulb temperature of the air and \( T_0 \) is the freezing point of ice. The capacitance term, \( C \), is considered to be that for spherical particles. Hence \( C = D/2 \). The ventilation factor, \( F \), is considered to be that given in the deposition and evaporation of ice term. The wet-bulb temperature is calculated using a numerical approximation containing the saturation deficit and the air pressure. The only subgrid-scale alteration made here from the grid box values is in the calculation of the saturation deficit for the wet-bulb temperature, where a subgrid-scale \((S_w - 1)_{\text{effective}}\) is used.

\[ (S_w - 1)_{\text{effective}} = (q - \alpha q_{\text{sat liq}}) / q_{\text{sat liq}}, \]  
(A.16)

where

\[ \alpha = RH_{\text{crit}}(1 - c_{f_{\text{liq}}}) + 1c_{f_{\text{liq}}}. \]  
(A.17)

This term is only applied if \( T_w > T_0 \).

(i) \textit{Melting of snow}

Since this term is essentially a diffusion term, although of heat instead of moisture, its form is very similar to that of the deposition and evaporation of ice term. The rate of change of ice mass of a melting particle is given by:

\[ \frac{dm}{dt} = -4\pi CF\left\{ k_a / L_m(T_w - T_0) \right\}, \]  
(A.15)

where \( L_m \) is the latent heat of melting of ice, \( T_w \) is the wet-bulb temperature of the air and \( T_0 \) is the freezing point of ice. The capacitance term, \( C \), is considered to be that for spherical particles. Hence \( C = D/2 \). The ventilation factor, \( F \), is considered to be that given in the deposition and evaporation of ice term. The wet-bulb temperature is calculated using a numerical approximation containing the saturation deficit and the air pressure. The only subgrid-scale alteration made here from the grid box values is in the calculation of the saturation deficit for the wet-bulb temperature, where a subgrid-scale \((S_w - 1)_{\text{effective}}\) is used.

\[ (S_w - 1)_{\text{effective}} = (q - \alpha q_{\text{sat liq}}) / q_{\text{sat liq}}, \]  
(A.16)

where

\[ \alpha = RH_{\text{crit}}(1 - c_{f_{\text{liq}}}) + 1c_{f_{\text{liq}}}. \]  
(A.17)

This term is only applied if \( T_w > T_0 \).

(j) \textit{Evaporation of rain}

This is another diffusion-based term, very similar to the evaporation of snow. The rate of change of mass of a raindrop is given by:

\[ \frac{dm}{dt} = \{4\pi C(S_w - 1)F\} / \left[\left\{ L_v / (RT) - 1\right\}L_v / (k_a T) + RT / (Xe_{\text{sat liq}}) \right\}. \]  
(A.18)

The capacitance is assumed to be equal to that of a spherical particle, so \( C = D/2 \); the ventilation factor is given by Beard and Pruppacher (1971) as:

\[ F = 0.78 + 0.31Sc^{1/3}Re^{1/2}. \]  
(A.19)

Integrating over the distribution of rain drops gives the rate of change of rain mass. The value of the supersaturation \((S_w - 1)\) is altered to take its subgrid form, using the liquid cloud fraction rather than ice cloud fraction. Since rain is considered to be a diagnostic quantity which all falls out in a single time step, a conversion is made between the rain mass (in kg of rain per kg of air) and the rainfall rate out of the layer.

\[ \Delta \text{Rain Mass} = (\Delta \text{Rain Rate} \Delta t) / (\rho \Delta z). \]  
(A.20)
(k) **Accretion of liquid water by raindrops**

As raindrops fall through liquid water cloud they can collect the droplets as they collide with them. Assuming that all collisions result in collection and that the collision cross-section is simply that of the falling raindrop, the expression for the rate of change of mass contained in rain is:

\[
\frac{dm}{dt} = \pi / 4 D^2 \nu_{\text{rain}}(D) \rho q_{\text{cl}},
\]

(A.21)

where \( \nu_{\text{rain}}(D) \) is the fall-speed of the raindrop. This can be integrated across the raindrop size distribution to obtain a rate of change of mass of the rain. To convert this into a change in rain rate one must use the diagnostic expression (A.20).

(l) **Autoconversion of rain from liquid water**

This term represents the coalescence/collection 'warm rain' process which can occur in thick clouds. Cloud droplets combine to form drops large enough to fall as rain. This process is extremely difficult to quantify and depends on many factors, such as the age of the cloud. The new scheme uses the formulation of Tripoli and Cotton (1980) in the calculation of the autoconversion rate. They write:

\[
\frac{dq_{\text{cl}}}{dt} = \frac{4 \pi g E_c \rho_{\text{cl}}^{4/3} q_{\text{cl}}^{7/3}}{18 (4 \pi / 3)^{4/3} \mu (N_c \rho_{\text{liq}})^{1/3}},
\]

(A.22)

where \( E_c \) is the collision/collection efficiency (assumed to be 0.55), \( g \) is the acceleration due to gravity, \( \mu \) is the dynamic viscosity of air, \( N_c \) is the number of droplets per cubic metre and \( \rho_{\text{liq}} \) is the density of water. Using the approximate value of \( N_c = 3 \times 10^8 \) m\(^{-3} \) gives an expression for the rate of change of water content due to autoconversion. This is adjusted slightly to take into account land and sea variations in the amount of cloud condensation nuclei. The minimum water content before autoconversion can occur is given by considering a critical drop size. When the radius of a typical drop exceeds \( 7 \times 10^{-6} \) m, autoconversion can occur. Autoconversion is not allowed to deplete the liquid water beyond this minimum liquid water content. Local values of \( q_{\text{cl}} \) are used since this term is particularly nonlinear.

(m) **tidying up the water contents**

A final term evaporates small values of ice back to vapour, to tidy up the water contents. If the mixing ratio of ice is less than \( 1 \times 10^{-8} \) kg kg\(^{-1} \) and either the temperature is greater than 0 °C or the grid-box relative humidity is less than \( RH_{\text{crit}} \) then the ice is evaporated. Ice which falls straight from one layer to another will be subject to an additional melting term, in order to make sure it melts before reaching the surface (assuming there is enough heat in the air to allow melting). Since this ice is never in a layer involving the microphysical transfer terms, at the very least a melting term must be applied to it.

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