Further development of a hybrid-isentropic GCM

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SUMMARY

The UK Universities' Global Atmospheric Modelling Programme hybrid-isentropic general-circulation model (HIGCM) uses a flexible \( \sigma-\theta-p \) vertical coordinate, where \( p \) is pressure, \( \theta \) is potential temperature, and \( \sigma = p/p_* \) where \( p_* \) is surface pressure. Three major improvements to the HIGCM are presented. The first improvement is a modification to the vertical–difference scheme so that spurious vertical motions in the isentropic domain are minimized. The second improvement is a modification to the implementation of the radiation scheme so that it is now able to damp, and does not itself create, noisy temperature profiles; this allows the model to be run without ad hoc extra vertical diffusion and so allows a cleaner comparison with \( \sigma-p \) simulations. The third improvement is to extend the isentropic domain up to the top of the model thus allowing \( \sigma-\theta \) or \( \sigma-\theta-p \) simulations to be performed.

Idealized baroclinic instability life-cycle experiments are used to investigate the impact of the new vertical scheme on the dynamical core of the HIGCM. The reduction in spurious vertical velocities is found to be substantial whilst the impact on the global conservation properties and overall evolution is found to be very small. These simulations also show that the commonly used \( \nabla^2 \) form of scale-selective dissipation can seriously compromise global energy conservation when model-layer thicknesses have significant horizontal gradients.

The impact of the isentropic coordinate on the climate of the full GCM is investigated by performing perpetual January simulations using \( \sigma-\theta \), \( \sigma-\theta-p \) and \( \sigma-p \) vertical coordinates. The most robust response to the isentropic coordinate is a warming of the southern hemisphere high-latitude lower stratosphere. In the northern hemisphere the largest changes in zonal mean temperature are in the polar stratosphere. The possible mechanisms by which the isentropic coordinate may yield these changes are described and investigated. The results strongly suggest that many of the potential benefits of the isentropic coordinate are realized, to some extent at least, with the HIGCM.

KEYWORDS: General-circulation model  Isentropic coordinate  Numerical techniques

1. INTRODUCTION

The use of potential temperature, \( \theta \), as the vertical coordinate for numerical modelling of the atmosphere offers several potential advantages over the more usual pressure, \( p \), or \( \sigma (= p/p_* \) where \( p_* \) is surface pressure (Phillips 1957)) vertical coordinates. In particular, when diabatic heating is small the flow becomes quasi-two dimensional, and the vertical velocity relative to isentropic levels is reduced almost to zero. In such a situation the use of an isentropic coordinate numerical model will reduce truncation errors associated with vertical advection almost to zero. Furthermore, the diagnostic relationship between diabatic heating and vertical velocity is brought out most clearly in isentropic coordinates irrespective of whether the diabatic heating is large or small. Thus, using an isentropic coordinate numerical model naturally leads to a more Lagrangian representation of the atmospheric circulation.

Isentropic coordinate models may also be especially suited to resolving fronts. The close packing of isentropes naturally leads to increased vertical resolution in such regions, and isentropic gradients of wind and temperature tend to be less sharp than isobaric gradients, so that in a model with a given horizontal resolution, features are better resolved with isentropic coordinates. Indeed, the use of the isentropic coordinate in the vertical yields simplifications analogous to using geostrophic momentum coordinates in the horizontal (Hoskins 1982).

Most numerical models use some form of scale-selective dissipation to control the build-up of information at small scales. A commonly used scheme is a \( \nabla^2 \) acting on

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the prognostic variables where $\nabla$ means the gradient along model levels. It is possible that when model levels are isentropes, the $\nabla^{2n}$ dissipation will more closely mimic the isentropic mixing of potential vorticity that arguably occurs in the real atmosphere. Indeed, Johnson (1997) has suggested that numerical mixing in climate models can be a spurious source of entropy, and that this may lead to systematic temperature biases. He showed that the global integral of entropy will be conserved when the mass in each isentropic layer is conserved, implying that an isentropic coordinate model, with scale-selective dissipation suitably formulated so as to conserve mass in isentropic layers, could avoid spurious entropy sources and hence any associated temperature errors.

Finally, isentropic coordinate diagnostics, such as isentropic maps of potential vorticity (e.g. Hoskins et al. 1985) or isentropic coordinate mean circulations (e.g. Townsend and Johnson 1985), are valuable tools for interpreting the behaviour of both the real atmosphere and numerical models. An isentropic coordinate numerical model will enable such diagnostics to be calculated more easily and accurately.

However, there are at least two major obstacles that must be overcome before an isentropic coordinate model may be successfully implemented. The first is to treat effectively the lower boundary since, in a purely isentropic model, the levels will intersect the ground. Bleck (1974) and Hsu and Arakawa (1990) circumvented this problem by extending the model levels horizontally as a series of layers of negligible mass when they reached the ground. However, this approach introduces discontinuities in the potential vorticity distribution, for example, which can become infinite in the massless parts of the layers. This in turn may contaminate the simulation in the interior of the domain. A second approach, which introduces a relatively shallow $\sigma$-coordinate domain near the ground, was employed by Deavin (1976), Uccellini et al. (1979) and Black (1987). Again, this technique may generate small-scale noise, in this case as grid points emerge above the interface between the $\sigma$-coordinate and $\theta$-coordinate domains.

More recently, Zhu et al. (1992, hereafter ZTHH) described a technique which allows pure $\sigma$-levels at the ground to blend smoothly into $\theta$-levels above and then into $p$-levels near the very top of the model. Their hybrid $\sigma-\theta-p$ vertical scheme is an extension of the vertical scheme described by Simmons and Burridge (1981) and was successfully implemented in the UK Universities' Global Atmospheric Modelling Programme (UGAMP) general-circulation model (GCM, and hereafter the UGAMP GCM being referred to as the UGCM) which is based on the European Centre for Medium-Range Weather Forecasts (ECMWF) cycle 27 forecast model. Hereafter, we shall refer to the $\sigma-\theta-p$ version of the UGCM (and in section 4 the $\sigma-\theta$ version) as the HIGCM. A similar type of hybrid coordinate with a smooth transition to $\theta$-levels has been proposed by Konor and Arakawa (1997). Indeed, their hybrid coordinate is more flexible than that of ZTHH since, for example, it allows a given model level to be partially terrain-following over high mountains but isentropic elsewhere. As a result, a greater proportion of their model domain can be on isentropic levels.

In extended climate simulations ZTHH and Thuburn (1993, hereafter JT93) encountered the second major problem associated with isentropic coordinate models. Since the pressure on a model level in the isentropic domain depends on the local temperature alone, the model levels can move up and down considerably. Thus, any numerical errors that lead to noisy temperature profiles result in irregularly spaced levels and, potentially, further inaccuracies. In the model used by ZTHH and JT93, noisy temperature profiles were caused by centred difference vertical advection of temperature in combination with interpolation in the radiation scheme that prevented radiative damping of grid-scale features. In this case the problem was exacerbated by the use of the Lorenz vertical grid, which meant that levels could even cross each other, causing the model to ‘blow
up'. The alternative Charney–Phillips vertical grid used by Konor and Arakawa (1997) should help to prevent this problem of levels crossing.

Since the work of ZTHH and JT93 the HIGCM has been improved in three ways: (i) the implementation of the radiation scheme has been improved so that it is now able to damp, and also does not itself create, small-scale noise in the vertical profile of temperature—this enables the model to run without the ad hoc increase of the vertical diffusion; (ii) the calculation of the vertical velocity has been improved so that in the isentropic domain it now more clearly corresponds to the diabatic heating; and (iii) the region of isentropic levels has been extended to the top of the model.

With the above-mentioned improvements in mind, the structure of the paper is as follows. In section 2 we describe the improvement to the calculation of the vertical velocity in the isentropic domain. Section 3 contains a description of a series of baroclinic instability life-cycle simulations performed with the HIGCM and with a σ-coordinate version of the UGCM. The new vertical-velocity calculation is shown to reduce the spurious cross-isentropic flow in the isentropic domain, but the overall impact on the simulation is actually rather small. These life-cycle simulations also highlight a problem with the global conservation of potential energy when the horizontal scale-selective temperature dissipation acts along model levels that have significant horizontal gradients of thickness.

In section 4 the results from a series of climate simulations using σ–θ, σ–θ–p and σ–p vertical coordinates are presented. The largest changes in zonal mean temperatures occur in the stratosphere, and it is argued that at least some of these changes are due to the potential advantages of the isentropic coordinate mentioned above. A summary of the main conclusions of this paper is given in section 5.

The configurations and names of the experiments to be described in the paper are listed in a table in appendix A. The names assigned to the experiments will be used to refer to them in sections 3 and 4.

2. A MODIFICATION TO THE VERTICAL SCHEME

One drawback of the vertical scheme of ZTHH is that the coordinate system vertical velocity formally does not vanish on an isentropic level when the diabatic heating $\dot{\theta}$ is zero. The resulting spurious vertical velocities could, for example, lead to the poor conservation of tracers in extended GCM simulations. Here we describe a modification to the scheme of ZTHH that greatly reduces these spurious vertical velocities. However, it does not appear to be possible, within the framework of the sort of scheme we are considering on the Lorenz vertical grid, to eliminate the spurious vertical velocities entirely.

The HIGCM vertical coordinate is a generalization of the hybrid $\sigma–p$ coordinate of Simmons and Burridge (1981). In the following, the notation is the same as employed by ZTHH. The pressure on a model level may depend on the local temperature $T$ as well as the surface pressure $p_s$, i.e.

$$p = p(p_s, T).$$

(2.1)

Four constants at each half-level $a_{k+\frac{1}{2}}, b_{k+\frac{1}{2}}, c_{k+\frac{1}{2}},$ and $d_{k+\frac{1}{2}},$ referred to as the vertical-coordinate table, are used to define the half-level pressures, $p_{k+\frac{1}{2}}$. At each horizontal
grid point
\[ p_{k+\frac{1}{2}} = \frac{a_{k+\frac{1}{2}} + b_{k+\frac{1}{2}} p_*}{d_{k+\frac{1}{2}} + c_{k+\frac{1}{2}} T_{k+\frac{1}{2}}^{-\frac{1}{3}}} \quad \text{for } k = 0 \text{ to } N \] (2.2)

where \( \kappa = R/C_p \) (\( R \) is the gas constant and \( C_p \) is the specific heat of air at constant pressure), \( N \) is the number of full model levels, and \( k \) runs from 0 at the top of the model to \( N \) at the ground. When \( d_{k+\frac{1}{2}} = 1 \) and \( c_{k+\frac{1}{2}} = 0 \) the levels are the usual \( \sigma-p \) model levels, and when \( b_{k+\frac{1}{2}} = d_{k+\frac{1}{2}} = 0 \) the levels are isentropic levels. By a suitable choice of the vertical-coordinate table, a smooth transition can be made from \( \sigma \)-levels at the ground to pure \( \theta \)-levels above.

Consider first the continuous primitive equations and let \( \eta \) be a vertical coordinate that ranges from 0 at \( p = 0 \) to 1 at \( p = p_* \). An expression for the pressure-normalized vertical velocity relative to model levels \( \tilde{\omega} = \eta (\partial p/\partial \eta) \) is obtained by integrating the mass continuity equation
\[ \tilde{\omega} = -\frac{\partial p}{\partial t} - \int_0^\eta \nabla \cdot \left( \nu \frac{\partial p}{\partial \eta} \right) \, d\eta \] (2.3)

where \( \nabla \) means the horizontal gradient at constant \( \eta \) and \( \nu \) is the horizontal vector. At the ground \( \tilde{\omega} = 0 \) so
\[ \frac{\partial p_*}{\partial t} = -\int_0^1 \nabla \cdot \left( \nu \frac{\partial p}{\partial \eta} \right) \, d\eta. \] (2.4)

Also, the pressure coordinate vertical velocity \( \omega = Dp/Dt \) is obtained by expressing \( Dp/Dt \) term by term in \( \eta \)-coordinates and using (2.3):
\[ \omega = -\int_0^\eta \nabla \cdot \left( \nu \frac{\partial p}{\partial \eta} \right) \, d\eta + \nu \cdot \nabla p. \] (2.5)

Differentiating (2.1) allows \( \partial p/\partial t \) to be expressed in terms of tendencies of prognostic variables:
\[ \frac{\partial p}{\partial t} = E \frac{\partial p_*}{\partial t} + F \frac{\partial T}{\partial t} \] (2.6)

where \( E = (\partial p/\partial p_*)|_\eta \) and \( F = (\partial p/\partial T)|_\eta \). Thus the vertical velocity in general depends on the temperature tendency through the \( \partial p/\partial t \) term in (2.3).

The thermodynamic equation is
\[ \frac{\partial T}{\partial t} = -\nu \cdot \nabla T + \kappa \omega T \frac{\partial T}{p} - \tilde{\omega} \frac{\partial T}{p} + \frac{Q}{C_p}, \] (2.7)

where \( Q \) is the diabatic heating rate. Thus the temperature tendency depends on the vertical velocity through the vertical-advection term, so that Eqs. (2.3) and (2.7) are coupled.

We can eliminate \( \partial T/\partial t \) from (2.3), (2.6) and (2.7) to obtain an expression for \( \tilde{\omega} \) in terms of known quantities:
\[ \left( 1 - F \frac{\partial T}{\partial p} \right) \tilde{\omega} = -E \frac{\partial p_*}{\partial t} - F \tilde{Q} - \int_0^\eta \nabla \cdot \left( \nu \frac{\partial p}{\partial \eta} \right) \, d\eta, \] (2.8)
where $\tilde{Q}$ is the contribution to the temperature tendency from all terms except vertical advection:

$$
\tilde{Q} = -\mathbf{v} \cdot \nabla T + \frac{\kappa T}{p} \left\{ - \int_0^\eta \nabla \cdot \left( \mathbf{v} \frac{\partial p}{\partial \eta} \right) \, d\eta + \mathbf{v} \cdot \nabla p \right\} + \frac{Q}{C_p}.
$$ (2.9)

Note that when $\eta$-surfaces are isentropes $E = 0$, $F = -p/\kappa T$, and $\nabla p = F \nabla T$, so that the horizontal-advection terms in (2.9) cancel and the vertical integral terms in (2.8) and (2.9) cancel, leaving $\tilde{\omega} = 0$ for adiabatic flow, as required.

We now consider the vertical discretization of $\tilde{Z}$THH and derive an expression for $\tilde{\omega}$ in an analogous way. The discrete forms of (2.3), (2.4), and (2.5) are

$$
\tilde{\omega}_{k+\frac{1}{2}} = -E_{k+\frac{1}{2}} \frac{\partial p_s}{\partial t} - F_{k+\frac{1}{2}} \frac{\partial T_{k+\frac{1}{2}}}{\partial t} - \sum_{r=1}^k \nabla \cdot \left( \mathbf{v}_r \Delta p_r \right),
$$ (2.10)

where $T_{k+\frac{1}{2}} = (T_k + T_{k+1})/2$,

$$
\frac{\partial p_s}{\partial t} = -\sum_{r=1}^N \nabla \cdot \left( \mathbf{v}_r \Delta p_r \right),
$$ (2.11)

and

$$
\left( \frac{\kappa \omega T}{p} \right)_k = \frac{\kappa T_k}{\Delta p_k} \left[ - \ln \left( \frac{p_{k+\frac{1}{2}}}{p_{k-\frac{1}{2}}} \right) \sum_{r=1}^k \nabla \cdot \left( \mathbf{v}_r \Delta p_r \right) + \alpha_k \nabla \cdot \left( \mathbf{v}_k \Delta p_k \right) \right] + \mathbf{v}_k \left\{ \ln \left( \frac{p_{k+\frac{1}{2}}}{p_{k-\frac{1}{2}}} \right) \nabla p_{k-\frac{1}{2}} + \alpha_k \nabla (\Delta p_k) \right\},
$$ (2.12)

where $\alpha$ is defined as:

$$
\alpha_k = 1 - \frac{p_{k+\frac{1}{2}}}{\Delta p_k} \ln \frac{p_{k+\frac{1}{2}}}{p_{k-\frac{1}{2}}} \quad \text{for} \quad k = 2, \ldots, N \quad \text{and} \quad \alpha_1 = \ln 2.
$$

The expression (2.12) was chosen for compatibility with the expression for the pressure-gradient term in the momentum equation to ensure that together they did not undermine energy conservation. The discrete thermodynamic equation is

$$
\frac{\partial T_k}{\partial t} = -\mathbf{v}_k \cdot \nabla T_k + \left( \frac{\kappa \omega T}{p} \right)_k
$$

$$
- \frac{1}{2\Delta p_k} \left[ \tilde{\omega}_{k+\frac{1}{2}} (T_{k+1} - T_k) + \tilde{\omega}_{k-\frac{1}{2}} (T_k - T_{k-1}) \right] + \frac{Q_k}{C_p}.
$$ (2.13)

Eliminating $\partial T_k/\partial t$ from (2.10) and (2.13) leads to an equation for $\tilde{\omega}$:

$$
\mathbf{H} \tilde{\omega} = \mathbf{R},
$$ (2.14)

where

$$
\tilde{\omega} = (\tilde{\omega}_{\frac{1}{2}}, \tilde{\omega}_{\frac{1}{2}}, \ldots, \tilde{\omega}_{N-\frac{1}{2}})^T,
$$ (2.15)
\( \mathbf{H} \) is a tri-diagonal matrix with elements

\[
\begin{align*}
H_{k,k} &= 1 - \frac{1}{4} F_{k+\frac{1}{2}} \left( \frac{1}{\Delta p_k} + \frac{1}{\Delta p_{k+1}} \right) (T_{k+1} - T_k) \\
H_{k,k-1} &= -\frac{1}{4\Delta p_k} F_{k+\frac{1}{2}} (T_k - T_{k-1}) \\
H_{k,k+1} &= -\frac{1}{4\Delta p_{k+1}} F_{k+\frac{1}{2}} (T_{k+2} - T_{k+1}) \\
H_{i,j} &= 0 \text{ otherwise,}
\end{align*}
\]

(2.16)

and

\[
\mathbf{R} = (R_{\frac{1}{2}}, R_{\frac{3}{2}}, \ldots, R_{N-\frac{1}{2}})^T.
\]

(2.17)

The elements of the column vector \( \mathbf{R} \) take the form

\[
R_{k+\frac{1}{2}} = -E_{k+\frac{1}{2}} \frac{\partial p_*}{\partial t} - \sum_{r=1}^{k} \nabla \cdot (\mathbf{v}_r \Delta p_r) - \frac{1}{2} F_{k+\frac{1}{2}} (\tilde{Q}_k + \tilde{Q}_{k+1}),
\]

(2.18)

where

\[
\tilde{Q}_k = -\mathbf{v}_k \cdot \nabla T_k + \left( \frac{\kappa \omega T}{p} \right)_k + \frac{Q_k}{C_p}.
\]

(2.19)

For adiabatic flow on an isentropic level (2.18) reduces to

\[
R_{k+\frac{1}{2}} = -\sum_{r=1}^{k} \nabla \cdot (\mathbf{v}_r \Delta p_r)
- \frac{1}{2} F_{k+\frac{1}{2}} \left\{ -\mathbf{v}_k \cdot \nabla T_k + \left( \frac{\kappa \omega T}{p} \right)_k - \mathbf{v}_{k+1} \cdot \nabla T_{k+1} + \left( \frac{\kappa \omega T}{p} \right)_{k+1} \right\}.
\]

(2.20)

In contrast to the case for the continuous equations, there is no longer any guarantee that the terms on the right-hand side of (2.20) will cancel and so there is no guarantee that \( \tilde{\omega}_{k+\frac{1}{2}} \) will vanish. The results presented in section 3 show that \( \tilde{\omega}_{k+\frac{1}{2}} \) can be as large as several hPa d\(^{-1}\).

This failure of the ZTHH vertical discretization to guarantee \( \tilde{\omega} = 0 \) for adiabatic flow on an isentropic level motivated the development and testing of an alternative vertical discretization. First note that for the continuous equations \( \tilde{Q} \) can be rewritten as

\[
\tilde{Q} = -\left( \frac{p}{p_0} \right)^\kappa \mathbf{v} \cdot \nabla \theta - \frac{\kappa T}{p} \int_0^\eta \nabla \cdot \left( \frac{\partial p}{\partial \eta} \right) \mathrm{d} \eta + \frac{Q}{C_p}
- \mathbf{v} \cdot \left( \frac{d}{d + cT^{-\frac{1}{k}}} \nabla T - \frac{b_k T}{p(d + cT^{-\frac{1}{k}})} \nabla p_* \right) - \frac{\kappa T}{p} \int_0^\eta \nabla \cdot \left( \frac{\partial p}{\partial \eta} \right) \mathrm{d} \eta + \frac{Q}{C_p},
\]

(2.21)

where \( p_0 \) is a standard surface pressure of 10\(^5\) hPa. This suggests an alternative vertical discretization in which \( \tilde{\omega} \) is given by

\[
\mathbf{H} \tilde{\omega} = \mathbf{R}'.
\]

(2.22)
where $\mathbf{H}$ is again defined by (2.16) and $\mathbf{R}'$ defined by

$$
R'_{k+\frac{1}{2}} = -E_{k+\frac{1}{2}} \frac{\partial p_*}{\partial t} - \left( \frac{F\kappa T}{p} - 1 \right)_{k+\frac{1}{2}} \sum_{r=1}^{k} \nabla \cdot (v_r \Delta p_r)
+ \left\{ F \left( \frac{p}{p_0} \right)^\kappa \nabla \theta \right\}_{k+\frac{1}{2}} - \frac{1}{2c_p} F_{k+\frac{1}{2}} (Q_k + Q_{k+1}),
$$

(2.23)

where

$$
\left( \frac{F\kappa T}{p} - 1 \right)_{k+\frac{1}{2}} = \left( \frac{d}{d + cT^{-\frac{1}{k}}} \right)_{k+\frac{1}{2}}
$$

(2.24)

and

$$
\left\{ F \left( \frac{p}{p_0} \right)^\kappa \nabla \theta \right\}_{k+\frac{1}{2}}
= \frac{1}{2} \left( \frac{cT^{-\frac{1}{k}}}{(d + cT^{-\frac{1}{k}})^2} \right)_{k+\frac{1}{2}}
\times \left\{ \left( \frac{pd}{\kappa T} \right)_{k+\frac{1}{2}} (v_k \cdot \nabla T_k + v_{k+1} \cdot \nabla T_{k+1}) - b_{k+\frac{1}{2}} (v_k + v_{k+1}) \cdot \nabla p_* \right\}.
$$

(2.25)

The expressions (2.24) and (2.25) are identically zero on isentropic half-levels because $b_{k+\frac{1}{2}} = d_{k+\frac{1}{2}} = E_{k+\frac{1}{2}} \equiv 0$. Thus, under adiabatic conditions on isentropic half-levels $R'_{k+\frac{1}{2}} \equiv 0$, as required.

With this new formulation, spurious cross-entrope flow is substantially reduced (see section 3). However, it is not eliminated entirely. Because the matrix $\mathbf{H}$ is tri-diagonal, a non-zero component of $\mathbf{R}'$ in the region of hybrid levels gives a non-zero contribution to $\hat{\omega}$ at every other level, including the pure isentropic levels. However, this contribution is small and decreases rapidly with increasing distance from the region of hybrid levels. This ‘contamination’ of the pure isentropic levels by the hybrid levels appears to be unavoidable on the Lorenz vertical grid, where $T$ is defined at full levels and $T_{k+\frac{1}{2}}$ must be obtained by vertical averaging, which is what leads to the tri-diagonal nature of the matrix $\mathbf{H}$.

After $\hat{\omega}$ has been calculated using (2.22) there are apparently two possible ways to calculate $\partial T/\partial t$: either using the mass continuity equation (2.10), or using the thermodynamic equation (2.13). However, although the equations (2.10), (2.13), and (2.14) are mutually consistent, the equations (2.10), (2.13), and (2.22) are not. Thus if we choose to calculate $\partial T/\partial t$ using the mass continuity equation (2.10) then $\hat{\omega}$ and $\partial T/\partial t$ are no longer consistent with the particular form of the thermodynamic equation (2.13), and, in particular, energy conservation is no longer guaranteed. On the other hand, if we choose to calculate $\partial T/\partial t$ using the thermodynamic equation (2.13) then $\hat{\omega}$ and $\partial T/\partial t$ are no longer consistent with the particular form of the mass continuity equation (2.10), and all formal conservation properties are lost. According to this argument it would seem preferable to use the mass continuity equation (2.10) to calculate $\partial T/\partial t$. Unfortunately, this is only possible on levels where $F_{k+\frac{1}{2}}$ is not zero and is thus not possible when
pure $\sigma$-levels are employed at the bottom of the domain or when pure pressure levels are employed at the top of the domain. Therefore, we have used the thermodynamic equation (2.13) to calculate the temperature tendency. Although the loss of formal conservation properties is potentially very serious, the results of section 3 suggest that the resulting errors in conservation are in fact very small.

3. Baroclinic Life-Cycle Experiments

In order that we may more clearly understand the impact of the new vertical scheme in climate simulations with the HIGCM, it is first worth investigating the impact of the new scheme on the model dynamics alone. Thus, in this section, we adopt the approach of JT93 and use the adiabatic baroclinic instability life-cycle experiments (Simmons and Hoskins 1978, 1980) as a test bed for the new vertical scheme.

Since the original investigations of Simmons and Hoskins the life-cycle experiments have been used by many authors to investigate various aspects of baroclinic instability. The initial state consists of a zonally symmetric mid-latitude jet with maximum velocity at $45^\circ$N and about 200 hPa. This experiment is now often referred to as the LC1 simulation following Thornicroft et al. (1993). The balanced-wind and potential-temperature fields of the initial state can be found in Fig. 1 of JT93. The most unstable zonal wave-number six normal mode is added to the basic flow with small amplitude and the model integrated for 15 days. The life cycle is characterized by an initial period of essentially linear dynamics which yields rapid exponential baroclinic growth. This phase ends when the wave saturates nonlinearly, first at low levels at about day 4 and then at upper levels at about day 7.5. Subsequently, at T42 horizontal resolution (spectral triangular truncation at total wave number 42), which is the resolution of all the runs to be described here, the wave amplitude diminishes as it undergoes barotropic decay. By day 15, the wave has essentially died away to leave a much narrower and stronger mid-latitude jet.

Five life-cycle simulations were performed in this study, and they augment a pair of simulations described by JT93. Thus, in total, we have seven simulations to compare. These simulations employed 15 levels in the vertical, with the HIGCM levels being defined by the vertical-coordinate table in Table A.1 of ZTHH. Thus there were six purely isentropic levels extending from the 310 K surface (the ninth model half-level) up to the 370 K surface (the fourth model half-level). The control experiments employed a $\sigma$-coordinate with the levels being defined by the values of $b$ in Table A.1 of ZTHH. In all simulations the only diabatic process included was a $\nabla^6$ scale-selective dissipation (where $\nabla$ is the horizontal gradient operator holding the model vertical coordinate constant). Unless otherwise stated, the dissipation was applied to the vorticity, divergence and temperature fields with a time-scale of four hours at the shortest retained scale. For brevity, for the remainder of this paper, we shall hereafter refer to the scale-selective dissipation in terms of the time-scale at the shortest retained scale and the operator alone, i.e. 4 hour $\nabla^6$ in this case.

In JT93, the main focus of the life-cycle study was to investigate the impact of the isentropic coordinate, and to identify whether any obvious improvements in accuracy were obtained. A comparison of experiments 1S and 1STP-old (referred to as experiment 11 there) yielded rather inconclusive results with, for example, the conservation of potential vorticity (PV) on isentropic surfaces appearing better on some surfaces but worse on others. The aim of the additional five simulations performed here is twofold. Firstly, we investigate the impact of the new vertical-difference scheme on the isentropic vertical velocities and on the conservation properties of the HIGCM. The isentropic velocities are indeed found to be substantially reduced whilst the loss of
formal energy conservation with the new vertical scheme is found to be insignificant relative to other errors in conservation. Secondly, these other errors in conservation are investigated in more detail by performing two additional (HIGCM, UGCM) pairs of experiments. The errors are shown to be associated with the scale-selective dissipation of temperature along model levels which, because of the non-zero horizontal gradient of model-level thickness, result in the dissipation scheme being particularly poor at conserving potential energy.

(a) Sensitivity to the new vertical scheme

As discussed in section 2, the new vertical scheme should reduce the spurious cross-isentropic flow in the isentropic domain of the HIGCM. However, the new scheme does not retain the usual conservation properties, and this is potentially a very serious problem. In this section, we investigate both these issues by comparing experiments 1STP and 1STP-old.

In general, the two simulations were found to be very similar. This conclusion was reached after a comparison of various diagnostics, such as instantaneous maps of PV on isentropic surfaces and time averaged zonal mean momentum fluxes. For instance, the peak momentum fluxes differed by less than 2% for the two experiments. These diagnostics (not shown) suggest that the new vertical scheme is behaving well or, at least, no worse than the original vertical scheme. However, because of the concern that the loss of formal conservation properties might cause problems, a more thorough investigation into the conservation properties of these two experiments was performed.

In both the UGCM and the HIGCM the globally integrated kinetic energy (KE) and globally integrated potential energy (PE) can be calculated at every time step. To investigate the issue of energy conservation, the KE and PE were monitored over the course of the life cycle. Figure 1 plots the KE, PE and also their sum, the total energy (TE), relative to the initial time for all seven experiments performed. The difference in TE between experiments 1STP and 1STP-old is at most 10% of the TE change in any of the seven experiments. In other words, Fig. 1(a) indicates that the loss of formal energy conservation with the new scheme has a much smaller impact than other non-conservative processes within the model. Indeed, this statement assumes that the difference in energy between these two simulations is entirely a direct consequence of the non-conservative nature of the new vertical scheme. The differences could equally be due to the indirect impact of the new vertical scheme, whereby the changes in the vertical velocities lead to changes in the life-cycle evolution. The other much larger conservation errors alluded to above will be further investigated in the next subsection.

The second aspect of the life-cycle simulation that is sensitive to the new vertical scheme is the cross-isentropic mass flow in the isentropic domain; by using the new vertical scheme this should be reduced to a minimum. To illustrate the impact on this aspect of the model simulation, Fig. 2 compares the time averaged vertical velocities across the 335 K and 310 K surfaces for the two experiments. These surfaces were chosen because they were in the middle and at the bottom of the isentropic domain respectively. A number of features are apparent from these plots.

Firstly, the spurious vertical motions in experiment 1STP-old, being of the order of several hPa d\(^{-1}\), are actually not that large. For instance, they are nearly two orders of magnitude smaller than the cross-isentropic vertical velocities typically associated with deep convection, such as those illustrated in Fig. 5 for the full GCM simulations to be discussed in section 4. Therefore, it is perhaps not too surprising that the spurious vertical motions have only a small effect on the evolution of the life cycle. However,
even though these spurious vertical motions are small, they could still have a significant impact on the tracer budget in an extended GCM simulation.

Secondly, the spurious vertical motions are indeed reduced in experiment 1STP. For example, for the 310 K surface (Figs. 2(b) and 2(d)) the extrema are reduced from about 5 hPa d\(^{-1}\) in experiment 1STP-old to about 2 hPa d\(^{-1}\) in 1STP. Plots at the time of maximum wave amplitude (day 7, not shown) exhibit a similar factor of 2 or 3 difference between the two schemes, the extrema being an order of magnitude larger than those in the time-mean.

A third point of note is that the corresponding plots for the 335 K surface (Figs. 2(a) and 2(c)) differ from each other much more than those for the 310 K surface. In experiment 1STP-old, the extrema on the 335 K surface are only very slightly less than those on the 310 K surface, whereas in experiment 1STP they are substantially reduced. Indeed, both instantaneous and time-averaged extrema are reduced by two orders of
magnitude in experiment 1STP. Therefore, these plots illustrate rather nicely that the contamination effect on the isentropic vertical velocities by the hybrid levels above and below dies away rapidly with increasing distance from the hybrid levels.

To summarize, the new scheme appears to have only a small impact on the baroclinic instability life-cycle simulation. The loss of formal energy conservation is insignificant when compared with other non-conservative processes in the model. However, just as initially hoped, the spurious vertical velocities in the isentropic domain are much reduced with the new scheme.

(b) Conservative errors and scale-selective dissipation

In the last subsection the difference in TE between experiments 1STP and 1STP-old was seen to be much smaller than the change in TE in any of the seven simulations. As already indicated in the introduction to this section, the two pairs of life-cycle simulations yet to be discussed were performed to investigate the impact of the model scale-selective dissipation on the global conservation of energy. In the first pair of simulations (experiments 1STP-s and 1S-s) the diffusion was increased to 15 minute $V^6$ with the aim of increasing the energetic impact of the horizontal diffusion on the life cycle. The $V^6$ scale selectivity is retained with the hope that the wave development
is not altered too significantly. The second pair of simulations (experiments 1STP-t and 1S-t) used the same diffusion as experiments 1STP and 1S except that no dissipation was applied to the temperature field. This pair of simulations was motivated by a pair of climate simulations performed by JT93 in which the global conservation of a tracer in the HIGCM was found to be rather poor. JT93 found that the lack of tracer conservation was due to the scale-selective dissipation terms acting on both tracer mixing ratio and temperature.

Returning again to Fig. 1, it is first worth noting the contrasting behaviour of the experiments which used the standard diffusion formulation. The HIGCM experiment (1STP) gains TE whilst the UGCM experiment (1S) loses TE over the course of the life cycle. It is clear from Figs. 1(b) and (c) that this behaviour is dominated by large differences in the PE evolution rather than by differences in the KE evolution. Since the globally integrated PE is proportional to the global mean temperature, this implies that the difference in TE between experiments 1STP and 1S is proportional to the difference in global mean temperature.

An analysis of the additional two pairs of experiments indicates that the global mean temperature is, in fact, regulated by the strength of the diffusion on temperature. This regulating effect results from the diffusion of temperature spuriously warming the model atmosphere. Consistent with this argument, Fig. 1(b) shows that the two experiments where this spurious warming cannot occur (experiments 1STP-t* and 1S-t) have a very similar PE throughout the life cycle. Relative to these experiments, the standard diffusion experiments (1STP and 1S) gain PE, whilst the strong diffusion experiments (1STP-s and 1S-s) gain even more PE. This gain in PE is much more pronounced in the HIGCM than in the UGCM. The changes in the KE evolution are much easier to understand; for the strong diffusion experiments (1STP-s and 1S-s) there is a reduction in KE, whilst for the experiments without temperature diffusion (1STP-t and 1S-t) there is little change compared with the standard diffusion experiments. The size of these changes is comparable in both models. Thus, it would appear that the diffusion applied to temperature acts to increase the PE, whilst the diffusion applied to vorticity and divergence acts to reduce the KE. It is the much larger increase in PE in the HIGCM that dominates the difference in TE between the two models.

The horizontal diffusion of temperature produces the observed changes in PE via the horizontal gradient of model-level thickness, \( \nabla \Delta p \). The PE tendency due to the horizontal diffusion of temperature is proportional to \( \int (\nabla^{2n}T)(\partial p/\partial \eta) \, d\eta \, dA \), where the \( \nabla \) operator acts at constant \( \eta \). Since \( \nabla \Delta p \) is not zero, \( \partial p/\partial \eta \) is not constant at each level; therefore the integrand is not an exact divergence and so the integral does not generally vanish. Figure 3 illustrates the kind of situation in which the temperature dissipation term can cause PE to increase. The thicker part of the layer is warmed whilst the thinner part is cooled, resulting in a net increase in PE.

In this section we have highlighted a problem with the widely used \( \nabla^{2n} \) scale-selective dissipation scheme; in a model where the gradient of model-level thickness may become large, the conservation of energy may be particularly poor. The gradient of model-level thickness may be much larger in the HIGCM than in \( \sigma \) or pressure based vertical-coordinate models and so the problem is much more noticeable in the HIGCM. However, experiment 1S-s suggests that even in the UGCM, where the model-level thickness may only vary due to variations in surface pressure, the problem is not small. Moreover, in a climate model with realistic orography, and hence large spatially

* Experiment 1STP-t 'blew up' at day 12 when the model levels crossed in the mid-troposphere in the tropics. However, up until that time the model simulation appeared to be well behaved.
fixed differences in surface pressure, the problem may be much larger. However, in that instance this effect is likely to be relatively unimportant because of the much larger impact of the physical diabatic processes represented in such a model.

4. Climate simulations

In this section we investigate the sensitivity to the isentropic coordinate of the full GCM by performing a series of climate simulations at both T21 and T42 horizontal resolutions. Twenty-four levels were employed in the vertical and, because of the limited computer resources, the simulations were limited to 120 days in duration. At T21 resolution a 45 minute time step was employed, the horizontal diffusion was 3 hour \( \nabla^8 \) and the last 90 days of simulation were compared, whilst at T42 resolution the time step was reduced to 30 minutes, the horizontal diffusion was 4 hour \( \nabla^6 \) and the last 60 days of simulation were compared. In order to maximize the significance of the results given the shortness of the simulations, the model was run in 'perpetual January' mode with the solar forcing, sea surface and deep soil temperatures all fixed at their climatological mean January values. The initial state was taken from ECMWF analyses at 12 GMT on 15 January 1987. The simulations included parametrizations of vertical mixing, condensation and surface processes as in cycle 27 of the ECMWF forecast model. A Betts–Miller convective adjustment scheme (Betts and Miller 1993) parametrized the effects of both deep and shallow convection whilst the Morcrette (1990) radiation scheme parametrized radiative processes. The gravity-wave drag (GWD) scheme employed was similar to that described by Palmer \textit{et al.} (1986). The only difference was that the gravity-wave stress was linearly reduced to 30\% of its surface value at \( \eta = 0.8 \). Above that level the stress was assumed not to vary with height until wave breaking was diagnosed, just as in the Palmer \textit{et al.} scheme. The modification was one of those proposed by Miller and Palmer (1989).
In the previous version of the HIGCM used by ZTHH and JT93, ad hoc extra vertical diffusion was required in order to achieve stable climate integrations. The root of the problem was noisy temperature profiles caused by the use of a centred difference vertical-advection scheme. Radiative processes would be expected to damp such small vertical scale noise sufficiently quickly to prevent a real problem occurring. However, the previous implementation of the radiation scheme employed by ZTHH involved a double vertical interpolation of temperature that masked the noise and prevented the radiation from damping it. Li (1994) removed this double interpolation from the radiation scheme, thus allowing it to damp grid-scale noise in the temperature profile. One of the aims of the integrations described here was to test whether the HIGCM could be run with the improved radiation scheme implementation without the need for ad hoc extra vertical diffusion. Eliminating the extra vertical diffusion in the HIGCM would allow cleaner comparisons with the UGCM.

A further problem with the implementation of the radiation scheme was subsequently discovered. This manifested itself as an eight zonal grid wave in temperature in the lower tropical stratosphere. This wave developed because the full radiation calculation was only performed at every fourth longitude and every three hours; a relatively simple horizontal interpolation in space and extrapolation in time of the short-wave transmissivities and long-wave emissivities was then performed to calculate the heating rates at each grid point. Investigation revealed that it was the spatial interpolation of the emissivities, in particular, which led to the spurious heating rates and thence to the noisy temperature profiles. By performing the full radiation calculation at every longitude and every three hours, stable integrations of the HIGCM have been performed without the ad hoc extra vertical diffusion. Indeed, the unevenness of the model-level thicknesses was routinely monitored during the simulations and instances of extreme unevenness were never encountered. It should be noted that at the relatively low horizontal resolutions used here, the additional radiation calculations led to only a very small increase in computational expense.

Our experience of the HIGCM with the modifications described above has shown it to be considerably more robust than the original version. However, with the model levels free to move up and down, the HIGCM is still less tolerant of extreme unphysical forcing than the UGCM. We believe that this will be true of $\theta$-coordinate models in general, rather than it being a feature peculiar to the HIGCM.

It is worth emphasizing at this point that the HIGCM climate integrations to be described later compare directly with those performed with the UGCM; the implementation of the radiation scheme is the same for both models and the HIGCM does not use any ad hoc increased vertical diffusion.

(a) Sensitivity to the new vertical scheme

In this subsection the sensitivity of the HIGCM climate to the new vertical scheme is investigated by comparing experiment 2STP with experiment 2STP-old. Experiment 2STP employed the new vertical scheme whilst experiment 2STP-old used the original vertical scheme. A $\sigma-\theta-p$ vertical coordinate was used, with the vertical levels being defined by Table B.1. Thus the purely isentropic domain extended over seven half-levels from the 344 K surface up to the 500 K surface. In terms of pressure, the zonal mean location of the model levels is very similar to that illustrated in Fig. 1 of ZTHH. Thus, in the tropics, the model levels ‘bow’ downwards below about 200 hPa (about 1.5 scale heights in Fig. 1 of ZTHH), whilst above this level they bow upwards. This leads to a much coarser resolution of the tropical tropopause than the high-latitude tropopause.
Consistent with the results of the baroclinic life-cycle experiments, the overall impact of the new vertical scheme is rather smaller than the impact of the isentropic coordinate relative to the pressure coordinate (see next subsection). For instance, the largest difference in the zonal mean temperature (Fig. 4) is a 4 K warming at 200 mb over the North Pole. Whilst this difference is not large, the warming is concentrated at the same level as that seen when comparing the $\sigma-\theta-p$ and $\sigma-p$ vertical coordinates (see next subsection and Fig. 6(a)), and perhaps suggests that the impact of the isentropic coordinate is enhanced with the new vertical scheme.

The other major difference is the colder tropical upper troposphere and lower stratosphere of experiment 2STP. This signal appears to be systematic and robust since it is persistent right through this pair of simulations and because a similar response was seen in another pair of simulations which compared the two vertical schemes. That pair of simulations were identical to the pair illustrated here except that the vertical coordinate only included four pure $\theta$-levels and the original Palmer et al. (1986) GWD parametrization was employed.

This difference in temperature appears to arise from the difference in vertical scheme in the following way. In the tropical upper troposphere there is a systematic area average warming contribution to the temperature tendency from the vertical-advection term $-\omega(\partial T/\partial p)$, even though the area average $\omega$ is close to zero, because $\partial T/\partial p$ is systematically larger in regions of ascent than in regions of descent. The old vertical scheme gives values of $\omega$ (both ascent and descent) that are typically 5–10% stronger than those given by the new scheme, and therefore the vertical-advection term warms the tropical upper troposphere more. The largest difference in the vertical-advection term is confined to the deep tropics, but its effect on the temperature field is felt over the whole band 30°S to 30°N because of the large Rossby radius of deformation in the tropics. Averaged over the tropical upper troposphere, the difference in the vertical-advection term is balanced by a difference in radiative cooling, with the balance occurring at a slightly cooler temperature in the case of the new vertical scheme. We have examined the vertical velocity, vertical-advection term, and radiative-heating term from experiments 2STP-old and 2STP (not shown) and they are consistent, in both pattern and magnitude, with the above mechanism.
The baroclinic life-cycle experiments showed that the accumulated mass flow diagnostic more closely corresponded to the (zero) diabatic heating field when using the new vertical scheme. A similar improvement is seen in this diagnostic in the full climate experiment 2STP. As an illustration of this improvement, Fig. 5 shows the accumulated mass flow and diabatic heating fields for the 360 K surface for both experiments. Both schemes appear to capture well the anti-correlation between vertical velocity and diabatic heating. Most strikingly, large ascent rates (negative values) associated with deep tropical convection are correlated with large diabatic heating rates. However, closer inspection reveals that experiment 2STP-old is in error in several regions, where ascent and cooling are coincident. This most obviously occurs in the band of ascent north of the equator in Africa and in the three regions of ascent south of 30°S. With the new scheme there is weak descent in these regions of diabatic cooling.

In summary, the new vertical scheme appears to have only a small impact on the climate of the HICGM. The main advantage of the new scheme is that it enhances the Lagrangian representation of the mean circulation in the isentropic domain, and thus leads to improved accuracy of associated diagnostics. However, as we shall see in the next subsection, the differences in the HICGM climate due to the new vertical scheme are very much smaller than the differences between the HICGM and the UGCM, i.e. the differences due to the introduction of the isentropic coordinate.

(b) Sensitivity of the climate to the vertical coordinate

Following ZTHH and JT93, the HICGM integrations discussed thus far have employed a hybrid $\sigma-\theta-p$ coordinate with a smooth transition from $\theta$ to $p$ levels at the model top. During the course of this study, we became aware of the work of Zhu and Schneider (1997) whose climate simulations employed a hybrid $\sigma-\theta$ coordinate, with the pure $\sigma$-levels close to the ground blending into nearly isentropic levels at the model top. In their study, Zhu and Schneider emphasized the importance of choosing a suitable reference-temperature profile when defining the vertical-coordinate table. Previously with the HICGM, it had been thought that reverting to pressure levels at the top of the model was necessary because of poor cancellation in the finite difference form of the pressure-gradient term as $\Delta p/p \rightarrow 1$, leading to spuriously large accelerations at the top model level. However, subsequent experimentation with different reference-temperature profiles has shown that this was not in fact a problem. Therefore, in this subsection we compare the use of a $\sigma-\theta$ coordinate with the $\sigma-\theta-p$ and more customary $\sigma-p$ vertical coordinates. The new vertical scheme is used in all the HICGM simulations discussed. Again, the reader is referred to appendix A for a description of the different experiments performed.

(i) T21 climate simulations. Firstly, the impact of introducing a number of purely isentropic levels into the interior of the model domain is investigated by comparing HICGM experiment 2STP discussed in the previous subsection with UGCM experiment 2SP. The model levels for experiment 2SP are defined by the values of $a$ and $b$ in Table A.1. Figure 6(a) shows the difference in zonal mean temperature between these experiments. The most striking difference is the warming of the northern hemisphere extratropical stratosphere, with experiment 2STP being more than 10 K warmer over the North Pole at all levels above 150 hPa. The warming is largest in the lower stratosphere and extends southwards to about 20°N where, both above and below, experiment 2STP is colder than experiment 2SP. A similar warming of the lower stratosphere is seen at high latitudes in the southern hemisphere, with experiment 2STP more than 4 K warmer at 65°S.
Figure 5. Mass flux across the 360 K isentropic surface for the last 90 days of experiment (a) 2STP and (b) 2STP-old (see text). Contours are plotted for the $-80, -40, -20, -10, -5, 0, 5$ and $10$ hPa d$^{-1}$ isolines and negative values are shaded. (c) and (d) The diabatic heating on the 360 K surface averaged over the last 90 days of experiment 2STP and 2STP-old respectively. Note that this field is simply calculated as the average of that output on the adjacent full levels. Contours are plotted at $-1, -0.5, 0, 0.5, 1, 2, 4,$ and $8$ K d$^{-1}$ and positive values are shaded.
The warming of the high-latitude lower stratosphere in the HIGCM coincides with the region where many Eulerian climate models, including the UGCM, are too cold (Boer et al. 1991). This response would seem to be a feature of the isentropic coordinate since a similar warming was seen in the simulations of Zhu and Schneider (1997). A reduction of this temperature bias has also been found in semi-Lagrangian climate models (Williamson and Olson 1994; Chen and Bates 1996). Spurious advection of water vapour in Eulerian climate models has been shown to initiate this cooling (Blackburn, personal communication), but it is not yet clear what maintains it in the long term. It is reasonable to speculate that the improvements found in isentropic coordinate GCMs and semi-Lagrangian GCMs occur through the same mechanism or mechanisms.

Perhaps the most obvious mechanism for the improved temperature fields is the reduction of errors in vertical advection, either of temperature or of moisture or of both. Another possible mechanism is through the improved representation of the vertical propagation and/or breaking of planetary waves; the wave-breaking drives descent and dynamical heating in the high-latitude winter stratosphere. The improved representation of the planetary waves may occur through either or both of two mechanisms. Firstly, the (quasi-adiabatic) vertical propagation of planetary waves involves significant displacements of isentropes relative to pressure surfaces. A measure of this is the large difference between the pressure-coordinate Eulerian mean meridional circulation ($\overline{v}$, $\overline{w}$)
and the transformed Eulerian mean meridional circulation \((\vec{v}^*, \vec{w}^*)\), which can be of opposite sign in the winter stratosphere (Dunkerton 1978). Planetary wave propagation therefore involves significant vertical advection in pressure coordinates, which could be inaccurately represented by traditional Eulerian advection schemes. Secondly, the horizontal-dissipation scheme in most pressure-coordinate models mixes vorticity, divergence and temperature along model levels. It is possible that when the dissipation is applied on the isentropic levels of the HIGCM it will more closely mimic the adiabatic mixing of PV than when it is applied on the pressure levels of the UGCM.

To investigate whether the planetary wave behaviour has changed in experiment 2STP, an Eliassen–Palm (EP) flux diagnosis following Edmon et al. (1980) has been performed. In the time mean, the convergence of EP flux vectors corresponds to the rate at which the mean flow is decelerated by the planetary wave breaking. Figures 7(a) and (b) show the EP flux divergence for experiments 2SP and 2STP. The plots show only the top 100 hPa since this is where the differences in flux divergence are most obviously associated with changes in wave-driven descent and hence changes in stratospheric temperatures. A comparison of the plots indicates that there is a general increase in the EP flux convergence in the northern hemisphere extratropical stratosphere above 40 hPa in experiment 2STP. The increased convergence would imply a stronger direct residual circulation and descent in higher latitudes consistent with the observed warming.

The other mechanism which may impact on the polar stratospheric temperatures is the GWD parametrization. The dynamical adjustment to stratospheric GWD is the same as that to planetary wave breaking, with it driving a mean meridional circulation which adiabatically warms the atmosphere polewards and below where the drag is exerted. Since the parametrized wave breaking is dependent on the local Richardson number, \(Ri\), in the isentropic domain of the HIGCM the GWD will depend on the model-level spacing because \(Ri \propto \partial \theta / \partial p\). Clearly, since the model-level spacing may change as part of the dynamical adjustment to the GWD, there exists the possibility of a feedback between the GWD and the HIGCM level spacing. However, although there are differences in the GWD in our simulations they are not as large as the differences in the EP fluxes and, moreover, the direct impact of these differences on the zonal mean circulation is not so readily apparent as those from the EP fluxes.

We now investigate the impact of extending the isentropic domain all the way up to the model top by comparing experiment 2ST with experiment 2STP. The vertical-coordinate table for experiment 2ST was identical to that for experiment 2STP except that the top four half-levels were defined to be purely isentropic. The vertical-coordinate definition for these four levels is as given in Table B.2. With the vertical coordinate so defined, experiment 2ST has 11 purely isentropic half-levels extending from the 344 K surface up to the 950 K surface.

Figure 6(b) shows the differences in zonal mean temperature between experiment 2ST and experiment 2STP. The first point of note is that the differences in the lower stratosphere are small; the impacts at the tropopause seen in experiment 2STP are not affected by the change in vertical coordinate at the model top. Consistent with the change in the vertical coordinate, the largest differences occur close to the model top. In the northern hemisphere extratropics, these temperature differences extend right down to the ground, with experiment 2ST colder from 30°N–60°N and warmer north of 60°N.

Clearly, with the extended isentropic domain of experiment 2ST, these temperature differences may be due to any of the mechanisms proposed above to explain the differences between experiments 2STP and 2SP. However, the differences between experiments 2ST and 2STP may also be related to the handling of the top boundary itself. Although the uppermost model half-level is nominally at \(p = 0\) and \(\theta = \infty\), there
Figure 7. Zonal mean Eliassen-Palm flux divergence fields for the (a) T21 \( \sigma-p \) (experiment 2SP), (b) T21 \( \sigma-\theta-p \) (experiment 2STP), (c) T21 \( \sigma-\theta \) (experiment 2ST), (d) T42 \( \sigma-p \) (experiment 3SP), (e) T42 \( \sigma-\theta-p \) (experiment 3STP), and (f) T42 \( \sigma-\theta \) (experiment 3ST) simulations. Note that only the top 100 hPa of the atmosphere is plotted. The contour interval is 0.5 m s\(^{-1}\) d\(^{-1}\). See text for further explanation of the experiments.
are only a finite number of model levels in the stratosphere, so that planetary waves can propagate up to the top boundary in a finite time and be reflected rather than dissipated in the upper part of the model domain. As discussed by Boville and Cheng (1988), this spurious reflection leads to a weaker wave-driven deceleration of the mean flow and, via thermal-wind balance, to a cooler polar stratosphere. It is possible that the use of isentropic levels all the way to the top of the model in experiment 2ST affects the reflection of planetary waves from the top boundary, for example through their numerical group velocity, and in this way affects the stratospheric temperatures.

This possibility may be investigated by comparing the EP flux convergence for experiment 2ST (Fig. 7(c)) with the EP flux convergence for experiments 2SP and 2STP (Figs. 7(a) and (b)). The most striking feature is that the flux convergence is much more coherent right up to the top level in experiment 2ST than in the other two experiments. This coherency alone suggests that isentropic coordinate models better represent planetary wave propagation and dissipation than pressure-coordinate models. The additional fact that the EP flux convergence is largest in experiment 2ST, thus implying that the planetary wave breaking is largest in this experiment, also indicates that the isentropic coordinate does improve the treatment of planetary waves close to the model top.

In summary, the sensitivity of the zonal mean temperature changes to the isentropic coordinate may be due to changes in the planetary wave propagation and/or breaking, or due to changes in the GWD, or associated with changes in the vertical advection of moisture and/or temperature. In the northern hemisphere it is not easy to determine which process is dominant. However, the EP flux diagnosis suggests that much of the response may be associated with changes in the planetary wave breaking. In the southern hemisphere the influence of planetary wave activity and GWD is much smaller; here it seems most likely that the warming of the lower stratosphere is indicative of the improved vertical advection of moisture and/or temperature with the isentropic coordinate.

(ii) \textit{T42 climate simulations}. A parallel set of experiments (3SP, 3STP, 3ST) was performed at T42 horizontal resolution to complement the T21 simulations just described. The vertical levels for these simulations were identical to those used in the T21 simulations, except that the transition from $\sigma$ to $\theta$-levels was less rapid for the two HIGCM simulations. The reasoning behind this, as pointed out by JT93, is that the Tibetan Plateau is significantly higher at T42 resolution than at T21 resolution. During northern summer, the warmth of the Tibetan Plateau limits the rate of transition to pure $\theta$-levels. In this study we have only performed perpetual January simulations and so there is no reason to make the transition so gradual, but in the first instance we were concerned with achieving stable HIGCM integrations. As an illustration of the more gradual transition, the vertical-coordinate table used to define the levels for experiment 3ST is shown in Table B.2. In this experiment the purely isentropic domain extended upwards from the 379 K surface rather than from the 344 K surface as in the corresponding T21 simulation.

There is one other important difference between the T42 and T21 resolution experiments. Since the model time step is reduced from 45 to 30 minutes whilst the horizontal resolution is doubled, the horizontal Courant number is more likely to exceed 1 at T42 resolution. The strongest winds simulated in the model are generally those associated with the stratospheric polar-night jet and these may, on occasions, lead to the Courant number exceeding 1. In the UGCM, the pragmatic approach adopted to inhibit this problem has been to increase the horizontal diffusion in the upper four levels of the
model by halving the time-scale at each successive level. Thus level 5 is the uppermost level with a diffusion time-scale of four hours, whilst for the top model level the time-scale is down to 15 minutes. As we shall see, the EP flux diagnosis suggests that the increased horizontal diffusion has a large impact on the stratospheric planetary wave breaking.

The differences in the zonal mean temperature between the three T42 experiments are shown in Figs. 6(c) and 6(d). Figure 6(c) shows that, as at T21 resolution, the main response to the isentropic coordinate in the southern hemisphere is the warming of the extratropical lower stratosphere. In the northern hemisphere the response is not so robust to the change in horizontal resolution. In particular, the lower stratosphere is cooler in the HIGCM (experiments 3STP and 3ST) than in the UGCM (experiment 3SP). This is the opposite response to that at T21 resolution. Additionally, Fig. 6(d) shows that experiment 3ST is colder than experiment 3STP through the depth of the polar stratosphere, which is again opposite to the response at T21 resolution.

One possible mechanism which may lead to the resolution sensitive response in the northern hemisphere lower stratosphere is the improved representation of baroclinic eddies at T42 resolution. At T21 resolution these eddies are barely resolved and so their explicit poleward transport of heat and momentum is greatly underestimated. Moreover, a plausible zonal mean climate is only obtained by employing a 3 hour $\nabla^8$ horizontal diffusion scheme which, in some senses, acts to parametrize the polewards transports of the baroclinic eddies. At T42 resolution, the improved resolution of the baroclinic eddies makes the model climate less sensitive to the horizontal-diffusion scheme. Indeed, Stephenson (1995) found that the response to changing the horizontal-diffusion scheme in the UGCM was rather different at T21 and T42 resolutions. We have already suggested that some of the temperature changes in our T21 resolution simulations may be due to the sensitivity of the horizontal-diffusion scheme to the isentropic coordinate. It is possible that the resolution sensitive aspect of the horizontal diffusion may be sufficiently large that the response to the isentropic coordinate is opposite at the two resolutions.

The other resolution sensitive response, namely the colder polar stratosphere of experiment 3ST, appears to be caused by the increased upper-level diffusion included in the T42 experiments. The impact of the increased upper-level diffusion on the planetary wave dissipation is much larger when the upper model levels are isobaric or almost isobaric compared with when they are isentropic. This is clearly borne out by comparing all the EP flux divergence plots in Fig. 7. The most striking features in these plots are the unrealistic minima just below the model top in experiments 3SP and 3STP. The unrealistic minima imply an unrealistically strong mean meridional circulation which drives the warming of the polar stratosphere in these experiments relative to experiment 3ST. In comparison with experiments 2SP and 2STP, the unrealistic minima imply a resolution sensitive change in the treatment of wave breaking in the pressure-topped models. In contrast, the similarity of the EP flux divergence field for experiments 2ST and 3ST indicates that the treatment of wave breaking is not strongly sensitive to the horizontal resolution when the uppermost model levels are isentropic. It is interesting to note that the absolute polar stratospheric temperatures (not shown) appear to be regulated by the EP flux divergences. Thus, the polar stratospheric temperatures are very similar in experiments 2ST and 3ST. However, they are much colder in experiments 2SP and 2STP than in experiments 3SP and 3STP. The resolution sensitive temperature response is therefore actually due to resolution sensitive changes in EP flux divergence occurring in the pressure-topped models.
The increased upper-level horizontal diffusion could lead to the differences in the planetary wave dissipation in the following way. As mentioned earlier, the vertical propagation of planetary waves is essentially a quasi-adiabatic process. Thus the unrealistically large amount of planetary wave breaking in experiments 3SP and 3STP suggests that the increased diffusion in these experiments leads to an unrealistically large diabatic mixing of the PV distribution. In contrast, the similarity of the planetary wave breaking in experiments 2ST and 3ST suggests that applying the increased horizontal diffusion along isentropic levels gives, at most, a small diabatic component to the mixing of the PV distribution. To put it another way, the planetary wave breaking differences described above suggest that the application of a horizontal-diffusion scheme to the vorticity, divergence and temperature fields along isentropic levels most closely mimics the diabatic mixing of PV that arguably occurs in the real atmosphere even though, in this instance, the numerical mixing occurs at much larger scales than the real mixing.

5. Conclusions

In this paper we have described three major improvements to the HIGCM. Firstly, we have modified the calculation of the vertical velocity so as to minimize finite-difference errors in the isentropic domain. Secondly, we have improved the implementation of the radiation scheme so that it is able to damp, and does not itself create, noisy temperature profiles. This has allowed us to compare directly the HIGCM with the UGCM. Thirdly, we have extended the isentropic domain up to the top of the model.

In idealized adiabatic baroclinic instability life-cycle experiments the spurious cross-isentropic vertical velocities were, as expected, substantially reduced with the new vertical scheme. The loss of formal conservation properties with the new scheme did not appear to compromise the simulation in any way. Indeed, in the life-cycle simulations it was found to be the horizontal-diffusion scheme that produced particularly large errors in conservation in the HIGCM. Investigation revealed that the diffusion of temperature could generate potential energy at about half the rate that potential energy was converted to kinetic energy by the baroclinic waves. This appeared to be because significant horizontal gradients of level thickness may exist in the HIGCM. Since the horizontal diffusion does not account for any change in mass along a model level, an increase in global mean temperature resulted. Clearly, a mass-weighted diffusion scheme should reduce these errors in conservation. More generally these results indicate that, for any model with non-zero horizontal gradients of level thickness, the use of such a horizontal dissipation scheme may result in much larger global energy-conservation errors than would be expected merely from the dissipative nature of the scheme.

In the future, there are plans to investigate the HIGCM dynamics scheme by implementing it in the framework proposed by Held and Suarez (1994) and Boer and Denis (1997). In such an investigation, the only model forcings are a relaxation of the temperature field towards a zonally symmetric state and a simple representation of boundary-layer friction. With no model parametrizations acting, the simulated climate is sensitive only to the model dynamics. The merits of the HIGCM 'dynamical core', relative to the other dynamics schemes already tested in this framework, may then be readily identified.

The impact of the new vertical scheme in a full climate simulation was consistent with the findings from the life-cycle experiments; the vertical velocities more closely corresponded to the diabatic heating fields whilst the overall impact on the model climate was rather small compared with the old hybrid-isentropic scheme. This fact, together with the fact that the introduction of the isentropic coordinate itself has a large
impact on the model climate, indicates that the spurious vertical motions the new scheme
is designed to minimize are relatively unimportant in determining the behaviour of the
dynamical core of the HIGCM.

A comparison of climate simulations made with $\sigma-p$, $\sigma-\theta-p$ and $\sigma-\theta$ vertical
coordinates suggested that some of the potential advantages of the isentropic coordinate
have been realized in the HIGCM. The warming of the southern hemisphere high-
latitude lower stratosphere, which represents a significant reduction in an error common
to many Eulerian GCMs, may be associated with the improved vertical advection of
moisture and/or temperature. In the northern hemisphere the dominant mechanism
by which the isentropic coordinate changed the climate simulation was through an
improvement in the vertical propagation and dissipation of the planetary waves. In
particular, an EP flux diagnosis indicated that simulations with the $\sigma-\theta$ version of
the HIGCM reduced the spurious reflection of planetary waves by the model lid.
Furthermore, at T42 resolution, the use of increased horizontal diffusion as a simple
sponge for the planetary waves had little impact in the $\sigma-\theta$ version of the HIGCM.
This was in marked contrast to the impact in the $\sigma-p$ coordinate UGCM where the
planetary wave breaking, as deduced from the EP flux diagnosis, was increased in an
unrealistic way just below the model top. Since the vertical propagation of planetary
waves is essentially a quasi-adiabatic process, this suggests that applying a horizontal-
diffusion scheme along an isentropic surface is more appropriate than applying it along
pressure levels; the mixing of vorticity, divergence and temperature along isentropic
levels appears to be more nearly adiabatic which means that the dissipation scheme more
closely mimics the adiabatic mixing of PV that arguably occurs in the real atmosphere.
Moreover, the results suggest that the T42 $\sigma-\theta$ version of the HIGCM could be run
without the need for the artificial increase in upper-level horizontal diffusion. This would
seem preferable to using pressure coordinates, where the artificial increase in upper-level
horizontal diffusion is required to counter the spurious treatment of the planetary waves.

ACKNOWLEDGEMENTS

This work benefited from helpful discussions with Mike Blackburn. The work was
supported by European Community grant EV5V-CT92-0125.

APPENDIX A

In the text, the various experiments are referred to by the names given to them
in Table A.1. Experiment number 1 is the set of baroclinic life-cycle simulations,
experiment number 2 is the set of T21 climate simulations, whilst experiment number 3
is the set of T42 climate simulations.

APPENDIX B

Below we give two examples of the vertical-coordinate table used to define the
half-level pressures in the HIGCM simulations. Table B.1 was used in the T21 $\sigma-\theta-p$
experiments (experiment 2STP and 2STP-old), whilst Table B.2 was used to define the
levels for the T42 $\sigma-\theta$ simulation (experiment 3ST). Note that the half-levels correspond
to the approximate $\sigma$ and $\theta$ values listed in the rightmost two columns and are identical
to those used in Table A.1 of JT93, except that the value of $\theta$ for level 1.5 is increased
from 850 K to 950 K. This change enabled us to use pure $\theta$-levels up to the model top in
the $\sigma-\theta$ experiments. Note that $(1 - d)/d$ is the ratio of the isentropic coordinate to the
TABLE A.1. A BRIEF DESCRIPTION OF THE EXPERIMENTS DISCUSSED IN THE TEXT TOGETHER WITH THE IDENTIFIER USED TO REFER TO THEM

<table>
<thead>
<tr>
<th>Experiment identifier</th>
<th>Vertical coordinate</th>
<th>Vertical scheme</th>
<th>Other comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S</td>
<td>$\sigma$</td>
<td>-</td>
<td>4 hour $\nabla^6$ scale-selective dissipation</td>
</tr>
<tr>
<td>1STP-old</td>
<td>$\sigma-\theta-p$</td>
<td>Old</td>
<td></td>
</tr>
<tr>
<td>1STP</td>
<td>$\sigma-\theta-p$</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>1S-s</td>
<td>$\sigma$</td>
<td>-</td>
<td>15 minute $\nabla^6$ scale-selective dissipation</td>
</tr>
<tr>
<td>1STP-s</td>
<td>$\sigma-\theta-p$</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>1S-t</td>
<td>$\sigma$</td>
<td>-</td>
<td>4 hour $\nabla^6$ scale-selective dissipation but not applied to temperature</td>
</tr>
<tr>
<td>1STP-t</td>
<td>$\sigma-\theta-p$</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>2SP</td>
<td>$\sigma-p$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>2STP</td>
<td>$\sigma-\theta-p$</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>2STP-old</td>
<td>$\sigma-\theta-p$</td>
<td>Old</td>
<td>3 hour $\nabla^8$ scale-selective dissipation</td>
</tr>
<tr>
<td>2ST</td>
<td>$\sigma-\theta$</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>3SP</td>
<td>$\sigma-p$</td>
<td>-</td>
<td>4 hour $\nabla^6$ scale-selective dissipation but with successively reduced time-scale for the top four model levels</td>
</tr>
<tr>
<td>3STP</td>
<td>$\sigma-\theta-p$</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>3ST</td>
<td>$\sigma-\theta$</td>
<td>New</td>
<td></td>
</tr>
</tbody>
</table>

TABLE B.1. VERTICAL-COORDINATE TABLE USED IN THE T21 $\sigma-\theta-p$ EXPERIMENTS (EXPERIMENTS 2STP AND 2STP-OLD)

<table>
<thead>
<tr>
<th>Level</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$\sigma$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.0</td>
<td>0.0000</td>
<td>0.0000E+00</td>
<td>1.000</td>
<td>0.0000</td>
<td>-</td>
</tr>
<tr>
<td>1.5</td>
<td>930.0</td>
<td>0.0000</td>
<td>0.2731E+08</td>
<td>0.889</td>
<td>0.0093</td>
<td>950</td>
</tr>
<tr>
<td>2.5</td>
<td>1860.0</td>
<td>0.0000</td>
<td>0.5768E+08</td>
<td>0.667</td>
<td>0.0186</td>
<td>705</td>
</tr>
<tr>
<td>3.5</td>
<td>2860.0</td>
<td>0.0000</td>
<td>0.9698E+08</td>
<td>0.444</td>
<td>0.0286</td>
<td>625</td>
</tr>
<tr>
<td>4.5</td>
<td>4060.0</td>
<td>0.0000</td>
<td>0.1232E+09</td>
<td>0.222</td>
<td>0.0406</td>
<td>550</td>
</tr>
<tr>
<td>5.5</td>
<td>5460.0</td>
<td>0.0000</td>
<td>0.1526E+09</td>
<td>0.000</td>
<td>0.0546</td>
<td>500</td>
</tr>
<tr>
<td>6.5</td>
<td>7060.0</td>
<td>0.0000</td>
<td>0.1474E+09</td>
<td>0.000</td>
<td>0.0706</td>
<td>460</td>
</tr>
<tr>
<td>7.5</td>
<td>8860.0</td>
<td>0.0000</td>
<td>0.1461E+09</td>
<td>0.000</td>
<td>0.0886</td>
<td>430</td>
</tr>
<tr>
<td>8.5</td>
<td>10960.0</td>
<td>0.0000</td>
<td>0.1440E+09</td>
<td>0.000</td>
<td>0.1096</td>
<td>403</td>
</tr>
<tr>
<td>9.5</td>
<td>13460.0</td>
<td>0.0000</td>
<td>0.1427E+09</td>
<td>0.000</td>
<td>0.1346</td>
<td>379</td>
</tr>
<tr>
<td>10.5</td>
<td>16360.0</td>
<td>0.0000</td>
<td>0.1448E+09</td>
<td>0.000</td>
<td>0.1636</td>
<td>360</td>
</tr>
<tr>
<td>11.5</td>
<td>19860.0</td>
<td>0.0000</td>
<td>0.1499E+09</td>
<td>0.000</td>
<td>0.1986</td>
<td>344</td>
</tr>
<tr>
<td>12.5</td>
<td>22018.8</td>
<td>0.0254</td>
<td>0.1422E+09</td>
<td>0.103</td>
<td>0.2456</td>
<td>329</td>
</tr>
<tr>
<td>13.5</td>
<td>23846.4</td>
<td>0.0641</td>
<td>0.1323E+09</td>
<td>0.212</td>
<td>0.3026</td>
<td>315</td>
</tr>
<tr>
<td>14.5</td>
<td>25095.3</td>
<td>0.1206</td>
<td>0.1215E+09</td>
<td>0.325</td>
<td>0.3716</td>
<td>303</td>
</tr>
<tr>
<td>15.5</td>
<td>25373.4</td>
<td>0.1979</td>
<td>0.1119E+09</td>
<td>0.438</td>
<td>0.4516</td>
<td>295</td>
</tr>
<tr>
<td>16.5</td>
<td>24333.0</td>
<td>0.2983</td>
<td>0.0985E+09</td>
<td>0.551</td>
<td>0.5416</td>
<td>289</td>
</tr>
<tr>
<td>17.5</td>
<td>21827.1</td>
<td>0.4176</td>
<td>0.8635E+08</td>
<td>0.657</td>
<td>0.6359</td>
<td>286</td>
</tr>
<tr>
<td>18.5</td>
<td>18089.9</td>
<td>0.5465</td>
<td>0.7069E+08</td>
<td>0.751</td>
<td>0.7274</td>
<td>285</td>
</tr>
<tr>
<td>19.5</td>
<td>15393.4</td>
<td>0.6750</td>
<td>0.5312E+08</td>
<td>0.832</td>
<td>0.8109</td>
<td>285</td>
</tr>
<tr>
<td>20.5</td>
<td>8970.1</td>
<td>0.7922</td>
<td>0.3506E+08</td>
<td>0.898</td>
<td>0.8819</td>
<td>285</td>
</tr>
<tr>
<td>21.5</td>
<td>4907.7</td>
<td>0.8875</td>
<td>0.1918E+08</td>
<td>0.948</td>
<td>0.9366</td>
<td>285</td>
</tr>
<tr>
<td>22.5</td>
<td>1961.9</td>
<td>0.9534</td>
<td>0.0766E+07</td>
<td>0.980</td>
<td>0.9730</td>
<td>285</td>
</tr>
<tr>
<td>23.5</td>
<td>320.9</td>
<td>0.9891</td>
<td>0.1254E+07</td>
<td>0.997</td>
<td>0.9923</td>
<td>285</td>
</tr>
<tr>
<td>24.5</td>
<td>0.0</td>
<td>1.0000</td>
<td>0.0000E+00</td>
<td>1.000</td>
<td>1.0000</td>
<td>-</td>
</tr>
</tbody>
</table>

See text for explanation of headings.

sigma coordinate in the definition of the model half-level. A level with $d = 0$ is purely isentropic so there are seven isentropic half-levels defined by this vertical-coordinate table.

Table B.2 differs from Table B.1 by making the transition from $\sigma$ to $\theta$-levels more gradually, and also by extending the purely isentropic levels up to the model top. Thus
### TABLE B.2. VERTICAL-COORDINATE TABLE USED IN THE T42 σ–θ SIMULATION (EXPERIMENT 3ST)

<table>
<thead>
<tr>
<th>Level</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>σ</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.0</td>
<td>0.0000</td>
<td>0.0000E+00</td>
<td>1.000</td>
<td>0.0000</td>
<td>-</td>
</tr>
<tr>
<td>1.5</td>
<td>930.0</td>
<td>0.0000</td>
<td>0.2692E+09</td>
<td>0.000</td>
<td>0.0093</td>
<td>950</td>
</tr>
<tr>
<td>2.5</td>
<td>1860.0</td>
<td>0.0000</td>
<td>0.2149E+09</td>
<td>0.000</td>
<td>0.0186</td>
<td>705</td>
</tr>
<tr>
<td>3.5</td>
<td>2860.0</td>
<td>0.0000</td>
<td>0.1897E+09</td>
<td>0.000</td>
<td>0.0286</td>
<td>625</td>
</tr>
<tr>
<td>4.5</td>
<td>4060.0</td>
<td>0.0000</td>
<td>0.1687E+09</td>
<td>0.000</td>
<td>0.0406</td>
<td>550</td>
</tr>
<tr>
<td>5.5</td>
<td>5460.0</td>
<td>0.0000</td>
<td>0.1526E+09</td>
<td>0.000</td>
<td>0.0546</td>
<td>500</td>
</tr>
<tr>
<td>6.5</td>
<td>7060.0</td>
<td>0.0000</td>
<td>0.1474E+09</td>
<td>0.000</td>
<td>0.0706</td>
<td>460</td>
</tr>
<tr>
<td>7.5</td>
<td>8860.0</td>
<td>0.0000</td>
<td>0.1461E+09</td>
<td>0.000</td>
<td>0.0886</td>
<td>430</td>
</tr>
<tr>
<td>8.5</td>
<td>10960.0</td>
<td>0.0000</td>
<td>0.1440E+09</td>
<td>0.000</td>
<td>0.1096</td>
<td>403</td>
</tr>
<tr>
<td>9.5</td>
<td>13460.0</td>
<td>0.0000</td>
<td>0.1427E+09</td>
<td>0.000</td>
<td>0.1346</td>
<td>379</td>
</tr>
<tr>
<td>10.5</td>
<td>15105.4</td>
<td>0.0125</td>
<td>0.1337E+09</td>
<td>0.077</td>
<td>0.1636</td>
<td>360</td>
</tr>
<tr>
<td>11.5</td>
<td>16759.3</td>
<td>0.0310</td>
<td>0.1265E+09</td>
<td>0.156</td>
<td>0.1986</td>
<td>344</td>
</tr>
<tr>
<td>12.5</td>
<td>18497.2</td>
<td>0.0606</td>
<td>0.1195E+09</td>
<td>0.247</td>
<td>0.2456</td>
<td>329</td>
</tr>
<tr>
<td>13.5</td>
<td>19965.6</td>
<td>0.1029</td>
<td>0.1108E+09</td>
<td>0.340</td>
<td>0.3026</td>
<td>315</td>
</tr>
<tr>
<td>14.5</td>
<td>20930.4</td>
<td>0.1623</td>
<td>0.1014E+09</td>
<td>0.437</td>
<td>0.3716</td>
<td>303</td>
</tr>
<tr>
<td>15.5</td>
<td>21071.0</td>
<td>0.2409</td>
<td>0.0929E+09</td>
<td>0.533</td>
<td>0.4516</td>
<td>295</td>
</tr>
<tr>
<td>16.5</td>
<td>20110.0</td>
<td>0.3405</td>
<td>0.0825E+08</td>
<td>0.629</td>
<td>0.5416</td>
<td>289</td>
</tr>
<tr>
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<td>0.718</td>
<td>0.6359</td>
<td>286</td>
</tr>
<tr>
<td>18.5</td>
<td>14797.9</td>
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<td>0.0573E+08</td>
<td>0.797</td>
<td>0.7274</td>
<td>285</td>
</tr>
<tr>
<td>19.5</td>
<td>11066.5</td>
<td>0.7002</td>
<td>0.0432E+08</td>
<td>0.864</td>
<td>0.8109</td>
<td>285</td>
</tr>
<tr>
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<td>0.8092</td>
<td>0.0284E+08</td>
<td>0.918</td>
<td>0.8819</td>
<td>285</td>
</tr>
<tr>
<td>21.5</td>
<td>3964.4</td>
<td>0.8970</td>
<td>0.0154E+08</td>
<td>0.958</td>
<td>0.9366</td>
<td>285</td>
</tr>
<tr>
<td>22.5</td>
<td>1581.0</td>
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<td>0.0617E+07</td>
<td>0.984</td>
<td>0.9730</td>
<td>285</td>
</tr>
<tr>
<td>23.5</td>
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<td>0.0100E+00</td>
<td>0.997</td>
<td>0.9923</td>
<td>285</td>
</tr>
<tr>
<td>24.5</td>
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<td>1.0000</td>
<td>0.0000E+00</td>
<td>1.000</td>
<td>1.0000</td>
<td>-</td>
</tr>
</tbody>
</table>

See text for explanation of headings.

This experiment has nine purely isentropic half-levels. The levels for the other two HIGCM experiments were obtained by switching between the two vertical-coordinate tables at level 6.5, i.e. in the purely isentropic domain for both tables. Thus, the vertical-coordinate table for the T21 σ–θ simulation was obtained by combining the bottom of Table B.1 with the top of Table B.2, whilst the vertical-coordinate table for the T42 σ–θ–p simulation was obtained by combining the top of Table B.1 with the bottom of Table B.2.

### REFERENCES


Bleck, R. 1974 Short-range prediction in isentropic coordinates with filtered and unfiltered models. Mon. Weather Rev., 102, 813-829


Miller, M. J. and Palmer, T. N. 1989 ‘Proposed revision and enhancement of the gravity wave drag parametrization scheme’. ECMWF Research Memorandum R2327/2421


