Momentum budgets over idealized orography with a non-hydrostatic anelastic model. I: Two-dimensional flows

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SUMMARY

Numerical simulations of two-dimensional (2D) mountain waves are carried out with a non-hydrostatic anelastic model to investigate the interaction between 2D nonlinear orographic effects and the synoptic flow. The problem is idealized to isolate the essential features of real orographic flows. For each simulation, the complete momentum equations of the model are summed in different domains of variable size representative of a grid mesh of a general-circulation model. The resulting budgets of momentum, instantaneous or averaged in time, are used to test the hypothesis of current drag parametrizations in three nonlinear regimes of 2D orographic flows. For mountain waves breaking in the troposphere, the mean flow is decelerated in the vertical between the ground and the theoretical breaking level. In the horizontal, the deceleration of the large-scale flow is uniformly distributed by non-resolved acoustic waves. When non-Boussinesq effects lead mountain waves to break in the lower stratosphere, the large-scale flow is decelerated on a vertical wavelength centred on the breaking level. In the case of non-breaking trapped lee waves, the deceleration of the mean flow is strongly dependent of the non-hydrostatic character of the primary propagating wave. Off-line tests of two drag parametrizations show that some adaptations are necessary to improve the prediction of the impact of subgrid-scale orographic effects on the large-scale flow for the set of investigated idealized orographic events.

KEYWORDS: Gravity-wave drag Mountain waves Parametrisation

1. INTRODUCTION

When a stable stratified atmosphere flows over an obstacle, orographically excited gravity waves transport positive momentum toward the obstacle to balance the momentum sink created by the pressure drag exerted by the obstacle on the airflow. When mountain waves are non-dissipative, the vertical flux of momentum exactly balances the pressure drag, and there is no interaction between the wave (typically smaller than 50 km) and the large scale (typically larger than 500 km), in agreement with the Eliassen and Palm (1960) theorem. In the presence of nonlinear processes such as wave breaking, the momentum sink due to pressure drag is only partly balanced by the vertical flux of horizontal momentum (e.g. Lilly and Kennedy 1973). Under those circumstances, a deceleration is exerted on the large-scale flow, and the impact of the subgrid-scale orography needs to be parametrized in a general-circulation model (GCM) in order to compute a realistic forecast.

Most gravity-wave drag parametrizations (e.g. Miller and Palmer 1986; Baines and Palmer 1990, henceforth referred to as BP90) are based on the concept of vertical transport of horizontal momentum by internal gravity waves, through the equation

$$\frac{\partial \overline{\rho u}}{\partial t} = - \frac{\partial \overline{\rho u' w'}}{\partial z},$$

where overbars denote horizontal average, z is height, \(\rho u\) is the horizontal momentum, and \(\overline{\rho u' w'}\) is the vertical flux of horizontal momentum. If these parametrizations have greatly improved the GCM forecasts (Boer et al. 1984; Palmer et al. 1986; Kim and Arakawa 1995), some aspects of the interaction between small-scale orographic effects and the large scale are still unclear. For instance, some uncertainties remain concerning the spatial distribution and the temporal evolution of the scale interaction, and the impact of non-hydrostatic effects on the mean flow needs to be investigated.

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Both observations (Lilly and Kennedy 1973) and numerical experiments (Peltier and Clark 1983) have identified wave breaking as an important source of divergence for the vertical flux of momentum. Durran (1995, hereinafter referred to as D95) studied this nonlinear regime through numerical simulations of realistic and idealized two-dimensional (2D) orographic flows with a high-resolution model. In the context of two events of breaking mountain waves, D95 used budgets of pseudo-momentum to show the importance of horizontal momentum transfers in the local momentum budget.

Decrease of density, ρ, with height implies the growth of internal gravity waves amplitude, in order to maintain constant the energy density. This growth induces wave overturning at a critical level in the stratosphere, a common feature of the observed flow (Kochanski 1964). Bacmeister and Schoeberl (1989) showed numerically that stratospheric overturning shifts the divergence of the vertical momentum flux aloft. This deceleration occurring at upper levels is present in current drag parametrizations through the Lindzen (1981) saturation principle. Nevertheless, some aspects of the scale interaction (spatial distribution, etc.) need to be investigated.

When the propagation characteristics vary with height, mountain waves can be trapped in a restricted layer of the atmosphere. According to the calculations presented by Bretherton (1969), the mean flow cannot be affected by trapped lee waves (t.l.w.). A great deal of work (Durran 1991; Keller 1994; Satomura and Bougeault 1994; D95; Lott 1998) has been dedicated to study the ability of t.l.w. to transfer momentum in the horizontal. The problem is still open and it is not clear whether t.l.w. may use a different scheme of momentum transfer than vertically propagating mountain waves.

The purpose of this paper is to test the basic concepts of current drag parametrizations with a high-resolution numerical model where the orographic flow is correctly resolved. Computation of exhaustive momentum budgets in boxes of variable size allows one to investigate the ability of current drag parametrizations to represent the impact of subgrid-scale orography on the flow of a large-scale model. By doing so, it is possible to isolate the essential budget terms which need to be parametrized in a GCM. For an easier comparison between the parametrization prediction and the results of the high-resolution model, the real-flow problem is simplified to an idealized problem of 2D orographic flow while conserving the main features representative of real-flow regimes.

The paper is organized as follows. Section 2 contains a brief summary of the characteristics of the numerical model, a presentation of the momentum-budget calculations, and a description of the experimental context of the numerical study. In the following sections, predictions of current drag parametrizations are compared with the results of a high-resolution model for three nonlinear regimes of orographic flow. Firstly, in section 3, the interaction between mountain wave breaking in the troposphere and the large-scale flow is thoroughly analysed. Secondly, section 4 deals with the mean-flow deceleration induced by mountain waves breaking in the stratosphere. Thirdly, in section 5, some typical events of t.l.w. are used to study the impact of non-hydrostatic effects on the large-scale flow. Discussion of the results leads us to draw some conclusions in section 6.

2. SET UP OF MOMENTUM-BUDGET CALCULATIONS IN NUMERICAL EXPERIMENTS

Momentum transfers in orographic flows are studied through numerical simulations of a stable stratified atmosphere over a bell-shaped 'Witch-of-Agnesi' mountain defined by

$$h(x) = \frac{h_0}{1 + (x/a)^2},$$

(1)
where $a$ and $h_0$ are respectively the half-width and the maximum height of the obstacle. Incident flow speed, $U_0$, and Brunt–Väisälä frequency, $N_0$, are uniform by layers far upstream of the obstacle. The atmosphere is dry, adiabatic, non-rotating and inviscid. Under these assumptions, the flow regime is controlled by three dimensionless parameters:

$$
\hat{h} = \frac{N_0 h_0}{U_0}, \quad \hat{a} = \frac{N_0 a}{U_0}, \quad \hat{\delta} = \frac{N_0 H}{U_0} = \frac{N_0 U_0}{g}.
$$

$\hat{h}$ measures the importance of the nonlinear effects in the momentum equations, $\hat{a}$ controls the non-hydrostatic effects, and $\hat{\delta}$ gives the importance of the non-Boussinesq effects ($g$ is the acceleration due to gravity and $H$ is the density scale height). This simplified configuration allows one to recover the main features of a real wave-breaking event and to conserve a minimum set of physical parameters controlling the regime of flow. In the non-hydrostatic regime, the square of Scorer’s (1949) parameter controls the wave propagation:

$$
I^2 = \frac{N^2}{U^2} - \frac{1}{U} \frac{d^2 U}{dz^2}.
$$

Its variation with height may induce wave trapping in a restricted layer of the atmosphere.

The numerical model used in this study is a 2D version of the non-hydrostatic model Meso-NH presented by Lefore et al. (1998). The formulation of the model is based on the Lipps and Hemler (1982) form of the anelastic approximation. This relies on the assumption that the atmosphere will not depart significantly from a reference state having a potential temperature $\theta$ slowly varying in the vertical. In a Cartesian domain and for a dry, non-rotating, adiabatic atmosphere, the Meso-NH equations reduce to

$$
\rho' = \rho_{\text{ref}} \left( \frac{C_p}{R} \frac{\Pi'}{\Pi_{\text{ref}}} - \frac{\theta'}{\theta_{\text{ref}}} \right),
$$

(2)

$$
\nabla \cdot (\rho_{\text{ref}} \mathbf{U}) = 0,
$$

(3)

$$
\frac{\partial}{\partial t} (\rho_{\text{ref}} \mathbf{U}) + \nabla \cdot (\rho_{\text{ref}} \mathbf{U} \otimes \mathbf{U}) + \rho_{\text{ref}} \nabla \Phi + \rho_{\text{ref}} g \frac{\theta - \theta_{\text{ref}}}{\theta_{\text{ref}}} - \rho_{\text{ref}} \mathbf{F}_m = 0,
$$

(4)

$$
\frac{\partial}{\partial t} (\rho_{\text{ref}} \theta) + \nabla \cdot (\rho_{\text{ref}} \theta \mathbf{U}) - \rho_{\text{ref}} \mathbf{F}_\theta = 0,
$$

(5)

where (2) is the linearized equation of state, (3) is the continuity equation, (4) is the momentum equation ($\mathbf{F}_m$ represent the turbulence terms), and (5) is the thermodynamic equation ($\rho_{\text{ref}} \mathbf{F}_\theta$ stands for the subgrid turbulence terms). In (4), terms are, from left to right, tendency, advection, pressure gradient, buoyancy force and subgrid-scale turbulent mixing. The pressure function is defined by

$$
\Phi = C_p \theta_{\text{ref}} \Pi'.
$$

(6)

The Exner function is defined by $\Pi = (P/P_{00})^{R/C_p}$ where $P$ is pressure, $P_{00}$ a reference value = 1000 hPa, $R$ is the specific gas constant and $C_p$ the specific heat at constant pressure. In the previous equations $\rho' = \rho - \rho_{\text{ref}}$, (where $\rho$ is density), $\Pi' = \Pi - \Pi_{\text{ref}}$, and $\theta' = \theta - \theta_{\text{ref}}$. The subscript ‘ref’ stands for the variables of the anelastic reference state, only depending on height $z$. A Boussinesq version of the model is easily obtained.
by enforcing a uniform density in the vertical for the reference state (Scinocca and Shepperd 1992).

Orography is introduced in the model through the use of the Gal-Chen and Sommerville (1975) terrain-following coordinate. The mountain is centred in the simulation domain. A rigid-lid condition is imposed at the upper boundary. To prevent non-physical reflection at the domain top, an absorbing layer is applied near the upper boundary. At the bottom boundary, a free-slip condition is imposed. The lateral boundary conditions (l.b.c.) are either periodic or open. In the case of open conditions, the model prognostic variables are relaxed towards their large-scale values by a combination of the Davies (1976) and Carpenter (1982) methods. A radiation condition is used to compute the normal velocity component \( u \) at the future time

\[
\frac{\partial C^*}{\partial t} + \rho_{\text{ref}} C^* \frac{\partial u}{\partial x} = 0,
\]

where \( C^* \) is the phase velocity of the waves, prescribed by the method proposed by Klemp and Wilhelmson (1978): \( C^* = c + u \), where \( c \) is a prescribed phase velocity.

The anelastic form of the continuity equation (3) represents a strong constraint on the wind field, which is enforced by solving an elliptic equation for the pressure. The spatial discretization is based on second-order finite differences on an Arakawa C grid. The temporal discretization is purely explicit, and a weak time filter (Asselin 1972) is applied to control the rapid oscillations generated by the leap-frog treatment of the equations. A fourth-order diffusion operator is applied to fluctuation fields, with a characteristic time-scale adapted for damping the \( 2\Delta x \) waves.

Subgrid turbulence is computed by a 'one-and-a-half' order closure scheme, allowing for the computation of different mixing lengths (Cuxart 1997). In the turbulence scheme, the eddy coefficients are related to the turbulent kinetic energy (t.k.e.) through

\[
K = C_k l_k \bar{\epsilon},
\]

where \( C_k \) is a constant, \( l_k \) is the mixing length, and \( \bar{\epsilon} \) is the t.k.e., computed by a prognostic equation. The Meso-NH model has been validated by simulations of typical 2D linear and nonlinear orographic flows, both in hydrostatic and non-hydrostatic regimes (Lafont et al. 1998).

In order to study the impact of small-scale orographic effects on the synoptic flow, a budget of momentum has been developed in the Meso-NH model, using the method developed by Stein (1992). The momentum equation (4) of Meso-NH is summed within a box of variable size. Typically, the basis of the budget box is representative of the horizontal grid mesh in a GCM (typically \( \Delta x = 100 \) km). This momentum budget is then discretized (see appendix A for details) to give the following equation

\[
\frac{\partial C^*}{\partial t} + \rho_{\text{ref}} C^* \frac{\partial u}{\partial x} + D \Phi_x \frac{\partial h}{\partial x} + E \rho_{\text{ref}} \Phi_x (Z) - \rho_{\text{ref}} F_x - \rho_{\text{ref}} N_x u_x = \text{Res}
\]

where subscript 's' stands for surface. In (8), the average operators are defined by

\[
\bar{F} = \int_{-L/2}^{+L/2} F \, dx, \quad \text{and} \quad \tilde{F} = \int_{h(x)}^{Z} F \, dz.
\]

\( L \) and \( Z \) are respectively the width and the height of the budget box. This momentum budget is integral, i.e. for a given height \( Z \), the budget represents the momentum transfers between the ground and the top of the box located at \( Z \). In (8), term A is the tendency of momentum in the box during a given number of time steps corresponding to the budget period. B corresponds to the difference of momentum flux between the
lateral sides of the box. $C$ accounts for the variation of momentum due to the horizontal pressure gradient between the lateral sides of the box. $D$ is the mountain pressure drag, and $E$ is the vertical flux of momentum through the upper side of the box. Term $F$ accounts for the variation of momentum due to turbulent mixing, and term $G$ includes the contribution of ‘numerical’ terms ($\mathcal{N} a_x$ represents the discrete form for numerical diffusion and upper relaxation in the Meso-NH momentum equation). Finally, ‘Res’ is the residual term of the budget, which must be small compared with the other terms to have a correct closure for the budget. This point has been checked for any budget of momentum presented in this paper. The special in-line calculation allows one to easily obtain either an instantaneous budget (i.e. calculated on one model time step) or a temporally averaged budget.

As these budgets include the ensemble of the terms existing in the model momentum equation, it is possible to isolate the essential terms required for a drag parametrization in a GCM. Furthermore, budget computation in boxes of variable size allows one to analyse the spatial distribution of the interaction between mountain waves and the large-scale flow. Such a flexible budget is well suited to compare the prediction of a drag parametrization with the momentum transfers resolved by a mesoscale model. The budget calculation is now applied to an idealized problem of 2D breaking mountain waves.

3. Drag induced by tropospheric breaking

(a) Study of the mean-flow deceleration for a typical breaking event

The orographic flow considered here has been investigated by numerous authors (see the reviews of Smith (1989) or Durrant (1990)). The atmosphere has uniform mean wind speed ($U_0 = 8$ m s$^{-1}$) and static stability ($N_0 = 0.012$ s$^{-1}$) far upstream of the obstacle. The mountain is specified by (1) with $h_0 = 667$ m and $a = 10$ km, so that it is similar to the obstacle used in D95. The dimensionless parameters $\hat{h}$ and $\hat{a}$ respectively are 1.0 and 10.0, corresponding to a nonlinear hydrostatic flow. As the characteristic scale of the physical fields perturbations is much smaller than the characteristic scale of the mean density variation, the Boussinesq hypothesis is assumed. The total domain is $410a$ wide and $18h_0$ deep, with half the resolution, but four times the domain width, employed in D95 for the same breaking event. L.b.c. are periodic. The phase speed of the fastest gravity waves which can propagate in the simulation domain of the anelastic Meso-NH model is given by $c_\phi = U_0 + N_0 H/\pi \approx 7U_0$, where $H$ is the height of the numerical domain. Regarding the wide simulation domain employed here ($410a$), the model solution at $t^* = tU_0/a = 32$ is not, therefore, altered by the imposition of periodic boundaries. To prevent non-physical wave reflection at the upper boundary, a numerical sponge zone is applied above $2\lambda_z$, where $\lambda_z = 2\pi U_0/N_0$ is the analytic value of the vertical wavelength for hydrostatic gravity waves. Subgrid turbulence is parametrized by the scheme developed by Bougeault and Lacarrère (1989). Table 1 gives an overview of the other numerical parameters employed for this simulation (2D01).

Initially, the atmosphere is in hydrostatic equilibrium, $u = U_0$ and $N = N_0$. The obstacle is abruptly inserted in the flow, and the model is integrated until $t^* = 32$, leading to the solution displayed in Fig. 1. In the potential-temperature field (Fig. 1(a)), a region of wave breaking is clearly visible just below the theoretical breaking level ($z_c = 0.75\lambda_z$), consistent with the results reported by D95 in his Fig. 11. In association with the breaking, strong turbulent mixing (see Fig. 1(b)) is located on $6a$ downstream of the obstacle, and below $z_c$ in the vertical. As one can see, t.k.e. is maximum (3.7 m$^2$s$^{-2}$) at an altitude of $2h_0$. Breaking features are also apparent in the $u$ field reported in
TABLE 1. OVERVIEW OF THE PARAMETERS FOR NUMERICAL SIMULATIONS
OF TROPOSPHERIC AND STRATOSPHERIC BREAKING

<table>
<thead>
<tr>
<th>Reference</th>
<th>Tropospheric breaking</th>
<th>Stratospheric breaking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2D01</td>
<td>2D02</td>
</tr>
<tr>
<td>$n_x \times n_z$</td>
<td>1024 × 36</td>
<td>32 × 36</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>4 km</td>
<td>128 km</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>333 m</td>
<td>333 m</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>60 s</td>
<td>60 s</td>
</tr>
<tr>
<td>$h_0$</td>
<td>667 m</td>
<td>0 m</td>
</tr>
<tr>
<td>$a$</td>
<td>10 km</td>
<td>0 m</td>
</tr>
<tr>
<td>Lateral boundary conditions</td>
<td>Cyclic</td>
<td>Cyclic</td>
</tr>
<tr>
<td>Obstacle shape</td>
<td>Witch</td>
<td>Witch</td>
</tr>
</tbody>
</table>

$n_x$ and $n_z$ respectively represent the number of points in the x and z directions. $\Delta x$, $\Delta z$ and $\Delta t$ are the mesh sizes for the spatial and temporal discretizations. See text for further explanation.

Fig. 1(c), where a zone of reverse flow in altitude exists, and a windstorm near the ground, extending to $6a$ downstream at $t^* = 32$, with winds blowing up to $2.5U_0$. Upstream of the obstacle, a zone of decelerated flow is visible in the low atmospheric levels.

In Fig. 2(a) the evolution of the pressure drag as a function of time is shown. After a rapid growth until $t^* = 15$, the drag continues to increase slowly until the end of the integration, to reach 3.5 times its theoretical linear value ($D_{\text{lin}} = \pi /4 \rho_{\text{ref}} U_0 N_0 h_0^2$) by $t^* = 32$. Comparison with the D95’s running-time integral of the pressure drag (see his Fig. 12) shows a good agreement, with a 7% error at $t^* = 32$. Despite a lower resolution, the results presented here therefore closely resemble their counterparts reported in D95.

The non-steadiness of the solution is revealed by the temporal variation of the domain-averaged momentum, shown in Fig. 2(b). The domain-averaged momentum is expressed as

$$
\langle \rho_{\text{ref}} u \rangle = \frac{1}{V} \int_{-W/2}^{+W/2} \int_{-h}^{H} \rho_{\text{ref}} u \, dx \, dz,
$$

where $H$, $W$, $V$ are respectively the height, the width and the volume ($dy = 1$) of the full numerical domain. The domain-averaged momentum continually decreases as a function of time, with a linear and more pronounced decrease after $t^* = 10$. The continual decrease of the domain-averaged momentum indicates the deceleration of the mean flow caused by breaking gravity waves. Here again, the agreement with the D95’s results is good (see his Fig. 12 for the temporal evolution of the domain-averaged perturbation of momentum). If the model integration was long enough so that perturbations traverse many times the periodic domain, the constant deceleration would lead the mean flow to stop progressively.

In order to study the distribution of the mean-flow deceleration in the physical domain, we have computed the budget of momentum for the model solution at $t^* = 32$ (Fig. 3). The different terms of the budget are defined in (8), and are normalized by $D_{\text{lin}}$. The momentum budget is calculated at different locations of the physical domain, in a box of size $80a$, representative of the horizontal extension of a GCM grid mesh. A supplementary term $E'$, defined by

$$
E' = \rho_{\text{ref}} u' \bar{w}' = (\rho_{\text{ref}} u - \langle \rho_{\text{ref}} u \rangle)(\bar{w} - \bar{w}) = \rho_{\text{ref}} u \bar{w} - \rho_{\text{ref}} \bar{u} \bar{w} = E - \langle \rho_{\text{ref}} u \rangle \bar{w},
$$
Figure 1. Meso-NH solution for an orographic airflow characterized by $\tilde{h} = 1$ and $\tilde{a} = 10$ (simulation 2D01, see text): (a) potential temperature (contour interval 2.5 K), (b) turbulent kinetic energy (contour interval 0.5 m$^2$s$^{-2}$), and (c) horizontal wind (contour interval 2 m s$^{-1}$, the stippled area represents negative values) at $r^* = rU_0/\alpha = 32$ (see text). In this and all the figures which follow, the airflow is from left to right.

has been added. $E$ is the exact vertical momentum flux required to close the momentum budget (8). In the conventional scheme where linear mountain waves transport positive momentum downward to balance the pressure drag exactly, the field perturbations vanish far from the obstacle and $\overline{w} = 0$. In the finite-domain solutions of a time-dependent nonlinear model, $\overline{w}$ is different from 0, and we have to calculate the Reynolds stress $E'$ to compare the results with the predictions of the Eliassen and Palm (1960) theorem. As the divergence of the vertical flux of momentum is related to the mean-flow deceleration, for a given budget, $E'$ represents the source of deceleration which would exist if the numerical domain was quasi-infinite.

The momentum budget for a box of size $80\alpha$ centred on the obstacle is plotted in Fig. 3(a). In this profile, the pressure drag is partly balanced by the vertical flux of horizontal momentum. According to the Eliassen and Palm theorem, this divergence involves a mean-flow deceleration. It is interesting to notice that the vertical fluxes $E$ and
Figure 2. Meso-NH solution for an orographic airflow characterized by $\tilde{R} = 1$ and $\tilde{U} = 10$ (simulation 2D01): (a) pressure drag, normalized by $D_{\text{lin}}$, and (b) domain-averaged momentum (kg m$^{-2}$s$^{-1}$) as a function of dimensionless time $t^* = tU_0/\varphi$. See text for further explanation.

$E'$ are almost equal, because the domain where the budget is computed is sufficiently large for $\overline{\varphi}$ to approach 0 in (11). The decrease of the vertical flux of momentum (here, and in the following, in terms of absolute value) is located between the ground and the breaking level $z_c$, with a magnitude of $2.5D_{\text{lin}}$. Above $z_c$, the residual vertical flux of momentum is equal to 25% of the flux near the ground. Furthermore, in this region, all the terms are uniform with height; this means that momentum transfers are located below the breaking level. Below $z_c$, the divergence of the vertical flux of momentum is mainly balanced by the pressure term $C$ and by the tendency term $A$. The contribution of term $C$ is three times larger than the contribution of term $A$. When the width of the budget box is extended to the domain limits, the lateral terms ($B$ and $C$) vanish, due to the imposition of periodic conditions, and the divergence of the vertical flux is entirely balanced by term $A$. In a periodic domain, term $A$ thus accounts for the deceleration of the mean flow, and term $C$ only acts to distribute the mean-flow deceleration in the
Figure 3. Nonlinear hydrostatic regime ($\h = 1$, $\v = 10$, simulation 2D01): budget of momentum at $t^* = 32$ in a 80$a$ wide box (a) centred on the obstacle, (b) localized upstream of the obstacle, and (c) localized downstream of the obstacle. The three boxes are contiguous. (d) Budget of momentum at $t^* = 32$ in a 14$a$ wide box centred on the obstacle. Unless otherwise noted, in this and in all the budgets of momentum which follow, all the terms are normalized by the linear theoretical drag $D_{lin}$. See text for further explanation.

physical domain. For this 80$a$ box, term A is equal to 20% of the divergence of the vertical momentum flux, and as a consequence, only 20% of the deceleration occurs in this central box.

Still in Fig. 3(a), it is worth noting the role played by the turbulent term, $F$, which tends to redistribute vertically the momentum, between the ground and the breaking level. More exactly, below $2h_0$ turbulence dissipates momentum, while between $2h_0$ and $z_c$ turbulence creates momentum. When looking at the horizontal wind field shown in Fig. 1(c), the turbulent contribution to the momentum budget is easy to understand. In the horizontal wind field an important vertical shear exists in the lee of the obstacle, associated with breaking in altitude and windstorm near the ground. The turbulence acts against this wind shear to restore the original wind profile: therefore, below
2h₀ turbulence acts to remove momentum, and above 2h₀ turbulence tends to create momentum. Above zₑ, the wind shear decreases and, accordingly, the contribution of subgrid-scale mixing becomes negligible. Concerning the small contribution of the lateral term B, it is related to local acceleration/deceleration of the flow below/above the breaking level. Below the breaking level, local acceleration involves a loss of momentum in the budget box, while above the breaking level, local deceleration involves a gain of momentum in the budget box. This lateral term redistributes momentum between the ground and the top of the simulation domain.

For this academic event of breaking mountain waves, D95 has shown, with a compressible model, that the momentum is removed in the remote part of the simulation domain. To specify the horizontal distribution of the mean-flow deceleration, the momentum budget has been calculated in two boxes of size 80a, contiguous to the central box, located upstream (see Fig. 3(b)) and downstream (see Fig. 3(c)) of the obstacle. If we compare this with the previous budget in the central box (see Fig. 3(a)), the three tendency terms (A) are found to be very close. As the drag is calculated in an asymmetrical terrain for these lateral boxes, a small contribution of this term is present in the budget profiles. This contribution, vanishing when integrating in a box located further from the obstacle, is partly balanced by the vertical flux and by the tendency term. As this drag is five times smaller than the largest value of the tendency term, it does not largely modify the profiles of tendency. As a consequence, the agreement between the different tendency profiles implies that the deceleration is equally distributed between the three boxes. Budget profiles (not shown) in the remaining 80a boxes located at the limits of the domain present the same profile for their respective tendency terms. Thus, the mean-flow deceleration is uniformly distributed in the horizontal. Looking at Fig. 3(a), one can see that this is the pressure term which acts to distribute uniformly the mean-flow deceleration. The deceleration in a given box of width L is equal to $T_D \ast (L/L_d)$, where $L_d$ is the width of the whole numerical domain, and $T_D$ is the ‘total deceleration’ equal to the divergence of the vertical flux of momentum.

It is interesting to isolate the process responsible for the uniform distribution of the mean-flow deceleration in the horizontal. In his numerical study with a compressible model, D95 suggests that rapid gravity waves and infrasound waves propagate the deceleration far from the obstacle. In the present study, the anelastic constraint imposes, through the elliptic solver, an infinite speed of propagation for acoustic waves. If we integrate the continuity equation (3) in a 2D domain limited in the horizontal between $X_1$ and $X_2$, and in the vertical between the ground h and the top of the simulation domain H, we obtain:

$$I = \int_{X_1}^{X_2} dx \int_{h(x)}^{H} dz \frac{\partial \rho_{ref} u}{\partial x} + \int_{X_1}^{X_2} dx \int_{h(x)}^{H} dz \frac{\partial \rho_{ref} w}{\partial z} = 0. \quad (12)$$

Using the lemma (A.2) in appendix A and the nonlinear boundary condition

$$w(h) = u(h) \frac{dh}{dx}, \quad (13)$$

we have

$$\int_{X_1}^{X_2} dx \int_{h(x)}^{H} dx \frac{\partial \rho_{ref} u}{\partial x} = \int_{X_1}^{X_2} dx \frac{\partial}{\partial x} \left( \int_{h(x)}^{H} dz \rho_{ref} u(x, z) \right) + \int_{X_1}^{X_2} dx \rho_{ref} w(h). \quad (14)$$
Considering the upper boundary condition $w(H) = 0$, the second term of Eq. (12) reads:

$$
\int_{X_1}^{X_2} dx \int_{h(x)}^{H} dz \frac{\partial \rho_{ref} u}{\partial z} = - \int_{X_1}^{X_2} dx \rho_{ref} w(h).
$$

(15)

Adding (14) and (15), Eq. (12) becomes:

$$
I = \left[ \int_{h(x)}^{H} dz \rho_{ref} u(x, z) \right]_{X_1}^{X_2} = 0.
$$

(16)

Thus, we have:

$$
\int_{h}^{H} \rho_{ref} u \, dz \text{ is uniform for any } x.
$$

(17)

This result means that each elementary column of the model contains the same amount of momentum at any time. Thus, when there is a variation of momentum in the model in a 2D configuration, the pressure solver uniformly distributes this variation in one time step by creating an adequate pressure gradient. As the anelastic constraint is integral, i.e. it concerns a complete column, it cannot be excluded that the propagation of the deceleration in the low levels may be due to rapid gravity waves. Further numerical experiments have not shown this point, and most of the propagation seems to be accomplished by acoustic waves.

Momentum-budget calculation in a narrow box ($L = 15a$) centred on the obstacle (see Fig. 3(d)) has shown that the source of deceleration for the whole numerical domain, i.e. the divergence of the vertical flux of horizontal momentum $E'$, is fully constructed on a few mountain half-widths. One can verify that the mean-flow deceleration (given by the tendency term) still obeys the spatial distribution mentioned before, i.e. $TD \ast (L/L_d)$. It is worth noting that for the 15a box, the vertical flux of momentum $E$ oscillates with the same vertical wavelength $\lambda_z$ as the hydrostatic mountain waves. A simple linear analysis (Elkhalif 1992) shows that this oscillation is created by a linear wave, with an amplitude proportional to $h_0/L$. As $\rho_{ref} u'w'$ is proportional to $h_0^2$, the oscillating behaviour becomes more predominant for airflow in the linear regime. As a consequence, momentum budgets have to be calculated in very large boxes in order to filter out these oscillations and to facilitate the interpretation of momentum budgets.

The previous analysis with a wave-resolving model shows that a restricted number of terms actually contribute to the momentum budget. This result leads one to ask the next question: what are the essential terms to introduce into a GCM to represent the subgrid-scale orographic effects?

(b) Response of a large-scale model to a parametrized orographic forcing

The Meso-NH model is now used to simulate the response of a large-scale model (LSM) to a parametrized orographic forcing. The low-resolution model is integrated in a limited area defined by 32 horizontal grid points with a 128 km mesh interval and with 36 layers each of 333 m depth. The atmospheric conditions are the same as in the first experiment 2D01. Other model parameters are as before, except for orography, which is removed (see simulation 2D02 in Table 1). To reproduce the effect of the subgrid-scale orography, a forcing obtained from the momentum budget of the nonlinear simulation taken every $t^* = 8$ and interpolated is introduced over one grid point of the LSM. With this processing, the LSM is synchronous with the meso-$\gamma$-scale model. From the
 budgets of momentum at $t^* = 8, 16, 24, 32$ in the previous simulation 2D01, the forcing is constructed by adding the pressure drag $D$ and the vertical flux of momentum $E$.

The LSM integration until $t^* = 32$ leads to the solution displayed in Fig. 4. Despite the lack of resolution, a gravity wave is visible in the field of potential temperature, with a vertical wavelength of nearly 3 km. The budget of momentum has been calculated for 7-point wide boxes (Fig. 5), i.e. as large as the 80a box of the simulation 2D01 with orography. When we compare the budget profiles (Fig. 5) with their counterparts calculated with the wave-resolving model (Fig. 3), there is a good accordance between the tendency and pressure terms for the different boxes. Despite the absence of orography and the coarse resolution, the LSM creates a mean-flow deceleration uniformly distributed in the horizontal. The propagation mechanism is therefore independent of the model resolution. This reinforces the previous conclusion that acoustic waves rather than rapid gravity waves are responsible for the propagation of the deceleration. Thus, when it is well parametrized, the vertical flux of momentum is the only term required by a GCM to reproduce accurately the mean-flow deceleration existing in a meso-scale anelastic model for this case of breaking mountain waves. Thus, there is no need to parametrize the lateral momentum fluxes, or additional budget terms, to obtain a realistic behaviour of the GCM.

With the previous budget analysis, it is now possible to explain the two steps in the decrease of the domain-averaged momentum flux reported before (Fig. 2(b)). This decrease is caused by two complementary processes: first, the progressive construction of the vertical momentum flux, and second the wave breaking. For the first mechanism, it is important to note that the pressure drag is larger than its theoretical linear hydrostatic value from the first times of the simulation (see Fig. 2(a)). As the formation of the waves which propagate momentum downward is longer than the formation of the pressure drag, the vertical flux of momentum is only constructed after a significant time (longer than $t^* = 30$ in our experiments). Other numerical experiments have confirmed that this feature exists also for linear mountain waves (Héreil 1996). The second process responsible for the more significant decrease of $\langle \rho c u \rangle$ is linked to wave breaking.
Figure 5. Simulation with a large-scale model without orography, and forced by the divergence of the vertical flux of momentum of the meso-scale model (simulation 2D02): budget of momentum at $t^* = 32$ in a 80a wide box (a) centred on the obstacle, (b) localized upstream of the obstacle, and (c) localized downstream of the obstacle.

The three boxes are contiguous. See text for further explanation.

and appears after $t^* = 10$ (Fig. 2(b)). As noted previously, wave breaking involves an enhancement of the drag correlated with the divergence of the vertical flux, resulting in an important deceleration of the large-scale flow. Afterwards, the enhanced deceleration due to breaking is always present in the simulation. As a consequence, it is important to carry out simulations of 2D breaking mountain waves until $t^* = 32$ in order to analyse the maximum deceleration linked to this strongly nonlinear regime.

The previous results show that to describe correctly the subgrid orography effects existing in an anelastic wave-resolving model, a parametrization just has to calculate the divergence of the vertical flux of momentum: then the GCM reacts to this forcing in the same way as a meso-scale anelastic model does. The question as to whether the mean-flow deceleration due to the time of construction of the vertical momentum flux needs to be parametrized is still open.
(c) Test of two typical drag parametrizations

Another interesting aspect is to investigate the ability of two typical drag parametrizations to represent the subgrid-scale orographic effects correctly on the large-scale flow. As shown for the large-scale simulation 2D02, the essential term for a parametrization is the divergence of the vertical flux of momentum. The degree of accuracy of two parametrizations is here evaluated by comparing the vertical momentum flux calculated by the wave-resolving model with the parametrized flux. The two parametrizations tested here have been successfully implemented in the ECMWF model. The first one was originally suggested by Miller and Palmer (1986), and revised in 1989 in a way described in BP90. The general principle of the revised scheme (hereinafter referred to as MP) consists in splitting the vertical flux into a linear part and a nonlinear part, only active when the dimensionless height of the obstacle is larger than a critical value. The vertical profile of the linear part takes into account wave ducting in the low levels, and is controlled in the upper levels by the saturation hypothesis of Lindzen (1981). The second scheme, proposed by Lott and Miller (1997, LM in the following), is currently operational in the ECMWF model. By representing the impact of the subgrid blocked flow, the LM scheme has improved the description of three-dimensional (3D) wave surface stress originally proposed by BP90. A short description of both schemes is given in appendix B.

The adjustable parameter $\beta$ of the MP scheme is set to 0.3, and the critical dimensionless height $\hat{h}_c$ is set to 2, in agreement with the values recommended in BP90. With the atmospheric profile of the case 2D01, the off-line prediction of the MP scheme gives the vertical flux displayed in Fig. 6. The flux $E^* = D + E' - D_{\text{max}}$ calculated from the momentum budget in the numerical model ($D_{\text{max}}$ is the maximum drag reached for $z > h_0$) is plotted for reference. This expression of the resolved flux facilitates the comparison with the prediction of the parametrizations where the flux near the ground is prescribed by the total drag.

As the case 2D01 is characterized by $\hat{h} = 1.0$, the linear part of the scheme is only active (Fig. 6). Consequently, the ground flux in the MP scheme is underestimated by more than a factor of 3. As the vertical profiles of wind and Brunt–Väisälä frequency are uniform for this simulation, the Lindzen (1981) principle, controlling the linear part of the MP scheme, does not lead to a saturation of the wave amplitude in the MP scheme (the minimum Richardson number calculated in the scheme stays above its critical value $Ric = 0.25$). The vertical decrease of the flux is only due to the parametrization of the source of deceleration due to low-level wave ducting. As this part of the scheme is active between the pressure levels $p_s$ (surface pressure) and $p' = 0.8 p_s$, independently of the atmospheric profile, this leads to a wrong prediction of wave stress when mountain waves propagate so as the vertical flux balances the drag. Here, the unrealistic representation of wave ducting leads to a height of interaction (1600 m corresponding to $p'$) lower than its explicit value ($z_c = 0.75 \lambda z$), and to a flux decrease underestimated by a factor of 4. In addition, the residual flux is equal to a third of the resolved flux. The MP scheme therefore fails in representing correctly the features of the resolved vertical flux of momentum for this typical event of breaking mountain waves.

The vertical flux foreseen by the LM parametrization is also shown in Fig. 6. The flux near the ground is equal to 65% of its explicit value. With a critical value of $h_c$ (0.5), the flux due to the blocked flow has a significant contribution (30%) to the total flux near the ground. In the vertical, the blocked flux returns to zero at the predicted blocked level, here equal to the first model level above the ground ($z = 333$ m). The linear flux is uniform between the ground and the blocked level, and then linearly decreases over a
quarter of a wavelength of hydrostatic mountain wave to a value dictated by a criterion on the minimum Richardson number. Above this region ($z > 1300$ m), the linear flux is uniform, due to the uniform vertical profile of the Boussinesq atmosphere for the case 2D01. The global decrease of the flux is underestimated, with only 70% of the explicit value. These results indicate that the prediction of the LM scheme is globally better than the MP scheme. It is important to note that, at the difference of the MP scheme, the parametrization qualitatively represents the orographic effects identified in the wave-resolving experiment (i.e. wave saturation aloft and local deceleration upstream of the obstacle). Despite these improvements, the LM scheme also fails in representing accurately the flux of momentum for this configuration of idealized breaking mountain waves. As the LM and MP schemes are suited to real synoptic flows where additional momentum sinks exist (friction for instance), some adaptations would be required to give optimal predictions in the idealized configuration of the present 2D flows where a free-slip condition is assumed.

(d) Impact of the lateral boundary conditions on the mean-flow deceleration

To reproduce the conditions of an infinite domain with a reasonable computing cost, it is interesting to replace periodic l.b.c. by open conditions. In order to study the impact of the l.b.c. on the mean-flow deceleration, we have run again the 2D01 case with open l.b.c. Accordingly, the signal near lateral boundaries is now advected outward. The other simulation parameters are the same as for the simulation with periodic l.b.c., except for the domain width, now reduced to 2560 km (see simulation 2D03 in Table 1). The model is integrated until $t^* = 32$. The physical fields and the drag evolution (not shown) for this simulation with open l.b.c. perfectly match their counterparts of the simulation
2D01 with periodic l.b.c. (see Fig. 1). In Fig. 7 the evolution of the domain-averaged momentum as a function of time for the simulations 2D03 and 2D01 is displayed. As it can be seen, the global momentum is nearly uniform for the simulation 2D03 while it linearly decreases for the simulation 2D01. Open l.b.c. therefore impose a strong constraint on the global momentum in the simulation domain, which is conserved during the simulation. As a consequence, the mean flow cannot be decelerated in the whole simulation domain with open l.b.c.

In order to verify this statement, the budget of momentum has been calculated for the simulation 2D03 (Fig. 8), for the same central box as for the periodic case 2D01 (see Fig. 3(a)). Comparison between the two budget profiles reveals that the forcing term (i.e. the divergence of the vertical momentum flux) and the turbulent term are not influenced by l.b.c. Concerning the tendency term (A), despite a nearly similar profile below $z_c$, it dramatically decreases, and even vanishes at the top of the physical domain. In this upper region, the vertical momentum flux divergence is entirely balanced by an increase of the pressure term. Open l.b.c. have created a supplementary horizontal pressure gradient which prevents the mean-flow deceleration at any place if the vertical extension of the domain is equal to the top of the simulation domain. This result can be anticipated from the model equations. The important point with open l.b.c. is that the incident flow remains constant far upstream of the obstacle throughout the simulation. This means that momentum is conserved far upstream of the obstacle in an elementary column extending from the ground to the top of the model. With the use of Eq. (17), we can deduce that the momentum is the same in any elementary column of our numerical model due to the anelastic constraint. Thus global momentum is conserved in a simulation with open
1.b.c. It is worth noting that this constraint due to open 1.b.c. has also been noticed by D95 in his simulations with a compressible model.

For the periodic domain, it has been shown that the deceleration A in a given box is equal to $T D \ast (L/L_d)$. Here, the present open 1.b.c. induce the same deceleration as an infinite periodic domain ($L_d$ tends towards $\infty$, and thus $A$ tends towards 0). In both cases, acoustic waves (by the intermediary of the pressure solver) are responsible for the propagation of the mean-flow deceleration. Nevertheless, open 1.b.c. are more realistic than periodic conditions since they do not modify the inflow condition. As stated before, the important parameter for a GCM is the correct prediction of the divergence of the vertical momentum flux. Then, the GCM reacts by transporting the deceleration to modify the global structure of the flow in a way depending on the 1.b.c. prescription (e.g. open 1.b.c. create a supplementary pressure gradient). As open 1.b.c. allow one to simulate nonlinear orographic flows accurately at low computing time, they are of particular interest for the exploration of the regime of 2D orographic flows, and even more so for the investigation of the 3D flows regime. As a consequence, they will be used for the next simulations.

4. Drag induced by stratospheric breaking

Conservation of wave energy induces the growth with height of the mountain wave amplitude, and may induce wave overturning at a critical level located in the stratosphere. To study a typical event of stratospheric breaking, the Boussinesq hypothesis has been replaced by the anelastic approximation (Lipps 1990), less restrictive by allowing the reference density to decrease with altitude. We consider a two-layer atmosphere, with an interface located at an altitude of $Z_1 = 20$ km. The Brunt–Väisälä frequency is uniform in each layer, with $N_1 = 0.01$ s$^{-1}$ in the lower layer, and $N_u = 0.0257$ s$^{-1}$.
in the upper layer. The incident flow speed is uniform with height in both layers \( (U_0 = 10 \text{ m s}^{-1}) \), and an isothermal profile is imposed above \( Z_i (T = 145 \text{ K}) \). The obstacle is defined by (1) with \( h_0 = 500 \text{ m} \), and \( a = 10 \text{ km} \). The domain is 100 km high and 512 km wide, with a numerical sponge applied above 70 km. The dimensionless parameter \( \tilde{z} \), giving the importance of non-Boussinesq effects, is here equal to 0.7. Additional model parameters are given in Table (1) for this simulation referred to as 2D04. The density of reference is given by \( \rho_{\text{ref}}(z) = \rho_0 e^{-z/Z_i} \). Due to the decrease of density with height, perturbations are amplified by a factor \( \sqrt{\rho_0/\rho_{\text{ref}}(z)} \). Linear theory suggests that a necessary condition for wave overturning is

\[
Nh_0/U_0 e^{z/2Z_i} > 1,
\]

satisfied when \( z \geq Z_i \).

The model is integrated until \( t^* = 100 \), so that gravity waves propagate far above the interface. In the simulated horizontal wind field (Fig. 9(a)), the wave amplification with height is clearly visible. Just below the interface, a region of intense turbulent mixing is apparent in the t.k.e. field (Fig. 9(b)), with a maximum of 3.2 m²s⁻¹ at \( z \approx 17.5 \text{ km} \). This turbulent activity, combined with the steepening of isentropes (Fig. 9(c)), indicates the presence of wave breaking with altitude. In Fig. 10, the temporal oscillation of the drag around a mean value of 1.2 reveals the unsteadiness of the model solution. The mean value is close to the drag calculated for the corresponding simulation carried out in a single-layer \( (N = 0.01 \text{ s}^{-1}) \) Boussinesq atmosphere (Hérel 1996). Contrary to the simulation 2D01 of tropospheric breaking, the pressure drag is not here modified by the breaking waves.

In the budget of momentum calculated for the simulation 2D04 at \( t^* = 100 \) in a 40a wide box (Fig. 11), the vertical flux of momentum is divergent in the region \( Z_1 \pm \lambda_{zh}/2 \), where \( \lambda_{zh} (= 2444 \text{ m here}) \) is the theoretical hydrostatic vertical wavelength for internal gravity waves in the upper layer. Above \( Z_1 + \lambda_{zh}/2 \), the residual flux is small, and the loss of momentum associated with drag is balanced by the pressure term, the other terms being small. We obtain a balance between the pressure term and vertical flux, as for the simulation of tropospheric breaking with open l.b.c. (see Fig. 8). From the budget analysis of 2D03, it can be deduced that mean-flow deceleration occurs in the region \( Z_1 \pm \lambda_{zh}/2 \) for the present simulation of stratospheric breaking employing open l.b.c. Due to the modest amplitude of the drag, the decrease of the vertical flux is lower than for the previous case of tropospheric breaking, and this despite a nearly vanishing residual flux above \( Z_1 + \lambda_{zh}/2 \). Furthermore, as this decrease extends on one \( \lambda_{zh} \) instead of a quarter of \( \lambda_{zh} \) for the tropospheric breaking, this considerably decreases the divergence of the vertical momentum flux. This smaller divergence counterbalances the decrease of density with height in the troposphere, and thus stratospheric breaking comparatively induces a weaker mean-flow deceleration per unit height than tropospheric breaking.

The off-line prediction of the MP parametrization for the case 2D04 is reported in Fig. 12. As for the previous case of breaking mountain waves, the dimensionless height is lower \( (\hat{h} = 0.5) \) than the critical value retained for the MP scheme, and the nonlinear part of the scheme is not active. The ground value of the flux is underestimated (20%) by the scheme. In the parametrized flux, we find again the low-level divergence representing the wave ducting between the ground \( (p_b) \) and the level \( p' = 0.8p_s \). This divergence is clearly inconsistent with the quasi-uniform numerical flux in the low levels, in agreement with the previous conclusions on the breaking regime. Above \( p' \) the flux is uniform with height up to \( z = 19 \text{ km} \), where the saturation hypothesis becomes active in the parametrization, due to the decrease of density with height. The
wave saturation induces a significant flux decrease centred around \( z = 20 \text{ km} \) over a quarter of \( \lambda_{zh} \). Above this region, the flux slowly decreases up to \( z = 35 \text{ km} \) where it is nearly equal to zero. In this region, the predicted flux is in good agreement with the resolved flux. In return, the extension of the region of interaction around \( z = 20 \text{ km} \) is clearly underpredicted, and the amplitude of the global flux decrease is underestimated, a consequence of the incorrect prediction of the flux in the low levels.

Concerning the LM scheme (Fig. 12), the blocked-flow drag is not active, due to the smallness of \( \widehat{h} \). In return, the ground value of the flux is overestimated (30\%) compared with the resolved flux. Due to the combination of this larger flux near the ground with a less restrictive criterion for the wave amplitude saturation (\( Ri_c \) equal to 1.0 instead of 0.25 for the MP scheme), the flux experiences successive decreases over a quarter of \( \lambda_{zh} \) in the region (5–20 km). Above this region, the change in static stability

Figure 9. Meso-NH simulation of hydrostatic mountain waves propagating in the stratosphere, and characterized by \( \widehat{h} = 0.5 \) and \( \widehat{a} = 10 \) (simulation 2D04): (a) horizontal wind (contour interval 2 m s\(^{-1}\)), (b) turbulent kinetic energy (contour interval 0.4 m\(^2\)s\(^{-2}\)), and (c) potential temperature (contour interval 2.5 K) at \( t^* = 100 \). See text for further explanation.
Figure 10. Meso-NH simulation of hydrostatic mountain waves propagating in the stratosphere, and characterized by $\hat{h} = 0.5$ and $\tilde{a} = 10$ (simulation 2D04); pressure drag, normalized by $\pi \rho_{ref} \hat{N}_1 \tilde{u}_0 \hat{h}_0^2/4$, as a function of dimensionless time. See text for further explanation.

Figure 11. Meso-NH simulation of hydrostatic mountain waves propagating in the stratosphere, and characterized by $\hat{h} = 0.5$ and $\tilde{a} = 10$ (simulation 2D04); budget of momentum at $r^* = 100$ in a 40α wide box centred on the obstacle. See text for further explanation.
involves an important flux decrease up to $z = 21$ km. Then the flux slowly decreases up to $z = 35$ km. An adjustment of the criterion for the wave amplitude saturation seems necessary to improve the prediction of the LM scheme for this idealized case of stratospheric breaking.

For this typical regime of stratospheric breaking, the results show that the momentum flux profile is partly described by both parametrizations. An important implication of this comparison is the necessary enlargement to one $\lambda_{zh}$ of the region of stress decrease described by both parametrizations when breaking occurs at upper levels. The study is now extended to another potential source of large-scale flow deceleration with the investigation of idealized cases of non-breaking t.l.w.

5. IMPACT OF TRAPPED LEE WAVES ON THE LARGE-SCALE FLOW

The interaction of non-hydrostatic effects with the mean flow is now investigated through the Meso-NH simulation of typical events of non-breaking t.l.w. The first event of t.l.w. is derived from D95. Most of the more recent work on t.l.w. has been performed with a cosine-shaped obstacle instead of a Witch-of-Agnesi shaped. To maintain continuity we keep the Witch-of-Agnesi shape. The basic flow has a wind speed which linearly increases from 10 m s$^{-1}$ at the surface to 40 m s$^{-1}$ at $z = 10$ km, and remains uniform above this height. An inversion layer is located in the region $2.1 < z < 3.1$ km, with a maximum Brunt–Väisälä frequency of 0.02 s$^{-1}$, below this layer $N = 0.005$ s$^{-1}$, and above $N = 0.01$ s$^{-1}$. The mountain is defined by (1), with $h_0 = 750$ m, and $a = 10$ km. With dimensionless parameters near the ground equal to $\hat{a} = 5.0$ and $\hat{h} = 0.4$, the orographic flow belongs to the nonlinear non-hydrostatic
regime. L.b.c. are open and the Boussinesq option is retained. Additional parameters for this simulation (2D05) are given in Table 2.

The Meso-NH solution at \( t^* = 30 \) is shown in Fig. 13. Isentropes of both potential temperature and vertical wind field show the superposition of a hydrostatic wave, propagating aloft, with t.l.w. propagating downstream, and having a maximum amplitude located near the inversion layer. The wavelength of the horizontally propagating waves is close to \( 2\pi U_0 / N_0 \), and the wave train propagates downstream at a speed of 8.5 m s\(^{-1}\), in agreement with the phase speed of t.l.w. calculated by Nance (1995).

For this regime of non-breaking t.l.w., the momentum budget has been calculated for a 40\( \alpha \) box centred on the obstacle. As nonlinearities involve unsteadiness of t.l.w. (Nance 1995), the budget calculation has been averaged on 100 time steps, which corresponds to the oscillation period of these waves. The corresponding budget is displayed in Fig. 14, where the normalization constant (0.03\( h_0^2 \)) is the value of the drag determined by a numerical simulation of the same case with a reduced mountain height. As it can be seen in this budget profile, the normalized drag is equal to 2.5: nonlinear t.l.w. significantly

<table>
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<th>Linear t.l.w.</th>
<th>Nonlinear t.l.w.</th>
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<td>2D07 250 \times 180</td>
<td>2D08 250 \times 90</td>
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<td>Cos</td>
<td>Witch</td>
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</table>

See Table 1 for further explanation.
enhance the drag compared with its linear value. The loss of momentum due to pressure drag is balanced by a uniform vertical flux \( E' = \rho_{ref} u' w' \). It must be pointed out that this uniformity is independent of the application of the time average on the momentum budget. This exact balance means that the mean flow is not decelerated. Therefore, the 2D t.l.w. simulated here do not significantly modify the conventional transport of momentum by vertically propagating mountain waves.

D95 used the existence of a flux of pseudo-momentum in the trough of the first lee wave to conclude that t.l.w. should be considered as a potentially important source of orographic drag, apparently in opposition with the present conclusions concerning an analogous regime of flow. In order to explain this discrepancy, we have carried out the numerical simulation of the exact D95 case of t.l.w., with a cosine-shaped obstacle defined by

\[
h(x) = \begin{cases} 
  h_0 \left[ 1 + \cos(\pi x/b) \right]^4/16 & \text{if } |x| \leq b, \\
  0 & \text{otherwise},
\end{cases}
\]

with \( b = 40 \text{ km} \) and \( h_0 = 750 \text{ m} \). The Boussinesq hypothesis is relaxed in the atmospheric profile, and the model resolution is the same as in D95 (see simulation 2D06 in Table 2). The Meso-NH solution (not shown) at \( t^* = 30 \) (\( t = 20000 \text{ s} \)) well agrees with its D95 counterpart (see Fig. 16(a) in D95). The computation of the momentum budget leads to the profile reported in Fig. 15. To describe the momentum budget in a quasi-infinite budget box, the lateral terms \( B \) and \( C \) of (8) have been put together to form the following term:

\[
(B + C)' = (B + C) + (E' - E').
\]

As it can be seen in the budget profile (Fig. 15), the drag has decreased compared with the case 2D05 because the response of the atmosphere to the new orographic forcing is more non-hydrostatic. The most important point is the decrease of the vertical
momentum flux in the trapping region \( z < 3000 \) m. We have verified in the momentum budget at \( t^* = 15 \) that the decrease of the vertical momentum flux is not due to the progressive construction of the vertical flux. This flux decrease, representing 1/6 of the drag value, clearly indicates that the deceleration of the mean flow is now possible. Thus we recover the agreement with the D95 conclusions. This experiment shows that the divergence of the vertical momentum flux, i.e., the source of deceleration for the mean flow, strongly depends on the spectra of excitation of the obstacle.

In order to complete this result, we have considered a second case of t.l.w. taken from the article of Lott (1998, L98 in the following). The obstacle is defined by (1), with \( a = 829 \) m, and \( h_0 = 15 \) m high. \( N \) is uniform \( (0.018 \, \text{s}^{-1}) \), and the vertical profile of \( U \) is defined by

\[
U(z) = U_0 + (U_\infty - U_0) \tanh \left( \frac{U_0}{U_\infty - U_0} \frac{z}{l} \right)
\]

where \( U_\infty = 5U_0 = 25 \, \text{m} \, \text{s}^{-1} \), and \( l = 552.5 \) m is the vertical variation of the incident flow (profile 15 with \( L = 2 \) and \( D = 3 \) in L98). Because of the modest height and width of the obstacle, the flow belongs to the non-hydrostatic linear regime. Other numerical parameters for this case are given in Table 2 (experiment 2D07).

The Meso-NH solution for \( w \) at \( t^* = 450 \) is shown in Fig. 16(a). Contrary to the case 2D05, the present t.l.w. are superimposed with a non-hydrostatic wave with a line of maxima slanted downstream. The pattern of the vertical wind field is in good agreement with the results of L98. For this linear non-hydrostatic regime, the momentum budget has been calculated for a \( 20a \) box centred on the obstacle. In this budget displayed in Fig. 16(b), the drag is equal to \( 0.7D_{\text{lin}} \), in agreement with the L98 results (difference below 10%). The important point here is the divergence of the vertical flux of momentum \( E' \) in the trapping region below \( z = 2 \) km. With a decrease of 50%
from the pressure-drag value, this divergence is underestimated compared with the L98 results where the flux experiences a 70% decrease. In Fig. 16(b), the flux divergence in the trapping layer is mainly balanced by the lateral terms and slightly by the tendency terms. The divergence of $E'$ above $z = 6$ km is due to the progressive construction of the vertical flux, as it has been verified by the study of its evolution with time. In another experiment where the trapping profile was removed, we verified that the non-hydrostatic primary wave alone does not decelerate the mean flow. These results indicate that the ability of t.l.w. to decelerate the mean flow seems to be linked to the non-hydrostatic character of the main wave.

To reinforce this statement, we carried out a further experiment. For this numerical simulation, the orographic airflow characteristics are the same as for the first t.l.w. experiment 2D05, except for the width of the obstacle which is now ten times smaller ($a = 1$ km, see experiment 2D08 in Table 2). This case is thus more non-hydrostatic ($\tilde{a} = 0.5$) than the case 2D05 ($\tilde{a} = 5$). The momentum budget of this case at $t^* = 30$ is displayed in Fig. 17. As it can be seen, the drag is reduced by 30% compared with its value in the experiment 2D05, an effect which is due to the non-hydrostatic character of the main wave (e.g. Laprise and Peltier 1989). The vertical flux $E'$ is now divergent in the region $z = (2 - 3$ km), with a residual flux equal to 20% of the pressure drag above $z = 3$ km. Therefore, the difference from experiment 2D05 is that there is now an interaction between the t.l.w. and the large-scale flow. As the unique difference between both simulations is the width of the obstacle, the results of the present experiment reinforce the point that this is the non-hydrostatic character of the waves that allows the deceleration of the mean flow by t.l.w.

If the 2D t.l.w. investigated in the case 2D05 with the Witch-of-Agnesi obstacle seem inefficient to modify the conventional scheme of momentum transfer, it cannot be excluded that the relaxation of the Boussinesq hypothesis in this case leads the primary vertically propagating wave to break in the stratosphere. As the drag computed for this case of breaking waves represents, in terms of physical units, the third of the drag determined for the case of breaking 2D01, these 2D t.l.w. may decelerate the mean flow.
in altitude, an effect which must be taken into account in a parametrization of subgrid-scale orographic effects. Furthermore, current parametrizations do not, accurately, take into account the non-hydrostatic character of the subgrid-scale orographic flow to calculate the mean-flow deceleration. This aspect is now investigated by testing two parametrizations for the case derived from D95.

As \( \tilde{a} \) is unknown in both drag schemes, we show in Fig. 18 the numerically resolved vertical flux of momentum \( E^* \) for both experiments 2D05 and 2D08 for which orographic flows only differ by \( a \). The vertical flux predicted by the MP scheme is also reported in Fig. 18. Here again, only the linear part of the parametrization is active and the decrease of the flux is linked to low-level wave ducting between the ground and \( p' \). Because of the less restrictive criterion in the saturation principle, the momentum flux foreseen by the LM scheme experiences a decrease of one unit, between the ground and \( z = 3500 \) m. When the numerically resolved vertical flux experiences a divergence, the flux near the ground and the amplitude of the decrease are better predicted by the MP scheme than by the LM scheme. In return, the maximum height of interaction is better foreseen by the LM parametrization. These results clearly show the need to take into account the non-hydrostatic character of the subgrid-scale flow accurately in current gravity-wave drag parametrizations.

6. Conclusions

This numerical study of different idealized orographic flows with a non-hydrostatic anelastic model has allowed the recovery or identification of different mechanisms affecting the mean flow in the context of an idealized problem of mountain waves. Moreover, in-line calculation of an exhaustive momentum budget has clarified some aspects of the mean-flow interaction with three typical 2D regimes of orographic flows.
Figure 18. Trapped lee waves: profiles of vertical momentum flux foreseen by the MP parametrization (dash-dot line) and by the LM parametrization (solid line) for the D95 atmospheric profile. The vertical momentum flux $E^*$ of the numerical model is given for the experiments 2D05 (stars, $\tilde{a} = 5$) and 2D08 (diamonds, $\tilde{a} = 0.5$). See text for further explanation.

In the case of mountain waves breaking in the troposphere, the mean flow is decelerated in the vertical between the ground and the height of interaction, equal to the theoretical breaking level. In the horizontal, acoustic waves are responsible for the transport of the deceleration in the simulation domain through the pressure solver of the anelastic model.

When non-Boussinesq effects lead mountain waves to break in the lower stratosphere, the interaction between orographic waves and the large-scale flow is transported far aloft, and located on a vertical wavelength centred on the breaking level. This broader extension of the zone of interaction leads to a reduced mean-flow deceleration compared with tropospheric breaking.

In the non-hydrostatic regime, a study of different events of non-breaking t.l.w. lead to the following conclusions. First, when the primary propagating wave is hydrostatic, the pressure drag is significantly enhanced but no mean-flow deceleration is observed. Second, additional experiments show that the non-hydrostatic character of the primary propagating wave is an essential factor to have a deceleration of the mean flow in the trapping layer. These non-hydrostatic effects are therefore important to take into account in drag parametrizations.

In the context of typical breaking mountain waves, the response of a large-scale model (LSM) to a drag parametrization has been investigated through momentum-budget calculations in a low-resolution simulation where orographic forcing is replaced by the prediction of a typical drag parametrization. The agreement between the behaviour of an LSM without orography and the response of a meso-scale model to an explicitly resolved orography, has shown that a realistic prediction of the vertical flux
of momentum due to subgrid-scale orography is sufficient for a GCM to recover the deceleration existing in a meso-scale anelastic model accurately.

Tests of two typical drag parametrizations on this set of idealized orographic flows have shown that some elements of the scale interaction are not correctly described by either scheme. For the MP scheme, the nonlinear part is always underestimated, and the imposed low-level divergence describing wave ducting is often unrealistic in our tests. For the LM scheme, the flux near the ground is generally better estimated, but the height of interaction and the flux divergence are still not correctly foreseen. It is worth noting that both parametrizations were developed for real orographic flows where other momentum sinks are present. Therefore, some adaptations would be necessary to give optimal predictions for the set of idealized orographic flows investigated in the present article. To illustrate this aspect, some sensitivity tests were carried out where we varied the values of the orographic constant ($K$ and $G$ for the MP and LM schemes, respectively) and of the subgrid orographic variance ($h_0^2$). The orographic constant only appears in the expression of the linear flux (see appendix B) whereas the variance is used to calculate both linear and nonlinear parts of the momentum flux. It has not been possible to determine an optimal couple of these parameters for which the predicted momentum flux closely fits its counterpart determined with the numerical model. Nevertheless, in Fig. 19, one can see that better agreement is obtained in the LM scheme when $G$ is multiplied by two for a test on the case of breaking mountain waves (2D01). In return, the region where the vertical flux decreases (one quarter of a hydrostatic wavelength) needs to be extended (here by a factor 2) to compare well with the flux of the numerical model. The consideration of a 2D orographic flow may be too restrictive compared with the dynamics of a real orographic flow where 3D nonlinear effects (e.g. flow splitting, lee vortices, vortex shedding, t.l.w.) play an important role. The 3D dynamics of the orographic flow are now taken into account in current drag parametrizations as, for instance, in the LM scheme where a representation of the blocked flow exists. In part II of this paper (Héreil and Stein 1999) the study is extended to the interaction of 3D nonlinear orographic effects with the large-scale flow, in order to test the hypothesis of current drag parametrizations in a more general framework.
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APPENDIX A

Computation of the momentum budget in the Meso-NH model

The momentum equation for the 2D version of the Meso-NH model is expressed as:

\[
\partial_t (\rho_{\text{ref}} u) + \partial_x (\rho_{\text{ref}} u^2) + \partial_z (\rho_{\text{ref}} u w) + \partial_x (\rho_{\text{ref}} \Phi) - \rho_{\text{ref}} F_x = 0.
\] (A.1)

The average operators defined by (9) are applied to (A.1), with the use of the following lemma

\[
\bar{\partial}_a f = \bar{\partial}_a f + \bar{f}(x, h) \bar{\partial}_a h,
\] (A.2)

where \( a = x, t \). We obtain the integral budget

\[
\partial_t \bar{\rho}_{\text{ref}} u + \partial_x (\bar{\rho}_{\text{ref}} u^2) + \bar{\rho}_{\text{ref}} u \bar{w}(Z) - \bar{\rho}_{\text{s}} \bar{u} \bar{w}_s + \partial_x (\bar{\rho}_{\text{ref}} \Phi) + \bar{\rho}_{\text{s}} \Phi_s \partial_x h - \bar{\rho}_{\text{ref}} F_x = 0.
\] (A.3)

As a free-slip condition is imposed at the lower boundary of the model which corresponds with the ground, the flux through the ground vanishes. The discretization of (A.3) leads to the final form of the integral momentum budget

\[
\frac{\partial_t \bar{\rho}_{\text{ref}} u}{A} + \frac{\partial_x \bar{\rho}_{\text{ref}} u^2}{B} + \frac{\partial_x (\bar{\rho}_{\text{ref}} \Phi)}{C} + \frac{\bar{\rho}_{\text{s}} \Phi_s \partial_x h}{D} + \frac{\bar{\rho}_{\text{ref}} u \bar{w}(Z)}{E} - \frac{\bar{\rho}_{\text{ref}} F_x}{F} = \text{Res.} - \frac{\bar{\rho}_{\text{ref}} N u_x}{G}
\] (A.4)

APPENDIX B

Description of drag parametrizations

(i) The MP scheme. In the MP parametrization evaluated for the set of idealized 2D simulations presented in this paper, the momentum flux is decomposed into a linear term and a nonlinear term accounting for the high-drag state:

\[
\tau(p) = \tau_w(p) + \tau_{nl}(p).
\] (B.1)

Near the ground \((p = p_s)\), \( \tau_w(p) \) is set equal to

\[
\tau_w(p_s) = -K \rho_{\text{ref}} U_0 N_0 \text{VAR},
\]

where \( K \) is a parameter depending on the obstacle sharpness. For a 2D Witch-of-Agnesi shape, \( K = \pi/4 \). We consider that the obstacle employed in the numerical simulations at fine scale is not resolved by the large-scale model where the parametrization is implemented. Accordingly, \( \text{VAR} \) is related to the variance of the subgrid-scale orography
by

$$\text{VAR} = \min \left\{ h_0^2, \left( \frac{\hat{h}_c * U_0}{N_0} \right)^2 \right\},$$

where $\hat{h}_c$ is the critical dimensionless height. In (B.1) the nonlinear term near the ground is given by

$$\tau_{nl}(p_s) = 4K \rho_{ref} U_0^3 \frac{(\hat{h} - \hat{h}_c)^2}{N_0}$$

if $\hat{h} > \hat{h}_c$,

$$\tau_{nl}(p_s) = 0$$

otherwise.

The vertical decrease of the momentum flux is specified for each term of B.1. The vertical distribution of the linear flux is expressed as

$$\tau_w(p) = \tau_w(p_s) \left( (1 - \beta) \frac{p - 0.8p_s}{p_s - p'} + \beta \right)$$

if $p > p'$, where $p' = 0.8p_s$

$$\tau_w(p) = \beta \tau_w(p_s)f(p)$$

otherwise;

where $\beta$ is an adjustable parameter and $f(p)$ depends on a Richardson number criterion for limiting internal wave amplitude. The vertical distribution of the nonlinear flux is given by

$$\tau_{nl}(p) = \tau_{nl}(p_s) \frac{p - p(Z_c)}{p_s - p(Z_c)}$$

if $\hat{h} > \hat{h}_c$,

$$\tau_{nl}(p) = 0$$

otherwise;

where $Z_c$ is the height of the critical level given by

$$\int_0^{Z_c} \frac{N(z)}{U(z)} dz = \frac{3\pi}{2}.$$

(ii) The LM scheme. One of the major concerns of the LM scheme is to deal explicitly with a low-level flow which is blocked when the effective height of the subgrid-scale orography is sufficiently important. In the LM scheme, the momentum flux is composed of two parts.

First, the gravity-wave drag is based on the work of Miller et al. (1989) and BP90 and takes into account some 3D effects of the flow. Under the hypothesis adopted in this paper, the mountain wave stress near the ground simplifies to

$$\tau_{wls} = -4\rho_{ref} U_H N_H \mu \sigma G B(\gamma)a^2,$$  \hspace{1cm} (B.2)

where $\sigma$ and $\mu$ respectively represent the standard deviation and the slope of the orography, and are related to the obstacle height by

$$\sigma a = \mu = \frac{h_0}{2}.$$

In (B.2), the subscript 'H' indicates an evaluation of the parameter between $\mu$ and $2\mu$. $G$ is a function of the mountain sharpness. For a Witch-of-Agnesi shape, $G \approx 1.23$ (Phillips 1984). $B(\gamma)$ is a function of the mountain anisotropy. For a 2D obstacle,
\( B(0) = 1, \alpha = 1 \), and (B.2) reduces to the simplified form:

\[
\tau_{\text{vis}} = - \rho_{\text{ref}} U H N H h_0^2 G. \tag{B.3}
\]

This term is uniform between the ground and the depth of the blocked layer \( Z_b \). Between \( Z_b \) and the model top, the stress is uniform unless the waves break, which is detected by the calculation of a local Richardson number, as for the MP scheme. When wave breaking occurs, the associated drag is distributed over a layer of thickness equal to a quarter of the vertical wavelength of mountain waves (\( \Delta z = \pi U_0 / 2 N_0 \)).

Second, the blocked flow drag at a given level \( z \) is given by

\[
D_b(z) = -C_d \rho_{\text{ref}} \frac{\sigma}{2\mu} \max \left[ 2 - \frac{1}{r}, 0 \right] \sqrt{\frac{Z_b - z}{z + \mu}} B(y) \frac{U|U|}{2} 4\alpha^2, \tag{B.4}
\]

where \( r \) is a function of the aspect ratio seen by the incident fluid and \( C_d = 2 \) for a 2D obstacle. The depth of the blocked layer is defined by

\[
\int_{Z_b}^{2\mu} \frac{N(z)}{U(z)} \, dz \geq \hat{h}_c.
\]

With our hypothesis, the drag coefficient is infinite, and (B.4) simplifies to

\[
D_b(z) = -2C_d \rho_{\text{ref}} \sqrt{\frac{Z_b - z}{z + \mu}} U|U|. \tag{B.5}
\]

Integration of (B.5) from \( z = 0 \) to \( z = Z_b \) leads to the total stress due to the blocked flow.

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