Cloud bands induced by isolated mountains

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Summary

The wave equation is solved analytically to obtain the solutions of diverging types of trapped waves, caused by an isolated hill, at the top of a convective boundary layer with an overlying stable layer above. Wind veering in the upper stable layer favours the wave trains to be to the left of the mean wind direction. The elevated lifting latent heat may make the crest line of the diverging wave a diabatic heating band, if the water vapour supply is sufficient.

Analytical solution is also used to analyse the effects of a heating band, which makes a small angle with the mean wind direction. The trapped components of the induced waves may spread horizontally and excite wave trains running parallel to the original heating band. The untrapped waves hold an in-phase relation with the heating. The resultants of trapped and untrapped waves possess multiscale structure and properties similar to those of cloud streets. These analyses may provide an explanation for the formation and propagation of cloud streets.

Keywords: Cloud streets Convection-waves interaction Trapped waves Wave-number selection

1. Introduction

Organized convective clouds, also called mesoscale shallow convection, are commonly observed in cold-air outbreaks. Their patterns may vary from cloud streets to open or closed cells. In addition, multiscale structures are also found in the convective system (Walter and Overland 1984). Mesoscale shallow convection has been the subject of intensive observational and theoretical studies. Several linear instability hypotheses, such as inflection point instability (Faller 1965; Brown 1970), parallel instability (Lilly 1966; Etling and Brown 1993), and thermal instability (Asai 1972) etc., have been developed to elucidate the mechanisms leading to the formation of shallow convection. However, none of the theories alone can appropriately explain the varied patterns and multiscale structures of these convective systems.

In addition to thermal and dynamic instabilities, Atkinson and Zhang (1996), in a review article, listed nine other mechanisms possibly related to the formation of the organized convection: large-scale vertical motion, anisotropy of eddy diffusivities, heating profile, cloud-top entrainment instability, mesoscale entrainment instability, boundary conditions, latent-heat release, up-scale transfer of turbulent energy, and gravity waves. Synthetic theories of combining those effects have not been established.

Instead, up till now the roles of those mechanisms are only analysed separately.

In the 1980s, several investigators (Clark et al. 1986; Kuettnert et al. 1987; Hauf and Clark 1989) advanced a new hypothesis for the formation of the shallow convection. In the hypothesis the roles of gravity waves and layer interaction were emphasized.

It was suggested that the boundary-layer thermals deform the capping inversion or interface region and cause ripples, which produce gravity waves. While the gravity waves propagate horizontally, they in turn initiate and tune up the convection activities in the convective boundary layer.

The mechanism described by the gravity-waves hypothesis has been proved in numerical simulation (Clark et al. 1986) and observation (Kuettnert et al. 1987). Sang (1991, 1993, 1997) developed a linear model to elucidate the role of gravity waves in the formation of shallow convection. In a two-layer model with a lower convective boundary layer and an overlying stable layer the analytical model provided the spatial distribution of waves and convections with the pattern similar to either cloud streets or

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convective cells determined by the wave-number selection relation. In the linear model the waves in the upper stable layer may be excited by thermals (Sang 1993), or by hills (Sang et al. 1997). Once the waves form, the trapped components of the waves will propagate downstream and adjust the pattern of the convections in the boundary layer below. The wave-number selection which determines the spacing and orientation of the waves is controlled by the atmospheric conditions in the two layers, such as temperature structures and mean wind speeds in the two layers, depth of the boundary layer, temperature jump at the interface, as well as wind-direction shear between the two layers. The linear model may explain the varied scale selections, such as cloud streets or cells, and the transition from one to another (Sang 1997).

The hypothesis of convection waves may give rise to some new questions. These are the subjects to be discussed in the present paper. First, if the waves can be induced by an isolated hill, why are the wave patterns different from those found in mountain waves? As analysed by Walter and Overland (1984) the convective bands could be traced upstream to individual mountain peaks, which may be taken as the mechanical source of the disturbances. The mountain-wave theory indicates that as a stratified airflow passes over an isolated mountain peak, trapped diverging waves can be produced on the leeside. The diverging waves are often symmetric and present a wedge angle, which points to the source, i.e. the mountain peak. According to the wave pattern we may find the wave origin. However, in cloud streets the convective bands are parallel to each other. We cannot trace their origins if there are any.

The second problem is associated with the first one. The cloud streets often spread a few hundred kilometers laterally across the mean wind direction, while the waves, induced by an isolated hill, are confined in a narrower region in the direction perpendicular to the mean wind. Thus the mechanism for the cloud streets to propagate widely in a lateral direction and how they acquire the energy to support their wide spreading should be illustrated.

In this paper we present two linear analyses. The first explains how an upstream mountain peak initiates primarily a few wave bands over the warm ocean surface during cold-air outbreaks. These parallel bands may become cloud streets since there is always a plentiful supply of water vapour over the warm ocean surface. The second describes how the primary band as a line heating source triggers more parallel bands and how those bands propagate laterally.

In section 2 of the paper the linear solutions of trapped waves over the leeside of an isolated mountain peak are obtained in a convective layer covered by an overlying stable layer. The effects of wind-direction shear on the wave pattern are emphasized. This shear condition favours only the wave propagating into the starboard side of the mountain peak.

The effects of band heating produced by latent-heat release in a wave ridge are described in section 3. Both the trapped and untrapped components of the wave induced by the heating propagate horizontally and overlap to form a multiscale band structure. Finally, a summary is provided.

2. INITIATION OF CONVECTION WAVES

(a) Linear analysis

The linearized governing equations for a steady-state perturbation in an incompressible Boussinesq airflow with diabatic heating can be written as

\[
U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -\frac{\partial \pi}{\partial x}
\]  

(1)
\[
U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} = -\frac{\partial \pi}{\partial y} \tag{2}
\]

\[
U \frac{\partial w}{\partial x} + V \frac{\partial w}{\partial y} = -\frac{\partial \pi}{\partial z} + \frac{\pi}{\theta} g \tag{3}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4}
\]

\[
U \frac{\partial}{\partial x} \left( \frac{\pi}{\theta} g \right) + V \frac{\partial}{\partial y} \left( \frac{\pi}{\theta} g \right) + w \frac{g}{\theta} \frac{\partial \theta}{\partial z} = \frac{g}{C_p T} q \tag{5}
\]

where \( U, V, \) and \( \bar{\theta} \) are the basic-state velocities and potential temperature respectively; \( u, v, w, \pi \) and \( \theta \) are the perturbation variables; \( q \) is the diabatic heating rate per unit mass; \( g \) is the gravitational acceleration; \( T \) is temperature; and \( C_p \) the specific heat at constant pressure. The above equations can be reduced to a single equation for the vertical velocity.

Making Fourier transformation of the vertical velocity from a physical domain into wave-number space as

\[
w(x, y, z) = \tilde{w}(k, l, z) \exp(ikx + ily) \tag{6}
\]

and

\[
q(x, y, z) = \tilde{q}(k, l, z) \exp(ikx + ily) \tag{7}
\]

we have the wave equation

\[
\frac{\partial^2 \tilde{w}}{\partial z^2} + \left\{ \frac{1}{(kU + lV)^2} \frac{g}{\theta} \frac{\partial \theta}{\partial z} - k(d^2U/dz^2) + l(d^2V/dz^2) \frac{1}{k^2 + l^2} \right\} \tilde{w} = \frac{g(k^2 + l^2)}{C_p T(kU + lV)^2} \tilde{q} \tag{8}
\]

The atmosphere is assumed to consist of two layers. The lower layer is an unstable boundary layer with the mean wind direction along the \( x \)-axis, i.e. \( V_1 = 0 \), while the upper layer is an overlying stable layer with a temperature jump between the two layers.

For the most part, the term including \( d^2U/dz^2 \) and \( d^2V/dz^2 \) in Eq. (8) can be neglected. Since in Eq. (8) the atmospheric conditions determining the characteristics of the disturbances are the potential-temperature gradient and wind speed, which may be represented by Scorer’s parameter \((1/U)((g/\bar{\theta})(d\bar{\theta}/dz)^{1/2})\), we set a parameter

\[
n^2 = (-1/U_1^2)(g/\bar{\theta})(d\bar{\theta}_1/dz) \tag{9}
\]

in the lower layer where \((g/\bar{\theta})(d\bar{\theta}/dz) < 0\), and \( m^2 = (1/U_2^2)(g/\bar{\theta})(d\bar{\theta}_2/dz) \) in the upper layer where \( d\bar{\theta}_2/dz > 0 \). In a near-neutral lower layer \( d\bar{\theta}_1/dz \approx 0 \), the parameter can be

\[
n^2 = -\frac{1}{U_1^2} \frac{g}{\bar{\theta}} \frac{d\bar{\theta}_1}{dz} + \frac{1}{U_1} \frac{d^2U_1}{dz^2} \frac{k^2}{k^2 + l^2}.
\]

In the lower layer Eq. (8) then becomes

\[
\frac{\partial^2 \tilde{w}_1}{\partial z^2} - (n^2 + k^2)(k^2 + l^2)/k^2 \tilde{w}_1 = \frac{g(k^2 + l^2)/k^2}{C_p T U_1^2} \tilde{q} \tag{9}
\]
and in the upper layer
\[
\frac{\partial^2 \tilde{w}_2}{\partial z^2} + \left( \frac{m^2}{k + \beta l} \right)^2 - 1 \right) (k^2 + l^2) \tilde{w}_2 = \frac{g(k^2 + l^2)/k^2}{C_p T U_2^2} \tilde{q}
\]  
(10)

where \( \beta = V_2/U_2 \) is also the direction shear of the mean wind between the two layers. The subscripts 1 and 2 represent the lower and upper layer, respectively.

The convection waves may be induced by upstream origins. The origin can be thermal, such as upstream thermals, forest fire and heat island, or mechanical, such as hills and islands. Without losing generality, we may assume that the convective bands are initiated by a circular mountain with a profile given by a bell-like form

\[
H(x, y) = H(r) = H_0 \frac{a^3}{(r^2 + a^2)^{3/2}}
\]  
(11)

which has Fourier transform

\[
\tilde{H}(k, l) = 2/\pi H_0 a^2 e^{-aK}
\]  
(12)

where \( r = (x^2 + y^2)_{1/2} \) and \( K = (k^2 + l^2)_{1/2} \). With a purely mechanical disturbing source, the diabatic heating is zero everywhere and then \( \tilde{q} = 0 \) in (9) and (10).

In the lower layer the solutions of Eqs. (9) and (10) have the form

\[
\tilde{w}_1 = A \cosh(\lambda z) + B \sinh(\lambda z)
\]  
(13)

and in the upper layer

\[
\tilde{w}_2 = C e^{-\mu z} \quad \text{if } (k + \beta l)^2 > m^2
\]

\[
\tilde{w}_2 = C e^{i\nu z} \quad \text{if } (k + \beta l)^2 < m^2
\]

where \( \lambda = [(n^2 + k^2)(k^2 + l^2)/k^2]^{1/2} \), \( \mu = [(1 - m^2/(k + \beta l)^2)(k^2 + l^2)]^{1/2} \) and \( \nu = [(m^2/(k + \beta l)^2 - 1)(k^2 + l^2)]^{1/2} \). The coefficients \( A, B \) and \( C \) can be determined according to the interface conditions and bottom boundary condition.

The interface between the two layers is set at \( z = 0 \) and the depth of the lower layer is \( h \). At the bottom boundary, i.e. the ground surface, \( z = -h \), the boundary condition is

\[
w_1 = U_1 \partial H/\partial x \quad \text{or} \quad \tilde{w}_1 = ik U_1 \tilde{H}.
\]  
(16)

The first interface condition is obtained from the continuity of vertical velocities at the two sides,

\[
w_1 = w_2 \quad \text{or} \quad \tilde{w}_1 = \tilde{w}_2 \quad \text{at } z = 0.
\]  
(17)

Similar to that by Smith and Lin (1982), the second interface condition is deduced from the discontinuity of potential temperature across the interface, which often appears at the top of the convective boundary layer. Assuming that the mean flows are continuous across the interface, integrating Eq. (8) from \(-\Delta z\) to \(+\Delta z\) near \( z = 0 \) and letting \( \Delta z \to 0 \), we have the second interface condition

\[
\frac{\partial \tilde{w}_2}{\partial z} + \frac{k^2 + l^2}{k^2} \gamma \tilde{w}_1 = \frac{\partial \tilde{w}_1}{\partial z}
\]  
(18)

where \( \gamma = (g/U^2)(\Delta \bar{\theta}/\bar{\theta}) \), \( \Delta \bar{\theta} = \bar{\theta}_2 - \bar{\theta}_1 \) is the temperature jump across the interface and \( U \) is the wind speed near the interface.
Applying conditions (16), (17) and (18) to eliminate \( A, B \) and \( C \) in (13)–(15), we have the solutions \( \tilde{w}_{1l} \) for \( k + \beta l < m \) and \( \tilde{w}_{1h} \) for \( k + \beta l > m \) in the lower layer and their counterparts \( \tilde{w}_{2l} \) and \( \tilde{w}_{2h} \) in the upper layer. \( \tilde{w}_{1l} \) and \( \tilde{w}_{2l} \) are the untrapped parts of the disturbances initiated by the hill. They propagate upward to the uppermost limit of the atmosphere and do not exert much influence on the flow fields far downstream, while \( \tilde{w}_{1h} \) and \( \tilde{w}_{2h} \) are the trapped components, which are confined in the lower atmosphere and propagate downstream.

For convenience we discuss the vertical displacement of streamlines \( \tilde{\zeta} \), instead of the vertical velocity \( w \). According to the relation \( \tilde{\zeta} = -i\tilde{w} / (Uk + Vl) \), we have, for example, the expression of \( \tilde{\zeta}_{1h} \) from the solution of \( \tilde{w}_{1h} \)

\[
\tilde{\zeta}_{1h} = 2H_0 / \pi a^2 \ e^{-a(k^2 + l^2)^{1/2}} \frac{\lambda \cosh(\lambda z) + [(k^2 + l^2)/k^2] \gamma - \mu} {G(k, l)} \sinh(\lambda z)
\]

(19)

where

\[
G(k, l) = \lambda \cosh(\lambda z) - \left( \frac{k^2 + l^2}{k^2} \gamma - \mu \right) \sinh(\lambda z).
\]

Similarly, we have

\[
\tilde{\zeta}_{2h} = 2H_0 / \pi a^2 \ e^{-a(k^2 + l^2)^{1/2}} \frac{\lambda \ e^{-\mu z}} {G(k, l)}
\]

(20)

\[
\tilde{\zeta}_{1l} = 2H_0 / \pi a^2 \ e^{-a(k^2 + l^2)^{1/2}} \frac{\lambda \cosh(\lambda z) + [(k^2 + l^2)/k^2] \gamma + iv} {\lambda \cosh(\lambda z) - [(k^2 + l^2)/k^2] \gamma + iv} \sinh(\lambda z)
\]

(21)

\[
\tilde{\zeta}_{2l} = 2H_0 / \pi a^2 \ e^{-a(k^2 + l^2)^{1/2}} \frac{\lambda \ e^{ivz}} {\lambda \cosh(\lambda h) - [(k^2 + l^2)/k^2] \gamma + iv} \sinh(\lambda h)
\]

(22)

Integrating \( \tilde{\zeta} \) in wave-number space, we have the vertical displacement of the streamlines in physical space, for example

\[
\zeta_{1l} = \text{Re} \left\{ \int_{-\infty}^{\infty} \int_{0}^{k_c} \tilde{\zeta}_{1l} \ e^{ikx + ily} \ dk \ dl \right\}
\]

(23)

and

\[
\zeta_{1h} = \text{Re} \left\{ \int_{-\infty}^{\infty} \int_{k_c}^{\infty} \tilde{\zeta}_{1h} \ e^{ikx + ily} \ dk \ dl \right\}
\]

(24)

where \( \text{Re} \) denotes the real part and \( k_c \) is the minimum wave number of \( k \) satisfying the relation

\[
k + \beta l \geq m.
\]

(25)

The components with a wave number higher than \( k_c \) will be trapped and propagate downstream; while those lower than \( k_c \) may propagate upward. Then \( k_c \) may be called the cut-off wave number.

Integrals (23) and (24) may be calculated by the numerical Fourier transform technique.

From (19) we can see that the integral (24) has singularities if the denominator of (19)

\[
G(k, l) = \lambda \cosh(\lambda h) - \left( \frac{k^2 + l^2}{k^2} \gamma - \mu \right) \sinh(\lambda h) = 0.
\]

(26)
Thus (24) can be integrated by the residue theorem
\[\xi_{1h} = \text{Re} \left[ 4H_0 a^{-2i} \int_{k_c}^{\infty} \frac{e^{-a\sqrt{k^2+l^2}} \lambda \sinh(\lambda(h+z)) e^{ikx+ily} \, dk}{(\partial/\partial l)G(k, l)} \right] \]  
(27)

where \( G(k, l) = 0 \).

The integral (27) converges since the integrand includes the factor \( e^{-a\sqrt{k^2+l^2}} \), which decreases rapidly with \( k \) and \( l \). It is indicated that the wave numbers of \( k \) and \( l \) composing the trapped waves must satisfy the relation (26), which may be regarded as the wave-number selection relation. The wave-number selection relation determines the pattern of the trapped waves, i.e. the spacing and orientation of the convective bands. The organization of the bands are now discussed further through four-case studies in the following subsection.

(b) Case-studies

Four cases are presented to indicate the effects of atmospheric conditions on the pattern of convective waves. In the first case the atmospheric conditions in the lower layer are chosen as \( d\theta_1/\sqrt{z} = -0.3 \text{ K per 100 m, } U_1 = 10 \text{ m s}^{-1} \) and depth \( h = 1000 \text{ m, } \) while those in the upper layer \( d\theta_2/\sqrt{z} = 0.35 \text{ K per 100 m, and } U_2 = 20 \text{ m s}^{-1} \). The wind directions in the two layers are the same, that is \( V_2 = 0 \) and \( \beta = 0 \). The temperature jump across the top of the boundary layer is set to be 2.5 K. Then we have the atmospheric parameters: \( n = 0.001 \text{ m}^{-1}, \ m = 0.0005 \text{ m}^{-1} \) and \( \gamma = 0.0003 \text{ m}^{-1} \). The terrain parameters are taken as \( H_0 = 300 \text{ m and } a = 2500 \text{ m.} \)

Integrating (23) numerically, we obtain the vertical displacements of streamlines of the untrapped waves at the interface, \( z = 0 \), produced by a mountain located at \( (0, 0) \).

Figure 1(a) shows the distribution of the vertical displacements \( \xi_1(x, y, 0) \). The pattern is similar to that of mountain waves caused by an isolated hill, derived by linear theories (e.g. Smith 1980) even though the lower layer is unstable in this study. This means that as the flow passes over the hill the air forced upward in the lower unstable layer may cause disturbances in the overlying stable layer. The disturbances propagate upward to form the untrapped waves. The wave pattern depends on the height and shape of the mountain as well as on the atmospheric conditions in both the upper and the lower layers. In this case the maximum upward motion is located over the mountain top, while the centre of the downward motion with value of about \(-170 \text{ m is at } 10 \text{ km downwind of the mountain peak. \ The descending area covers most of the leeside with a few ascending patches along the } x-\text{axis. Since the untrapped part of the waves is a near-field solution the amplitudes of the disturbances decay away from the mountain.} \)

In the following paragraphs we discuss the trapped waves. For convenience we set a polar coordinate system for wave-number space. In this system the total horizontal wave number is \( K = (k^2 + l^2)^{1/2} \) and the azimuthal angle \( \psi = \arctan(l/k) \).

According to (25) and (26) the cut-off wave numbers are \( k_c = 0.0005 \text{ m}^{-1} \) and \( l_c = 0.0018 \text{ m}^{-1} \). The total horizontal wave number is \( K_c = (k_c^2 + l_c^2)^{1/2} = 0.00187 \text{ m}^{-1} \). The azimuthal angle \( \psi_c = \arctan(l_c/k_c) = 74.5^\circ \), which is the angle between the wave-number vector \( K \) and the \( x-\)axis. As \( k \) increases, \( l, K \) and \( \psi \) increase correspondingly. For example, if \( k \) increases to 0.0008 \text{ m}^{-1}, \( l \) satisfying (26) is then \( \pm 0.0005 \text{ m}^{-1} \), and \( K = 0.0051 \text{ m}^{-1}, \psi = 80.9^\circ \). However, the amplitude of the wave-number component decreases rapidly with \( K \). Taking \( e^{-aK} \) as the measure of the amplitude of \( \xi_{1h} \) in (19), we may find that if \( k = 0.0005 \text{ m}^{-1} \), then \( e^{-aK} = 0.15, \) while if \( k = 0.0008 \text{ m}^{-1}, \)
Figure 1. Horizontal distributions of vertical displacement of streamlines, caused by an isolated hill in the \((x, y)\) plane at the interface \(z = 0\), in Case 1. The terrain parameters are: \(a = 2500\) m and \(H_0 = 300\) m. The atmospheric conditions are: \(\frac{\delta \theta_1}{\delta z} = -0.3\) K per 100 m, \(\frac{\delta \theta_2}{\delta z} = 0.35\) K per 100 m, \(U_1 = 10\) m s\(^{-1}\), \(U_2 = 20\) m s\(^{-1}\), \(V_2 = 0\), \(\Delta \bar{\theta} = 2.5\) K and \(h = 1000\) m, i.e. the parameters \(n = 0.001\) m\(^{-1}\), \(m = 0.0005\) m\(^{-1}\), \(\gamma = 0.0003\) m\(^{-1}\) and \(\beta = 0\).

(a) The displacements induced by untrapped waves. The interval is 20 m. (b) The displacements induced by trapped waves. Only positive displacements, i.e. \(\zeta > 0\), are shown. (c) The displacements of composite waves.

See text for further explanation.
then $e^{-aK} = 0.006$. Thus the contribution of the wave-number components to the trapped waves comes from a very narrow azimuthal spectrum with $\psi$ close to $\psi_c$.

Figure 1(b) shows the distribution of the vertical displacements of streamlines of the trapped waves at $z = 0$, i.e. at the top of the lower layer. For convenience we only show the positive displacements, i.e. $\zeta > 0$. The wave bands are symmetric about the $x$-axis. The angle $\alpha$ between the bands and the $x$-axis is about $14^\circ$, approximately close to $90^\circ - |\psi_c| = 15.5^\circ$. The spacing of the bands is about 3 km, which is approximately equal to $2\pi/K_c$, the cut-off wavelength. So the cut-off wave numbers $(k_c, l_c)$ and $K_c$, $\psi_c$ are important parameters determining the characteristics for the wave bands. They depend on the atmospheric conditions in the two layers, such as $m$, $n$, i.e. $U_1$, $U_2$, $d\theta_1/dz$, $d\theta_2/dz$, as well as $\gamma$ and $h$.

The maximum displacement is 71 m. If the air in the lower layer is humid enough, the lift may cause condensation of the water vapour. It is likely the case of a marine atmospheric boundary layer.

Figure 1(c) shows the composite waves composed of the untrapped and trapped waves in Case 1. In this figure we can see that in the vicinity of the mountain top the pattern of the composite waves is similar to that of the untrapped waves shown in Fig. 1(a), since the untrapped waves are the dominant disturbances near the source. With decaying of the untrapped waves far downstream the trapped waves emerge and gradually become dominant, and the wave pattern is more like that of Fig. 1(b).

In this case the atmospheric conditions are set to be similar to that of a marine atmospheric boundary layer during a cold-air outbreak. The waves produced, which make an angle of $14^\circ$ with the $x$-axis, are like the diverging mode of atmospheric ship waves. They propagate outward into both the starboard and the port sides, and are not like the cloud streets morphologically.

A process of cold-air outbreak over the Bering Sea in winter time has been analysed by Walter and Overland (1984). In their studies of the structure of the longitudinal roll vortices two events are noted, which might be the conditions related to the formation of the cloud streets. First, some bands over the sea were thought to be initiated by upstream topography over the Chukatka Peninsula. Second, there was a rotation of about $18^\circ$ in the wind direction through the top of the boundary layer. The second condition will be considered in the next case-study.

The second case illustrates the effect of wind-direction shear on the wave pattern. The atmospheric conditions are assumed to be the same as those in Case 1, but the wind direction in the upper layer turns right to the $x$-axis. For instance, $V_2 = -5$ m s$^{-1}$ and so $\beta = V_2/U_2 = -0.25$. In this case the parameters, such as $m$, $n$, $\gamma$, and $h$, are all the same as those in Case 1 except for $\beta$. Figure 2(a) shows the distribution of the untrapped waves in Case 2. Its pattern is similar to that of the untrapped waves of Case 1. However, the direction shear distributes the waves asymmetrically about the $x$-axis. The downward motions extend farther downstream, especially on the starboard side.

With $\beta \neq 0$ the trapped wave numbers satisfying the wave-number selection relations (25) and (26) are no longer symmetric about $\psi = 0$ in the wave-number space. For $\psi < 0$, $k_c = 0.0003$ m$^{-1}$, $l_c = -0.0011$ m$^{-1}$, $K_c = 0.0012$ m$^{-1}$, $\psi_c = -75.6^\circ$ and $e^{-aK} = 0.3$, while for $\psi > 0$, $k_c = 0.0011$ m$^{-1}$, $l_c = 0.009$ m$^{-1}$, $K_c = 0.0091$ m$^{-1}$, $\psi_c = 83^\circ$ and $e^{-aK} = 1.1 \times 10^{-4}$. Note that the amplitudes of the wave mode in wave-number space of $\psi < 0$ are at least $10^3$ times larger than those of $\psi > 0$. Thus the former dominates the wave pattern. Figure 2(b) shows the distribution of vertical displacements of streamlines caused by the trapped waves at the top of the lower layer in Case 2. It indicates that the bands with $\alpha > 0$ are in control of the wave pattern, while those with $\alpha < 0$ are almost invisible. The bands present an angle of $\alpha = 15^\circ$, close to
Figure 2. Case 2. As in Fig. 1 except for $V_2 = -5 \text{ m s}^{-1}$, i.e. $\beta = -0.25$. 
\[ 90^\circ - |\psi_c| = 14.5^\circ. \] The maximum displacement is 87 m, larger than that in Case 1, since the wave energy is concentrated on only the mode with \( \psi < 0 \).

Six bands can be seen in Fig. 2(b). Only the central two or three have stronger amplitudes. The average spacing, i.e. the lateral wavelength, is about 5 km, slightly narrower than that of the cut-off wavelength \( 2\pi|l_c| = 5.4 \) km.

Figure 2(c) shows the composite waves in Case 2. Similar to that in Case 1, the wave pattern in the vicinity of the mountain top is determined by the untrapped components. The strong downward motions of the untrapped waves in the area of 10 to 20 km downwind offset the upward motions of the trapped wavebands. Beyond the descending area the upward-motion bands emerge and become the main feature of the vertical-motion distribution. We can see three distinct bands and two or three faint ones; they are all parallel to each other.

In the above analyses the formation of the parallel bands caused by a disturbing source is attributed to the trapped waves. The characteristics of the trapped waves are determined by the wave-number selection relation (26), which in turn depends on the stratifications and wind speeds in the upper and lower layers, the temperature jump at the interface, i.e. the density difference between these two layers, the depth of the lower layer as well as the wind-direction shear through the top of the boundary layer. The effects of these factors have been analysed in previous studies (Sang 1991, 1993, 1997). In the present study the roles of directional shear and the unstable stratification in the lower layer are further emphasized. Because of the effect of the Ekman layer the wind veering from the boundary layer to the free atmosphere is a common phenomenon. A comparison of Cases 1 and 2 indicates that the directional shear, though not large, is a necessary condition for the formation of the parallel bands. Dorman (1994) analysed a cloud-band event forming in the lee of Guadalupe Island. As a cold front passed over the island the lower marine boundary layer was unstable and the wind direction backed from north in the marine boundary layer to north-west above the inversion layer. A cloud band roughly parallel to the mean wind in the marine layer appeared on satellite photos, conforming to the condition of directional shear. The effects of the stratification and the wind profile curvature in the boundary layer are discussed in the following paragraphs.

In Case 2 the parameter \( n^2 = (-1/U_1^2)(g/\bar{\theta})(d\bar{\theta}_1/dz) > 0 \) is an important condition for the formation of the trapped waves. However, observations show that the cloud streets may form in a near-neutral boundary layer, i.e. \( d\bar{\theta}_1/dz \sim 0 \). In this situation \( n^2 \) would be mainly determined by the shear gradient \( d^2U_1/dz^2 \) and/or \( d^2V_1/dz^2 \). In most cases in the boundary layer the wind profiles are convex, i.e. \( d^2U_1/dz^2 < 0 \). Then we have \( n^2 < 0 \). If there is a piece of concave profile, we may have \( d^2U_1/dz^2 > 0 \) on the average in the boundary layer, i.e. \( n^2 > 0 \). In both the above situations \( n^2 \) has a small absolute value. Sang (1993) analysed the cut-off wave numbers \( k_c \) and \( l_c \) of the trapped waves as the minimum wave numbers satisfying relations (26) and (28)

\[ k_c + \beta l_c \geq m. \quad (28) \]

The relation between \( l_c \) and \( n^2 \) may be written as (Sang 1993)

\[ l_c^2 = (k_c^2 + n^2)/\gamma^2 \coth(\lambda h) - k_c^2. \]

It means that small positive or negative values of \( n^2 \) result in small \( l_c^2 \). For example, in case-study 3 the lower layer is set to be neutral with a convex wind profile while the other atmospheric conditions and topography features are the same as those in Case 2. Then we have the same parameters as those in Case 2 except for \( n^2 = -4 \times 10^{-8} \) m\(^{-2}\). Accordingly, we have the cut-off wave numbers \( k_c = 0.0004 \) m\(^{-1}\) and \( l_c = \)
-0.0005 m\(^{-1}\), the azimuthal angle of the wave-number vector \(\psi_c = \arctan(l_c/k_c) = -51^\circ\), the orientation angle of the waveband \(\alpha_c = 39^\circ\), and the lateral wavelength \(L_y = 2\pi/|l_c| = 12.6\) km. Figure 3 shows the distribution of the trapped waves in Case 3. It indicates that the pattern of the trapped wave bands is mainly determined by the cut-off wave numbers. In Fig. 3 the induced trapped waves are characterized by weak, short and widely spaced bands with a large orientation angle to the mean wind direction of the lower layer. This is not a favourable situation for the formation of cloud streets. We will discuss this in the next subsection.

If in the near-neutral boundary layer the wind profile is partly concave, i.e. \(d^2U_1/dz^2 > 0\), the parameter \(n^2\) may be set to be positive, but small. In addition, in a near-neutral condition the temperature jump across the top of the boundary layer is usually small. In Case 4, in which the boundary layer is neutral with a concave wind profile, the parameters are considered to be the same as those in Case 2 except for \(n = 0.0001\) and \(\gamma = 0.0001\). Figure 4(a) shows the distribution of the vertical displacement of streamlines at the interface induced by the untrapped waves. The wave pattern is similar to that in Case 2, but the downward motions in the starboard side are stronger and extend far downstream. The descending centre is located at the leeside with a value of \(-171\) m. Figure 4(b) is the same as Fig. 4(a) but for trapped waves. In this case the cut-off wave numbers are \(k_c = 0.00028\) and \(l_c = -0.00129\). Accordingly, the azimuth of wave-number vector \(\psi_c = -76.9^\circ\), the orientation angle of the waveband \(\alpha_c = 13.1^\circ\), and the lateral wavelength \(L_{yc} = 2\pi/|l_c| = 5.2\) km. In this figure we may see that the average lateral wavelength is about 4 km with orientation angle of 10\(^\circ\). The maximum upward displacement of the streamlines is only 18 m, much smaller than that in Case 2. Figure 4(c) shows the distribution of the composite waves in Case 4. Over the leeside the upward bands of the trapped waves almost disappear in the strong downward area caused by the untrapped waves. It might be imagined that as the untrapped waves fade away far downstream the trapped bands emerge. However, unless there is a strong disturbing source, a high mountain for instance, the bands are usually weak.
Figure 4. Case 4. As in Fig. 1 except for $n = 0.0001 \text{ m}^{-1}$, $\gamma = 0.0001 \text{ m}^{-1}$ and $\beta = -0.25$. 
(c) Discussion

During cold-air outbreaks the unstable or near-neutral boundary layer with an
overlying stable layer may form over the warm sea surface. Because of the strong mixing
the shears of wind speed and direction in the boundary layer are usually small, while
across the top of the boundary layer wind-direction veering or backing may occur. In this
situation the trapped waves produced by an upstream point disturbing source, thermal
or dynamic, e.g. an isolated mountain or an island, may result in a few parallel upward-
motion bands as shown in Fig. 2(b).

In the marine atmospheric boundary layer, cloud may form along the updraught
bands. Then each band becomes a thermal line source because of the latent-heat release.
It in turns excites more trapped wave bands. Since these bands are composed of the same
wave-number components they are all in phase with each other.

As seen in Case 2 the unstable stratification with an adequate strength in the
boundary layer is favourable to the formation of the trapped wave bands, which have
strong updraughts and proper spacing.

The near-neutral stratification and convex wind profile in the boundary layer as in
Case 3 would make the band spacing too wide. Then convective activities may appear
between two adjacent bands. The convections would finally be distributed randomly.

If the wind profile is concave in a neutral boundary layer the dynamic effect of the
wind-shear curvature on the band initiation is similar to the thermal effect of the unstable
stratification as shown in Case 4. Since a large value of \((1/U)(d^2U/dz^2)\) rarely occurs
in the boundary layer the possibility of dynamic initiation would be less than that of
thermal initiation.

In summary, in a suitable condition a few bands may form over the leeside at the
initial stage as we have seen in Cases 2 and 4. We suppose that each band could become
a new thermal line source because of the latent-heat release. It in turn further excites
more trapped and untrapped waves. The trapped wavebands may be aroused one by one
and propagate laterally.

This is a nonlinear process. If we examine the heating effects of one single band
only, however, the linearized model can also be used.

3. Band Heating

The variety of diabatically forced mesoscale circulation has been investigated an-
alytically since the eighties (for instance, Smith and Lin 1982; Lin and Smith 1986;
Bretherton 1988; Hsu 1987) and reviewed by Lin and Stewart (1991). In this section we
will study the effects of a waveband similar to that in Case 2. The two-layer model in the
last section is still used except that a diabatic band heating around the interface, instead
of a mountain at the ground surface, is taken as the disturbing source.

The diabatic source, \(q\) in Eq. (5), is represented by a finite long and thin band, which
presents an angle of \(\alpha = 14.5^\circ\) to the \(x\)-axis facing downstream. It can be expressed as

\[
q(x, y, z) = q_H(x, y) \delta(z)
\]

where

\[
q_H = \begin{cases} 
Q_0 b^2 / (b^2 + y'^2), & c < x < a \\
0, & x > a \ or \ x < c.
\end{cases}
\]

\(Q_0\) is the heating rate of a thin layer, \(b\) is the half-width of the heating band, \(y'\) is the
axis of a new coordinate system \((x', y')\) which is made up by counterclockwise rotation
of system \((x, y)\) with an angle \(\alpha\), that is
\[
\begin{align*}
x' &= x \cos \alpha + y \sin \alpha \\
y' &= -x \sin \alpha + y \cos \alpha \\
x &= x' \cos \alpha - y' \sin \alpha
\end{align*}
\]
and
\[
y = x' \sin \alpha + y' \cos \alpha.
\]

The Fourier transform of \(q_H\) is
\[
\hat{q}_H(k, l) = Q_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{b^2}{b^2 + (y \cos \alpha - x \sin \alpha)^2} e^{-ikx} e^{-ily} \, dx \, dy
\]
\[
= Q_0 b e^{-|l| \cos \alpha - k \sin \alpha |b|} \frac{e^{-i(l \sin \alpha + k \cos \alpha)a'} - e^{-i(l \sin \alpha + k \cos \alpha)c'}}{l \sin \alpha + k \cos \alpha}
\]
where \(a'\) and \(c'\) are the ends of the band on the \(x'\)-axis, \(a' = a / \cos \alpha\) and \(c' = c / \cos \alpha\), respectively.

Similar to (18) the second interface condition is obtained by integrating (8) from \(-\Delta z\) to \(+\Delta z\) around \(z = 0\). Then we have
\[
\frac{\partial \tilde{w}_2}{\partial z} - \frac{\partial \tilde{w}_1}{\partial z} + \frac{g}{U^2} \frac{\Delta \theta}{\theta} \frac{k^2 + l^2}{k^2} \tilde{w}_1 = \frac{g}{C_p \cdot T U^2} \frac{k^2 + l^2}{k^2} \tilde{q}_H.
\]
(31)

The bottom boundary condition is now
\[
\tilde{w}_1 = 0 \quad \text{at} \quad z = -h
\]
(32)
since we have moved the bottom boundary from the mountain terrain to the downstream flat area, say, the sea surface.

Transforming the system \((x, y)\) and \((k, l)\) into \((r, \theta)\) and \((K, \psi)\), respectively, where as already mentioned in physical domain
\[
r = (x^2 + y^2)^{1/2}, \quad \theta = \arctan(y/x)
\]
\[
x = r \cos \theta, \quad y = r \sin \theta
\]
and in wave-number space
\[
K = (k^2 + l^2)^{1/2}, \quad \psi = \arctan(l/k)
\]
\[
k = K \cos \psi, \quad l = K \sin \psi
\]
we obtain the vertical velocity from (9), (10) and conditions (32), (17), and (31), for untrapped waves
\[
w_{1l} = \Re \left\{ -i \frac{gQ_0}{C_p \cdot T U^2} b \int_{-\pi}^{\pi} \frac{1}{\cos^2 \psi} e^{-bK|\sin(\psi - \alpha)|} \right. \\
\times \left. \frac{e^{-iKa' \cos(\psi - \alpha)} - e^{iKc' \cos(\psi - \alpha)}}{\cos(\psi - \alpha)} \cdot \frac{e^{iKr \cos(\psi - \theta)} \sinh[\lambda(h + z)]}{\lambda \cos(\lambda h) - \left(\frac{K}{\cos^2 \psi + iv}\right) \sinh(\lambda h)} \right\} \\
\]
where \( \lambda = \left(n^2 + K^2 \cos^2 \psi\right)^{1/2}/\cos \psi \), \( \nu = \left(m^2/(\cos \psi + \beta \sin \psi)\right)^2 - K^2\right)^{1/2} \) and for trapped waves\
\[
w_{1h} = \Re \left\{ \frac{2\pi g Q_0}{C_p T U^2} \beta \int_{K_{\Sigma}^2}^{\infty} \frac{1}{\cos^2 \psi} e^{-bK} |\sin(\psi - \alpha)| \right. \\
\left. \times e^{-iK' \cos(\psi - \alpha)} - e^{-iK' \cos(\psi - \alpha)} \frac{e^{iK' \cos(\psi - \theta)} \sinh(\lambda h + z)}{\delta G/\delta \psi} \frac{dK}{dK} \right\} (34)\]

where \( G = \lambda \cosh(\lambda h) - (\gamma/\cos^2 \psi - \mu) \sinh(\lambda h) = 0 \) and \( \mu \) is \( \{K^2 - m^2/(\cos \psi + \beta \sin \psi)^2\}^{1/2} \). Similarly, we may obtain, \( w_{2j}, w_{2h} \) as well as the other variables in the two layers.

In the following we present a case-study to illustrate the characteristics of the band heating. In Case 5 the atmospheric conditions are set to be the same as those in Case 2 of the last section. That is, \( n = 0.001 \) m\(^{-1}\), \( m = 0.0005 \) m\(^{-1}\), \( \gamma = 0.0003 \) m\(^{-1}\), \( h = 1000 \) m and \( \beta = -0.25 \). The heating parameters are as follows: \( Q_0 = 30 \) J m (kg s\(^{-1}\)), \( b = 1000 \) m, \( a' = 100 \) km and \( c' = -100 \) km, i.e. a heating band with length of 200 km, half-width of 1000 m and depth of zero.

Figures 5, 6 and 7 show the horizontal distributions of vertical velocity near the origin induced by the untrapped, trapped and composite waves, respectively.

From these figures we see that the convective bands of either untrapped waves or trapped waves have nearly the same orientation with an angle to the left of the \( x \)-axis, close to the orientation of the original heating band.

Figure 5 shows that the induced updraught band is in phase with the original heating band, which passes through the origin \((0,0)\). The maximum updraught is about 0.2 m s\(^{-1}\). Outside the band there are weak downdraughts. This figure indicates that the untrapped wave is a near-field phenomenon. Its influences are mainly confined within a narrow updraught region, roughly coinciding with the heating band. Outside the region there are very weak downdraught and updraught bands alternately.
There are seven trapped wave bands in Fig. 6. For convenience only the updraughts of \( w > 0.2 \text{ m s}^{-1} \) are shown. Figure 6 indicates that the trapped waves induced by the band heating can propagate laterally into both the starboard and port sides. The maximum vertical velocity is 0.58 m s\(^{-1}\), corresponding to the vertical displacement of about 100 m, stronger than that of the untrapped waves. Since there is always enough water vapour in the marine atmospheric boundary layer the cloud bands with clear sky in between would be produced by lifting condensation. Meanwhile the latent-heat release would make each band a heating line source.

Each heating band may excite more trapped waves propagating horizontally. As the processes repeat the upper boundary layer would be full of resonance wave bands.

The composite waves are shown in Fig. 7. Since the amplitude of the trapped waves is larger than that of the untrapped ones the pattern of the composite waves is similar to that of the trapped waves shown in Fig. 6. However, since there is a
phase difference between the trapped and untrapped waves the composite ones present multiscale structure, as observed in some rolls (Walter and Overland 1984).

Figure 8 shows the distribution of the vertical velocities caused by trapped waves in the \((y, z)\) section at \(x = 10\) km. The updraughts and downdraughts align alternately in the section.

The band spacing, i.e. the distance between the centres of two adjacent updraughts, ranges from 2.6 to 3.4 km with a mean value of about 3 km, corresponding to a mean wave number of \(l = 0.0021\) m\(^{-1}\). The trapped waves are evanescent in the upper stable layer and meanwhile their amplitude attenuates downward to zero at the surface. However, since the waves are all in phase in the vertical through the whole layer they may play remarkable roles in tuning up the convections in the lower layer. Because of the significant temperature difference between the cold air and the warm sea surface there must be strong convective activity over the sea surface. And the convections must be randomly distributed since the opportunity for their taking place over the homogeneous sea surface is equal. The selection is made by the waves. The convections will be reinforced in the updraught belts and suppressed in the downdraught belts.

The reinforced convections, if they are strong enough, would feed back the wave-selection system by impinging on the overlying inversion. The convective activities are nonlinear and unsteady. Thus in order to solve the whole processes we should rely on a numerical model.

4. CONCLUDING REMARKS

In this study a dynamic analysis is suggested to explain the formation and propagation of convective cloud streets. The first band appearing at the top of the convective boundary layer may be just one of the trains of the diverging waves, a kind of atmospheric ship wave, caused by an isolated hill. The trapped parts of the induced waves propagate downstream. Their patterns are determined by the atmospheric conditions.
Among these conditions the veering or backing of the wind direction plays a special role on the alignment of the wave trains. For example, veering favours the wave bands lining up to the left of the mean wind direction in the lower layer, as shown in Fig. 2(b). Since veering and backing usually exist the wave pattern like that in Fig. 2(b) might be commonly observed.

The updrafts of the waves may produce condensation and latent heating, which most likely occur as the cold-air outbreaks over the warm ocean surface, while the downdraughts may cause evaporation and cooling. As the heating and cooling bands become thermal perturbation sources, new convective bands could be further induced.

Section 3 of this study describes the effects of a heating band, oriented at a small angle to the left of the mean wind direction. The heating induces both trapped and untrapped waves, and the wave trains are parallel to the heating band. The untrapped waves are near-field phenomena and have an in-phase relation with the diabatic heating. They propagate mainly upward, while the trapped waves with stronger components spread horizontally without attenuation. Thus every ridge line of the trapped waves may become a new heating band.

The convective cloud streets may enhance each other since the bands are composed of the same wave-number components and have similar spacings. These interacting processes are nonlinear and should be further studied by using a numerical model.

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