Parametrization of effective sizes of cirrus-cloud particles and its verification against observations

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SUMMARY

A simplified parametrization of effective sizes of cirrus-cloud particles is developed in terms of the ice-water content and the cloud temperature based on some earlier studies. The effective particle sizes of the cirrus clouds determined using the simplified parametrization are compared with the results determined from observations. The results are generally encouraging.

The simplified parametrization is further tested by incorporating it into a new ice-cloud optical-property parametrization scheme and performing radiative-transfer calculations. The results show that this scheme in conjunction with the ice-cloud optical-properties scheme can generate a reasonably good estimate of cirrus-cloud radiative properties compared with the observations.

KEYWORDS: Cirrus clouds  Crystal effective size  Scattering properties

1. INTRODUCTION

Cirrus clouds cover approximately one third of the globe and are resident in the upper troposphere. Therefore, these clouds have a large influence on the heating and cooling in the upper troposphere and an impact on net global heating and cooling. Studies of cirrus clouds and their influence on the radiation budget are an important task in improving numerical weather prediction (NWP) and general-circulation climate models. A good parametrization of optical properties of cirrus clouds is fundamental to the accuracy of radiation schemes used in NWP and climate models. Unlike lower-level water clouds, cirrus clouds are composed of a majority of non-spherical ice crystals with sizes ranging from a few microns to millimetres. Theoretical study (e.g. Takano and Liou 1989) and observations (Stackhouse and Stephens 1991; Francis 1995) have shown that the radiative properties of cirrus clouds are significantly different from those of water clouds.

The radiative properties of ice clouds required for radiative-transfer calculations in NWP and climate models are the extinction coefficient, single-scattering albedo, and asymmetry factor. These properties are normally parametrized in terms of ice-water content and effective ice-crystal sizes, based on detailed scattering calculations using sophisticated methods such as the Mie theory (Rockel et al. 1991), the anomalous-diffraction theory (Stephens et al. 1990) and the geometric-optics method (Takano and Liou 1989). The liquid- or ice-water content can either be predicted by using a sophisticated prognostic scheme (Smith 1990; Das et al. 1995) or diagnosed from large-scale model variables using a simple regression scheme (Lemus et al. 1997) or a more complicated scheme (Heymsfield and Donner 1990). While there have been many attempts at improving cloud and cloud-water content schemes used in NWP and climate models, the ability to predict cloud particle sizes remains problematic. The current solution is to prescribe cloud effective sizes if required (Morcrette 1990) or to employ an optical-properties scheme without invoking the effective size (Rockel et al. 1991; Sun and Shine 1995; Platt 1997). Since cirrus particle sizes vary over a very wide range it may not be adequate to use a prescribed size.

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In order to improve the ability to represent the influence of cirrus clouds in NWP and climate models several extensive field experiments have been carried out in recent years. These experiments have provided valuable information on the cirrus-cloud properties which are essential for developing a physical parametrization for cirrus. Using the data collected during the Central Equatorial Pacific EXperiment (CEPEX), McFarquhar and Heymsfield (1997) (hereafter referred to as MH97) have developed a parametrization of ice-crystal size distribution in terms of cloud temperature and ice-water content. This parametrization is essentially an extension to the earlier parametrization developed by Heymsfield and Platt (1984), (hereafter referred to as HP84). The HP84 parametrization is based on mid-latitude observations whereas MH97 is based on measurements over tropical regions. The advantage of the MH97 parametrization is that the size spectra for small particles are based on actual observations while the size spectra of the HP84 parametrization, due to lack of accurate observations for small ice crystals, extends only down to 20 μm. Since the effects of small ice crystals on radiative transfer are very significant (Sun and Shine 1995; Heymsfield and McFarquhar 1996) the MH97 parametrization is thus considered useful in improving the representation of cirrus clouds in NWP and GCMs, especially over the tropical regions.

In addition to the observational studies, there have been many advances in the theoretical studies of cirrus clouds in recent years. The conventional method commonly used to calculate scattering properties for non-spherical particles is a geometric ray-tracing technique (Takano and Liou 1989). This method is estimated to be valid for size parameters (ratio of particle size to wavelength) larger than about 30. Therefore, it may not be appropriate for small ice crystals in the near infrared. Yang and Liou (1995) have developed an improved ray-tracing method which can be used to calculate the scattering properties of hexagonal particles for size parameters as small as 15. Another significant feature in the phase function determined using the improved method is that, as the size parameter decreases, the forward and backscattering peaks become wider and weaker than those from the conventional program. Macke et al. (1998) calculated the single-scattering properties for a hexagonal column, plate and polycrystal. They found that while the hexagonal particle shows a number of scattering features like pronounced forward and backscattering peaks as well as halos, the phase function of polycrystal is essentially featureless which is more or less consistent with some laboratory experiments (Volkovitskiy et al. 1980) and aircraft observations (Francis 1995). The asymmetry factor is much more sensitive to the choice of particle shape than to the size distributions. They concluded that cirrus-cloud scattering properties may be better determined in terms of the total number density and particle shape than by the particle sizes.

Based on the Yang and Liou (1995) study, Fu (1996) has developed an accurate parametrization of the solar radiative properties of cirrus clouds. Fu et al. (1998) have also developed a parametrization of the infrared radiative properties of cirrus by comparing the results from Mie theory, anomalous-diffraction theory, the geometric-optics method and the finite-difference time-domain technique (Yang and Liou 1996). In order to develop parametrizations that are applicable to a wide range of cases Fu et al. (1998) used 28 ice-crystal size distributions based on in-situ aircraft observations from several field experiments over the mid-latitude and tropical regions. The effects of small ice crystals were taken into account by extrapolating the size distributions to an appropriate point (Fu et al. 1998). These parametrizations have a good physical basis and relatively high accuracy compared with reference results. To apply these parametrizations to NWP and climate models the ice-water content and effective particle size must both be available.
The MH97 parametrization provides a means of predicting particle size distributions of cirrus clouds in terms of large-scale model variables. However, to apply this scheme to NWP and climate models some further work needs to be done. This includes (a): testing the scheme to see if it is applicable to areas other than the tropics as this scheme is designed particularly for the tropical areas; (b): verifying the scheme to see if it leads to a good estimate of the radiative properties of cirrus clouds (this involves the cirrus optical-properties scheme); and (c): simplifying the scheme such that it is easy to implement in the models. In this study, we will first simplify the MH97 parametrization such that the effective particle size can be estimated directly using the cloud temperature and ice-water content. We then test this approach by applying it to Fu’s parametrizations and compare the results with observations.

2. MH97 PARAMETRIZATION

The details of the MH97 parametrization are described in their paper. The parametrization is based primarily upon distributions of mass-equivalent spheres with diameter, $D_m$, defined by

$$D_m = \sqrt[3]{\frac{6M}{\pi \rho_i}} \tag{1}$$

where $M$ is the mass of the ice crystal determined from the maximum crystal length and area ratio, and $\rho_i$ is the density of the solid ice sphere. The size distributions for large and small ice crystals are parametrized separately. For small ice crystals with sizes less than 100 $\mu$m they use a first-order gamma distribution given by

$$N(D_m) = \frac{6IWC_{<100} \alpha_{<100}^5 D_m}{\pi \rho_i \Gamma(5)} \exp(-\alpha_{<100} D_m), \tag{2}$$

where $\Gamma(5)$ is the gamma function, $\alpha_{<100}$ is a parameter and $IWC_{<100}$ is the mass of all crystals smaller than 100 $\mu$m in the observed size distribution. These two values are determined using data measured with the Video Ice Particle Sampler (VIPS), (McFarquhar and Heymsfield 1997). For particle sizes greater than 100 $\mu$m they use a lognormal function to represent the size distributions,

$$N(D_m) = \frac{6IWC_{>100}}{\sqrt{2\pi^3 \rho_i \exp(3\mu_{>100} + 4.5\sigma_{>100}^2)D_m\sigma_{>100}D_0^3}} \exp\left\{ -\frac{1}{2} \left( \frac{\log(D_m/D_0) - \mu_{>100}}{\sigma_{>100}} \right)^2 \right\}, \tag{3}$$

where $IWC_{>100}$ is the mass of all ice crystals with $D_m$ greater than 100 $\mu$m in the observed size distributions, $\sigma_{>100}$ the geometric standard deviation of the distribution, and $\mu_{>100}$ the location of the mode. $D_0 = 1 \mu$m is a parameter used to ensure that the equation does not depend on the choice of units for $D_m$. The coefficients involved in this expression are determined using the data measured by a two-dimensional cloud probe (2DC). The measurements are grouped into six temperature bins. The coefficients of the size distribution function are determined in each temperature group and then expressed as a function of temperature and ice-water content.

Comparison with observation shows that the crystal size distribution for a given temperature and ice-water content can be well reproduced using this parametrization.
(see Fig. 11 in MH97). However, to directly apply this parametrization to NWP or climate models may not be appropriate as it will introduce too much of a computational burden on the models. In practice, the effect of crystal sizes on the radiative properties of the cirrus clouds can be well described by a mean effective particle size (commonly known as the effective radius) which is the mean ratio of particle volume to its geometric cross-sectional (or projected) area. Therefore, we extend the MH97 detailed crystal size parametrization into a parametrization of the effective size to enable it to be used in the atmospheric models.

3. EFFECT OF CRYSTAL HABIT

To calculate effective sizes we need to know crystal habits so as to compute their volumes and projected areas. Aircraft observations have shown that there is a wide variety of crystal shapes in cirrus clouds. The most common particle shapes are bullet rosettes, columns, plates and aggregates (Heymsfield et al. 1990). Mitchell and Arnott (1994) (hereafter referred to as MA94) calculated the effective sizes for several ice-particle shapes as a function of the particle maximum dimension. Their results show that the effective size can vary by a factor of two from bullet rosettes to hexagonal columns. This large variability leads to multiple definitions of the effective sizes for non-spherical particles. McFarquhar and Heymsfield (1998) summarized a number of different definitions of effective sizes for distributions of ice crystals used in radiative parametrization schemes. Two of those related to the present study are introduced here. One is defined by Foot (1988) and later by Francis et al. (1994) as

\[ r_e = \frac{3}{4\rho_i} \frac{IWC}{A_c}, \]  

(4)

where \( IWC \) is ice-water content determined by equivalent-volume spheres, and \( A_c \) the projected area per unit volume of the distribution. The other is defined by Fu (1996) for hexagonal crystal columns as

\[ D_e = \frac{\int W^2 Ln(L) \, dL}{\int (WL + \sqrt{3}/4W^2) n(L) \, dL} \]  

(5)

where \( n(L) \) denotes the ice size distribution, \( L \) is the crystal length and \( W \) is the width. The numerator of Eq. 5 is proportional to the hexagonal crystal volume while the denominator is proportional to its surface area. This definition is slightly different from the ratio of a hexagonal particle volume to its projected area by a factor of \( 2\sqrt{3}/3 \), i.e.

\[ D_e = \frac{2\sqrt{3}}{3} D_r, \]  

(6)

where \( D_r \) represents the ratio of hexagonal volume to its projected area. Fu has shown that \( D_e \) can be determined using \( IWC \) and \( A_c \) by

\[ D_e = \frac{2\sqrt{3}}{3\rho_i} \frac{IWC}{A_c}, \]  

(7)

where \( IWC \) is assumed to be determined by the total volume of hexagons. If observed \( IWC \) and \( A_c \) are both available then the effective sizes can be calculated from Eq. 4 or 7.
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regardless of particle shape. Combining Eqs. 4 and 7, the $D_e$ can be related to $r_e$ by

$$r_e = \frac{3\sqrt{3}}{8} D_e.$$ 

(8)

One of the advantages of Eqs. 4 and 7 is that they both preserve the measured ice-water content and projected area. Fu (1996) has shown that the extinction coefficient and single-scattering albedo are largely dependent on $IWC$ and $A_e$. Therefore, the use of Eqs. 4 or 7 may lead to more accurate estimates of these two parameters.

If the effective sizes are determined using a size distribution function like the MH97 parametrization without information about crystal habit, it is necessary to make an assumption about the crystal shape. Since the MH97 size parametrization was based on a distribution of mass-equivalent spheres, any assumed particle shape must be converted into the equivalent spheres for the calculations of the size distributions. Now we use the MH97 size distribution function to perform a few sensitivity studies to explore the effect of crystal shape and related size distributions on the effective sizes. We calculate the effective sizes for hexagonal columns, plates and bullet rosettes having different numbers of branches. For the hexagonal particles, the effective sizes are calculated using Eq. 5. The aspect ratio of columns and plates is taken from MA94 based on the studies of Auer and Veal (1970). For bullet rosettes, two methods were suggested in MA94. One is to use the geometry of their branches. The other is to use relationships between volume, projected area and maximum dimension of rosettes. These relationships have been presented by Mitchell and Arnott (1994) based on the studies of Arnott et al. (1993). We use the second method to calculate the effective sizes for rosettes having 5 and 11 branches. The calculations are performed for each crystal with different maximum dimensions. The effective sizes for hexagonal columns are also calculated, using the relationship between volume, projected area and maximum dimension as given in MA94, for comparison. The results are shown in Fig. 1. As expected, the results for rosettes with 5 and 11 branches are just reproductions of MA94, but the results for hexagonal columns calculated using Eq. 5 are systematically larger than those from MA94. This is partly due to the definition difference between $D_e$ and $D_t$ as indicated by Eq. 6 and largely due to the fact that Eq. 5 is for regular hexagonal columns whereas the results of MA94 are determined using the relationship between the volume, projected area and maximum dimension of observed columns. The point from this comparison is that the effective sizes for hexagonal columns determined using the method of MA94 may be more realistic but may not be consistent with the associated method of deriving the ice-cloud optical scattering properties for regular hexagonal columns. It is seen that the effective size is sensitive to the particle shapes. The values for regular hexagonal columns are larger than those of other shapes.

We then converted these particle shapes into the equivalent-mass spheres to see the shape effect on size distributions. Figure 2 shows the size distributions as a function of the particle maximum dimension for these crystal shapes. The calculations assume an ice-water content of 0.024 g m$^{-3}$ and a temperature of $-45$ °C. It is seen that the particle shape assumption influences the size distribution if the MH97 parametrization is used. This is because the diameter of an equivalent-mass sphere will be different if a different particle shape is assumed. The diameter of a sphere determined from a hexagonal column is generally larger than that from the other particle shapes, leading to a lower number density for the same maximum dimension.

The integrated effective sizes are calculated using the size distributions shown in Fig. 2 for an $IWC$ range between 0.0003 and 0.3 g m$^{-3}$ with a fixed temperature of $-45$ °C. The results are plotted in Fig. 3. Again, the values determined using the
Figure 1. Effective sizes for single crystals with the shape of a hexagonal column, hexagonal plate and bullet rosette with 5 and 11 branches.

Figure 2. Size distributions determined using the McFarquhar and Heimsfield (1997) parametrization for four crystal types. The calculations assumed a temperature of $-45 \, ^\circ C$ and an ice-water content of 0.024 g m$^{-3}$.

Regular hexagonal columns are larger than those from other shapes by a factor of two for larger ice-water content. It should be noted that there is a slight inconsistency in the above comparisons. The effective size for regular hexagonal columns is calculated using Eq. 5 and is not an exact ratio of particle volume to its projected area. For a consistent comparison the values for the hexagonal columns should be reduced by a factor of $2\sqrt{3}/3$ (see Eq. 6). However, this does not account for the difference shown in the figures. The particle shape, therefore, is an important factor in the determination of the effective sizes for non-spherical particles. Caution must be exercised when calculating the effective sizes by considering the particle shapes as they will have a large impact on cloud optical properties if they are inappropriately defined.
4. COMPARISON WITH OBSERVATIONS

The aim of this study is to simplify the MH97 parametrization so that it can be used in NWP and climate models. It is consistent to calculate the effective sizes using the MH97 parametrization by assuming particle shapes of hexagonal columns as we intend to use the cirrus scattering parametrization developed by Fu et al. (1998) based on detailed scattering calculations for hexagonal columns. Before doing this, we compare the effective sizes determined using the MH97 parametrization with those obtained from observed particle size distributions to explore the behaviour of this parametrization.

Kinne et al. (1997) analysed cirrus microphysical properties measured during the second cirrus field program of the First International Satellite Cloud Climatology Project (ISCCP) Regional Experiment (FIRE-II) and presented effective radius and ice-water content determined using the observed ice-crystal size distributions. The results were derived based on 2DC measurements. They assumed that all ice crystals have the shape of a single hexagonal column, with the exception of complex ice crystals. The complex crystals were treated as a set of many sub-sized hexagonal columns. The scattering properties for complex crystals were not determined by their overall size but by the size of their sub-sized branches. The distributions of hexagonal columns generated in this way were transformed into distributions of equivalent-surface-area spheres characterized by their effective radii. The quantitative comparison of these observations with the results determined using the MH97 parametrization is impossible because of the difference in definition and the lack of temperatures corresponding to observed effective radii. Therefore, this comparison should be regarded as a qualitative comparison. The ice-water content and effective radius listed in Tables 5–9 in the Kinne et al. (1997) paper are plotted in Fig. 3 as asterisks. The result determined using the MH97 parametrization with the transformation from hexagonal columns to equivalent-surface-area spheres is plotted as a long dash-triple-dotted curve. This curve should be more consistent with the observed data because of the way the data were derived. The calculations were performed for a fixed temperature of −45 °C. It can be seen that the
modelled curve for equivalent-surface-area spheres is much closer to that of hexagonal columns than the other curves. This may be used to justify the comparison to some extent. The second feature we can see is that the curves for rosettes and hexagonal plates are all below the observed data points. This indicates that using the assumption of bullet rosettes or plates underestimates the effective sizes defined by Kinne et al. (1997). The third feature is that there is a large variability in the observed effective sizes which cannot be accounted for by ice-water content alone. This variability is partly due to the variety of vertical locations of the measurements (which can probably be accounted for by cloud temperature to some extent (see later)), but is mainly due to the inhomogeneity of clouds as Kinne et al. (1997) described. The curve modelled from hexagonal columns more or less represents an average over these observed data points.

The second set of observed data are taken from Fu (1996) who calculated the effective sizes using Eq. 5 for 28 crystal size distributions collected from several field experiments based in both the mid latitudes and tropics. We still cannot make a direct comparison due to the lack of observed cloud temperatures. Therefore, we use the same plots as for the Kinne et al. (1997) data to show this comparison which is given in Fig. 4. It is seen that the agreement between the modelled results for hexagonal columns and the observations seems to be better than for those shown in Fig. 3. Using bullet rosettes and plates again leads to the underestimation of the effective sizes defined by hexagonal columns. In Fig. 4, the observed results from the mid latitudes and tropics are plotted using different symbols. It seems that there is no systematic difference in effective sizes between these two regions. However, observations do show that the crystal habits in tropical regions differ from those in the mid latitudes. Heymsfield et al. (1990) found that bullet rosettes predominate at each height level in mid-latitude cirrus clouds, whereas McFarquhar and Heymsfield (1996) noted a greater fraction of plates and columns from the CEPEX measurements. Macke et al. (1998) calculated the effective radius defined as a ratio of the third to the second moment of the distribution for 185 samples collected from mid latitudes and 26 from the tropics. The results show that the effective radii in tropical regions, on average, are larger than in the mid latitudes.
Both Figs. 3 and 4 show a large variability in the observations. To examine if this can be explained by variation of cloud temperature, we calculate the effective sizes using the MH97 parametrization for temperatures of $-10$ and $-60 \, ^{\circ}C$ and plot the results in Fig. 5. The calculations assume three crystal shapes. The upper curve for each shape corresponds to a temperature of $-10 \, ^{\circ}C$ and the lower curve to $-60 \, ^{\circ}C$. It is seen that the scatter of the observed effective sizes can be partly accounted for by the temperature variation, but the range cannot be fully explained. The hexagonal columns seem to cover the scatter best. The modelled results also show that the effect of temperature on the effective size varies with crystal shape. The magnitude of change in size due to the temperature change is larger for hexagonal columns than for the other shapes. This reflects the growth rate difference between volume and projected area among these crystal shapes.

5. SIMPLIFIED PARAMETRIZATION

The results explored in the last two sections suggest that using hexagonal columns leads to a better representation of the effective sizes defined by that shape if the MH97 parametrization is used. This is also consistent with the scattering parametrization derived from detailed scattering calculations for hexagonal crystals. Therefore, we perform a simplification of the MH97 parametrization based on the calculations of the effective sizes for hexagonal columns. Now that the ice-crystal size distribution can be determined by cloud temperature and ice-water content this simplification should be rather straightforward. We simply calculate the effective size using the MH97 parametrization for a reasonable number of cloud temperatures and ice-water contents and then fit the results in terms of a simple expression as a function of the temperature and ice-water content. The calculations were performed for temperatures between $-5$ and $-70 \, ^{\circ}C$ and ice-water contents between 0.0001 and 0.3 g m$^{-3}$. To fit the results in terms of a simple expression, we take two steps. First we perform a fit to the temperature variation and then to the ice-water content variation. As can be seen from MH97, the temperature dependence of the crystal size distribution is linear. Therefore, we perform
Figure 6. Hexagonal effective sizes determined using the simplified parametrization compared with that from the McFarquhar and Heymsfield (1997) (MH97) parametrization.

a linear fit of the effective size, $D_e$, versus temperature for a given ice-water content. The equation has the form,

$$D_e = a(IWC) + b(IWC)(T + 190),$$  

(9)

where $T$ is the temperature in Celsius and $D_e$ is in $\mu m$. The coefficients $a(IWC)$ and $b(IWC)$ are further fitted to $IWC$ using the following expressions,

$$a(IWC) = 26.1571 \log_{10}(10^{-12} + IWC)^{-0.5995},$$  

(10)

$$b(IWC) = 0.6402 + 0.1810 \log_{10}(10^{-12} + IWC).$$  

(11)

Here $IWC$ is in $g \, m^{-3}$. The value of $10^{-12}$ is used to avoid a zero in the logarithm function. Figure 6 shows the fitted $D_e$ for temperatures of $-10$ and $-60^\circ C$, where the solid curves represent the results determined using the MH97 detailed parametrization and the dotted curves are from Eqs. 9–11. It is seen that the results determined by the MH97 parametrization can be well reproduced by the simple Eq. 9.

McFarquhar and Heymsfield (1997) have shown a reasonably good representation of the observed crystal size distribution from CEPEX using their parametrization. They also emphasized that this parametrization was determined using measurements from the tropics and should be only used in this region. It would be of great benefit if the scheme could be used in mid latitudes. We have shown in the previous section that the general relationship between the effective size, ice-water content and cloud temperature can be reproduced by the MH97 scheme regardless of location (see Fig. 5). Therefore, it is possible to extend this scheme to mid latitudes. For this purpose we further compare the effective sizes determined by this scheme with observations obtained in the mid latitudes. The results of observations are those presented by Francis et al. (1994) who analysed observational data collected by the C-130 aircraft of the UK Meteorological Research Flight during the International Cirrus Experiment (ICE). The aircraft missions were known as ICE215 and ICE217. The detailed measurements were given in Francis et al. (1994). In that paper Francis et al. calculated the effective radius using Eq. 4. For a consistent comparison, the values of $r_e$ presented in Francis et al. (1994) are converted
into $D_e$ using Eq. 8. We also calculated effective sizes using the observations provided by Heymsfield and Platt (1984). The effects of small ice crystals are considered by extrapolating the size spectra to 10 $\mu$m. Both ice-water content and temperature are available in these two datasets so we are able to make a direct comparison between the modelled results and the observations. Figure 7(a) shows a comparison between the observations and the results determined using Eqs. 9–11. Note that the observed effective sizes near the cloud base for the case of ICE215 are very large due to the horizontal inhomogeneity of the cloud and therefore are not used in this plot. It is seen that there is a general agreement between observations and the parametrization, although more than half of the estimated values are larger than the observations. This is probably due to the difference in crystal habits and sizes between the tropics and mid latitudes. As mentioned previously the columns and plates are the most common crystal shape in the tropics while bullet rosettes are favoured in the mid latitudes. Therefore, we would expect that effective sizes are larger in the tropics than in mid latitudes if they are based on populations of crystal habits. Although the observed results in Fig. 4 do not support this argument, the results from Macke et al. (1998) which are derived from a large amount of observed samples do support it. Considering the large variability of effective sizes shown in Figs. 3 and 4, the results in Fig. 7(a) should be acceptable. However, if the discrepancy is due to a systematic difference between the two regions it is worthwhile correcting this error. Figure 7(b) shows the ratio of the observed values to those estimated as a function of temperature. In spite of a rather scattered distribution the ratio generally increases with temperature, indicating that the effective sizes are overestimated in the low-temperature region. The solid line in Fig. 7(b) shows a least square fit to the scattered points with the equation,

$$ f = 1.2351 + 0.0105T, \quad (12) $$

where $T$ is in Celsius. We adjust Eq. 9 by multiplying it by Eq. 12 and the results are shown in Fig. 7(c). It is seen that the overestimated effective sizes at low temperatures are corrected. We will show below that this adjustment may be necessary as it leads to a better estimation of the radiative properties of cirrus clouds.

6. RADIATION MODELS

The radiation code used to calculate the radiative properties of cirrus clouds is the Edwards and Slingo (1996) scheme. The gaseous absorption by different species is treated using the exponential sum-fitting technique applied to the HITRAN (High Resolution Transmission) molecular absorption database (Rothman et al. 1992). The radiation code is based on two-stream equations in both the short-wave and long-wave regions of the spectrum. The code has been modified at the Bureau of Meteorology Research Centre as described by Sun and Rikus (1999). The modified code employs four spectral bands in the solar spectrum and seven bands in the infrared. The band limits are shown in Table 1. Two parametrization schemes for cirrus optical properties have been implemented in the code. One is the Sun and Shine scheme (Sun and Shine 1995) and the other is the Fu scheme (Fu 1996; Fu et al. 1998). In Fu's scheme the cirrus optical properties are parametrized in terms of the ice-water content and hexagonal effective sizes. In the short-wave region of the spectrum, the geometric ray-tracing program (Takano and Liou 1989) is used to calculate the single-scattering properties of ice crystals with size parameters larger than 200. For the size parameters less than 200, the improved ray-tracing method (Yang and Liou 1995) is used. In the long-wave region of the spectrum, Fu et al. (1998) compared the absorption efficiency
Figure 7. Comparison between hexagonal effective sizes determined by the simplified parametrization and those calculated using the observations. (a) Estimated sizes against observations; (b) ratio of observations to estimates as a function of temperature; (c) adjusted effective sizes against observations.
determined using Mie code, anomalous-diffraction theory, and the geometric-optics method with that determined using the finite-difference time-domain technique (Yang and Liou 1996). They found that for non-spherical particles the results of the absorption efficiency from Mie theory with equivalent spheres may be overestimated while the results from the anomalous-diffraction theory and the geometric-optics method may be underestimated relative to the reference results. Based on these comparison studies they further developed a composite method which combines Mie theory, anomalous-diffraction theory and the geometric ray-tracing program to approach the reference results determined by the finite-difference time-domain technique. In these detailed scattering calculations they assumed that the ice crystals are hexagonal in shape. The results are then parametrized in terms of IWC and $D_e$ as

$$\beta_e = \text{IWC}(a_0 + a_1/D_e),$$  \(13\)
$$1 - \omega = b_0 + b_1 D_e + b_2 D_e^2 + b_3 D_e^3,$$  \(14\)
$$g = c_0 + c_1 D_e + c_2 D_e^2 + c_3 D_e^3,$$  \(15\)

for the short wave and

$$\beta_e = \text{IWC}(p_0 + p_1/D_e + p_2/D_e^2),$$  \(16\)
$$\beta_a = \frac{\text{IWC}}{D_e}(q_0 + q_1 D_e + q_2 D_e^2 + q_3 D_e^3),$$  \(17\)
$$g = r_0 + r_1 D_e + r_2 D_e^2 + r_3 D_e^3,$$  \(18\)

for the long wave. In the above equations, $\beta_e$ and $\beta_a$ represent the extinction and absorption coefficients, $\omega$ is the single-scattering albedo and $g$ is the asymmetry factor. The physical basis of these parametrizations were described by Fu (1996) and Fu et al. (1998). The coefficients involved in Eqs. 13–18 were obtained by numerical fitting to the single-scattering properties computed from the reference light scattering and absorption program.

In order to implement the above scheme into the modified Edwards and Slingo (1996) radiation code, the reference single-scattering properties calculated for the 28 ice-crystal size distributions at high spectral resolution by Fu et al. (1998) are averaged across each spectral band used in the radiation codes (Sun and Rikus 1999) in the same way as described by Fu (1996) and then parametrized in the same forms as above except for the long-wave absorption coefficient which is parametrized in the same form as the extinction coefficient, i.e.

$$\beta_a = \text{IWC}(s_0 + s_1/D_e + s_2/D_e^2).$$  \(19\)

We found that the accuracy of this equation is almost the same as Eq. 17 but the order of the polynomial drops to 2. The coefficients involved in Eqs. 13–19 for each spectral band are listed in Tables 1–3.

7. Validation of Parametrizations

Now we examine the radiation field determined using Fu’s scheme in conjunction with the parametrized effective sizes. This is a test of both the parametrizations of the cirrus optical properties and the effective size since Fu’s schemes have been compared with reference calculations but have not been examined against observations. The observed radiation data used to test the parametrizations are also from ICE. These data
### TABLE 1. VALUES OF THE COEFFICIENTS IN Eqs. 13 AND 16 FOR THE PARAMETRIZATION OF THE EXTINCTION COEFFICIENT (M$^{-1}$)

<table>
<thead>
<tr>
<th>Spectral limits</th>
<th>$a_0$</th>
<th>$a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short wave (µm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2–0.69</td>
<td>-1.30817E-04</td>
<td>2.52883E+00</td>
</tr>
<tr>
<td>0.69–1.19</td>
<td>-6.39479E-05</td>
<td>2.52393E+00</td>
</tr>
<tr>
<td>1.19–2.38</td>
<td>-6.74730E-06</td>
<td>2.52056E+00</td>
</tr>
<tr>
<td>2.38–5.0</td>
<td>1.62674E-04</td>
<td>2.49823E+00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long wave (cm$^{-1}$)</td>
<td>$p_0$</td>
<td>$p_1$</td>
</tr>
<tr>
<td>0–400</td>
<td>3.48509E-03</td>
<td>2.53323E+00</td>
</tr>
<tr>
<td>400–560</td>
<td>-1.29859E-02</td>
<td>4.00652E+00</td>
</tr>
<tr>
<td>560–800</td>
<td>-6.61401E-03</td>
<td>3.29237E+00</td>
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<tr>
<td>800–990</td>
<td>-2.81481E-03</td>
<td>2.88729E+00</td>
</tr>
<tr>
<td>990–1070</td>
<td>-8.76230E-03</td>
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</tr>
<tr>
<td>1070–1200</td>
<td>-8.25118E-03</td>
<td>3.40830E+00</td>
</tr>
<tr>
<td>1200–2200</td>
<td>-4.45625E-03</td>
<td>3.03798E+00</td>
</tr>
</tbody>
</table>

### TABLE 2. VALUES OF THE COEFFICIENTS IN Eqs. 14 FOR THE PARAMETRIZATION OF THE SHORT-WAVE SINGLE-SCATTERING ALBEDO (1 − $\omega$) AND IN Eqs. 19 FOR THE PARAMETRIZATION OF THE LONG-WAVE ABSORPTION COEFFICIENT (M$^{-1}$)

<table>
<thead>
<tr>
<th>Spectral limits</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short wave (µm)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>0.2–0.69</td>
<td>-1.55357E-08</td>
<td>1.95793E-07</td>
<td>-2.31234E-10</td>
<td>1.12247E-12</td>
</tr>
<tr>
<td>0.69–1.19</td>
<td>-7.90657E-07</td>
<td>7.79991E-06</td>
<td>2.90894E-10</td>
<td>-2.02818E-12</td>
</tr>
<tr>
<td>2.38–5.0</td>
<td>2.25112E-01</td>
<td>3.05017E-03</td>
<td>-2.54236E-05</td>
<td>8.49116E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long wave (cm$^{-1}$)</td>
<td>$s_0$</td>
<td>$s_1$</td>
<td>$s_2$</td>
<td></td>
</tr>
<tr>
<td>0–400</td>
<td>2.11300E-03</td>
<td>1.07441E+00</td>
<td>-5.22156E+00</td>
<td></td>
</tr>
<tr>
<td>400–560</td>
<td>3.15325E-03</td>
<td>8.58573E-01</td>
<td>-4.59409E+00</td>
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</tr>
<tr>
<td>560–800</td>
<td>-2.57276E-03</td>
<td>1.48590E+00</td>
<td>-2.79960E+00</td>
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</tr>
<tr>
<td>800–990</td>
<td>-6.18238E-04</td>
<td>1.30122E+00</td>
<td>-2.14834E+00</td>
<td></td>
</tr>
<tr>
<td>990–1070</td>
<td>1.73157E-03</td>
<td>1.08293E+00</td>
<td>-4.94940E+00</td>
<td></td>
</tr>
<tr>
<td>1070–1200</td>
<td>1.12814E-03</td>
<td>1.13189E+00</td>
<td>-4.97006E+00</td>
<td></td>
</tr>
<tr>
<td>1200–2200</td>
<td>4.13595E-03</td>
<td>1.22740E+00</td>
<td>-4.52789E+00</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 3. VALUES OF THE COEFFICIENTS IN Eqs. 15 AND 18 FOR THE PARAMETRIZATION OF THE ASYMMETRY FACTOR

<table>
<thead>
<tr>
<th>Spectral limits</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short wave (µm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2–0.69</td>
<td>7.39781E-01</td>
<td>9.10564E-04</td>
<td>-4.62479E-07</td>
<td>-1.05910E-08</td>
</tr>
<tr>
<td>0.69–1.19</td>
<td>7.52355E-01</td>
<td>1.06211E-03</td>
<td>-2.45770E-06</td>
<td>-3.03712E-09</td>
</tr>
<tr>
<td>1.19–2.38</td>
<td>7.85607E-01</td>
<td>1.73564E-03</td>
<td>-8.92191E-06</td>
<td>1.97757E-08</td>
</tr>
<tr>
<td>2.38–5.0</td>
<td>8.30812E-01</td>
<td>2.62788E-03</td>
<td>-2.43196E-05</td>
<td>8.23543E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long wave (cm$^{-1}$)</td>
<td>$r_0$</td>
<td>$r_1$</td>
<td>$r_2$</td>
<td>$r_3$</td>
</tr>
<tr>
<td>0–400</td>
<td>6.05386E-01</td>
<td>8.96396E-03</td>
<td>-9.74635E-05</td>
<td>3.65031E-07</td>
</tr>
<tr>
<td>400–560</td>
<td>7.22700E-01</td>
<td>3.06925E-03</td>
<td>-1.80401E-05</td>
<td>4.37845E-08</td>
</tr>
<tr>
<td>560–800</td>
<td>7.74964E-01</td>
<td>4.19530E-03</td>
<td>-4.28989E-05</td>
<td>1.53761E-07</td>
</tr>
<tr>
<td>800–990</td>
<td>8.73987E-01</td>
<td>2.56756E-03</td>
<td>-2.86045E-05</td>
<td>1.08210E-07</td>
</tr>
<tr>
<td>990–1070</td>
<td>8.91347E-01</td>
<td>1.80543E-03</td>
<td>-1.58497E-05</td>
<td>5.21892E-08</td>
</tr>
<tr>
<td>1070–1200</td>
<td>8.61983E-01</td>
<td>2.15823E-03</td>
<td>-1.68998E-05</td>
<td>4.93175E-08</td>
</tr>
<tr>
<td>1200–2200</td>
<td>8.56094E-01</td>
<td>2.49487E-03</td>
<td>-2.18124E-05</td>
<td>6.95232E-08</td>
</tr>
</tbody>
</table>
have been used by Francis et al. (1994) and Sun and Shine (1995) to verify their cirrus radiative parametrizations. Both short-wave and long-wave observations from flights ICE215 and ICE217 are used here to test the new parametrizations. As pointed out by Francis et al. (1994) during the measurements of ICE215 and ICE217 there were low-level boundary-water clouds present but no in-situ observations were made in these water-cloud layers. This problem is treated in the same way as described in Francis et al. (1994), i.e. we constrain the upwelling-flux measurements to the mean values measured in the lowest level by varying the amount of water cloud in the models until agreement is reached.
In the model simulations three calculations are performed. The first uses Fu’s parametrization with the observed effective sizes. The second uses Fu’s parametrization with the parametrized effective sizes determined by Eq. 9. The third calculation uses the adjusted effective sizes.

The short-wave comparisons are shown in Fig. 8. The observations from aircraft are plotted as symbols, with the horizontal bars representing the observation error. The agreement between the two calculations is reasonable. It is seen that Fu’s scheme with the observed effective sizes fits the observations reasonably well. Using the parametrized effective sizes leads to a slight overestimate of the downward irradiance for the ICE217 case and this is due to the overestimate of effective sizes at the cloud top. After adjusting
the effective sizes in accord with Eq. 12 this discrepancy is ameliorated. We also see that the comparison errors around 500 hPa for the case of ICE215 are relatively large and this is due to the horizontal inhomogeneity of the cloud as emphasized by Francis et al. (1994).

The comparisons for the long-wave results are shown in Fig. 9. For the cloud of ICE215 (Fig. 9(a)) the downwelling irradiances calculated using Fu’s scheme with three specifications of effective sizes are all less than the observations near the cloud top and larger than the observations at the cloud base. However, the upwelling irradiances are well modelled. Agreement between the modelled results is reasonably good.

The ICE217 comparison shows that the downwelling irradiances determined using the observed effective sizes are overestimated near cloud top whereas the upwelling irradiances are slightly underestimated. The modelled results using the adjusted effective sizes are much closer to the observations.

These comparisons indicate that the accuracy of the Fu parametrization is reasonably good. The application of the parametrized effective sizes to Fu’s scheme also generates relatively good radiative properties within cirrus clouds. This result is more important as it may suggest that the simplified parametrization for particle sizes might be used in climate models to improve the representation of the radiative properties of cirrus clouds. Since comparisons were only performed for two separate datasets more observational data are needed to further verify these parametrizations.

Note that the observed effective sizes from Francis (1995) are converted from \( r_e \) using Eq. 8. The successful comparisons shown above imply that the simplified parametrization may also be applicable to the ice-cloud optical parametrization based on the effective radius defined by Eq. 4 with the aid of Eq. 8 even though it was derived for hexagonal columns.

8. DISCUSSIONS AND CONCLUSIONS

The sensitivity of the effective size to ice-crystal habits is investigated using an analytical size distribution function (the MH97 parametrization). The results show that the effective sizes are very sensitive to crystal shape and must be determined in accordance with the associated ice-crystal optical properties.

The parametrization of hexagonal effective sizes is developed based on the studies of McFarquhar and Heymsfield (1997) and compared with some observations. The results show that this parametrization may be applicable to both the mid latitudes and the tropics although the data used to develop the parametrization were collected over the tropics only. However, some correction may be necessary in the mid latitudes to improve the results.

The purpose of developing such a parametrization is to apply it to the new radiative parametrization for cirrus clouds developed by Fu (1996) and Fu et al. (1998) which depends on the ice-water content and the hexagonal effective sizes. This radiative parametrization has a sound physical basis and an excellent accuracy relative to the reference results and, therefore, could be used in NWP and climate models. However, before doing so it is worthwhile further testing this scheme against observation, as has been done in this study. We have found that the effective sizes input to Fu’s scheme must be consistent with the original definition. Our results show that Fu’s parametrization can generate reasonably good radiative properties for cirrus compared with the aircraft observations. The important result from this study is that the application of the parametrized effective sizes to Fu’s scheme is very successful. There is no significant difference between the modelled irradiances determined using the observed
effective sizes and the parametrized sizes. Using the adjusted parametrization leads to improved results. The effective size is not a quantity predicted by NWP and climate models. To apply the radiative parametrization scheme that depends on such a quantity, one has to use an assumed value of the effective sizes for ice clouds. We have seen from the observations that the effective sizes of cirrus clouds can vary over a very wide range. The use of any assumed constant value will introduce large uncertainty into the calculations.

The parametrizations for both effective sizes and optical properties presented in this paper are confined to regular hexagonal columns. In real cirrus clouds, almost all particles are irregular in shape and more complicated than hexagonal columns. Although the extinction coefficient and single-scattering albedo are largely determined by the effective sizes as demonstrated by Fu (1996) the asymmetry factor is more sensitive to the habit of ice crystals. The sensitivity studies performed in the present work show that the effective size is also highly dependent on the habit of ice crystals. Therefore, it is desirable to develop the parametrization of optical properties for different ice-crystal habits. The overall effect from a mixture of different crystal habits can be determined by a weighted average of the individual optical properties. The scattering program for irregular particles has been developed (Macke et al. 1998) and research work in this direction is under way (Fu, personal communication).

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