Multiple potential-vorticity inversions in two FASTEX cyclones

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SUMMARY

This paper investigates pre-existing synoptic-scale disturbances responsible for cyclogenesis, using the manipulation of initial conditions through quasi-geostrophic potential vorticity (QGPV) and its inversion. The use of a QGPV inversion together with a T63 model allows us to study the sensitivity of some FASTEX events to several QGPV patterns present within the analysis. It has been found that Ertel potential-vorticity modifications of the analysis are preserved by the dynamical initialization process. The QGPV inversion method is applied to cyclones occurring in Intensive Observation Periods (IOPs) 11 and 12 of FASTEX (Fronts and Atlantic Storm-Track EXperiment) to demonstrate which potential-vorticity anomalies, present within the atmosphere prior to the development, are involved in the cyclogenesis. The choice of the boundary condition when an upper-level precursor is removed from the initial conditions has little impact on the cyclone development. The linear interactions (advection/deformation of the anomaly by its environment and downstream development) between these upper-level precursors and the surrounding flow dominate the dynamics of IOP11 cyclone development, whereas nonlinear processes are particularly strong in the IOP12 case. The cyclone development is only weakly sensitive to low-level initial structures unless they are shaped like adjacent model sensitivities; collocation with another shape has little influence on the development.

KEYWORDS: Cyclogenesis Potential vorticity

1. INTRODUCTION

Weather variability in mid latitudes is dominated by cyclones and anticyclones. Since the early work of Kleinschmidt (1950a,b) and the landmark paper of Hoskins et al. (1985), an atmospheric system can be interpreted as a superposition of several potential-vorticity (PV) and surface potential-temperature anomalies—surface sheets of PV (Hoskins et al. 1985). Petterssen et al. (1955) emphasize the importance of such pre-existing features for cyclone development. Sutcliffe (1947) suggests the baroclinic interaction as a possible explanation for cyclones triggered by upper-level troughs—or PV anomalies. One of the main objectives of the Fronts and Atlantic Storm-Track EXperiment (FASTEX; Joly et al. 1997, 1999) was to pick up such events in the North Atlantic wintertime using dropsondes, in order to improve our understanding of mid-latitude cyclones triggered by pre-existing finite-scale features. One way to take advantage of such well-documented real cases in this perspective is to use PV and its inversion.

The invertibility principle states that the full state of the atmosphere can be recovered for a given equilibrium, knowing the PV field and a boundary condition. Three kinds of PV-inversion frameworks have recently been used. The quasi-geostrophic (QG) PV (QGPV) inversion (Robinson 1988; Holopainen and Kaurula 1991; Black and Dole 1993; Hakim et al. 1996; Fehlmann and Davies 1997; Arbogast and Joly 1998a) is the simplest. Its linearity makes the solution of the piecewise inversion unique. However, the piecewise inversion using the Charney (1955) balance equation, first proposed by Davis and Emanuel (1991) and Davis (1992), and successfully used by others up to now (Huo et al. 1998; Demirtas et al. 1999), is more appropriate to flows with Rossby numbers close to unity. Other nonlinear inversions are also designed in Birkett and Thorpe (1997), Thorpe (1997), and Arbogast and Joly (1998b).

In this paper, we study the dynamics of two FASTEX cases following a strategy based on predictions from different initial conditions. QGPV inversion is used to define

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several initial conditions by local PV modification. The concept of local modification to initial conditions has been proposed by Hollingsworth et al. (1982). They performed experiments where they transplanted one analysis into another one, inside a latitude–longitude rectangle, and showed large differences in the forecasts associated with these localized differences in the analyses. This technique, however, requires a multivariate transplant appropriately balanced. Since Demirtas et al. (1999), Fehlmann and Davies (1997), Huo et al. (1998) and Arbogast and Joly (1998a), PV inversion is now commonly used to modify initial conditions in a subjective way without the spurious signal shown in Hollingsworth et al. (1982). Unlike the first three papers, our goal is not to improve the initial conditions to provide a better forecast, but rather to investigate the sensitivity of some FASTEX cyclone life cycles to finite-amplitude PV features, in order to shed some light on tropopause and low-level precursors if they exist.

In a first part, we focus on both the design of a QGPV inversion method and the weaknesses and advantages of the latter methodology for studying real cases. In the second part two FASTEX cyclones are discussed: a case of explosive deepening in Intensive Observing Period (IOP) 12; and a case with one rapid development phase, IOP11. A main goal of the paper is to investigate the linearity of the response of the model to PV perturbations. Model forecasts starting with various PV distributions provided by the QGPV inversion framework are used to highlight low-level and tropopause-based precursors. Sensitivity of the cyclone development to low-level and upper-level perturbations of similar amplitude are also investigated and compared with the sensitivity to adjoint model outputs.

2. ON THE USE OF QGPV INVERSION FOR THE STUDY OF REAL CASES

In this section, the design of a QGPV (Hoskins et al. 1985; Pedlosky 1987) inversion method will be outlined. Another goal of the section is to demonstrate the ability of QGPV inversion to modify the initial conditions in a primitive-equation model. The strategy for implementing the PV ‘surgery’ is as follows:

- all ‘coarse grains’ of the upper-level PV field are identified;
- for each PV element, the ‘balanced’ part of the flow is deduced using a new, variational implementation of PV inversion (Arbogast and Joly 1998a,b);
- the balanced part of the flow corresponding to the PV element of interest is removed from the initial conditions of the ARPEGE* model;
- a prediction with the revised fields is then performed; the model does not require the dynamical initialization step.

(a) The PV-inversion framework

Our purpose is to find an invertibility principle which applies at low Rossby numbers on the sphere, and which is not too computationally demanding. One way could be to derive the Lagrangian invariant from balanced dynamics relevant for the sphere. Since the QG system on the sphere used by Simmons and Hoskins (1976) does not possess any obvious Lagrangian invariant, we choose the very common form of QGPV on the $\beta$-plane. The balance equation is the zero-order truncation of the ARPEGE divergence equation whereas no simplification of the Coriolis parameter is assumed. Unlike QGPV, a balance equation relevant to the sphere exists. Then, the set of partial differential equations corresponding to the invertibility principle, compatible with the horizontal and vertical representation of variables in the ARPEGE/IFS model (see Courtier et al.

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(1991) for more details), takes the following form:

\[ QGPV: q_g - f = \nabla^2 \Psi + f_0 \frac{\partial}{\partial p} \left( \frac{\Theta}{\partial \Theta_r/\partial p} \right), \]

balance equation: \( \nabla^2 (\Phi + RT_f \ln p_s) - \nabla \cdot f \nabla \Psi = 0, \)

hydrostatic relation: \( \frac{\partial \Phi}{\partial p} = \frac{R}{p} \left( \frac{p}{p_0} \right)^{R/C_p} \Theta, \)

where \( q_g \) is the QG pseudo-PV anomaly. The unknowns of (1) are \( \Psi \) the stream function anomaly, \( \Theta \) the potential-temperature anomaly, \( \Phi \) the geopotential anomaly and \( p_s \) the surface pressure. \( p = p_0 a(s) + b(s) p_s \) is the pressure, where \( s \) stands for the vertical coordinate of the model; \( f \) is the Coriolis parameter with \( f_0 \) its value taken at a reference latitude; \( \partial \Theta_r/\partial p \) is a static stability profile relevant to the mid latitudes; \( T_r = 283 \text{ K}, \)

where \( R = 1000 \text{ hPa}, R \) is the gas constant and \( C_p \) the specific heat at constant pressure. The term \( RT_f \ln p_s \) is due to the use of a generalized vertical coordinate.

The QGPV and the balance equations form a set of two second-order partial-derivative equations with \( \Phi \) and \( \Psi \) as unknowns. Thus, this problem is solved as a boundary-value problem. Therefore, to solve the system, a surface pressure anomaly must be known and an appropriate boundary condition—either \( \Theta \) or \( \Psi \) at the top and the bottom of the model—has to be defined:

- The Eulerian evolution of the surface pressure is due to its advection and to the vertical integral of the divergence. Using the scaling that is already used to derive (1) the integral of the divergence is found to be negligible (Simmons and Hoskins 1976). Therefore \( \partial p_s/\partial t = 0 \) is the pressure equation consistent with the QG assumption, and the balance state of the atmosphere can be expressed in terms of QGPV, a boundary condition and surface pressure. All the results presented in this paper are performed with a zero surface pressure anomaly.

- The main reason for using potential temperature as boundary conditions is related to the fact that this variable is conserved following an air parcel under the adiabatic assumption. One way to compute the flow associated with a tropopause disturbance is to invert the QGPV anomaly near the tropopause and the anomaly in surface potential temperature. However, in the case of a surface low with a tropopause anomaly just above, one cannot know which part of the surface potential-temperature pattern belongs to the tropopause anomaly and which part belongs to the low. For this reason, we arbitrarily assume that tropopause-based QGPV anomalies do not exhibit any potential-temperature signature at the surface and at the top of the model (where \( p = 0 \)). Although the most commonly used boundary condition is potential temperature, this choice is arbitrary and turns out to be a weak point of the method. The impact in the forecast of the choice between vorticity and temperature boundary conditions will be addressed in the next section. No lateral boundary conditions are involved in this boundary value problem since it is solved on the sphere. But an additional constraint arises from the global integration of the first equation in (1). The global integral of \( q_g - f \) is equal to the surface integral of \( \Theta \) divided by the reference static stability. One common way to ensure this new property of the QGPV inversion solution is to assume the mean surface potential-temperature anomaly and the mean QGPV anomaly to be zero.

Equation (1) is solved using the variational method introduced by Arbogast and Joly (1998). Let \( \mathbf{P} \) be the vector of QGPV in spectral space corresponding to the right-hand side (r.h.s.) of the first equation in (1), and \( \mathbf{B} \) the vector of departure from balance obtained by applying the r.h.s. of the second equation in (1). Both are functions of the
nondivergent variable of ARPEGE $x$, namely ($\Psi$, $\Theta$). The solution of (1) is also the minimum of the function:

$$J = \frac{1}{2} (P - \delta q_g)^T (P - \delta q_g) + \frac{1}{2} B^T B,$$

(2)

where $\delta q_g$ is a known QGPV field and index $T$ stands for transposition. The minimum of $J$ is reached using a quasi-Newton-based method (Gilbert and Lemarechal 1989). The development required consists of writing the definition of PV and the balance equation in operator form, together with their adjoint.

(b) About the convergence of the method

On one hand, the variational method outlined above enables the use of different balance equations, depending on the scale and balance assumption considered; on the other, the weak point of the method is related to the convergence of the descent algorithm. The behaviour of the algorithm is displayed in Fig. 1. Let $g$ be the norm of the gradient of $J$ with respect to $x$:

$$g = \left\| \frac{\partial J}{\partial x} \right\|.$$

$g$ reduces to 1% of the initial value after about 25 iterations. Unfortunately, no significant decrease of $g$ occurs after that step (see the flat part of the curve in Fig. 1 between iterations 30 and 100). In order to improve the accuracy of the solution, the initial guess for the descent algorithm is chosen carefully. It is, in fact, the solution of the QGPV inversion with $f$ and $p_s$ taken to be constant in space; (1) leads to a single equation for the geopotential:

$$q_g - f = \nabla^2 \Phi + f_0 \frac{\partial}{\partial p} \left\{ \frac{1}{\sigma(s)} \frac{\partial \Phi}{\partial p} \right\},$$

(3)

where, $\sigma(s)$ depends only on the vertical. The geopotential may also be expressed in spectral space:

$$\Phi(\lambda, \mu) = \sum_{m,n} \tilde{\Phi}(m, n) P^m_n(\mu) e^{im\lambda}.$$
and

\[ q_g(\lambda, \mu) = \sum_{m,n} Q_n^m P_n^m(\mu) e^{im\lambda} \]

where the \( P_n^m \) are Legendre functions, \( \lambda \) is the longitude and \( \mu \) the sine of the latitude. Equation (3) can be rewritten for each spectral coefficient of the T63 truncation:

\[ Q_n^m = -m^2 \tilde{q}_n^m + f_0 \frac{\partial}{\partial p} \left\{ \frac{1}{\sigma(s)} \frac{\partial \tilde{q}_n^m}{\partial p} \right\} . \tag{4} \]

The discrete form of the r.h.s. of (4) is a tri-diagonal set of equations, which can be directly solved by a factorization into upper- and lower-triangular matrices. This strategy has already been adopted by Holopainen and Kaurola (1991) and Nielsen-Gammon and Lefevre (1996) for QGPV inversion in spherical geometry.

When a guess solution of the QGPV inversion is obtained using this algorithm, horizontal variations of \( f \) as well as \( p_s \) are taken into account by the variational algorithm. The solution obtained in this way is closer to the exact minimum of \( J \) than the solution obtained without an initial guess for the minimization (see Fig. 1).

(c) Definition of anomalies and horizontal-scale separation

For many reasons, it is not useful to deal with the state variable of the model at full vertical and horizontal resolution. One reason is that the QC assumption is intrinsically only relevant to large-scale flows. A second reason is related to the fact that only 'coarse-grain' QGPV anomalies, i.e. those whose size exceeds the Rossby radius (see Hoskins et al. 1985), are key features for mid-latitude cyclogenesis. For these reasons we choose a T63 spectral resolution and 19 vertical levels. The corresponding horizontal resolution in physical space (a few hundred km) may be compared with the density of upper-air measurements used in the global model analyses.

We also assume that 'coarse-grain' QGPV anomalies and the 'planetary-scale' signal—defined by the wave numbers 0 to 7—are independent. Let us now show the anomaly organization in the case of an anomaly embedded within a strong westerly jet with Rossby number close to unity. The anomaly of interest is located near 50°N, 40°W on the cyclonic-shear edge of a strong jet (see Fig. 2(a) and (b)). The planetary-scale part of the flow, unaltered by the inversion, is shown in Fig. 2(c). The dipole of quite weak amplitude over the Atlantic may be directly related to the presence of the zonal regime. Hereafter, QGPV will stand for a band-pass—wave numbers 8 to 63—QGPV. Figure 2(a) and (b) reveals a strong correlation between 300 hPa QGPV and the geopotential of the 2 PVU* surface. In particular, the coarse-grain tropopause disturbance located near 50°N, 40°W also exhibits a strong signature in Fig. 2(b). Another representation of this signature is given by the cross-section in Fig. 3(a) and (b) where the tropopause jump appears as a QGPV dipole.

Davis (1992), among others, suggests that time filtering is well suited to the definition of anomalies. In this paper we adopt the strategy based upon a horizontal scale-separation as in Nielsen-Gammon and Lefevre (1996). A diagram of the different steps required to define a coarse-grain QGPV anomaly is shown in Fig. 3(b); an approximate anomaly position (the vertical axis) as well as a maximum size must be prescribed. The QGPV anomaly field is set to zero outside the hatched area in Fig. 3(b) as well as when it is lower than a prescribed threshold (heavy line in Fig. 3(b)). The presence of closed contours of QGPV gives rise to a straightforward use of this simple algorithm.

* 1 PVU = 10^{-6} \text{ m}^2 \text{ s}^{-1} \text{ K kg}^{-1}.
Figure 2. ARPEGE analysis for 0600 UTC 9 February: (a) relative vorticity and geopotential (every 500 m) of the 2 PVU surface; (b) 300 hPa wind (>60 m s⁻¹) and 300 hPa QGPV for wave numbers 8 to 63 every 0.1 QGPVU (the zero isoline is not shown, dashed lines are for negative values; 1 QGPVU = 2 × 10⁻⁴ s⁻¹); (c) 300 hPa 'planetary' QGPV every 0.05 QGPVU; (d) 300 hPa QGPV anomaly (every 0.05 QGPVU). See text for further details.

(d) Advantages and weaknesses of the method

The method outlined above is supposed to isolate the contribution of each QGPV element. Now our purpose is to show how a coherent QGPV pattern modification alters the Ertel PV. We also check that this modification is maintained by the dynamical initialization of ARPEGE.

(i) Ertel PV changes. At this stage, the input of the QGPV inversion is the anomaly defined along the lines of the last paragraph and displayed in Fig. 3(c). The vorticity and potential-temperature anomalies provided by the inversion of the QGPV anomaly are then removed from the analysis. Again, the Ertel PV is calculated and displayed in Fig. 3(d). The remaining smooth tropopause jump is related to the strong mean jet stream (perpendicular to the cross-section) but no anomaly is present on the figure. The difference between Fig. 3(a) and (c) is plotted in Fig. 4. The only undesirable effect of the inversion is the positive Ertel PV appearing in an area where the QGPV anomaly was supposed to be zero (see Fig. 4 near 1000 hPa below the anomaly). This behaviour might be due to the fact that the term which operates on Θ in QGPV is only dependent on the reference state stability, unlike in the semi-geostrophic (Thorpe 1997) and nonlinear-balance-based inversion framework (Davis and Emanuel 1991; Davis 1992; Raymond 1992).

(ii) The implicit balance of primitive equations versus geostrophic balance. The solution of QGPV inversion shown here does not fulfil the implicit balance of a
Figure 3. Cross-sections following the bold line shown in Fig. 2(b). (a) Ertel potential vorticity (PV) in the analysis every 0.5 PVU. (b) Truncated QGPV from wave numbers 8 to 63, every 0.1 QGPVU (1 QGPVU = $2 \times 10^{-4}$ s$^{-1}$) and sketch of the anomaly definition. The approximate location of the anomaly centre is displayed by the vertical line, its size by the hatched area and finally a threshold (0 in this case) by the bold isoline. The QGPV anomaly is set to zero at the surface and just above (below 850 hPa). (c) QGPV anomaly every 0.1 QGPVU defined following (b). (d) Ertel PV of the analysis without the part of the flow associated with the anomaly (and obtained by quasi-geostrophic inversion). See text for further details.

Figure 4. Ertel potential vorticity anomaly, the difference between Fig. 3(a) and (c).

primitive-equation model that is provided by the nonlinear normal-mode initialization. If the implicit balance of the model and the balance of the inversion are too different, coherent changes in the initial conditions might suffer from geostrophic adjustment and the emission of spurious gravity waves. In order to assess whether that is so, the model state without the anomaly, displayed in Fig. 3(d), is initialized. The modification to the solution added by the initialization process does not modify Ertel PV and relative vorticity (Fig. 5) in a significant way. The modification concerning both fields is about 1%. The effects of the initialization process are larger above the tropopause, where the Rossby-order terms in the divergence equation might be of the same order, and near the
ground. This behaviour might be due to the difference between the static-stability profile involved in the QGPV form and the actual local static stability.

The conclusion of this section is that QGPV anomaly definition and inversion are considered coherent and useful for modifying initial conditions of a primitive-equation model.

3. PRESENTATION OF TWO FASTEX CASES

In each case a summary of the synoptic-scale evolution is provided by maps of the dynamic tropopause defined by the 2 PVU surface to show the 'topography' of the tropopause. Vorticity and pseudo wet-bulb-potential temperature at 850 hPa are also displayed.

(a) Intensive Observation Period 11

The presentation of the IOP11 case is based on a 72 hour ARPEGE forecast starting from the analysis at 1200 UTC 3 February 1997 (see Fig. 6), covering the triggering of the cyclone and the main part of the development. The analysis indicates an upper-level short wave appearing as a minimum at the tropopause level. The cyclogenesis begins as the upper-level feature crosses the baroclinic region by 40°N, 60°W between 1200 UTC 3 February and 1200 UTC 5 February. While the cyclone moves northeastward, the vorticity maximum reaches $1.7 \times 10^{-4}$ s$^{-1}$ between 1200 UTC 4 February and 1200 UTC 6 February. This is consistent with the conceptual picture that an upper-level feature can initiate, in the presence of a surface front, the development of a new cyclone. One can note a shift of a few hundred kilometres between the location of the low in the forecast and in the analysis at 1200 UTC 6 February. This behaviour of the low-resolution model is satisfactory, and the forecast is taken as a reference for the following sensitivity study.
Figure 6. FASTEX IOP11: (a) The ARPEGE analysis at 1200 UTC 3 February 1997, showing: 850 hPa relative vorticity (shading interval: $2.5 \times 10^{-5}$ s$^{-1}$), wet-bulb potential temperature (thin lines, every 4 K); and geopotential (heavy lines, every 1000 m) of the 2 PVU surface. S locates the low-level precursor at the initial time and afterwards the low; T locates the upper-level precursor. (b) As (a) but for the T63 48 h forecast based on the ARPEGE analysis at 1200 UTC 3 February 1997. (c) As (b) but for the T63 72 h forecast. (d) As (a) but for the operational analysis (T149) at 1200 UTC 6 February 1997. See text for further details.

(b) Intensive Observation Period 12

For IOP12, a 36 h forecast based on the ARPEGE analysis of 1200 UTC 8 February is considered (see Fig. 7). At the initial time, a deep PV anomaly is located over the St Laurent Bay. Between 1200 UTC 8 February 1997 and 0000 UTC 9 February 1997, a warm 850 hPa tongue and a weak tropopause short wave, just above, move eastward while the amplitude of the warm tongue increases significantly. One possible explanation for this is the thermal advection by the flow at a distance attributable to a weak upper-level short wave. Between 0000 UTC and 1200 UTC, a baroclinic interaction between the warm tongue and the leading tropopause disturbance takes place. A vorticity maximum appears at 1200 UTC near 50°N, 30°W. This incipient stage is followed by a strong baroclinic deepening phase. Again, the low in the coarse-resolution model is close to the actual low given by the analysis.
4. Sensitivity to initial conditions using QGPV inversion

The aim of this section is to point out the precursors of both the IOP11 and the IOP12 cyclones. Particular attention will be paid to the behaviour of the precursors in the model, and to interactions between them and the surrounding flow. The way this is achieved is by performing ARPEGE forecasts with and without some synoptic-scale patterns present in the analysis.

(a) Sensitivity of the IOP11 cyclone to upper-level features

(i) Upper-level precursor depiction. Experiment (exp.) 11A (see Fig. 8) is a forecast carried out without the tropopause maximum at 1200 UTC 3 February marked on Fig. 6. The 72 h forecast does not exhibit further cyclogenesis; only a weak frontal wave crosses Ireland. This experiment confirms the crucial role of this upper-level precursor which we suspected.

(ii) Linear versus nonlinear aspects of the interaction of a tropopause disturbance with the ambient flow. In order to study the genesis of moving troughs, Nielsen-Gammon and Lefevre (1996) use an extension of PV inversion; instead of inverting PV anomalies, they invert PV-advection terms associated with different dynamical processes. The state of the atmosphere is partitioned into a perturbation attributable to an upper-level PV anomaly \( q_u' \), and the environmental flow ascribable to its PV distribution \( \bar{q}_u \). Thus, the
total PV is

\[ q = \bar{q}_u + q_u'. \]  

(5)

The authors paid particular attention to the advection of the environmental QGPV by the flow attributable to the QGPV anomaly. This linear term is the so-called downstream-development term (Simmons and Hoskins 1979; Orlanski and Sheldon 1995). Nielsen-Gammon and Lefèvre (1996) compute the flow attributable to a QGPV anomaly by inversion, and are able to compute each advection term of the tendency equation for \( q_u' \). Our approach is quite different. It is based on the comparison between a forecast with a QGPV anomaly within the initial conditions and a forecast with the same anomaly but with opposite signs (see Fig. 9). Let us consider a particular QGPV anomaly present in the analysis of 1200 UTC 3 February.

Figure 9 shows exp. ANA, which is the forecast based on the analysis at 1200 UTC 3 February 1997. The cyclone development is represented by the model in this experiment. Exp. 11A is the forecast without the upper-level precursor and without the cyclone development. A third initial condition is established by the removal of the same QGPV anomaly from the initial state of exp. 11A; the forecast based on these new initial conditions is called exp. 11B. Thus, we are able to compare the behaviour of the QGPV both added to, and subtracted from exp. 11A. The behaviour of the anomaly and its opposite will be close to each other in the case of QG linear dynamics (Fig. 9(a)), since cyclonic and anticyclonic development are symmetric in the QG model (Rotunno et al. 1994). If large differences appear in some areas between exps. ANA–11A and 11A–11B, patterns present in those areas will be related to non-QG linear dynamics (Fig. 9(b)).

Although the QGPV of exps. ANA–11A and 11A–11B are exactly the same at the initial time, some differences appear in the Ertel PV field (Fig. 10). These differences are mainly due to nonlinearities involved in the Ertel formulation of PV. Figure 11 shows the difference between exps. ANA and 11A (upper panel) and the difference between exps. 11A and 11B (lower panel) after 72 hours. The QGPV anomaly itself (labelled U1 in Fig. 11) and the pattern due to downstream development (U2) are present in both panels, with comparable shapes. Therefore, linear QG dynamics seems to explain the displacement of the anomaly, whereas the evolution through downstream development gives rise to an anomaly of the opposite sign just ahead. However, some patterns are
Figure 9. A schematic representation of the three experiments (ANA, 11A and 11B) used to compare linear and nonlinear interactions between the anomaly $q'_U$ and the environment. The state of the model is represented by the amplitude of $q'_U$. Case (a) interactions are dominated by linear processes; case (b) interactions are dominated by nonlinear processes. See text for discussion.

Figure 10. FASTEX IOP11: Two different initial states at 1200 UTC 3 February 1997. (a) Experiment ANA–11A; (b) experiment 11A–11B. Lines are at 2 PVU, geopotential difference, interval 500 m; negative values are dashed. See text for further details.

present in one panel and not in the other; L and U3, which are directly related to the FASTEX low, are not present in the lower panel. The development of the IOP11 cyclone is, therefore, associated with non-QG linear dynamics. U3 is a direct consequence of the low, in the sense that this negative pattern may be associated with an outflow driven by latent-heat release which is particularly strong within the ascending branch of the ageostrophic motion associated with the low (Cammas et al. 1999).

(b) Sensitivity of the IOP12 cyclone to upper-level features

First, precursors of the IOP12 cyclogenesis will be identified. Then, the impact of the boundary condition of QGPV inversion will be addressed. Again, particular attention is paid to the behaviour of the precursors in the model and to interactions between them and the surrounding flow. Finally, the sensitivity to QGPV and surface potential-temperature changes are compared with the sensitivity to changes that follow objective sensitivity fields.
Figure 11. FASTEX IOP11: 72 h forecast based on two different initial states (see Fig. 10) at 1200 UTC 3 February 1997. (a) Experiment ANA—11A; (b) experiment 11A—11B. Bold lines stand for 2 PVU geopotential difference (interval 500 m). Thin solid lines stand for relative vorticity difference (interval $5 \times 10^{-8} \, s^{-1}$). Negative differences of vorticity are shaded. See text for further details.
(i) **Upper-level precursor depiction.** In exp. 12A (see Fig. 12), the initial state is the analysis for 1200 UTC 8 February without the tropopause anomaly near 50°N, 65°W defined by its QGPV and with no surface potential-temperature anomaly. Using these new initial states 36 h forecasts do not show the low any more, but only a weak frontal wave below the jet stream with a vorticity maximum of $8 \times 10^{-5}$ s$^{-1}$. This experiment demonstrates that the upper-level pattern is the leading precursor of the cyclone development.

(ii) **Impact of the boundary condition at the surface.** As pointed out earlier, the choice of boundary condition is arbitrary. In this subsection, we investigate the sensitivity of the deepening of the IOP12 low to the flow attributable to the upper-level pattern defined by its QGPV, and by either homogeneous Dirichlet boundary conditions (no surface vorticity anomaly) or homogeneous Neumann boundary conditions (no surface temperature anomaly). In exp. 12B, the initial state is the analysis with the same QGPV anomaly removed and no vorticity anomaly at the surface. By looking at the difference between both initial states (Fig. 13, left panel) we observe a large difference in 850 hPa potential temperature (about 5 K) due to the type of boundary condition used. Whereas the tropopause disturbance moves downstream and triggers the low, the lower part of the modification moves slowly to the south-east and remains far upstream with respect to the IOP12 cyclone—see the broad pattern near 40°N, 40°W on Fig. 13(b). Actually, the most significant difference in vorticity between the two 36 h forecasts mainly concerns the area where the difference in low-level potential temperature is a maximum. As regards the IOP12 cyclone, the two 36 h forecasts are comparable: they do not show any significant differences in the region previously affected by the cyclone. The unimportance of the boundary condition when the question of the sensitivity of a cyclone development to QGPV patterns is addressed, is a promising conclusion which has to be confirmed with other case-studies.

(iii) **Linear versus nonlinear aspects of the interaction of a tropopause disturbance with the ambient flow.** Results obtained here are presented in the same manner as the results for IOP11 (see Fig. 14). The initial state of exp. 12A is the operational analysis (exp. ANA) truncated to T63 without the upper-level precursor of the cyclogenesis (see Fig. 14, upper panels). The initial state of exp. 12C is the initial state of exp. 12A without
that same QGPV anomaly. Again, the importance of QG linear dynamics is investigated through a comparison between exps. ANA—12A and 12A—12C. The differences at the initial time are shown in Fig. 14. Figure 15 displays the differences after 36 hours. In spite of the short integration time (with respect to the 72 h forecasts previously used), large differences appear between the two panels of Fig. 15. The location and the shape of the QGPV anomaly (U1 in the upper panel) and its opposite (lower panel) are quite different. Also, the contribution U2 due to downstream development is different in both panels. It may be inferred from the upper panel that the low L1 at 0000 UTC 10 February is a direct consequence of baroclinic interaction with the upper-level QGPV anomaly defined 36 h before. The comparison between the two panels also shows that the location and the shape of U1 are consequences of the development at the surface. In this case, the dynamics of the upper-level precursor seem to be dominated by non-QG linear dynamics. Though the total PV is conserved by adiabatic motion, the shape and the amplitude of a particular anomaly can be significantly modified. The only comparable feature in this case is the multiple extrema called U3. This feature is the consequence of
Figure 15. FASTEX IOP12: 36 h forecast for 0000 UTC 10 February based on different initial states (see Fig. 14) at 1200 UTC 8 February 1997. (a) Experiment ANA—12A; (b) experiment 12A—12C. Bold lines stand for 2 PVU geopotential difference (interval: 500 m). Thin solid lines stand for relative vorticity difference (interval: $5 \times 10^{-5}$ s$^{-1}$). Negative differences of vorticity are shaded. For discussion of numbered U and L anomalies and further details see text.
Figure 16. As Fig. 15 but for adiabatic simulations. (a) Forecast of experiment ANA–12A by nonlinear simulations; (b) ANA–12A by tangent-linear forecast. The model is linearized about the trajectory of the model given by experiment 12A.
the advection of a jet streak (see Fig. 7, lower-left panel) by northerly winds attributable to the anomaly.

The previous results are now compared with nonlinear adiabatic simulations and with a tangent-linear forecast. Figure 16 (upper panel) shows the difference between exps. ANA and 12A. The development associated with the IOP12 cyclone is still present here (U1/L1) but is weaker compared to the nonlinear simulation (see Fig. 15, upper panel). Diabatic processes determine the amplitude of the cyclone but not its existence. The lower panel of Fig. 16 shows the 36 h tangent-linear forecast of the flow associated with the upper-level QGPV anomaly at the initial time (see Fig. 14). It may be inferred from this panel that U1 triggers the low L2 in the linear simulation in the same way as in exp. 12A–12C. Thus, the existence of the IOP12 cyclone is determined by nonlinear processes. Furthermore, the study of the linear behaviour of a QGPV anomaly by comparing a forecast with the anomaly to a forecast without it, is relevant.

As a provisional conclusion, tropopause disturbances give rise to downstream development and also lead to significant upstream modifications if other synoptic-scale disturbances at the tropopause, like jet streaks, are present there. Therefore, attributing a part of the wind and mass field at one moment to a part of the QGPV field one, two, or 3 days previously is a difficult task, requiring the use of a numerical model together with a PV-inversion tool. When an upper-level precursor takes part in a baroclinic development at the surface, QG linear dynamics is no longer able to explain the behaviour of the upper-level QGPV anomaly.

Finally, when it appears that a low is going to develop somewhere along the frontal zone, the upper-level modifications control its position at first, and then its intensity.

5. Sensitivity to low-level features

The purpose of this section is to find potential low-level precursors of the IOP12 cyclone. Sensitivity of the cyclogenesis to upper-level features of comparable energy and to perturbations that project onto sensitivity fields will also be addressed. The amplitude of the modification for each experiment is measured through its total energy (in a QG form) and its QG potential enstrophy. The values are reported in Table 1.

(a) How to define 'low-level precursor'

The sensitivity to pre-existing finite-amplitude low-level patterns is investigated using the following strategy. Low-level QGPV anomalies and surface temperature anomalies are removed from the analysis in a domain of arbitrarily defined, but constant, surface and shape (see Fig. 17).

The sensitivity of the cyclone development to the low-level patterns captured in this way is measured through a comparison of the forecast with the patterns of interest, and the forecast without them. Figure 17 displays a diagram of the eight experiments performed along these lines. Results in Table 1 (exps. 12E to 12L) point to the fact that the modifications exhibit comparable potential enstrophy and total energy. Figure 18 presents the location and the value of the vorticity maximum corresponding to the low after a 36 h forecast. The actual low-level precursor arises from this systematic sensitivity study. The pre-existing low-level signature to which the cyclone development is most sensitive, is the signature captured in the initial conditions of exp. 12I (see Fig. 18). It is also the anomaly mentioned in the case-study as a potential-temperature tongue. The effect of QGPV inversion on the analysis is visible at 850 hPa in Fig. 20(b). The amplitude of the anomaly is about 1.5 K. Only a small part \(3 \times 10^{-5} \text{ s}^{-1} \) in
### TABLE 1. IOP12: DEFINITION OF EXPERIMENTS.

<table>
<thead>
<tr>
<th>Experiment (IOP/letter)</th>
<th>Nature of the modification ((\delta q_g^{UP}): upper level, (\delta q_g^{LO}): low level)</th>
<th>Total energy of the modification</th>
<th>Potential enstrophy of the modification</th>
</tr>
</thead>
<tbody>
<tr>
<td>12A</td>
<td>100% (\delta q_g^{UP}) ((T_{bd} = 0))</td>
<td>356.3</td>
<td>53.2</td>
</tr>
<tr>
<td>12B</td>
<td>100% (\delta q_g^{UP}) ((T_{bd} = 0))</td>
<td>415.9</td>
<td>50.1</td>
</tr>
<tr>
<td>12C</td>
<td>200% (\delta q_g^{UP}) ((T_{bd} = 0))</td>
<td>1500.0</td>
<td>208.0</td>
</tr>
<tr>
<td>12D</td>
<td>30% (\delta q_g^{UP}) ((T_{bd} = 0))</td>
<td>8.1</td>
<td>1.2</td>
</tr>
<tr>
<td>12M</td>
<td>(\delta q_g^{UP}) (following gradient fields)</td>
<td>9.1</td>
<td>6.5</td>
</tr>
<tr>
<td>12E</td>
<td>(\delta q_g^{LO}) ((T_{bd}))</td>
<td>2.1</td>
<td>1.7</td>
</tr>
<tr>
<td>12F</td>
<td>(\delta q_g^{LO}) ((T_{bd}))</td>
<td>3.0</td>
<td>3.2</td>
</tr>
<tr>
<td>12G</td>
<td>(\delta q_g^{LO}) ((T_{bd}))</td>
<td>24.7</td>
<td>4.9</td>
</tr>
<tr>
<td>12H</td>
<td>(\delta q_g^{LO}) ((T_{bd}))</td>
<td>10.9</td>
<td>7.5</td>
</tr>
<tr>
<td>12I</td>
<td>(\delta q_g^{LO}) ((T_{bd}))</td>
<td>7.8</td>
<td>2.0</td>
</tr>
<tr>
<td>12J</td>
<td>(\delta q_g^{LO}) ((T_{bd}))</td>
<td>27.4</td>
<td>11.6</td>
</tr>
<tr>
<td>12K</td>
<td>(\delta q_g^{LO}) ((T_{bd}))</td>
<td>8.8</td>
<td>3.0</td>
</tr>
<tr>
<td>12L</td>
<td>(\delta q_g^{LO}) ((T_{bd}))</td>
<td>11.8</td>
<td>4.8</td>
</tr>
<tr>
<td>12N</td>
<td>(\delta q_g^{UP}) (following gradient fields)</td>
<td>4.3</td>
<td>3.7</td>
</tr>
</tbody>
</table>

If the symbol \(\alpha \% \delta q_g^{UP}\) is present, then \(\alpha \%\) of the entire upper-level QGPV anomaly is removed from the analysis. The kind of boundary condition is given in parentheses: (\(T_{bd} = 0\)) stands for zero vorticity anomaly at the lowest level; (\(T_{bd} = 0\)) is for zero temperature anomaly at the lowest level (i.e. the high-pass filtered field, between wave numbers 8 and 63, is removed within an area). If the symbol \(\delta q_g^{LO}\) is present then low-level QGPV is removed from the analysis. \(\delta q_g^{LO}\) (following gradient fields) means that the modification follows gradient fields below 700 hPa and is set to zero above; \(\delta q_g^{UP}\) (following gradient fields) means that the modification follows gradient fields above 700 hPa and is set to zero below 700 hPa. Changes in energy are expressed in \(10^{12}\) J.

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**Figure 17. FASTEX IOP12: Locations of different initial-conditions modification at the lowest levels, showing experiment numbers (see Table 1).**
36 h) of the development seems to be due to this high-potential-temperature tongue. It contributes to the development without having any effect on its existence.

(b) Sensitivity to tropopause perturbations of weak amplitude

In the previous section, we demonstrated that the leading precursor of the IOP12 cyclone is an upper-level PV anomaly. Is this sensitivity due to the larger amplitude of the changes when the upper-level PV is modified? To address this, the critical upper-level precursor is modified with an amplitude similar—in terms of energy—to that employed at low-levels (Table 1, exp. 12D, and Fig. 19(b)). As a result, this change is hardly visible by eye. The 36 h forecast using these new initial conditions suggests that the weak-amplitude upper-level modification is not crucial for the development, yet it also clearly shows that the impact of this modification is comparable to the impact of low-level precursors.
(c) Sensitivity to perturbations following adjoint-model output

The purpose of this section is to compare the sensitivity of the cyclone development to pre-existing finite-scale and amplitude disturbances, and to disturbances following so-called gradient fields. Gradient fields are the gradient of the objective function, \( J \), at time, \( t \), with respect to the initial conditions (Rabier et al. 1992; Hello et al. 2000). The objective function used here is the average of the surface pressure inside an area encompassing the low at 0000 UTC 10 February (52°–62°N, 20°–10°W). The adjoint-model output has to be re-scaled to become a perturbation applicable to the initial conditions.

In exp. 12M, the analysis is modified following the gradient fields above 700 hPa. This modification, displayed in Fig. 19(a), tends to split the precursor into two parts. Once more the amplitude of the initial perturbation is made comparable to the perturbations discussed in the previous section. The cyclone location is found to be highly sensitive to these changes. The maximum value of the vorticity at 850 hPa is now very close to Iceland.

In exp. 12N, the analysis is modified following the low-level gradient fields (i.e. below 700 hPa). The amplitude of the perturbation is set so that the maximum of potential temperature at 850 hPa and the total energy of the perturbation are comparable with those of the preceding experiments. Figure 20(a) highlights a potential-temperature perturbation maximum of 3 K at 850 hPa: the corresponding energy amount of this perturbation is half the energy amount of the low-level precursor (see Table 1). On one hand, the 36 h forecast for exp. 12N does not exhibit any low inside the area used to compute the objective function; but on the other hand, two strong vorticity maxima appear outside this area, near the northern and southern boundaries of the area used to compute \( J \). The perturbations delivered by adjoint computations do not inhibit the baroclinic interaction between the upper-level precursor and the surface. This behaviour might be attributed to the small size of the box used to compute the cost function with respect to the size of the precursor. As long as the upper-level precursor is present, there is a development. Its location, however, is fundamentally controlled by the low-level structures depicted by the sensitivity fields.

The initial conditions of the experiment called 12G (see Fig. 17 and Table 1) result from the suppression of the QG PV and boundary temperature patterns within the area defined by the strong signature of the gradient fields (Fig. 20(a)) at low levels. In spite
of the fact that the modification is collocated with the most sensitive area, and that its energy norm is six times as large as the energy norm of the previous experiment (12N), no significant impact is found in Fig. 18. Therefore, one can note the weak sensitivity of the storm to changes in the low-level initial conditions when these modifications are not shaped exactly like adjoint-model output; their location, on its own, is not a sufficient condition.

6. CONCLUSIONS

The QG perspective of PV inversion turns out to be an efficient tool in the modification of initial conditions of a primitive-equation model. The linear balance does not produce too many spurious gravity waves. Furthermore, the initialization that ensures a reduction in the gravity wave activity, changes the modified PV only marginally. The QGPV inversion method is applied to the FASTEX IOP11 and IOP12 cyclones to demonstrate which PV anomalies, present within the atmosphere prior to the development, are involved in the cyclogenesis. The main findings of this paper can be summarized as follows:

- QGPV inversion turns out to be a very efficient way to depict the upper-level precursor of cyclone development.
- The choice of the boundary condition when a precursor is removed from the initial conditions has little impact on the subsequent cyclone development.
- The QG linear interactions (advection and downstream development) between the QGPV anomaly and the surrounding flow dominates the behaviour of the upper-level precursor of the IOP11 cyclone development. The coupling between QG attribution and numerical forecasts helps us document the integrated effect in time of these linear processes. The comparison of nonlinear forecasts with QGPV anomalies, their opposite, and without them, is one possible way to highlight processes that are not related to the QG linear dynamics. Such processes are so strong in the IOP12 cyclone development that they determine the existence of the low. This latter result suggests that a four-dimensional-variational assimilation scheme with no precursor in the background would have been unable to simulate the IOP12 cyclone properly.
- Upper-level precursors actually determine the existence of lows such as the IOP12 cyclone.
- The cyclone development is only weakly sensitive to low-level initial structures, unless they are shaped like gradient fields; collocation with another shape has little influence on the development. Moreover, upper-level and low-level initial-condition modifications of comparable energy have a comparable impact on the forecast. Two consequences arise from this: first, low-level cyclogenesis precursors are hard to identify, assuming that they exist; second, targeted localized observations based on adjoint products will be a valuable strategy only if the analysis increments provided by those observations follow the functional shape given by adjoint calculations.

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