The dynamical consequences for tropopause folding of PV anomalies induced by surface frontal collapse

By M. Z. ZIEMIAŃSKI and A. J. THORPE

1Institute of Meteorology and Water Management, Poland
2University of Reading, UK

(Received 12 November 1999; revised 22 June 2000)

SUMMARY

It has been noted previously that during frontal collapse dynamical processes lead to the formation of potential vorticity (PV) anomalies in the vicinity of the surface front. These processes can either be associated with diffusion in the presence of the tight temperature gradients or with intrusion into the atmosphere of the vanishingly thin layer adjacent to the surface. This paper explores the dynamical consequences of these PV anomalies on the parent baroclinic wave cyclone. The method used is to find the vertical motion attributable to these anomalies. The location and magnitude of this vertical motion is clearly key to the dynamical influence of the anomalies. As we are dealing with a three-dimensional evolving cyclone with an emergent tropopause fold, the calculation of vertical motion needs to be capable of accounting for the role of the highly deformed tropopause. Hence we develop and use a nonlinear balance model approach. The result shows that the PV anomalies near the surface front induce downward vertical motion at the tropopause fold, and thus they act to amplify this feature. This role of diffusion associated with surface frontal collapse is the key finding of this paper. An analysis is made of the role of upper- and lower-level PV anomalies on the overall rate of the wave development.

KEYWORDS: Frontal collapse Potential vorticity Vertical motion

1. INTRODUCTION

The semi-geostrophic (SG) simulation of frontogenesis leads to frontal collapse, as was shown first by Hoskins and Bretherton (1972). Namely, after a finite time period the horizontal gradient of temperature in the frontal zone becomes unbounded and the SG simulation becomes singular, as there is no longer a one-to-one mapping between the geostrophic and physical spaces. This difficulty can be overcome in two ways. First, an inflow of an infinitesimally thin layer of air, originally located at the surface, into the atmosphere is allowed for in the physical space (Cullen and Purser 1984; Cho and Koshyk 1989; Koshyk and Cho 1992). That atmospheric layer forms an infinitesimally thin surface of temperature discontinuity (front). Secondly, explicit diffusion can be introduced into the simulation (Blumen 1992).

The problem of frontal collapse also appears in primitive equation (PE) simulations of frontogenetic dynamics; e.g. in two-dimensional simulations performed for comparison with the SG simulations (Cullen 1983; Snyder et al. 1993) and for the Eady wave simulations (Nakamura and Held 1989), as well as in a three-dimensional study of a baroclinic wave evolution by Rotunno et al. (1994; hereafter RSS). For all these simulations it was necessary to introduce explicit diffusion to overcome numerical problems associated with the emergence of large temperature gradients in the frontal zones.

Both SG and PE simulations of frontogenesis lead to the creation of localized, but strong potential vorticity (PV) anomalies, which appear in the vicinity of frontal zones in the lower troposphere soon after commencement of the frontal collapse. As discussed by Nakamura and Held (1989), these anomalies can be understood as resulting from a lifting into the atmosphere of a vanishingly thin atmospheric layer, initially adjacent to the surface (see Fig. 8 in their paper). Such a lifting results from mixing with the surrounding air, or is introduced explicitly to eliminate the singularity in the SG transformation. As shown by Bretherton (1966), the surface temperature distribution is equivalent to the
PV distribution in the atmosphere. Therefore, an injection into the atmosphere of the
vanishingly thin near-surface air layer with specific temperature characteristics results
in the creation of appropriate PV anomalies in the lower troposphere.

Nakamura and Held (1989) discussed the role of such PV anomalies, resulting from
the frontal collapse, in the developing baroclinic system. They argue that the main
role of these anomalies is in generating a stably stratified atmospheric layer near the
surface. This would result in a decoupling of low-level and high-level interacting Rossby
waves, and finally in the decay of the instability. In this paper, we analyse the possible
dynamical role of these low-level PV anomalies in the development of the baroclinic
wave, via consideration of their influence on the vertical velocity field. We do this
by calculating the vertical velocity field attributed to the anomalies and assessing its
dynamical significance. The necessary data for our analysis are taken from the results of
a high-resolution PE simulation of the baroclinic wave development performed by RSS
for an adiabatic, frictionless and Boussinesq atmosphere on an \( f \)-plane.

In that simulation, the frontal collapse starts around day 6 of the wave development,
and on day 7 it leads to the formation of a pronounced PV dipole in the lower tropo-
sphere in the vicinity of the surface low centre. At this stage of the wave development
the process of tropopause folding is fairly advanced, causing its significant deformation.
To obtain credible diagnosis of the vertical velocity structure for that highly disturbed
tropopause and for localized small-scale low-level PV anomalies we apply the method
of vertical velocity diagnosis based on a nonlinear balance (NLB) omega equation. This
was first introduced by Pedersen and Grönskii (1969) and discussed later by Gent and
McWilliams (1983). The equation was applied in the context of the PV-based diagnosis
of a mid-latitude weather system by Davis and Emanuel (1991; hereafter DE).

An outline of the paper is as follows. In section 2 we present briefly the results
of the PE simulation of the baroclinic wave by RSS, and especially the structure of
the low-level PV anomaly resulting from the frontal collapse. Section 3 discusses the
attribution methods we use. In section 4 we present the NLB model used in our study.
In section 5 we compare the vertical velocity diagnosis performed by the NLB model
with the PE results taken from the RSS simulation, in order to demonstrate the accuracy
of the NLB model. Section 6 shows the vertical velocity field attributed to the low-
level PV anomalies, and section 7 discusses its dynamical effect on the baroclinic wave
development. Finally, section 8 summarizes the results.

2. PE SIMULATION OF THE BAROCLINIC WAVE

RSS performed the simulation of a baroclinic wave evolution in a Boussinesq atmos-
phere comprised of a troposphere and stratosphere separated by a flexible tropopause.
Initially, the troposphere and stratosphere are characterized by constant PV values, equal
to 0.4 and 3.6 PVU\(^*\), respectively. The evolution of the PV distribution during the wave
development is shown in Fig. 1. It is a three-dimensional view of the 1 PVU surface,
representing the position and shape of the tropopause, as seen from the south-east (SE)
towards the north-west (NW). Also, the surface distribution of the perturbation geopo-
tential is shown every 400 m\(^2\)s\(^{-2}\). Figure 1(a), (b) and (c) show days 4, 6 and 7 of the
wave development, respectively. The frames each show the area from the surface up to
12 km; and horizontally they show the middle 4000 km of the calculation domain in the
meridional direction and the whole 4000 km of the calculation domain in the zonal
direction.

\(^*\) 1PVU is \(10^{-6} \text{m}^2\text{s}^{-1}\text{K}\text{kg}^{-1}\).
Figure 1. The structure of potential-vorticity distribution during the primitive-equation evolution of the baroclinic wave presented as a three-dimensional view of the 1 PVU (see text) surface; additionally the perturbed geopotential on the ground is shown (negative values are dashed); (a) day 4 of the wave development, (b) day 6, and (c) day 7. The picture is viewed from the south-east corner towards the north-west corner of the model domain.

In Fig. 1(c), in addition to the tropopause, the small dome of 1 PVU air near the surface is seen. This is a part of the low-level PV anomaly associated with the frontal collapse. Another view of that PV anomaly is shown in Fig. 2(a) via the surface distribution of PV; the area shown coincides with the horizontal extension of the domain of Fig. 1. The presence of the strong positive PV anomaly near the low centre, already seen in Fig. 1(c), is evident in Fig. 2(a), but positive PV anomalies are also located along the surface fronts. The anomaly associated with the cold front (see Fig. 2(b) showing the surface distribution of the potential temperature) is stronger than the one associated with the warm front. Also negative PV anomalies are present, the strongest one is located south of the maximum of the positive PV anomaly, and the other is seen north of the cold front. Figure 2(c) shows a vertical cross-section of the PV distribution.
in the domain shown in Fig. 1. The cross-section goes from north to south, across the extremum value of the negative low-level PV anomaly and very close to the positive PV anomaly maximum. It is seen that while the main body of the low-level PV anomaly is restricted to the lowest 2 km, a weak ‘tail’ of the positive low-level PV anomaly penetrates above the lowest level of the weak downward penetration of the tropopause PV anomaly.

The maximum PV value in the low-level anomaly reaches 1.8 PVU, and the minimum is 0.18 PVU. The possible importance of the negative PV anomaly for the vertical velocity diagnosis has already been shown by Thorpe and Pedder (1999). They demonstrated that the minimum SG PV values in the anomaly were actually negative, indicating the possible formation of an internal instability of the flow and, subsequently,
leading to the overestimation of the magnitude of the rising motion. Application of the Ertel–Rossby PV analysis confirms, however, that the PV anomaly minimum is well above zero in this location.

3. Attribution

One of the main advantages of the PV-based analysis of atmospheric dynamics lies in the possibility of assessing the role of particular features of the PV distribution for atmospheric processes. Using the attribution method it is possible to diagnose flow and mass (temperature) fields associated with particular features (anomalies) of the PV distribution, and to assess the influence of those fields on the evolution of other PV anomalies (Hoskins et al. 1985). The attribution method is conceptually simple for the quasi-geostrophic (QG) model, due to the linear form of the QG PV definition. The attribution method has also been applied for the QG diagnosis of the vertical velocity (Clough et al. 1996; Thorpe 1997).

The attribution methodology becomes more complicated for the NLB model because, due to the nonlinearity involved, there is no unique flow and mass field which can be attributed to a specific PV anomaly. Nevertheless, the attribution method for the NLB models was introduced by DE (see also Davis 1992) and discussed further by Thorpe (1997). A comparative study of different attribution methods (Davis 1992) proved the reliability of the approach. The study showed that the different methods give very similar results, except in the immediate vicinity of the PV anomaly. Similar results were also obtained by Birkett and Thorpe (1997) for the SG model.

In terms of practical applications, the PV attribution for the NLB model has been mainly used to determine the horizontal velocity field associated with (induced by) particular PV anomalies. In this paper we extend the attribution method to retrieve the vertical velocity induced by a particular PV anomaly. We are especially interested in the role of the PV anomaly which developed at day 7 of the baroclinic wave development in the lower troposphere as a result of frontal collapse.

Here, we base the attribution on perhaps the simplest method, both conceptually and computationally. That is, for any meteorological field, its component induced by a particular PV anomaly is defined as the difference between the field determined by the full distribution of the PV and the field determined by the PV field modified in such a way that the PV anomaly in question is replaced by an appropriate average (background) value of the PV. Additionally, to check our results we apply an alternative attribution method, which results from the linearization of the equations for the attributed fields, according to Schneider (1988) and Davis et al. (1993).

Another source of the non-uniqueness of the vertical velocity attribution is the choice of the boundary conditions for the inversion of the PV distribution with the removed low-level PV anomaly. As discussed in the appendix, for the vertical velocity diagnosis it is natural to use the Neumann boundary conditions defined by the potential temperature (θ) distribution at the bottom and top of the model domain. It follows the idea of the PV–θ approach (Hoskins 1991), which describes atmospheric processes via the conservative properties of the flow. We applied such boundary conditions for both attribution methods. Namely, the boundary potential temperature is the same for both PV inversions, with and without low-level PV anomalies, which is equivalent to the assumption that there is no surface temperature anomaly attributed to the low-level PV anomaly we are interested in.

Additionally, we perform an alternative diagnosis of the attributed vertical velocity, applying Dirichlet conditions on the horizontal boundaries for the inversion of the PV
distribution without the low-level PV anomalies. These Dirichlet conditions are defined as the geopotential field resulting from the inversion of the full PV distribution. That choice of boundary conditions is equivalent to the assumption that there is no surface flow anomaly attributed to the low-level PV anomaly.

Finally, we assess the sensitivity of the attributed vertical velocity to the magnitude of the PV anomaly.

4. THEORETICAL FRAMEWORK

Following the RSS simulation of baroclinic wave development, our study is performed for an adiabatic, frictionless and Boussinesq atmosphere on an $f$-plane in physical coordinates $x, y, z$. To obtain the high-accuracy diagnosis of the vertical velocity in the presence of the highly curved tropopause and very localized PV anomalies, we use the NLB omega equation (Pedersen and Grønike 1969; Gent and McWilliams 1983):

$$f (f + \nabla^2 \psi) w_{zz} + \nabla^2 (w \phi_{xz}) - f w \nabla^2 \psi_{zz} - f (\nabla w \cdot \nabla \psi) z$$

$$= f (v \cdot \nabla (f + \nabla^2 \psi))_z - \nabla^2 (v \cdot \nabla \phi) - 2 \{\psi_{xx} \psi_{yy} - (\psi_{xy})^2\} z_t,$$  

where $w$ is the vertical velocity, $f$ is the Coriolis parameter, $\psi$ is a stream function and $\phi$ is a perturbation of a function

$$\Phi = c_p \theta_o \left( \frac{p}{p_o} \right)^\kappa$$

which, for short, we will call geopotential, following RSS. Here, $p$ and $p_o$ are pressure and reference pressure, respectively, $c_p$ is a specific heat at constant pressure, $\theta_o$ is a reference potential temperature, and $\kappa$ is the ratio of the gas constant for dry air to $c_p$. The advection terms on the right-hand side (r.h.s.) of the NLB omega equation, Eq. (1), involve both rotational and divergent components of horizontal wind velocity:

$$v = v_\psi + v_\chi = k \times \nabla \psi + \nabla \chi,$$

where $\chi$ is a velocity potential. To solve Eq. (1), $\psi$ and $\phi$ are calculated via inversion of the Ertel–Rossby PV with the NLB equation (see DE and Raymond (1992)):

$$(f + \nabla^2 \psi) \phi_{zz} - \phi_{xz} \psi_{xz} - \phi_{yz} \psi_{yz} = \frac{\rho_o g}{\theta_o} q$$

$$\nabla^2 \phi = f \nabla^2 \psi + 2 \{\psi_{xx} \psi_{yy} - (\psi_{xy})^2\},$$

where $q$ is the Ertel–Rossby PV and $\rho_o$ is a reference density. The divergent part of the horizontal flow is needed to evaluate the forcing term on the r.h.s. of Eq. (1). This is calculated from the continuity equation:

$$\nabla^2 \chi = -w_z.$$  

Finally, Eq. (1) requires the calculation of the time derivative of $\psi$. This is found via a one-step time integration of the balanced atmospheric system using conservation equations for the PV and the deviations of potential temperature:

$$\frac{D\theta}{Dt} = \theta_t + v \cdot \nabla \theta + w \theta_z = 0,$$

$$\frac{Dq}{Dt} = q_t + v \cdot \nabla q + w q_z = 0,$$
where the horizontal wind is composed both of rotational and divergent components as in Eq. (3), and the potential temperature deviation is related to $\phi$ via the hydrostatic relation:

$$\phi_z = g \frac{\theta}{\theta_0}.$$  \hspace{1cm} (9)

The above equations are highly implicit, and the iterative method used for their numerical solution is presented in the appendix.

5. Accuracy of the Vertical Velocity Diagnosis

Now, we demonstrate the accuracy of the vertical velocity calculation with the NLB omega equation. This is done by calculating the vertical motion determined by the Ertel–Rossby PV distribution in the atmospheric domain, using Eq. (1), and comparing that vertical velocity with the result of the PE diagnosis performed by RSS. We present results of such a test for day 7 of the baroclinic wave development, as our attribution analysis is performed for that instant. The results are presented in a scatter diagram and a table of mean and absolute errors of the NLB diagnosis of the vertical flow. Prior to these analyses the vertical velocity data were smoothed using a simple three-dimensional filter.

A scatter plot of results of the NLB diagnosis of the vertical velocity versus the PE diagnosis is presented in Fig. 3. The figure demonstrates the very high accuracy of the method, especially in the region of the downward branch of the circulation and to a lesser extent in the upward branch. The spread of scatter points significantly diminishes in the
area of the vertical velocity extrema and around zero, suggesting a good consistency of the method in these intervals.

More detailed analysis is presented in Table 1, showing mean and absolute errors of the NLB diagnosis of the vertical velocity for its subsequent intervals.

Table 1 confirms the analysis of the scatter diagram in Fig. 3. It also allows a comparison of our NLB diagnosis of the vertical velocity with an analogous study on the performance of the SG and QG diagnoses of the vertical velocity distribution for this same baroclinic wave and this same instant by Pedder and Thorpe (1999) and Thorpe and Pedder (1999). The results of our Table 1 are analogous to those presented by Thorpe and Pedder (1999) as Table 2. The comparison shows that the absolute errors of the NLB diagnosis are significantly smaller than the absolute errors for the QG method, and smaller than such errors for the SG method. This is also generally the case for biases, with the exception of the bias of the SG diagnosis in the vertical velocity interval between $-6$ and $-4$ cm s$^{-1}$ in the upper part of the domain, and—to a lesser degree—for the biases of the QG diagnosis in small vertical velocity intervals in the upper part of the domain. However, the absolute-error analysis and the more compact spread of scatter points confirms a better consistency of the NLB diagnosis.

Generally, therefore, the NLB diagnosis of the vertical velocity associated with the developing baroclinic wave, based on the NLB omega equation, has a better performance than the QG and SG diagnoses. Also, the results of the NLB diagnosis are generally very close to the PE results, enabling us to use the method to analyse the influence of different PV features, resulting from the PE simulation of the wave, on its vertical velocity field.

6. Attribution results

Now we present the vertical velocity field attributed to the low-level PV anomaly, which emerged on day 7 of the wave development as a result of the frontal collapse. This is based on the results of the simple attribution method, wherein the attributed field results from the subtraction of the vertical velocity fields obtained with and without these PV anomalies. We apply the Neumann boundary conditions for the inversion of the PV distribution without the PV anomalies. Additionally, we briefly compare these results with those of the alternative diagnoses, described in section 3.
(a) General structure of the attributed vertical velocity

The vertical velocity induced by the low-level PV anomaly consists of an area of ascent, the maximum reaching 0.9 cm s$^{-1}$, and an area of descent with the extreme value reaching $-3.4$ cm s$^{-1}$. As these areas have a complicated structure with many small-scale details, we present a three-dimensional view of the attributed vertical velocity structure in Fig. 4. The domain shown coincides with the smaller square in Fig. 2(b); the area extends vertically from the surface to 9 km. As a background, the shape of the 1 PVU surface is shown. The surface is coloured blue in the area where the full vertical velocity (resulting from the NLB diagnosis based on the complete PV distribution) is negative (full descent area), and it is orange and red in the area where the full vertical velocity is positive (full ascent area). The more intense the colour, the greater is the value of the ascent or descent. Figure 4(a) also shows the shape of the surface on which the attributed vertical velocity equals 0.2 cm s$^{-1}$. This surface, encapsulating the significant attributed ascent area, is shown in yellow. The view is taken from the NW toward the SE, opposite to that in Fig. 1. Figure 4(b) shows the shape of the surface on which the attributed vertical velocity equals $-0.2$ cm s$^{-1}$. This surface encapsulates the significant attributed descent area, and is painted green. The view direction is as in Fig. 4(a). Figure 4(c) is a superposition of Fig. 4(a) and (b), showing the relationship between areas of significant ascent and descent. Additionally, we present a view of these surfaces along the tropopause fold, that is from W toward E in Fig. 4(d). The figure shows that the areas of significant ascent and descent attributed to the low-level PV anomaly penetrate through the tropopause into the stratosphere, especially in the case of the attributed descent.

Figure 4 suggests that the attributed vertical velocity structure, apart from some small-scale features, can be described as a quadrupole, with two broader areas of ascent situated ahead of the maximum of the positive low-level PV anomaly, one taller in the region where the tropopause is relatively high, and one smaller below the relatively low tropopause. There is also a narrow ‘tail’ of descent, divided into two segments in the middle troposphere and situated just above and behind the maximum positive PV anomaly. It is stretched along the lowest part of the tropopause fold. Note, that the attributed vertical velocity distribution is not three-dimensionally filtered for the presentation. It is only averaged onto the mean levels on which the PV is defined, since the calculation of the vertical velocity is performed on vertically staggered mid-levels.

Another view of the structure of the attributed vertical velocity is presented in Fig. 5. Figure 5(a) shows a horizontal cross-section of the attributed vertical velocity at the 5 km height level, in the domain given by the rectangle in Fig. 2(b). The field shows the compact area of ascent, situated downstream of the tropopause fold, and the area of descent which, to a great extent, coincides with the area of the tropopause fold. Also, two local extrema inside the areas of ascent and descent reflect the quadrupole structure of the attributed vertical velocity field.

The extremal values of attributed ascent and descent are located very close to each other, which we link with the very small scales of the low-level PV anomaly. The extrema are separated by one grid point horizontally and by three grid points vertically. The vertical cross-section along these extrema is presented in Fig. 5(b). It is seen that the area of extremal values of descent forms a compact vertical column above the area of the positive PV anomaly maximum. In terms of the attributed ascent the situation is different, as there are a few relatively close but separated grid points for which the attributed vertical velocity has the value between 0.7 and the maximum 0.9 cm s$^{-1}$. It is also reflected in Fig. 5(b), where there are two separate low- and mid-level
attributed velocity extrema in the ascent region. The area of the maximum attributed ascent, therefore, does not have as compact a structure as the maximum descent area. Figure 5(b) also confirms the dipole nature of the attributed descent area.
Figure 5. Vertical velocity attributed to the low-level potential-vorticity anomaly: (a) horizontal cross-section at 5 km every 0.1 cm s\(^{-1}\); (b) vertical cross-section along the attributed vertical velocity extrema, line AB in frame (a), every 0.25 cm s\(^{-1}\). The thick lines show the intersection of the cross sections with the 1 PVU (see text) surface; negative values are dashed.

The shape of the attributed vertical velocity suggests the link between the attributed field and the general distribution of the PV in the whole atmospheric domain. Figure 4(d), for instance, suggests that as the southern edge of the attributed descent coincides closely with the slope of tropopause, the vertical velocity component attributed to the low-level PV anomaly depends also (at least to some degree) on that general distribution of the PV in the atmospheric domain. This can be regarded as a clear illustration of the general analysis of the attribution methodology by Thorpe (1997), summarized in the statement that for such an attribution method “the results depend on the properties of the surrounding atmosphere”.

(b) The attributed vertical velocity and the tropopause fold

As shown in Fig. 4, the rear part of the significant attributed descent collocates with the area of the tropopause fold. Moreover, the descent extends well above the tropopause, and coincides with the area for which the full vertical velocity is negative and which is, therefore, responsible for tropopause folding. It is clear, therefore, that this part of the vertical velocity field induced by the low-level PV anomaly participates in the process and, in fact, augments it. To better analyse the contribution of the attributed vertical velocity to the process of the tropopause folding we show, in Fig. 6, both the attributed vertical velocity and the full NLB vertical velocity fields in the vicinity of the fold. Figure 6(a) and (b) are vertical cross-sections along the axis of the dipole of the negative attributed vertical velocity, which coincides with the lowest part of the tropopause fold. The axis is shown in Fig. 5(a) as the line EF. Figure 6(a) shows the attributed vertical velocity and Fig. 6(b) shows the full vertical velocity. Figure 6(c) and (d) present vertical cross-sections along the north–south direction and across the lowest point of the tropopause fold, with (c) showing the attributed vertical velocity and (d) the full vertical velocity. It is seen that, while the maximum value of the attributed vertical velocity on the tropopause level reaches 0.35 cm s\(^{-1}\) and the maximum value of the
Figure 6. The structure of the vertical velocity attributed to the low-level potential vorticity anomaly compared with the structure of the nonlinear balance, NLB, vertical velocity in the vicinity of the tropopause fold. (a) And (b) present vertical cross-sections along the axis of the negative part of the attributed vertical velocity (EF in Fig. 5(a)): (a) attributed vertical velocity every 0.1 cm s\(^{-1}\); (b) NLB vertical velocity every 1 cm s\(^{-1}\). (c) And (d) present vertical cross-sections through the line crossing the lowest point of the tropopause (CD in Fig. 5(a)): (c) attributed vertical velocity every 0.1 cm s\(^{-1}\); (d) NLB vertical velocity every 1 cm s\(^{-1}\). The thick lines show the intersection of the cross sections with the 1 PVU (see text) surface; negative values are dashed.
full vertical velocity on the fold reaches 7 cm s\(^{-1}\), the attributed vertical velocity on the folded tropopause level lies in the range of 5 to 10\% of the full vertical velocity.

All other attribution methods used for the study, indicate in the region of the tropopause fold the presence of a branch of downward motion induced by the low-level PV anomalies. The linearized attribution method for both Dirichlet and Neumann boundary conditions even suggest slightly bigger values of the attributed descent on the fold (maximum between 0.4 and 0.5 cm s\(^{-1}\)), while the simple attribution method applied to the Dirichlet boundary conditions implies a value of descent around one third of that presented above. We can, therefore, state generally that the low-level PV anomalies resulting from the frontal collapse induce descent in the area of the tropopause fold, and the magnitude of that attributed descent lies in the range of magnitudes predicted by the simple attribution method, discussed above (that is around 0.35 cm s\(^{-1}\)).

In terms of the effect over 24 hours, descent with velocity of 0.35 cm s\(^{-1}\) results in the lowering of the tropopause fold by 300 m, which can be regarded as a significant value from the viewpoint of the wave dynamics. In practice we should expect the effect to be greater, as the magnitude of the low-level PV anomaly increases with time, leading to an increase of the attributed descent. We have checked that a two-fold increase in the value of the low-level PV anomaly leads to the doubling of the attributed descent on the tropopause fold. Similarly, reducing the value of the PV anomaly to half its original value results in the reduction of the attributed descent by a factor of two. This estimated effect is realistic, as the phase of the tropopause fold and the surface low does not change significantly, at least during the next 24 hours of the wave development.

7. THE ROLE OF THE PV ANOMALIES IN THE WAVE EVOLUTION

It is interesting to find out the dynamical significance of the tropopause fold, lowered due to the vertical velocity component induced by the low-level PV anomaly, for the development of the baroclinic wave. We can argue that the lowering of the tropopause fold would result in bringing the interacting high- and low-level Rossby waves closer to each other. But the lowering of the tropopause fold also results in bringing stratospheric, very stable air into the mid-tropospheric region. Although the stability of that air would diminish significantly in the stretching field of descent (see Fig. 6(b) and (d)), causing a significant increase of the relative vorticity of the flow, it is possible that the process would result in the net increase of the static stability in the mid-tropospheric region. That would counteract the effect of the lowering of the upper-level Rossby wave. To assess which of these two mechanisms prevails, we have compared the tendencies of the surface geopotential of the original baroclinic system with a modified structure, for which the tropopause fold was lowered slightly by 250 m (the vertical grid spacing). The fold was defined as the area of PV greater than 0.8 PVU in the layer between 3.5 and 5 km. The phase of the fold relative to the surface temperature field was not changed, and neither was the low-level distribution of PV, including the anomaly. The tendencies are calculated using a single forward time-step of the NLB model.

Following the standard forecasters’ practice, we take the values of the surface geopotential tendencies in the vicinity of the low centre as the indicator of the system development. If the tendencies are negative (the geopotential falls) then the low is deepening and the system is intensifying, if the tendencies are positive (the geopotential rises) then the low is filling and the system is weakening. The result is that the geopotential tendencies for the wave with the unaltered position of the tropopause are negative around the low centre, reaching \(-14 \times 10^{-3} \text{ m}^{2} \text{ s}^{-3}\) at the grid point closest
to the low centre. It indicates intensification of the system, in agreement with the PE model (and also with the effect of the time integration of the NLB model). For the system with the lowered tropopause these falls have even greater magnitudes, by around $3.5 \times 10^{-3} \text{m}^{-2} \text{s}^{-3}$ at that grid point. That means that even this slight lowering of the tropopause fold results in a significant increase in the rate of development of the system. We relate this to the greater effect of a stronger coupling of the low- and upper-level Rossby waves rather than the countereffecting effect of the increase in stability.

This same method, based on the comparison of the tendencies of the surface geopotential, can be applied to assess the direct effect of the low-level PV anomaly on the wave development, which has been discussed by Nakamura and Held (1989). We have calculated (using the NLB model) the surface tendencies of the wave with the low-level PV anomaly replaced by the background PV values. The result is that the falls in the vicinity of the low centre have their magnitudes increased by 3 to $4 \times 10^{-3} \text{m}^{-2} \text{s}^{-3}$, compared with the unaltered wave. Therefore, the system with the low-level PV anomalies present has a smaller rate of development. This is evidence of the stabilizing effect of these PV anomalies, induced by frontal collapse, on the three-dimensional development of the baroclinic wave, as anticipated by Nakamura and Held (1989). It seems that at the surface this effect is stronger than any possible increase of coupling of the Rossby waves forced by the existence of these PV anomalies. This difference in the dynamical role of upper- and lower-level PV anomalies in the wave development remains to be fully explored.

We have shown, therefore, that the dynamical role of the low-level PV anomalies on the wave evolution is two-fold. First, they have a direct stabilizing effect on the rate of the wave evolution, in accordance with Nakamura and Held (1989). But they also have a secondary effect via induced vertical motion, which leads to the amplification of the upper-level PV anomaly and, consequently, to the amplification of the rate of wave development. This secondary effect is, therefore, opposite to the direct one. We can expect that the actual relation between these two effects depends on the stage of wave evolution, and this remains to be explored.

8. CONCLUDING REMARKS

This paper presents methods for attribution of the vertical velocity field to anomalies of the PV distribution within the framework of the NLB model. The methods are applied for the PV anomalies resulting from the simulation of the frontal collapse. It is shown that the vertical velocity field induced by these low-level PV anomalies enhances the process of tropopause folding. This leads to a stronger coupling of the interacting low- and high-level Rossby waves, and to an increase of the rate of the system development.

This result has more general implications. It demonstrates that the ‘classic’ analysis of the dynamics of baroclinic instability, resulting from the mutual amplification of two Rossby waves due to appropriate two-dimensional horizontal flows (Hoskins et al. 1985; DE) using the attribution method, can be extended to the full three-dimensional picture, where the dynamical role of the vertical velocity fields induced by particular PV anomalies for the baroclinic processes can be accounted for.

The paper also presents a quantitative assessment of the performance of the NLB diagnosis of the vertical motion in the baroclinic wave. The performance of the NLB diagnosis is significantly better than the performance of the QG diagnosis, and also better than the SG diagnosis. The method therefore offers a good estimation of the vertical velocity field.
ACKNOWLEDGEMENTS

The work of M.Z.Z. on vertical velocity diagnosis was supported by the Polish State Committee for Scientific Research through Grant 6P04D 041 14. The study was performed as a part of the PhD studies of M.Z.Z. at the University of Reading sponsored by WMO under the VCP programme. We would like to thank Dr Richard Rotunno for providing the PE model and Dr Piotr Smolarkiewicz for providing the semi-Lagrangian advection routines.

APPENDIX

The calculations with the NLB model were performed with the resolution of the PE simulation by RSS. That is to say, the model domain consisted horizontally of 80 points (in the meridional direction) by 40 (in the zonal direction) separated uniformly by 100 km. In the vertical the model consisted of 60 levels, separated uniformly by 250 m.

(a) PV inversion

The method of PV inversion was developed from the DE approach (Ziemiański 1994). The main differences from the DE method are as follows. First, while forming the three-dimensional Poisson-like differential equation for \( \phi \) by adding the PV and the NLB, Eqs. (4) and (5), we do not formally non-dimensionalize them, but use a flexible scaling of the PV Eq. (4). The scaling is designed to obtain similar magnitudes of the terms involving the second-order vertical and horizontal derivatives of \( \phi \), in the new three-dimensional equation for \( \phi \). The term \( (f + \nabla^2 \psi) \phi_{zz} \) in Eq. (4) scales like \( f \cdot \frac{\Delta \phi}{H^2} \), and the term \( \nabla^2 \phi \) in Eq. (5) scales like \( \frac{\Delta \phi}{L^2} \), where \( H \) and \( L \) are, respectively, characteristic height- and length-scales and \( \Delta \phi \) is a characteristic scale of the geopotential perturbation. If, therefore, the PV Eq. (4) is multiplied by the scaling factor \( c = \frac{H}{L} \left( \frac{H}{L} \right)^2 \), and added to the NLB Eq. (5), the three-dimensional differential equation for \( \phi \) follows:

\[
\nabla^2 \phi + c(f + \nabla^2 \psi) \phi_{zz} - c \phi_{xz} \psi_{xz} - c \phi_{yz} \psi_{yz} = c \frac{\rho_0 g}{\theta_0} q + f \nabla^2 \psi + 2(\psi_{xx} \psi_{yy} - (\psi_{xy})^2).
\]

(A.1)

The second difference is that we solve directly the NLB Eq. (5) for the stream function \( \psi \). We solve it using an Arnason’s (1958) linearization:

\[
f \nabla^2 \psi^{(n+1)} + \psi_{yy}^{(n+1)} \psi_{xx}^{(n+1)} + \psi_{xx}^{(n+1)} \psi_{yy}^{(n+1)} - 2 \psi_{xy}^{(n)} \psi_{xy}^{(n+1)} = \nabla^2 \psi
\]

(A.2)

(where index, \( n \), designates the number of an iteration step), which ensures its convergent iterative solution.

The third difference is that the vertical gradient of \( \psi \) on the horizontal boundaries of the domain, necessary to solve Eq. (A.1), is calculated inside the domain using the four-point procedure for a one-sided derivative, instead of using the geostrophic approximation.

The set of equations Eq. (A.1) and Eq. (A.2) is solved iteratively using the strongly implicit method and applying under-relaxation for both \( \phi \) and \( \psi \). The scaling parameter, \( c \), is calculated assuming \( H = 10 \) km and \( L \) equals the horizontal size of a weather system. In practice the value of \( L \) is adjusted to achieve the convergence of the iterative procedure. Once the convergence is achieved the results are not sensitive to the value of
this scaling parameter. It significantly influences the convergence rate of the inversion, however.

The NLB Eq. (5) is of the Monge-Ampere type and has the following ellipticity condition:

\[ \nabla^2 \phi + 0.5 f^2 > 0 \]  
(A.3)

(see for instance Arnason (1958)). However, as discussed for instance by Knox (1997), a violation of that ellipticity condition in the atmospheric domain does not necessarily indicate the presence of an internal instability of the flow. Such an instability is indicated rather by a negative value of \( f \times \text{PV} \). As suggested by Knox (1997), following arguments by Lynch (1989), a violation of the ellipticity condition (Eq. (A.3)) results from a too coarse truncation of the PE divergence equation leading to the NLB equation. Because of the truncation, the terms ensuring ellipticity of that PE equation, as the equation for \( \psi \), are rejected. It is interesting that for the current study of the developing baroclinic wave, the perturbed geopotential resulting from the inversion of the positive Ertel PV (taken from the PE simulation of the baroclinic wave) violates the ellipticity condition in some places. This does not, however, fatally affect the procedure of the PV inversion, which remains convergent.

As the PE simulation by RSS was performed in a periodic zonal channel with rigid walls on northern and southern boundaries, the boundary conditions for the PV inversion were chosen to define an analogous configuration. Thus, we apply the periodic boundary conditions for \( \phi \) and \( \psi \) in the zonal direction. On the vertical walls, the Dirichlet boundary conditions for the perturbed geopotential are applied, and are calculated from the PE diagnosis of the surface perturbed geopotential and the potential temperature on the walls, using the hydrostatic relation Eq. (9). As there is no across-wall component of the flow, the horizontal wind is one-dimensional there and is geostropically balanced. Consequently, the Dirichlet boundary conditions are applied there for \( \psi \), calculated from \( \phi \) using the geostrophic relation. At the top and the bottom of the model domain, the potential temperature taken from the PE simulation is used as the Neumann boundary condition for \( \phi \). The distribution of the Ertel–Rossby PV in the domain is calculated using the results of PE diagnoses of \( v \), \( w \) and \( \theta \).

(b) Solution of the NLB omega equation

The NLB omega equation, Eq. (1), is implicit, and is solved iteratively so that after every iteration step the divergent horizontal velocity is diagnosed using the continuity equation. This velocity field is used to update the advection terms on the r.h.s. of the equation, which are calculated using the semi-Lagrangian advection procedure (Smolarkiewicz and Grell 1992; Smolarkiewicz and Pudykiewicz 1992).

The local time derivative of \( \psi \) is calculated by performing a forward time-step integration of the system for a small time increment (in practice 900 s), using the PV and potential temperature conservation equations, Eqs. (7) and (8), and applying the semi-Lagrangian advection procedures. The advected distribution of the PV is subsequently inverted to give the new distribution of the stream function, used to estimate the necessary local time derivative. The boundary conditions for that inversion are recalculated using the advected potential temperature.

The NLB omega equation is solved on a grid staggered in the vertical. The homogeneous Dirichlet horizontal boundary conditions are applied on the half-level above the highest and the half-level below the lowest of the main model levels, on which the PV inversion is performed. These boundary conditions are analogous to those of the PE simulation. On the vertical edges, limiting the model domain north and south, the
homogeneous Dirichlet boundary conditions are imposed, while on western and eastern sides the periodicity condition is applied.

The Poisson equation, Eq. (6), for the velocity potential, \( \chi \), is solved on the main model levels with the homogeneous boundary conditions on the southern and northern edges and with the periodicity condition in the zonal direction. For both equations the strongly implicit solver was applied.

The overall iterative procedure of solving the NLB omega equation is performed until the maximum difference of each component of the divergent wind between subsequent iterations is less than 1 mm s\(^{-1}\). This results in the maximum difference for the vertical velocity field between the iterations being less than 5 \( \times \) \( 10^{-3} \) mm s\(^{-1}\). This accuracy can be regarded as sufficient to perform the vertical velocity attribution analysis. To reach that accuracy, 17 iterations were performed to calculate the vertical velocity for day 7 of the baroclinic wave development.

References


Rotunno, R., Skamarock, W. C. and Snyder, C.  

Schneider, E. K.  

Smolarkiewicz, P. K. and Grell, G. A.  

Smolarkiewicz, P. K. and Pudykiewicz, J. A.  

Snyder, C., Skamarock, W. C. and Rotunno, R.  

Thorpe, A. J.  

Thorpe, A. J. and Pedder, M. A.  

Ziemiański, M. Z.  
1994  ‘Potential vorticity inversion’. JCMM Report No. 39, available from the Joint Centre for Mesoscale Meteorology, University of Reading, UK