Analytical and numerical modelling of jet streaks: Barotropic dynamics

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(Received 24 August 1999; revised 22 May 2000)

SUMMARY

Observations suggest that vortex dipoles of mesoscale dimensions may provide a simple yet realistic representation of the structure and dynamics of jet streaks in the extratropical upper troposphere. Moreover, the effects of horizontal divergence in the vicinity of jet streaks often may be of secondary dynamical importance, suggesting that a nondivergent barotropic framework may provide a logical starting point for an idealized investigation of jet streaks.

Analytical solutions of barotropic vortex dipoles are shown to exhibit characteristic signatures similar to those identified in observational case-studies of jet streaks. In addition to the dipole of relative vorticity, these signatures include: (i) a localized maximum in fluid speed (i.e. a jet streak), (ii) ageostrophic flow that is directed towards lower pressure in the entrance region and towards higher pressure in the exit region of the jet streak, (iii) a four-cell pattern of ageostrophic vorticity that is cyclonic in the entrance and exit regions and anticyclonic on the flanks of the streak, and (iv) a translation speed that is significantly slower than the maximum fluid speed. On the basis of these similarities, it is suggested that vortex dipoles provide a plausible dynamical representation of the structure and motion of jet streaks.

Nevertheless, vortex dipoles in isolation are unable to account for certain observed features of jet streaks, such as the anisotropy of the wind field in the along-stream direction and the asymmetry in the relative-vorticity field, in which the cyclonic vortex typically is stronger than the anticyclonic vortex. Moreover, jet streaks generally are not isolated, but are embedded in a larger-scale jet stream, which may be zonally varying or wavelike. Analytical and numerical solutions of barotropic vortex dipoles in the presence of a variety of non-uniform background flows characteristic of the large-scale extratropical circulation are shown to account for the above features absent from dipoles in isolation. These solutions are also shown to provide idealized depictions of the life cycles of jet streaks in the extratropical upper troposphere.

KEYWORDS: Jet streaks Nondivergent barotropic model Vortex dipoles

1. INTRODUCTION

Jet streaks, defined as localized wind speed maxima situated along the axis of a jet stream at the level of maximum wind (Palmén and Newton 1969, p. 199), have assumed a prominent role in modern synoptic meteorology, largely in recognition of their association with rapid cyclogenesis, heavy precipitation, and severe convective storms. In this paper, we aim to investigate the dynamics of the well-documented class of jet streaks found on the polar-front jet stream referred to as ‘polar-front jet streaks’ (e.g. Palmén and Newton 1969, sections 4.2, 8.1 and 8.3; Uccellini 1990, sections 6.3.1–6.3.2; Carlson 1991, sections 15.1–15.5; Bluestein 1993, sections 2.7.1, 2.7.4 and 2.8).

There exist numerous conceptual models of polar-front jet streaks that describe the characteristic vertical circulations and associated divergence patterns accompanying these systems in various flow configurations, the most common being the ‘four-quadrant model’ of a straight streak due to Bjerknes (1951). Salient features of this conceptual model, so called because the upstream (‘entrance’) and downstream (‘exit’) regions of the streak are divided into left and right quadrants by the jet axis (where left and right are defined facing downstream), are illustrated in Fig. 1. These features include: ageostrophic flow directed towards lower (higher) geopotential height in the entrance (exit) region, accompanied by horizontal convergence in left-entrance and right-exit regions and by horizontal divergence in right-entrance and left-exit regions (Fig. 1(a)); a direct (an indirect) transverse ageostrophic circulation in the entrance (exit) region centred below the jet streak in the middle troposphere (Fig. 1(b)); and a cyclonic–anticyclonic dipole of relative vorticity straddling the jet maximum (Fig. 1(c)).

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Despite the popularity of the four-quadrant model, isolated, straight jet streaks such as that shown in Fig. 1 are observed relatively infrequently in the atmosphere. Hence, a number of modifications to this model have been proposed that account for flow curvature (e.g. Beebe and Bates 1955; Shapiro and Kennedy 1981; Moore and VanKnowe 1992), along-jet thermal advection (e.g. Shapiro 1982; Keyser and Shapiro 1986), lateral interactions between two jet streaks (e.g. Uccellini and Kocin 1987), and vertical interactions between upper-level jet streaks and lower-level jets and frontal zones (e.g. Uccellini and Johnson 1979; Shapiro 1982). More recently, Ziv and Paldor (1999) have examined the modifications to the patterns of divergence associated with straight and curved jet streaks that arise both from translation and from intensification or weakening of the streak. This array of phenomenological models has proved useful in the qualitative kinematic diagnosis of jet-streak circulations, and numerous observational and numerical studies have documented the role of these
circulations in rapid cyclogenesis and severe convection. Nevertheless, relatively few investigations have examined the underlying dynamics of jet streaks; these will be reviewed below.

Newton (1959, 1981) described jet streaks in terms of ‘partial-inertial oscillations’, in which parcels travelling through a jet streak experience cyclclical velocity oscillations whereby the geostrophic and ageostrophic winds vary in a related manner. This description was intended to account for Newton’s observation that wind speed maxima and minima are supergeostrophic and subgeostrophic, respectively, and the structure to which it is applied requires anticyclonic curvature of the axis of maximum wind speed and of the isochas pattern constituting the jet streak (see Fig. 3 in Newton (1959)). However, polar-front jet streaks do not appear to show a preference either for supergeostrophy or for anticyclonic curvature (e.g. Achtor and Horn 1986; Cammas and Ramond 1989). Consequently, Newton’s description appears more applicable to the quasi-stationary maxima of the winter subtropical jet stream, which are of synoptic- and planetary-scale dimensions (with characteristic length scale $L \geq 4000$ km) and which reside in the ridges of the stream’s meanders (e.g. Krishnamurti 1961), than to mobile polar-front jet streaks, which typically are of mesoscale and subsynoptic-scale dimensions ($L \approx 1000$–2000 km).

A number of previous idealized investigations have examined the unbalanced nature of jet streaks; for example, Houghton et al. (1981), Van Tuyl and Young (1982), and Weglarz and Lin (1997) consider the process of geostrophic adjustment and the role of inertia-gravity waves in the evolution of jet streaks. However, in the present study we will consider jet streaks to be balanced phenomena, such that inertia-gravity-wave activity is unimportant for the evolution of the streak. A balanced framework is adopted not only because of the simplifications in analysis that it affords, but also because there is evidence that rapid extratropical cyclogenesis and the attendant jet streaks may lie within the realm of balanced dynamics (e.g. Davis et al. 1996). Hence, although it is acknowledged that unbalanced flow may be important for some observed jet streaks, the investigation of such flow conditions is deferred to future study.

Adopting a nongeostrophic but balanced framework, Garner (1991) proposes a linear model for jet streaks that is comprised of a sum of neutral Eady plane waves. His model reproduces some of the salient features of polar-front jet streaks, including the characteristic divergent ageostrophic circulations and the slow translation with respect to the maximum wind speed in the core of the streak. An additional property of his model is the presence of supergeostrophic along-jet flow, obtained as a higher-order correction to quasi-geostrophy, without the requirement for anticyclonic curvature of the jet axis; however, as noted above, observations of polar-front jet streaks do not appear to show such a preference for supergeostrophy. Further evidence for interpreting jet streaks in terms of linear wave dynamics is provided by Orlanski and Sheldon (1995), who employ a local energetics approach to develop a conceptual model of a growing baroclinic wave, highlighting the presence of kinetic energy centres located in the north-westerly and south-westerly flow inflections of the wave (see their Figs. 2 and 3). These energy centres are interpreted to correspond to jet streaks, and the location of these jet streaks with respect to the wave, along with their sequential development in the downstream direction, are attributed to the propagation of energy via ageostrophic geopotential fluxes.

The models of Garner (1991) and Orlanski and Sheldon (1995) appear to provide viable dynamical descriptions of polar-front jet streaks that are associated with synoptic-scale wavellike flow. However, there exists observational evidence for a different class of polar-front jet streaks that are associated with the superposition of large-amplitude,
tropopause-based disturbances of mesoscale dimensions with the enhanced Ertel potential vorticity (EPV) gradients that constitute the extratropical tropopause and that coincide with the polar-front jet stream (Cunningham 1997, section 1; Pyle 1997, sections 3.3, 5.3 and 6.3; Hakim 2000). These tropopause-based disturbances typically are manifested as closed contours of EPV on tropopause-intersecting isentropes and as closed contours of potential temperature on surfaces of constant EPV defining the dynamic tropopause. Consistent with this observational evidence, a number of previous investigations (e.g. Mattocks and Bleck 1986; Takayabu 1991; Clough et al. 1996; Davies and Rossa 1998) also have noted a close correspondence between jet streaks and localized anomalies in the EPV field. The presence of closed contours of a materially conserved quantity such as EPV is indicative of significant nonlinearity and is a defining characteristic of coherent vortices. Associated indications of nonlinearity and coherence are trapping of fluid parcels and longevity of the disturbance with respect to an eddy-turnover time. With regard to the former, Hakim (1997, sections 5.1.4 and 5.2.4) analysed parcel trajectories for several large-amplitude tropopause-based disturbances and showed that these features may indeed trap fluid parcels. With regard to the latter, observed life cycles of selected tropopause-based disturbances have been documented that are in excess of two weeks (e.g. Bosart et al. 1996; Pyle 1997, section 3.3), during which time they may be associated with multiple distinct jet streaks.

An example of this class of jet streak is shown in Fig. 2, which displays selected fields associated with a wind speed maximum located over the north central United States at 1200 UTC 3 November 1995, obtained from an uninitialized, 1.125° resolution global analysis generated at the European Centre for Medium-Range Weather Forecasts. The core of maximum wind in this case is located at 300 hPa and is traversed by the 328 K isentropic surface; on the latter surface (Fig. 2(a)), the jet streak, which displays significant anisotropy in the along-stream direction, is accompanied by cyclonic relative vorticity on its poleward flank and by anticyclonic relative vorticity on its equatorward flank. The dipolar nature of the relative-vorticity field is reminiscent of that seen in the idealized depiction of Fig. 1(c), although a difference is found in the asymmetry of this dipole such that the cyclonic member is significantly stronger than the anticyclonic member. Similar structure is evident on the 300 hPa surface (not shown) and, moreover, these properties of wind-field anisotropy and vorticity-field asymmetry appear to be common to many observed jet streaks. The EPV field at 328 K is shown in Fig. 2(b), and exhibits a relatively strong local maximum on the poleward flank of the jet streak and a local minimum on the equatorward flank. These extrema are superposed on a region of enhanced EPV gradient that apparently is associated with the synoptic-scale trough in which the jet streak is embedded. Hence, it is proposed that the jet streak in this case is associated with (or ‘induced by’) an asymmetric dipolar vortex superposed on a larger-scale background flow.

Continuing the diagnosis of this jet streak, Fig. 3 shows the total ageostrophic wind at 300 hPa (Fig. 3(a)), along with its decomposition into harmonic (Fig. 3(b)), divergent (Fig. 3(c)), and rotational (Fig. 3(d)) parts, following the method described by Lough et al. (1995). The harmonic and divergent velocity potentials and the rotational stream function associated with the partitioned ageostrophic winds are shown in Figs. 3(b), (c) and (d), respectively. Comparison of the three components of the ageostrophic wind shows that the rotational part is dominant in the vicinity of the jet streak. The divergent velocity potential (Fig. 3(c)) reveals horizontal divergence in the right-entrance and left-exit regions, and convergence in the left-entrance region. Nevertheless, there is no indication of convergence in the right-exit region, so that the horizontal divergence does not exhibit the classic four-cell pattern described in Fig. 1(a). The rotational stream
Figure 2. ECMWF analyses valid at 1200 UTC 3 November 1995 on the 328 K isentropic surface of: (a) relative vorticity (contour interval $4 \times 10^{-2}$ s$^{-1}$, negative values dashed); and (b) Ertel potential vorticity (EPV) (contour interval 1 PV unit). In both panels total wind speeds greater than 50 m s$^{-1}$ are shaded as indicated.

function (Fig. 3(d)) exhibits a four-cell pattern that is cyclonic in the entrance and exit regions, and anticyclonic on the flanks of the streak. Moreover, it is emphasized that the extremum of this field is significantly larger in magnitude than that of the divergent velocity potential.

Given the observed dominance of the rotational component of the ageostrophic wind over the divergent component, it is conceivable that, to a first approximation, a
two-dimensional nondivergent system may capture the essential dynamics of this jet streak. The approximation of nondivergence may be considered to contradict conventional wisdom in synoptic meteorology, which states that horizontal divergence and vertical circulations are of primary importance for jet streaks (e.g. Carlson 1991, section 15.2; Bluestein 1993, section 2.8). Nevertheless, support for adopting a nondivergent system may be obtained by appealing to a scale analysis of the vorticity equation (e.g. Haltiner and Williams 1980, section 3.3), in which the effects of horizontal divergence are quantified by the ratio of the vortex-stretching term (i.e. the absolute vorticity multiplied by the horizontal divergence) to the vorticity-advection term. Provided that the magnitude of the relative vorticity does not exceed the Coriolis parameter, this ratio is given by $R_1/Ro$, where $R_1$ is the ratio of the magnitude of the divergent component of the horizontal wind velocity to the magnitude of the rotational component (equivalent to the ratio of the horizontal divergence to the relative vorticity), and $Ro$ is the Rossby number, expressed as the ratio of the relative vorticity to the Coriolis parameter. Since the ratio of the ageostrophic vorticity to the relative vorticity is $Ro$, the ratio of the horizontal divergence to the ageostrophic vorticity is also given by $R_1/Ro$. Hence, on
the basis of this scale analysis, it is suggested that when the divergent component of the ageostrophic wind is small in comparison with the rotational component, the vortex-stretching term also is small in comparison with the vorticity-advection term such that the effects of horizontal divergence may be neglected to a first approximation.

In the vicinity of the jet streak shown in Figs. 2 and 3, the relative vorticity and the ageostrophic vorticity (not shown) at 300 hPa are of similar magnitude and comparable with the Coriolis parameter, such that \( Ro \approx 1 \), while the horizontal divergence (not shown) is significantly smaller in magnitude than the former two quantities, such that \( R_1 \) and \( R_1/Ro \approx 0.2-0.4 \). Moreover, in this region the ratio of the vortex-stretching term to the vorticity-advection term varies between 0.2 and 0.4, consistent with the above scale analysis. It is also noteworthy that an order-of-magnitude estimate of terms in the vorticity equation by Carlson (1991, Table 3.1, p. 67) applicable to midlatitude synoptic-scale systems indicates that at 300 hPa this ratio is typically about 0.3. Finally, observational support for the dominance of the rotational over the divergent component of the ageostrophic flow in the vicinity of jet streaks is found in a number of previous investigations (e.g. Krishnamurti 1968; Keyser et al. 1989, 1992; Loughe et al. 1995; Pyle 1997, sections 3.2 and 5.2). Based on the foregoing evidence, it is suggested that the effects of horizontal divergence may be neglected to a first approximation and that a nondivergent barotropic system may be suitable for an idealized investigation of jet-streak dynamics. An additional, conceptual justification for adopting this system is provided by the desire to explain as many attributes of jet-streak structure, motion and evolution as possible within a simple dynamical framework, and to lay a foundation on which to build upon these explanations hierarchically with models of increasing complexity and generality.

The foregoing discussion raises a number of issues regarding the structure and dynamics of jet streaks that need to be addressed. Of particular interest is whether there exist coherent vortex structures that may be employed as idealized representations of jet streaks exhibiting the characteristic features illustrated in Fig. 1 and accounting for the observed signatures identified in Figs. 2 and 3. The extent to which these structures are able to describe the life cycles of jet streaks in relation to the larger-scale environment in which they are embedded also will be considered. The objective of this investigation is to address as many of the above issues as possible within the simple dynamical framework of the nondivergent barotropic system. Moreover, it is hoped that the results of the present investigation will allow the incorporation of jet streaks into the dynamical framework of the so-called potential vorticity (PV) paradigm (e.g. Hoskins et al. 1985).

Given the above justification for adopting the nondivergent barotropic framework, and motivated by the dipolar structure of relative vorticity suggested in Fig. 2, we investigate the possibility that the behaviour of vortex dipoles in a nondivergent barotropic fluid may provide a useful starting point for the study of jet-streak dynamics. With this aim in mind, we examine idealized analytical and numerical solutions of barotropic vortex dipoles, both in isolation and in the presence of large-scale background flows representative of the extratropical upper troposphere. In the following section, we describe the barotropic framework that will be employed, and illustrate an analytical solution of a vortex dipole that may be used as a simple dynamical representation for the structure and motion of isolated, straight jet streaks. In section 3 we employ both analytical solutions and numerical simulations to consider the effects on vortex dipoles of spatially non-uniform jetlike and wavelike background flows, and the resulting implications for the structure, motion, and evolution of the associated jet streaks. Finally, in section 4 we discuss the results presented herein and speculate on potential avenues for future research.
2. REPRESENTATION OF JET STREAMS IN TERMS OF VORTEX DIPOLES IN UNIFORM BACKGROUND FLOWS

As noted in the introduction, a class of jet streaks in the extratropical upper troposphere appears to be associated with coherent vortex structures in the form of mesoscale vortex dipoles. Moreover, there appears to be justification for neglecting to a first approximation the horizontal divergence associated with these features and thus for adopting a nondivergent barotropic system: this system will be described in section 2(a). In section 2(b), we examine the extent to which an analytical solution of a vortex dipole in isolation on an $f$-plane may be employed to derive a basic understanding of the dynamics of jet streaks.

(a) Nondivergent barotropic system

The nondivergent barotropic system is governed by the vorticity equation

$$\frac{\partial \zeta}{\partial t} + J(\psi, \zeta + f) = 0,$$  \hspace{1cm} (1)

where $\psi$ is the stream function, $\zeta = \nabla^2 \psi$ is the relative vorticity, $J$ is the standard Jacobian operator, and $f$ is the Coriolis parameter. In this paper, we shall consider cases both in which $f = f_0$ (an $f$-plane) and in which $f = f_0 + \beta y$ (a $\beta$-plane), where $f_0$ and $\beta$ are constants. In all cases to be considered, it is assumed that the central latitude of the domain, corresponding to $y = 0$, is at 40°N; hence $f_0 = 9.37 \times 10^{-5}$ s$^{-1}$ and $\beta = 1.75 \times 10^{-11}$ m$^{-1}$s$^{-1}$.

Since we are concerned with the dynamics of jet streaks, it is of interest to examine the nature of localized fluid speed maxima in the barotropic system and the circumstances under which such maxima may arise. This issue may be addressed by formulating a diagnostic equation in the barotropic system that relates the nondivergent velocity $\mathbf{V}$ to gradients of relative vorticity:

$$\nabla^2 \mathbf{V} = \mathbf{k} \times \nabla \zeta,$$ \hspace{1cm} (2)

where $\mathbf{k}$ is the unit vector in the vertical direction. From this relationship, it is evident that $\mathbf{V}$ is associated with gradients of $\zeta$, with higher values located to the left of $\mathbf{V}$. It follows that local maxima in fluid speed correspond to similar maxima in gradients of relative vorticity, and it is apparent that a vortex dipole possesses such a maximum in the region between its component vortices (e.g. refer to Fig. 1(c)). Furthermore, if the dipole is superposed on a background flow that itself possesses vorticity gradients, such as a jet stream, a strong jet streak may result.

The ageostrophic wind often is emphasized in the jet-streak literature; thus it is desirable to be able to diagnose the ageostrophic flow in the barotropic system. Such a diagnosis requires knowledge of the pressure field, and in the barotropic model this field may be obtained by forming a 'divergence equation' from the momentum equations applicable to a nondivergent barotropic fluid. The equation so obtained is a form of the nonlinear balance equation, given by

$$\frac{1}{\rho_0} \nabla^2 p = f_0 \nabla^2 \psi + 2J(\psi_x, \psi_y) + \beta(\psi_y + y \nabla^2 \psi),$$ \hspace{1cm} (3)

where $p$ is the pressure and $\rho_0$ is the (constant) density*.

* It should be noted that a leading-order truncation of both the shallow-water and the stratified primitive equations for small Froude number and Rossby number $O(1)$ or larger also results in equations for two-dimensional flow governed by (1) coupled with (3) (e.g. Vallis 1996).
relative vorticity, whereby \( \zeta_{ag} = \nabla^2 \psi - (1/\rho_0 f_0) \nabla^2 p \); thus, (3) may be rewritten as

\[
\zeta_{ag} = -\frac{2}{f_0} J(\psi_x, \psi_y) - \frac{\beta}{f_0} (\psi_y + y \nabla^2 \psi).
\]

By inverting a Poisson equation that relates a stream function \( \psi_{ag} \) to \( \zeta_{ag} \), the ageostrophic velocity may readily be obtained from the relation \( \mathbf{V}_{ag} = \mathbf{k} \times \nabla \psi_{ag} \).

(b) Representation of jet streaks in terms of an analytical vortex dipole solution

Analytical solutions of vortex dipoles may be found for a variety of dynamical frameworks; these dipoles are exact nonlinear solutions to the relevant governing equations and are often referred to as ‘modalons’ (see Flierl (1987) for further discussion of this class of solutions). The dipole solution to (1) for a \( \beta \)-plane, due to Larichev and Reznik (1976), is formulated in polar coordinates \( (r, \theta) \) and is assumed to be steadily translating in the \( x \)-direction with constant speed \( c \). In this case, the stream function is given by

\[
\psi = \begin{cases} 
ca \sin \theta \left( \frac{k_e^2}{k_i^2} \frac{J_i(k_i r)}{J_1(k_i a)} - \left( 1 + \frac{k_e^2}{k_i^2} \right) \frac{r}{a} \right), & r < a, \\
-ca \sin \theta \frac{K_i(k_e r)}{K_1(k_e a)}, & r > a,
\end{cases}
\]

where \( a \) is the ‘radius’ of the dipole, separating interior \( (r < a) \) and exterior \( (r > a) \) regions, and \( J_1 \) and \( K_1 \) are ordinary and modified Bessel functions of order one, respectively. The interior wave number \( k_i \) may be found from the relation

\[
\frac{1}{k_i a J_1(k_i a)} = \frac{1}{k_e a K_1(k_e a)},
\]

where \( k_e = (\beta/c)^{1/2} \) is the exterior wave number. We shall refer to this solution as a barotropic modon.

A solution analogous to the barotropic modon for an \( f \)-plane (i.e. \( \beta = 0 \)) may be obtained by investigating the limiting behaviour of (5)–(6) as \( k_e \to 0 \) (e.g. Cunningham 1997, section 3.2). In this case,

\[
\psi = \begin{cases} 
ca \sin \theta \left( \frac{2}{k_i a} \frac{J_1(k_i r)}{J_0(k_i a)} - \frac{r}{a} \right), & r < a, \\
-\frac{ca^2}{r} \sin \theta, & r > a,
\end{cases}
\]

where it is required that \( J_1(k_i a) = 0 \), such that the smallest nonzero value of \( k_i a \) is 3.83. This solution was described initially by Lamb (1932, Art. 165), and thus is referred to here as a Lamb dipole. For simplicity in the discussion that follows in this section, we shall examine only the Lamb dipole, since consideration of the barotropic modon does not alter qualitatively the results obtained. However, in section 3(b), which in part addresses the motion and evolution of jet streaks in the presence of a Rossby wave, the barotropic modon solution will be employed.

Figure 4 shows the structure of a typical solution of the form given by (7). For illustration, we choose \( c = 4.5 \text{ m s}^{-1} \) and \( a = 750 \text{ km} \); this choice results in a maximum relative-vorticity amplitude of approximately \( 6.6 \times 10^{-5} \text{ s}^{-1} \). The dipole of relative
vorticity is evident in Fig. 4(a); consistent with the interpretation of (2), a localized maximum in the fluid speed is located in the region between the component vortices (Fig. 4(b)). The ageostrophic velocity (Fig. 4(c)), which is directed to the cyclonic-shear (anticyclonic-shear) side of the dipole in the entrance (exit) region of the fluid speed maximum, conforms to that expected in conceptual models of straight jet streaks (e.g. Fig. 1(a)) and to that seen in observations (Fig. 3(a)). Moreover, the ageostrophic vorticity (Fig. 4(d)) displays a four-cell pattern that is cyclonic in the entrance and exit regions and anticyclonic on the flanks of the fluid speed maximum, and that bears some resemblance to the ageostrophic stream function diagnosed in the observed jet streak (Fig. 3(d)). An interpretation of this four-cell structure may be obtained by rewriting the
expression for the ageostrophic vorticity, given by (4), as follows:

$$\zeta_{ag} = \frac{1}{2f_0}(S_1^2 + S_2^2 - \zeta^2),$$

(8)

where \(S_1 = -2\psi_{xy}\) and \(S_2 = \psi_{xx} - \psi_{yy}\) are the respective components of the strain*, and \(f\)-plane geometry is assumed, consistent with the derivation of the Lamb dipole solution. Hence the ageostrophic vorticity may be interpreted as providing a measure of the relative magnitudes of strain and relative vorticity. It is thus evident that the four-cell pattern seen in Fig. 4(d) represents a dominance of strain over relative vorticity in the entrance and exit regions of the fluid speed maximum, and a dominance of relative vorticity over strain in the vicinity of the vortices. Further interpretation of the ageostrophic vorticity in the context of a stratified quasi-geostrophic system is provided by Xu (1992, see his Fig. 4). In this system, the ageostrophic vorticity is defined as in (8), but with the components of the strain and the relative vorticity replaced by their geostrophic counterparts.

The ageostrophic vorticity also holds implications for the Lagrangian rate of change of vorticity gradients in a barotropic fluid, and hence from (2) for the Lagrangian rate of change of velocity. Applying the horizontal gradient operator \(\nabla\) to (1) results in

$$\frac{d}{dt} \nabla \zeta = -J(\nabla \psi, \zeta),$$

(9)

where again we have assumed \(f\)-plane geometry. Following the argument of McWilliams (1984), which requires that the strain and vorticity are slowly varying in a Lagrangian frame of reference when compared with the vorticity gradient, (9) has solutions of the form

$$\nabla \zeta = (\nabla \zeta)_0 \exp(\pm Q^{1/2}t),$$

(10)

where \(Q = (S_1^2 + S_2^2 - \zeta^2)/4\), such that via (8) \(\zeta_{ag} = 2Q/f_0\), and \((\nabla \zeta)_0\) is the vorticity gradient at \(t = 0\) for a particular fluid element under consideration at time \(t\). Hence the evolution of the vorticity gradient is intimately related to the sign of the ageostrophic vorticity; when \(\zeta_{ag} > 0\) the vorticity gradient will increase (or decrease) exponentially with time, whereas when \(\zeta_{ag} < 0\) the time evolution is oscillatory. Analysis of the stability of Lagrangian particle trajectories (Benzi et al. 1988; Schubert et al. 1999) also reveals the parameter \(Q\), and thus \(\zeta_{ag}\), to be of fundamental importance. In regions for which \(\zeta_{ag} > 0\), the distance between two neighbouring particles will decrease (or increase) exponentially in time; however, if \(\zeta_{ag} < 0\), initially neighbouring particles will remain close, which would appear to indicate the trapping of fluid parcels in a localized region and thus the presence of a coherent vortex.

It is emphasized that the foregoing discussion is consistent with the observation of a four-cell pattern of ageostrophic vorticity (Fig. 4(d)) associated with the fluid speed maximum. Regions of positive \(\zeta_{ag}\), characterized by an increase or decrease of the vorticity gradient following a fluid particle and by a decrease or increase in the separation between neighbouring fluid particles, correspond to the entrance and exit regions, respectively; regions of negative \(\zeta_{ag}\), characterized by an oscillation of the vorticity gradient and by the trapping of fluid particles, correspond to the vortices on the flanks of the fluid speed maximum. Hence, the configuration of the ageostrophic vorticity is of interest not only with regard to the structure of the ageostrophic wind

* In a meteorological context, the quantity \((S_1^2 + S_2^2)^{1/2}\) is often referred to as the resultant deformation.
field, but also with regard to the evolution in a Lagrangian framework of localized fluid velocity maxima in the barotropic model, the latter associated through (2) with the evolution of localized maxima in the vorticity gradient (i.e. ‘vorticity frontogenesis’). An extension of this concept to the stratified case involves the evolution of localized PV gradients (i.e. ‘PV frontogenesis’; see Davies and Rossa (1998) for a discussion of the dynamics of this process in the atmosphere).

Figure 4 illustrates that the Lamb dipole solution exhibits features commonly attributed to jet streaks that are evident in both the conceptual model (Fig. 1) and in observations (Figs. 2 and 3). However, the above description of the solution gives no insight into the dynamics of the motion of this feature. In this regard, a useful technique is that of piecewise vorticity inversion, in which the relative-vorticity field is divided into pieces that are inverted individually to yield the flow field associated with each piece. Such a technique allows consideration of the instantaneous motion of the dipole due to self- and mutual advections of the cyclonic and anticyclonic dipole members. In the case of the steady Lamb dipole solution, this instantaneous motion is equal to the specified translation speed \( c \). Since the relative vorticity is the perturbation PV for the nondivergent barotropic model, this technique may be considered a form of piecewise PV inversion (e.g. Davis and Emanuel 1991; Davis 1992).

The relative-vorticity field is partitioned into two pieces, \( \zeta_c \) and \( \zeta_a \), where the subscripts ‘c’ and ‘a’ denote the cyclonic and anticyclonic components of the dipole’s vorticity, respectively. Such a partition is simple in the case of the Lamb dipole, in which the relative vorticity vanishes everywhere for \( r > a \). Each piece is inverted separately, subject to a suitable boundary condition, to yield the stream-function fields associated with the respective pieces \( \psi_c \) and \( \psi_a \). Homogeneous Dirichlet boundary conditions are employed in these inversions, and although such conditions are not strictly consistent with those of the analytical solution, that is \( \psi \to 0 \) as \( r \to \infty \), the calculation domain is chosen large enough (a square region of side \( 20a \)) that such inconsistency has a minimal effect on the solutions obtained. Figure 5 shows the velocity fields associated with the cyclonic and anticyclonic members of the dipole, \( V_c \) and \( V_a \), respectively, obtained from the piecewise inversions, and their sum the total velocity \( V \). Only a portion of the total calculation domain used for the inversions is shown. These velocity fields are superposed on the total relative-vorticity field to allow assessment of the contributions to dipole motion associated with self- and mutual advections of the component vortices.

It is apparent from Figs. 5(a) and (b) that the predominant component of dipole motion is associated with the mutual advection of each vortex by the flow field of its dipole partner, with self-advection of the vortices being significantly smaller in magnitude. It has been verified that these two components of advection combine to produce a flow across the centres of the component vortices of the dipole of 4.5 m s\(^{-1}\), the specified translation speed. A noteworthy observation regarding the motion of the dipole is that this translation speed is considerably less than the maximum fluid speed of 15.7 m s\(^{-1}\) associated with this feature (Fig. 4(b)), a property that is commonly observed of jet streaks in the upper troposphere. Since the fluid speed maximum is induced by the dipole, the former will move at the speed of the latter. Thus, by viewing the jet streak as a feature in the relative-vorticity (i.e. perturbation PV) field, rather than in the wind field, the difference between the translation speed and the maximum fluid speed may be explained in a balanced framework without the need to invoke an interpretation requiring horizontal divergence (e.g. Carlson 1991, section 15.2).

It was noted in the introduction that the fields of horizontal divergence and vertical motion often are emphasized in the diagnosis of jet streaks. In a preliminary attempt
Figure 5. Fields associated with vorticity inversion of the Lamb dipole solution (7): (a) $V_c$; (b) $V_a$; and (c) $V_c + V_a$ (see text). Total relative vorticity is shown in all panels as in Fig. 4(a). Vectors are plotted every four grid points and the vector wind scale in m s$^{-1}$ is shown at bottom right. Negative values are shown dashed.

to examine the role of horizontal divergence in vortex dipoles of the kind described above, Cunningham (1997, section 3.2.1) employed a numerical shallow-water model. Taking the modon solution to the $f$-plane equivalent barotropic (i.e. quasi-geostrophic shallow-water) model described by Flierl et al. (1980) as input for the model’s initialization procedure (the dynamic initialization scheme of Temperton (1976)), the resultant structure is a vortex dipole (not shown) that, although not an exact solution to the shallow-water equations, displays a quasi-steady evolution in a numerical simulation of the model. The divergence associated with this shallow-water dipole exhibits a four-cell pattern very similar to that described in Fig. 1(a). Numerical experiments (not shown) for a range of Froude numbers characteristic of the mesoscale and synoptic-scale upper troposphere show that the magnitude of this divergence is at least an order of magnitude smaller than the ageostrophic vorticity and reveal the vortex-stretching term in the
shallow-water vorticity equation to contribute less than 5% to the vorticity tendency. Both of these findings, which are consistent with the scale analysis referred to in the introduction, indicate the apparently minor role of the divergence in this solution. This outcome provides additional support to the suggestion that for jet streaks in the upper troposphere associated with vortex features, the divergent circulation may be considered to be a secondary response to the jet streak, rather than a fundamental requirement for the existence and maintenance of the streak.

3. STRUCTURE AND EVOLUTION OF JET STREAKS IN NON-UNIFORM BACKGROUND FLOWS

The results of the previous section suggest that the fluid speed maxima associated with vortex dipoles exhibit characteristic signatures of structure and motion similar to those found in observed jet streaks, and thus that the analytical solutions so far described provide a useful starting point for the dynamical interpretation of the structure and motion of jet streaks. Nevertheless, vortex dipoles in isolation are unable to explain certain characteristic features that appear common to upper-tropospheric jet streaks, such as the anisotropy of the wind field and the asymmetry in the relative-vorticity field, both noted in the introduction. Furthermore, jet streaks typically are not isolated, but are embedded within a larger-scale jet stream, which often is zonally varying or wavelike in character. In this section, we consider the effects on vortex dipoles of large-scale background flows representative of the extratropical upper troposphere, with a view to explaining the observed jet-streak attributes cited above, and to describing simple jet-streak life cycles.

(a) Jet-like background flows

To gain preliminary insight into the effect of a jet-like background flow on a vortex dipole, we shall examine an analytical solution for a dipole of point vortices in a nondivergent barotropic fluid on an $f$-plane. The classical theory of point vortices applied to such a fluid (e.g. Batchelor 1967, section 7.3) describes the evolution of a population of singularities in the relative-vorticity field, and typically assumes that the relative vorticity of any background flow is uniform. However, this uniformity condition may be relaxed to include more general basic states that have piecewise uniform relative vorticity (e.g. DiBattista and Polvani 1998). Although the analysis in such a configuration is potentially complex, a significant simplification results from restricting attention to a steadily translating point-vortex dipole in the presence of a simple zonal background flow that has two regions of uniform relative vorticity separated by a discontinuity. This zonal flow is specified by

$$
\bar{u}(y) = U_0 - \zeta_0 |y|,
$$

where $U_0$ and $\zeta_0$ are constants, and describes a linear jet profile, with maximum speed $U_0$ collocated with the relative-vorticity discontinuity at $y = 0$. Given that the absolute vorticity is the PV for the nondivergent barotropic system, this discontinuity may be considered to provide a rudimentary representation of the extratropical tropopause, since it separates regions of large PV ($= f_0 + \zeta_0$) for $y > 0$ from regions of small PV ($= f_0 - \zeta_0$) for $y < 0$. The idealization of the tropopause as a PV discontinuity is based on the observation that the tropopause is characterized by a band of large EPV gradients separating the high EPV values in the stratosphere from the low EPV values in the
troposphere, and has been employed previously in barotropic frameworks to investigate the behaviour of large-scale waves in the extratropical upper troposphere (e.g. Platzman 1949; Verkley 1994).

A point-vortex dipole may be superposed on the zonal flow given by (11) and a steadily translating solution obtained, provided that the vortices are equidistant about the vorticity discontinuity at \( y = 0 \) and have circulations of equal magnitude but opposite sign. The antisymmetry of the dipole, along with the symmetry of the zonal flow (11), ensures that the vortices translate in the \( x \)-direction uniformly at the same speed and that there is no normal flow at \( y = 0 \), preventing the excitation of waves supported by the discontinuity. The stream function for this configuration is given by

\[
\psi(x, y, t) = -U_0 y + \frac{1}{2} \zeta_0 y |y| + \frac{\Gamma}{4\pi} \log \left( \frac{(x - ct)^2 + (y - d/2)^2}{(x - ct)^2 + (y + d/2)^2} \right),
\]

where \( c \) is the translation speed of the dipole, \( d \) is the distance between the cyclonic and anticyclonic point vortices, and \( \Gamma \) is the magnitude of the circulation of the vortices. Since a point vortex moves with the flow at its location, it can be shown from (12) that the translation speed \( c \) of both vortices, and hence of the dipole, is entirely in the \( x \)-direction and is \( c = U_0 - \zeta_0 d/2 + \Gamma/2\pi d \). Figure 6 shows the stream function \( \psi \) and relative stream function \( \psi + cy \) for \( U_0 = \zeta_0 = 0 \) (Figs. 6(a) and (b)), and for \( U_0 = 15 \, \text{m s}^{-1} \) and \( \zeta_0 = 4 \times 10^{-5} \, \text{s}^{-1} \) (Figs. 6(c) and (d)). In both cases, \( \Gamma = 1.7 \times 10^7 \, \text{m}^2\text{s}^{-1} \) and \( d = 600 \, \text{km} \), resulting in a translation speed \( c \) of 4.51 m s\(^{-1} \) in the first case and 7.51 m s\(^{-1} \) in the second case. It is apparent from comparison of Figs. 6(a) and (c) and of Figs. 6(b) and (d) that the presence of the background flow enhances the anisotropy of the stream function both in a stationary frame and in a frame moving with the dipole. Nevertheless, the singularities present in this solution render the velocity field unrealistic with respect to observation, so that further insight into the role of jet-like background flows will be sought using continuous distributions of vorticity.

The importance of the background flow for the anisotropy of jet streaks suggested by the point-vortex solution is illustrated further by superposing the Lamb dipole, with parameters identical to those in section 2(b), on a jet-like zonal flow \( \tilde{u}(y) \) of the form

\[
\tilde{u}(y) = U_0 \sech^2 \left( \frac{y}{y_0} \right),
\]

and investigating the subsequent interaction in the context of an initial-value problem solved numerically. The numerical model solves (1) using an enstrophy-conserving finite-difference scheme, following Smith et al. (1990). Although \( f \)-plane geometry is adopted in this simulation, the \( \beta \)-effect may be incorporated into the model, in which case the scheme conserves absolute enstrophy. The grid spacing is 20 km, and a leap-frog time-integration scheme with a time step of 240 s is employed. A weak Robert-Asselin time filter (Robert 1966; Asselin 1972) is applied to control the computational mode associated with the leap-frog scheme, and weak biharmonic damping is included to prevent aliasing. The model geometry is that of a midlatitude channel, where \(-L_x < x < L_x\) and \(-L_y < y < L_y\), solid-wall boundary conditions of \( \psi_x = 0 \) are specified at \( \pm L_y \), and periodicity is imposed in the \( x \)-direction. As with the vorticity inversions described in section 2(b), the inconsistency between the boundary conditions of the model and those of the Lamb dipole solution are minimized by adopting a computational domain much larger than the dipole; in this and subsequent simulations we take \( L_x = 4000 \, \text{km} \) and \( L_y = 3000 \, \text{km} \). We choose \( U_0 = 25 \, \text{m s}^{-1} \) and \( y_0 = 750 \, \text{km} \) in (13) as values representative of an extratropical jet stream. For
Figure 6. Fields associated with the point-vortex dipole solution (12): (a) stream function, and (b) relative stream function for the point-vortex dipole in zero background flow ($U_0 = \zeta_0 = 0$; contour interval $1 \times 10^6$ m$^2$s$^{-1}$, negative values dashed); (c) stream function, and (d) relative stream function for the point-vortex dipole in nonzero background flow ($U_0 = 15$ m s$^{-1}$, $\zeta_0 = 4 \times 10^5$ s$^{-1}$; contour interval $1.5 \times 10^6$ m$^2$s$^{-1}$, negative values dashed). Plus signs indicate the locations of the vortices. See text for further explanation.

This choice of parameters, the zonal flow is barotropically unstable to infinitesimal perturbations (e.g. Lipps 1962; Kuo 1973); however, the duration of the numerical simulation (120 h) is comparable with two e-folding times of the most unstable normal mode, and thus growth of this mode from numerical noise is considered unimportant. Simulations of longer duration (360 h) show negligible growth of any modes, further supporting this assertion.

Although both the Lamb dipole and the jet profile (13) individually are steady nonlinear solutions to (1) on an $f$-plane, it is not expected that a superposition of the two also will yield a steady solution. Nevertheless, steady solutions of vortex dipoles in background flows with shear have been found for an equivalent barotropic model by Haupt et al. (1993) using numerical techniques, and it is hypothesized that the same might be true for the case considered here. The numerical simulation in this case exhibits unsteady behaviour that is characterized by an oscillation of the aspect ratio of the vortices, apparently associated with the competing processes of advective shearing by the background flow and of vortex axisymmetrization accompanied by filamentation (e.g. Melander et al. 1987; Smith and Montgomery 1995; Montgomery and Kallenbach 1997). Hence, it is suggested that a steady solution may exist, about which the unsteady numerical solution is oscillating. Such behaviour has been noted in
Figure 7. Fields associated with the Lamb dipole solution superposed on the zonal background flow given by (13): (a) relative vorticity (contour interval $2 \times 10^{-5}$ s$^{-1}$, negative values dashed); (b) fluid speed (contour interval 4 m s$^{-1}$); (c) ageostrophic velocity (vectors plotted every 10 grid points, vector wind scale in m s$^{-1}$ shown at bottom right); and (d) ageostrophic vorticity (contour interval $2 \times 10^{-6}$ s$^{-1}$, negative values dashed). The zero contour has been suppressed in panels (a) and (d) and the value of the first plotted contour is half that of the contour interval.

other cases of interactions between two monopolar vortices by McWilliams (1983) and by Couder and Basdevant (1986).

In an effort to isolate this steady structure, a time-averaging technique proposed by McWilliams (1983) is applied to the above numerical simulation. The time average is taken over a four-day period between 24 h and 120 h of simulation time, and, in an attempt to reduce errors associated with the technique, the structure obtained from application of the time averaging is adopted as the initial condition for a second simulation in which the time average is taken over a three-day period between 0 h and 72 h. (It may be noted that although these averaging periods are chosen arbitrarily, the results to be shown are relatively insensitive to the periods chosen.) The structure obtained from application of the time averaging to the second simulation is shown in Fig. 7. Taking this structure as an initial condition for a third numerical simulation (not
shown) reveals that the dipole translates in the positive x-direction at an approximately uniform speed of 25.5 m s\(^{-1}\) with little change in form.*

It is apparent that the dipole in Fig. 7 is significantly more anisotropic than the Lamb dipole in isolation (cf. Fig. 4). The component vortices of the dipole (Fig. 7(a)) exhibit greater ellipticity than those of the Lamb dipole, and the structure of the fluid speed maximum (Fig. 7(b)) better reflects the localized along-stream variation characteristic of a jet streak than does a dipole in quiescent flow. Moreover, as with the dipole in isolation, the fluid speed maximum translates at a speed (25.5 m s\(^{-1}\)) that is significantly slower than the maximum fluid speed (41.5 m s\(^{-1}\)), consistent with observations of jet streaks. It is also apparent that the ageostrophic circulations (Figs. 7(c) and (d)) possess the structure evident in observations. To illustrate further the anisotropy of this dipole, we show in Fig. 8 the relative vorticity of the structure shown in Fig. 7 with the jet profile (13) subtracted. The resulting structure exhibits a dipole that is contained within an elliptical envelope, whereas for the Lamb dipole (see Fig. 4(a)) this envelope is circular. The small-amplitude ‘tail’ extending upstream in Fig. 8 and the corresponding asymmetry in the zonal direction evident in Fig. 7(a) appear to be artifacts of the numerical technique, since the structure of these features varies with the spatial resolution of the model and with the time interval chosen for the averaging procedure.

\(\zeta \quad (s^{-1})\)

(b) Wavelike background flows

The analytical and numerical evidence presented above suggests that the imposition of a jet-like background flow has the effect of increasing the anisotropy of the wind field for jet streaks associated with vortex dipoles. As noted previously, however, the larger-scale flow in which jet streaks are embedded often exhibits significant zonal asymmetry

* More sophisticated numerical techniques than that employed here are available, such as the Newton–Kantorovich method used by Haupt et al. (1993). Nevertheless, these techniques typically require a severe truncation in spatial resolution for computational efficiency, rendering their implementation impractical for the present application. Given this consideration, the ad-hoc method devised here is deemed satisfactory, since it is the basic structure of the dynamical fields that is of interest, rather than the identification of an exactly steady solution.
and may have a wavelike structure. It is unlikely that steady solutions of vortex dipoles exist in such zonally varying flows, but examination of the resulting unsteady evolution may provide insight into the asymmetry in the relative-vorticity field characteristic of jet streaks and also may suggest an interpretation of jet-streak life cycles. As in the case of jet-like background flows, this unsteady evolution is investigated in the context of an initial-value problem solved using the numerical model described in section 3(a).

The classical paradigm for large-scale wave motion in the extratropical upper troposphere is the Rossby wave (e.g. Pedlosky 1987, section 3.18; Holton 1992, section 7.7.1); a single Rossby mode is a steady nonlinear solution to (1) on a $\beta$-plane. The general solution for Rossby-wave motion in a uniform zonal flow $U_0$ in a channel with periodic boundaries at $x = \pm L_x$ and solid-wall boundaries at $y = \pm L_y$ is given by

$$\psi(x, y, t) = \text{Re} \left\{ \Psi_0 \sin \left( \frac{n\pi}{2L_y} (y + L_y) \right) e^{ik(x-c_{ph}t)} \right\},$$

(14)

where $\Psi_0$ is a constant, $n$ is a positive integer, $k = 2\pi/2L_x$ is the wave number in the $x$-direction, and $c_{ph} = U_0 - \beta/K^2$ is the phase speed in the $x$-direction, with $K^2 = k^2 + n^2\pi^2/4L_y^2$ the total wave number. If $n = 1$ (meridional wave number one) the flow is an idealization of a midlatitude wave train, whereas if $n = 2$ (meridional wave number two) the wave exhibits regions of confluence and diffuence similar to those found in planetary-scale jet stream entrance and exit regions (e.g. Blackburn 1985). Here we consider two simulations that describe the evolution of initial configurations consisting of a vortex dipole (in this case the barotropic modon given by (5)–(6), with $c = 2$ m s$^{-1}$, $a = 750$ km, $k_0a = 4.10$, and $k_0\alpha = 2.22$) superposed on the waveform prescribed by (14) for $n = 1$ and $n = 2$. Although highly idealized, these simulations represent a first step in the investigation of the evolution of mobile jet streaks in the presence of background flows characteristic of the large-scale extratropical circulation, since they allow examination of the evolution of the dipole and its attendant jet streak through a large-scale wave ($n = 1$) or a large-scale jet stream ($n = 2$).

We examine first the $n = 1$ case, and choose $\Psi_0 = 3 \times 10^7$ m$^2$s$^{-1}$ and $U_0 = 10$ m s$^{-1}$; since $L_x = 4000$ km, $L_y = 3000$ km, and $\beta = 1.75 \times 10^{-11}$ m$^{-1}$s$^{-1}$, the above expression for the total wave number yields $K = 9.44 \times 10^{-7}$ m$^{-1}$ and thus $c_{ph} = -9.64$ m s$^{-1}$. The wavelength in the $x$-direction in this case is 8000 km, which corresponds approximately to zonal wave number 4 in midlatitudes. Selection of this background flow as an idealization of planetary-scale wavelike flow would appear to be consistent with the results of Blackmon et al. (1984), who showed that fluctuations with intermediate time-scales (10–30 days) exhibit zonal wave numbers between 4 and 6, and appear to be controlled by two-dimensional Rossby-wave processes. Although waves of such scales in general are quasi-stationary in the atmosphere, it should be noted that the nondivergent barotropic system is invariant to the addition of a uniform zonal flow; hence the choice of $U_0$ does not affect the behaviour seen in the interaction between the Rossby wave and the dipole.

Fields at various times in the simulation are illustrated in Fig. 9; the initial condition in this case (Fig. 9(a)) consists of a barotropic modon superposed on the wave in the crest of the ridge, which subsequently will progress through the wave pattern. The localized maximum in fluid speed associated with the dipole is evident in the initial condition, and henceforth we shall refer to this feature as the jet streak. Also evident are additional maxima that are associated with the Rossby wave and that are located in the inflections between the trough and ridge axes; the existence of these maxima is consistent with the study of Orlanski and Sheldon (1995) cited in the introduction. The presence of localized fluid speed maxima in the wave inflections suggests the possibility that different classes
Figure 9. Fields associated with the $\beta$-plane nondivergent barotropic model initialized with the Rossby-wave solution given by (14) with meridional wave number one: relative vorticity ($10^{-5}$ s$^{-1}$, shaded as indicated), stream function (thin lines, contour interval $5 \times 10^6$ m$^2$s$^{-1}$, negative values dashed), and fluid speed (thick lines, contour interval 5 m s$^{-1}$, value of first plotted contour is 20 m s$^{-1}$) at (a) $t = 0$ h, (b) $t = 24$ h, (c) $t = 48$ h, and (d) $t = 72$ h.

of jet streaks may exist, with some associated with mesoscale coherent vortices and others an inherent part of the planetary-scale wave pattern. Also apparent from Fig. 9(a) is the observation that the large-scale anticyclonic relative vorticity of the Rossby wave enhances the magnitude of the anticyclonic member of the dipole (minimum relative vorticity $-9.44 \times 10^{-5}$ s$^{-1}$) and diminishes the magnitude of the cyclonic member (maximum relative vorticity $5.72 \times 10^{-5}$ s$^{-1}$), such that the dipole appears asymmetric.

After 24 h (Fig. 9(b)), the dipole is located in the inflection between the ridge and the trough, and the jet streak represents a superposition of the fluid speed maxima associated with the dipole and the wave. Since the background relative vorticity at this location is approximately zero, the component vortices of the dipole possess similar relative-vorticity magnitudes ($7.64 \times 10^{-5}$ s$^{-1}$ and $-7.16 \times 10^{-5}$ s$^{-1}$ for the cyclonic and anticyclonic members, respectively). At 48 h (Fig. 9(c)), the dipole is located close to the base of the trough, and it is evident that the maximum relative vorticity of the cyclonic vortex ($9.49 \times 10^{-5}$ s$^{-1}$) is considerably larger in magnitude than the minimum relative vorticity of the anticyclonic vortex ($-5.33 \times 10^{-5}$ s$^{-1}$), due to the contribution of the
large-scale cyclonic relative vorticity associated with the Rossby wave. The maximum fluid speed in the jet streak has decreased from 33.8 m s\(^{-1}\) observed 24 h previously to a value of 26.5 m s\(^{-1}\) at 48 h. This decrease in maximum fluid speed appears to result in part from the lack of coincidence at this time of the dipole with the fluid speed maximum associated with the Rossby wave, and in part from the separation of the constituent vortices due to their self-induced motion associated with the \(\beta\)-effect. This self-induced component of vortex motion, which is often referred to as the \(\beta\)-drift, is north-westward and south-westward for cyclonic and anticyclonic vortices, respectively, and occurs throughout the duration of the simulation. Figure 9(d) reveals that at 72 h the structure of the dipole differs considerably from that seen initially, with the cyclonic member relatively axisymmetric and the anticyclonic member significantly elongated. Nevertheless, the dipole is collocated with the fluid speed maximum in the inflection between the trough and ridge of the Rossby wave, and the total maximum of 33.4 m s\(^{-1}\) at this time is stronger than that seen at 48 h.

It is evident in this simulation that as the dipole travels through the wave, its cyclonic and anticyclonic components are strengthened selectively depending on whether the dipole is located in the trough or the ridge. This behaviour suggests a simple way to view the asymmetry in the relative-vorticity field: as with the anisotropy of the wind field, it is strongly influenced by the structure of the background flow. Moreover, it is apparent that as the jet streak travels through the wave, the maximum in fluid speed fluctuates, primarily in response to the superposition of the velocity fields induced by the dipole and by the wave. Hence, although the jet streak diminishes in intensity as it travels through the base of the trough, the vortex feature that induces the streak is still present, but configured with respect to the larger-scale flow such that the streak is somewhat weakened compared with its intensity when located in the wave inflections. The tendency for jet streaks to be observed relatively infrequently in the bases of troughs may be related to this behaviour.

Since the scale of the Rossby wave is significantly larger than that of the dipole, there is little evidence of the latter feature in the stream-function field: the predominant process governing this evolution is the advection of the dipole by the wavelike background flow, with the wave affected little by the presence of the dipole. Nevertheless, the dipole does not behave as a passive tracer, and several processes common to vortex evolution on a \(\beta\)-plane are observed in this simulation. The first of these is the \(\beta\)-drift noted previously, which would be expected to lead to an increase with time of the separation between the member vortices of the dipole (compare Figs. 9(a) and (c) for potential evidence of this effect). The second is axisymmetrization accompanied by filamentation of the vortices as they are sheared both by the background flow and by each other, behaviour that is readily apparent throughout the evolution, particularly in Figs. 9(c) and (d). The third process is dispersion due to Rossby-wave radiation, a signature of which is the wake apparent in the region surrounding the vortices in Fig. 9(d). Although this process would be anticipated to result in a gradual decrease in the relative-vorticity magnitudes of the member vortices, in the absence of the capability to extract quantitatively the dipole from the background flow, such a decrease in magnitude cannot be confirmed.

In Fig. 10 we show the \(n = 2\) case for which we choose \(\Psi_0 = 1.5 \times 10^7\) m\(^2\)s\(^{-1}\). In this case \(K = 1.31 \times 10^{-6}\) m\(^{-1}\); for illustrative purposes, we take the wave to be stationary (i.e. \(c_{\text{ph}} = 0\)), requiring \(U_0 = 10.21\) m s\(^{-1}\). Regions of confluence and diffuence and an attendant large-scale fluid speed maximum associated with the wave pattern are evident in Fig. 10(a). The barotropic modon in the initial condition is specified to be in the region of confluence upstream of the speed maximum associated
Figure 10. Fields associated with the β-plane nondivergent barotropic model initialized with the Rossby-wave solution given by (14) with meridional wave number two; relative vorticity ($10^{-5}$ s$^{-1}$, shaded as indicated), stream function (thin lines, contour interval 5 $\times$ 10$^{-6}$ m$^2$ s$^{-1}$, negative values dashed), and fluid speed (thick lines, contour interval 5 m s$^{-1}$, value of first plotted contour is 20 m s$^{-1}$) at (a) $t = 0$ h, (b) $t = 24$ h, (c) $t = 48$ h, and (d) $t = 72$ h.

with the wave; the weaker fluid speed maximum associated with the dipole is apparent. After 24 h the dipole has been sheared considerably, and the component vortices are elongated zonally (Fig. 10(b)). Moreover, the fluid speed pattern shows that the superposition of the dipole and the background flow results in a jet streak that is considerably stronger (37.8 m s$^{-1}$) than that associated with the dipole or the wave in isolation (14.0 m s$^{-1}$ and 26.0 m s$^{-1}$, respectively). At 48 h (Fig. 10(c)), the dipole is located in the diffluent exit region of the large-scale jet, and the associated fluid speed maximum has decreased dramatically to a value of 25.1 m s$^{-1}$ compared with 37.8 m s$^{-1}$ observed 24 h previously. In addition, the meridional elongation of the vortices is readily apparent at this time, and at 72 h (Fig. 10(d)) the components of the dipole have been separated and sheared meridionally almost beyond recognition.

Both of the above simulations highlight the importance of examining the relative-vorticity field for the presence of mesoscale coherent vortices that may interact favourably with the large-scale flow to yield a jet streak. In this regard, it is apparent from Figs. 9 and 10 that the inability of the stream function to resolve such small-scale
features, a consequence of the smoothing properties of the inverse Laplacian operator relating the stream function to the relative vorticity, renders the former less appropriate than the latter for visualizing the dynamical features important for jet streaks. Pressure or geopotential-height fields in the atmosphere also suffer from similar difficulties in representing small-scale features, and thus would be expected to be less discriminating than the perturbation PV field in representing the structure and in elucidating the dynamics of jet streaks. Further support for the use of relative vorticity in the barotropic system, or perturbation PV in more general frameworks, for the identification of dynamical features that may induce jet streaks is provided in section 4.

(c) General background flows

Although the Rossby wave has provided much insight into the nature of the extratropical circulation, planetary-scale waves in the extratropics often are observed to propagate along the waveguide provided by the localized PV gradients associated with a large-scale jet stream. Many investigators refer to such features also as Rossby waves; however, we shall restrict this terminology to waves supported solely by the planetary vorticity gradient, and shall refer to waves supported by localized PV gradients as ‘PV waves.’ The interaction between PV waves, which may potentially be unstable, and coherent vortices is likely to be complex, and a detailed examination of the evolution in such situations is beyond the scope of the present study. Nevertheless, it is worthwhile investigating the characteristic behaviour in a case representative of such an interaction.

As noted in the introduction, the vortex dipoles associated with jet streaks typically are asymmetric; moreover, the large-scale jet streams on which these dipoles are superposed typically are also asymmetric, with stronger cyclonic than anticyclonic shear. Several simulations have been performed using the nondivergent barotropic model on both $f$- and $\beta$-planes for initial conditions comprised of symmetric and asymmetric dipoles superposed on an asymmetric jet-like zonal flow (Cunningham 1997, section 4.3). This zonal flow has the form

$$
\tilde{u}(y) = U_0 \begin{cases} 
\text{sech}^2 \left( \frac{y}{y_+} \right), & y < 0, \\
\text{sech}^2 \left( \frac{y}{y_-} \right), & y > 0,
\end{cases}
$$

(15)

where $y_+$ and $y_-$ control the magnitudes of the cyclonic and anticyclonic shear vorticities, respectively. We choose $U_0 = 20$ m s$^{-1}$, $y_- = 1000$ km, and $y_+ = 650$ km; for these parameter values this zonal flow is barotropically stable for $\beta$-plane geometry but unstable for $f$-plane geometry. In an attempt to illustrate the general case involving the interaction between an asymmetric dipole and an unstable zonal background flow, we present here a simulation using the $f$-plane nondivergent barotropic model for an initial condition of a strongly asymmetric vortex dipole superposed on the zonal flow given by (15). The dipole solution employed is described by Flierl et al. (1983); this solution is valid on an $f$-plane and represents a modification to the Lamb dipole. This modified dipole allows for asymmetry between the component vortices and in isolation travels in a circular path of radius $b$. The degree of asymmetry is related to a parameter $\epsilon = a/b$, such that for $\epsilon = 0$ the solution reduces to the symmetric Lamb dipole (7). Parameters are chosen to be identical to those for the Lamb dipole, described in section 2(b), with $\epsilon = 1$.

The initial condition for this case is shown in Fig. 11(a), and fields at 48 h intervals of the subsequent evolution are shown in Figs. 11(b), (c), and (d). Evident in the
Figure 11. Fields associated with the $f$-plane nondivergent barotropic model initialized with a strongly asymmetric vortex dipole superposed on the zonal background flow given by (15): relative vorticity (10$^{-5}$ s$^{-1}$, shaded as indicated), stream function (thin lines, contour interval 5 $\times$ 10$^8$ m$^2$s$^{-1}$, negative values dashed), and fluid speed (thick lines, contour interval 5 m s$^{-1}$, value of first plotted contour is 15 m s$^{-1}$) at (a) $t = 0$ h, (b) $t = 48$ h, (c) $t = 96$ h, and (d) $t = 144$ h.

initial condition is the localized fluid speed maximum associated with the dipole. As the simulation proceeds, an undulation can be seen to develop and amplify in the stream-function field, dominating the large scales of the flow at the end of the simulation. This amplifying undulation apparently corresponds to an unstable PV wave that is growing at the expense of the kinetic energy of the zonal flow. Another notable aspect of this evolution is the separation of the component vortices of the dipole. Such dipole destruction appears to be common to dipoles in the presence of moderate-to-large background shear (e.g. McWilliams et al. 1981). Nevertheless, the component vortices of the dipole still are present and independently are associated with localized maxima in fluid speed, suggesting that monopolar vortices also may be useful in understanding jet-streak behaviour. Further discussion of the relevance of monopolar vortices to jet-streak dynamics will be presented in the following section. The evolution in this simulation is difficult to verify observationally, suggesting a requirement for comprehensive life-cycle studies of jet streaks and the features that induce them in various flow configurations. In anticipation of such studies, atmospheric water-vapour
imagery offers graphic observational evidence suggestive of the structures seen in Fig. 11. In this regard, Fig. 11(d) may be compared favourably with the water-vapour image for 2016 UTC 14 April 1988 presented by Weldon and Holmes (1991, p. 151).

4. SUMMARY AND DISCUSSION

In this paper, we have examined the possibility that characteristic features of a class of polar-front jet streaks can be explained in a balanced framework in terms of the superposition of mesoscale vortex dipoles with a larger-scale background flow. Moreover, motivated by the secondary dynamical importance of the effects of horizontal divergence in the vicinity of these jet streaks, we have investigated the extent to which a nondivergent barotropic framework is capable of describing various aspects of jet-streak dynamics. It was shown that barotropic vortex dipoles are associated with fluid speed maxima that share many characteristics in common with observed polar-front jet streaks. In this regard, Fig. 12 depicts schematically a straight jet streak represented in terms of a barotropic vortex dipole superposed on a uniform zonal background flow. A four-cell pattern of ageostrophic vorticity (Fig. 12(a)), inherent to vortex-dipole features, is able to account for the cross-contour ageostrophic flow in entrance and exit regions seen in observations of jet streaks. Furthermore, the motion of jet streaks relative to the background flow may be explained in terms of the flow induced by the component vortices of the dipole (Fig. 12(b)); assuming that self-advection of the respective vortices is negligible, each vortex moves with the flow at its centre induced by its dipole partner, while between the vortices the jet streak itself is induced by the combined flow due to both members of the dipole. Consequently, the translation speed of the jet streak is the same as that of the dipole, and this speed is considerably less than the flow speed in the core of the streak, a commonly observed property of upper-tropospheric jet streaks. The schematic shown in Fig. 12 may be considered to be the nondivergent barotropic counterpart of the commonly accepted conceptualization of a straight jet streak depicted in Fig. 1, and in this regard the similarity between Fig. 12(b) and Fig. 1(c) deserves emphasis.

In light of the properties summarized above that are common both to observed jet streaks and to vortex dipoles in a nondivergent barotropic framework, it is suggested that such vortex features provide a plausible dynamical representation of the structure and motion of jet streaks. Nevertheless, vortex dipoles in isolation or in uniform zonal background flows are unable to explain certain characteristic features of observed jet streaks, such as the anisotropy of the wind field and the asymmetry in the relative-vorticity field, both of which are apparent in Fig. 2. The results presented in section 3 suggest that these characteristic features may be explained, at least in part, by the larger-scale background flow: a jet-like background flow enhances the along-stream anisotropy of the fluid speed maxima associated with vortex dipoles, whereas a wavelike background flow affects the symmetry properties of the dipole. In addition, the interactions between vortex dipoles and these background flows, as described by numerical simulations of the nondivergent barotropic model, appear to exhibit similarities to the life cycles of observed jet streaks as they progress through planetary-scale jet streams and waves.

The approach adopted in this paper involving use of the barotropic model has been successful in describing the essential dynamics of jet streaks; nevertheless, it is evident that a number of issues relating to jet-streak behaviour have yet to be addressed. In particular, it is immediately apparent that the barotropic model does not incorporate horizontal divergence or vertical structure. Hence, it is suggested that consideration of vortex features with vertical structure may yield further insight into the dynamics of jet
streaks and allow the incorporation of divergent and vertical circulations into the vortex interpretation proposed in this paper. Moreover, whereas descriptions of the motion and evolution of coherent vortices in the presence of a variety of background flows are applicable to jet streaks in a barotropic framework, these descriptions may differ significantly for jet streaks in a stratified atmosphere. Of particular interest in this regard is the evolution of coherent vortices and the jet streaks that they induce in vertically sheared environments such as are commonly observed in the extratropics, and the role of divergent circulations in maintaining vertical coherence in such environments. These limitations of the present study are currently under investigation using a stratified quasi-geostrophic model.

A further limitation of the present study concerns its restriction to the consideration of vortex dipoles. Although dipolar structures are observed, it is common for these features to be significantly asymmetric (e.g. Fig. 2), with strong cyclonic but weak anticyclonic components, and in some instances the relative-vorticity field associated
with jet streaks is seen to have a monopolar structure (e.g. Hakim 2000). In this regard, it is suggested that symmetric vortex dipoles should be interpreted as idealized representations of jet streaks, since such symmetry relies on the existence and subsequent pairing of cyclonic and anticyclonic vortices of similar magnitude in the presence of a symmetric zonal background flow. Given the noted disparity in magnitudes between cyclonic and anticyclonic vortices, it is proposed that the infrequency of straight jet streaks in the extratropical upper troposphere noted in the introduction may be related to the small probability of a symmetric dipole pairing. Furthermore, as is evident from the simulations described in sections 3(b) and (c), vortex dipoles often are decoupled in the presence of non-uniform background flows. Hence it is suggested that a more realistic description of jet streaks might be formulated in terms of the interaction between monopolar vortices and non-uniform background flows, with dipole pairing possible but likely to be asymmetric and transient. This interpretation is consistent with the observation that polar-front jet streaks typically are found in association with 'short-wave troughs,' which appear to correspond to cyclonic monopolar vortices (Hakim 2000).

An important implication of the present study is that an understanding of jet-streak dynamics may be gained in the PV framework by viewing jet streaks as being induced by the dipole in the relative-vorticity field (i.e. the perturbation PV applicable to the nondivergent barotropic system). This interpretation leads to the speculation that the life cycles of a class of polar-front jet streaks may be understood in terms of the life cycles of coherent vortices and their interactions with jet-like and wavelike background flows, suggesting a shift in dynamical emphasis from jet streaks to the vortex features that induce them. It is proposed that tracking vortex features in the upper troposphere using the quasi-conserved PV field may be preferable to the traditional method of tracking jet streaks using the nonconserved wind and geopotential height fields. In this regard, the often observed transient nature of jet streaks may in part result from their representation in terms of these nonconserved fields, whereas the possibility exists that the corresponding vortex features in the quasi-conserved PV field are considerably longer lived.

The possibility that the extratropical upper troposphere is populated by mesoscale coherent vortices, which in turn may be the dynamical features responsible for some jet streaks and short-wave troughs, also has ramifications for the dynamics of the origin of the latter two features. Indeed, the origin of jet streaks and short waves has been a topic of long-standing interest in synoptic meteorology given their role as upper-level precursors to surface cyclogenesis (e.g. Petterssen 1956, section 16.7; Palmén and Newton 1969, section 11.2; Petterssen and Smebye 1971). It is suggested that by viewing jet streaks as the manifestation in the wind field of interactions between monopolar or dipolar vortices and their background flows, the question of the origin of jet streaks may be redirected to predicting such interactions. Predicting these interactions would appear to require consideration of the origin of coherent vortices in the extratropical upper troposphere and lower stratosphere, a complex issue that has at its root the theory of geophysical turbulence. The emergence of coherent vortex structures in turbulent flows in numerous idealized dynamical frameworks has been documented extensively (e.g. Fornberg 1977; McWilliams 1984; McWilliams et al. 1994; Bartello et al. 1994; Vallis et al. 1997), and remains a subject of intensive investigation in the turbulence community. Accordingly, the interpretation of jet-streak life cycles in a general framework of turbulence is beyond the scope of the present study, but is believed to provide a promising direction for future research.

Also of interest to jet-streak dynamics, but not considered here, is the role and importance of unbalanced flow in the form of inertia-gravity waves. A number of
investigations have asserted that jet streaks are characterized by strongly unbalanced flow and that inertia-gravity waves are central to the dynamics of these features in the atmosphere (e.g. Kaplan and Paine 1977; Houghton et al. 1981; Van Tuyl and Young 1982; Uccellini and Koch 1987; Kaplan et al. 1997; Weglarz and Lin 1997). Indeed, considerable observational evidence suggests that jet streaks are preferred regions for significant inertia-gravity-wave activity (e.g. Uccellini and Koch 1987). Despite this evidence, there appear to be few dynamical studies that examine the generation of unbalanced motions from balanced initial states and the subsequent influence of these motions on the evolution of the balanced flow. It appears from the present study that the fundamental dynamics of jet streaks can be explained in a balanced framework, and it is possible that even rapidly evolving jet streaks in the upper troposphere may be described meaningfully by balanced dynamics. Nevertheless, the extent to which inertia-gravity waves are responsible for large departures from balanced evolutions, and the flow regimes in which such departures are likely to occur, remain the subject of future research.

ACKNOWLEDGEMENTS

Stimulating discussions with Dr Gregory Hakim during the course of this work are gratefully acknowledged. We thank Drs Roger Smith and Wolfgang Ulrich for providing the barotropic numerical model employed in this study. We are also grateful to Drs Michael Montgomery and Louis Uccellini and to an anonymous referee for comments on an earlier version of this paper that clarified the presentation of our results. This research was supported by the National Science Foundation through Grants ATM-9421678 and ATM-9818088, awarded to the University at Albany, State University of New York.

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Unknown, page number 1 of 1
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