Reply to comments by Greg M. McFarquhar on ‘Parametrization of effective sizes of cirrus-cloud particles and its verification against observations’
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In his comments, McFarquhar (2000) (McF00) claimed that the parametrization of cirrus-cloud effective size presented by Sun and Rikus (1999) (SR99) based on the parametrization of ice-crystal size distributions developed by McFarquhar and Heymsfield (1997) (MH97) is inconsistent with both the MH97 scheme and the observed data on which MH97 is based. The basis for this argument is that (i) the MH97 scheme is based on direct measurements of cross-sectional area and estimates of ice-crystal mass obtained from identified particle shapes; (ii) that direct use of the MH97 scheme to determine the effective size of cirrus particles can conserve both the volume and cross-sectional area of particles; and (iii) that the SR99 scheme is obtained by applying the MH97 size distribution to ice particles having a single hexagonal shape and the results will not conserve the observed volume and area of particles. For this reason McFarquhar created his parametrization of effective size in terms of ‘a straightforward application of MH97’. He claimed that this scheme is more physically meaningful than SR99 and recommended its use.

This issue is important as it involves interpretation of both the MH97 parametrization and the SR99 scheme. An inappropriate interpretation of either may cause misunderstanding and confusion. In order to clarify this issue, in this reply a brief overview of both schemes may be necessary. In MH97, the observed crystal size distributions are expressed as a function of the diameter of mass-equivalent spheres determined from identified particle shapes and then parametrized in terms of ice-water content (IWC) and cloud temperature. A ratio of the integrated cross-sectional area from observations to that from the parametrized size distribution based on the equivalent spheres is further parametrized as a function of IWC and temperature. Thus, the cross-sectional areas determined using the size distribution of equivalent spheres can be corrected, in terms of this ratio, to approximate the observed values. If the effective sizes of cirrus-cloud particles are determined using IWC and the corrected cross-sectional areas as suggested by McFarquhar the results will conserve both the volume and surface area in the sense of the measurements.

In the SR99 approach, we calculate the effective sizes defined by Fu (1996) using the MH97 size distribution function by assuming that all crystals are hexagonal, and then parametrize the results in terms of IWC and temperature. This parametrization is developed specifically for use in conjunction with a radiative parametrization scheme for hexagonal crystals. It may be applied to other schemes but an appropriate adjustment must be made to allow for the difference in particle shape. If ice crystals were all hexagonal in the real world, then our treatment would be perfectly consistent with the MH97 parametrization and the observed data. Since the observed proportion of hexagonal columns is only about 18% of the total particles as found by McFarquhar, he considers this approach to be inconsistent with their parametrization and the original data.

It is certainly true that the cloud radiative properties are proportional to the ratio of the observed IWC to cross-sectional area. However, they are even more sensitive to the cloud particle shapes, especially with respect to the asymmetry factor (Macke et al. 1998). It is impossible to take realistic ice crystal shapes into account within the present theoretical capability. Using idealized particle shapes for the calculation of radiative properties of ice clouds is a common practice (Takano and Liu 1989; Rockel et al. 1991; Ebert and Curry 1992; Sun and Shine 1995; Fu 1996). If a radiative parametrization scheme uses the effective size of ice crystals as a predictor, the shape of the crystals corresponding to the effective size must be consistent with that on which the parametrization is based. In other words, an effective size for spherical particles cannot be directly used with a radiative parametrization scheme for hexagonal particles. Although the effective sizes determined using the method suggested by McFarquhar may be more realistic they may not be appropriate for use in conjunction with a particular radiative parametrization scheme, such as that for hexagonal particles, and this is the key issue to be clarified in this reply.

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The main purpose of the SR99 study is to apply the MH97 parametrization together with a radiative parametrization scheme developed by Fu (1996) for hexagonal particles to numerical weather prediction (NWP) and climate models. Our first attempt was exactly the same as suggested by McFarquhar in his comments. We found, however, that this ‘direct application of the MH97 scheme’ does not provide satisfactory results (see below). We also tried to convert the cross-sectional area of equivalent spheres to that of hexagonal columns and then calculate the effective sizes. These results were not encouraging either (these have been shown in SR99). Finally, we chose to use the MH97 function to calculate the effective sizes for hexagonal particles. In this case we only use the crystal size distributions and ignore the actual particle shapes. The use of a particle size distribution function in this way is very common in studies of radiative properties of cirrus clouds (Roeckel et al. 1991; Ebert and Curry 1992; Fu 1996). Although this treatment does not conserve the observed areas as McFarquhar emphasized, it does for ‘hexagonal particles’ as the definition of the effective size permits this (Fu 1996). More importantly, it is consistent with the radiative parametrization scheme we are using.

We have shown sensitivity studies and experimental results in SR99 to justify our approach. It may be more helpful to compare the radiative properties determined using both the McF00 and SR99 schemes to demonstrate the importance of keeping the effective sizes consistent with the radiative parametrization scheme. For this reason, the effective sizes are calculated using the McF00 parametrization for the observational case of the International Cirrus Experiment (ICE217) (see Francis et al. (1994) for details). The coefficients involved in the McF00 scheme (Eq. (1) in his comments) are determined by linear interpolation from cloud temperature. These effective sizes are input to the single-scattering parametrization for hexagonal particles described in SR99 to calculate the cloud optical properties of ICE217. The results of irradiances from this test calculation together with those from the SR99 scheme are shown in Fig. 1. It is seen that the use of the McF00 scheme (MH97) clearly overestimates the cloud reflection. This is mainly because the effective sizes determined using the McF00 scheme are systematically smaller than those determined by the SR99 scheme. This result is consistent with McFarquhar’s finding in his comments. In Table 1 we list the observed cloud temperature, IWC and effective radius ($r_e$) defined by Francis et al. (1994) (Eq. (4) in SR99). We refer to $r_e$ as the effective radius because the definition of IWC is based on a sphere. $r_e$ can be converted to the effective size of hexagons using Eq. (8) in SR99. We list this converted value in column 5 of Table 1. The SR99 results are given in column 6 and those of McF00 in column 8. It can be seen that the McF00 results are close to the observed effective radius $r_e$ shown in column 4, which is not surprising as this is what they are designed to be. But compared with the effective size of hexagonal particles their values are too small, causing more reflection if the optical properties for hexagonal particles are used. In SR99 we have shown that the observed $r_e$ has to be converted to $D_e$ in order to obtain accurate irradiances in the model calculation, which is evidence that even the observed effective radius cannot be directly applied to the radiative properties scheme for hexagonal particles. It is clear that if we apply the same conversion factor to the McF00 scheme the modelled results in column 8 and the irradiances in Fig. 1 will be significantly improved. The justification for such a conversion, however, is beyond the scope of this reply.

It should be emphasized that the above comparison may not be fully justified as the McF00 scheme is only applicable to the tropics, whereas, the ICE217 cloud was measured in mid latitudes. However, the observational results presented by Macke et al. (1998) showed that the effective sizes of ice clouds are generally larger in the tropics than in mid latitudes. The application of the McF00 scheme to the mid latitudes in this case should result in an overestimate of the effective sizes. The reason for the underestimate shown above is clearly not due to the spatial region. It is instead due to the inconsistency between the definition of the effective size and the single-scattering properties.

McFarquhar criticized the method we used to test our approach against some observations and pointed out that the MH97 parametrization cannot be applied to single-crystal habit separately. It is a good suggestion to perform a comparison between the modelled results and observations in a consistent manner. Unfortunately, we are unable to work in this way as the observed size-distribution data are not available. Nevertheless, we have already made our treatment as consistent with the measurements as possible. For instance, we converted all assumed particle shapes to the equivalent-area spheres in the comparison with the observed results of Kinne et al. (1997). McFarquhar produced a Fig. 1 showing a comparison of size distribution determined using the MH97 parametrization of the size distributions of columns, rosettes, poly crystals and small crystals determined from measurements. He used this figure to demonstrate that the MH97 parametrization cannot be applied to single crystal habit. This implies that the applications of the MH97 parametrization in SR99 are invalid. However, their results do not convince the author as the number distribution from the parametrization represents that of total particles whereas those of a single habit denote the distributions for the sub-group. It may be more appropriate to convert all observed particles into each of these habits and compare with the MH97 parametrization for the same habit. Such a diagram will be useful to verify the treatment of SR99.
TABLE 1. EFFECTIVE SIZES DETERMINED FROM OBSERVATION AND ESTIMATED USING
DIFFERENT PARAMETRIZATIONS

<table>
<thead>
<tr>
<th>Height</th>
<th>T</th>
<th>IWC</th>
<th>r_e(obs.)</th>
<th>D_e(obs.)</th>
<th>D_e(SR)</th>
<th>D_e(MSR)</th>
<th>D_e(McF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>km</td>
<td>K</td>
<td>g m⁻³</td>
<td>μm</td>
<td>μm</td>
<td>μm</td>
<td>μm</td>
<td>μm</td>
</tr>
<tr>
<td>8.68</td>
<td>232.3</td>
<td>2.49e⁻⁰²</td>
<td>29.90</td>
<td>46.03</td>
<td>57.95</td>
<td>66.80</td>
<td>25.98</td>
</tr>
<tr>
<td>7.76</td>
<td>238.1</td>
<td>3.05e⁻⁰²</td>
<td>50.50</td>
<td>77.75</td>
<td>66.82</td>
<td>72.09</td>
<td>25.96</td>
</tr>
<tr>
<td>6.87</td>
<td>244.4</td>
<td>6.43e⁻⁰²</td>
<td>49.70</td>
<td>76.52</td>
<td>85.84</td>
<td>88.99</td>
<td>44.27</td>
</tr>
<tr>
<td>6.87</td>
<td>243.8</td>
<td>6.33e⁻⁰²</td>
<td>55.00</td>
<td>84.68</td>
<td>84.77</td>
<td>88.41</td>
<td>44.29</td>
</tr>
<tr>
<td>5.31</td>
<td>253.6</td>
<td>8.74e⁻⁰²</td>
<td>56.60</td>
<td>87.14</td>
<td>104.77</td>
<td>99.87</td>
<td>44.07</td>
</tr>
<tr>
<td>5.31</td>
<td>252.6</td>
<td>1.40e⁻⁰¹</td>
<td>61.10</td>
<td>94.07</td>
<td>113.19</td>
<td>111.61</td>
<td>71.67</td>
</tr>
<tr>
<td>4.10</td>
<td>259.7</td>
<td>2.78e⁻⁰¹</td>
<td>94.10</td>
<td>144.88</td>
<td>144.88</td>
<td>136.12</td>
<td>71.02</td>
</tr>
<tr>
<td>4.10</td>
<td>257.9</td>
<td>2.07e⁻⁰¹</td>
<td>98.70</td>
<td>151.96</td>
<td>132.30</td>
<td>125.66</td>
<td>71.18</td>
</tr>
</tbody>
</table>

Also shown are measured ice-water content (IWC) and temperature. See text for further explanation.

Figure 1. Comparison between observed and modelled shortwave irradiances using two effective size (D_e) parametrizations: from Sun and Rikus (1999) (SR99) and from McFarquhar and Heymsfield (1997) (MH97). Also shown is the modified SR99 parametrization, MSR99.

In addition to the issue of consistency, McFarquhar also found a singular point in the SR99 scheme at an IWC of 1.0 g m⁻³ and a temperature dependency in the SR99 scheme which is opposite to that of the McF00 scheme. The finding of the singular point is highly appreciated. Although the IWC value of 1.0 g m⁻³ is rare in the atmosphere and our parametrization is valid in the range of IWC between 0.0001 and 0.3 g m⁻³ as indicated in SR99 it is a deficiency in the scheme if it is extended beyond 0.3 g m⁻³. This singular point occurs to the coefficient a(IWC) due to the use of a power law with a logarithm function as an argument. To correct this deficiency we have replaced both coefficient functions a and b by a power law with IWC as the argument and performed a new fitting with the results given below.

\[
a(IWC) = 45.8966(IWC)^{0.2214},
\]

\[
b(IWC) = 0.7957(IWC)^{0.2535}.
\]

The rest of the parametrization remains the same. The accuracy of the modified scheme is slightly degraded compared with the MH97 parametrization. But compared with the observed effective sizes and radiative irradiances it still produces satisfactory results. The effective sizes determined using the modified scheme are listed in column 7 of Table 1 and the irradiances modelled for the case of ICE217 are shown in Fig. 1. The plot of D_e from the modified SR99 versus that of McF00 is shown in Fig. 2 for a comparison with Fig. 3 in McF00.

The effective sizes in the SR99 scheme increase with cloud temperature for a constant IWC; this feature is the direct result of the MH97 parametrization if the effective sizes are determined using that
function in conjunction with assumed particle shapes. This temperature dependency is clearly shown in Fig. 6 of SR99. The McF90 scheme depends on the cross-sectional area ratio of \( A_C \) to \( A_S \), where \( A_C \) is from observation and \( A_S \) from the MH97 size distribution function. This ratio increases with cloud temperature for the same IWC, leading to a decrease in the effective size with temperature. This paradox is the second reason we rejected the use of the cross-sectional area ratio of MH97 in the development of the SR99 scheme as we prefer to accept our results based on knowledge that the large ice crystals tend to fall to the lower portion of cloud where the temperature is usually high.

Finally, we want to repeat the emphasis we have given in the SR99 paper that our studies were confined to the regular hexagonal columns only, although we have suggested that the SR99 scheme may be applicable to an ice-cloud optical parametrization based on the spherical effective radius with the aid of Eq. (8) of SR99. To apply the SR99 scheme to other types of optical parametrization is entirely up to the user. We have also concluded that the MH97 parametrization may be applicable to the mid latitudes with some correction based on our test results. This does not necessarily mean that ‘the original MH97 scheme can be extended to the mid latitudes’ as McFarquhar quoted. There is certainly a need to verify this scheme with more observed data.

**REFERENCES**


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