Estimation of entrainment rate in simple models of convective clouds

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SUMMARY
A method for estimating lateral entrainment rate in clouds is suggested, linking its magnitude to buoyant production of vertical kinetic energy within a cloud updraught. In single-column model studies the formulation captures the high values of entrainment seen in shallow convection and the values an order of magnitude lower in cases of deep convection, as suggested by observational studies and large-eddy model simulations, together with a realistic vertical variation. Comparison is made with other recent attempts to parametrize entrainment.

KEYWORDS: Convection Entrainment Parametrization

1. INTRODUCTION

The concept of an entrainment rate† is commonly used to describe the inflow of air into cumulus clouds and is a crucial parameter in the mass-flux approach to the parametrization of moist convection. However, its specification has been rather ad hoc, referring back to studies of plumes in water-tank experiments and commonly being taken to be inversely proportional to cloud radius (Simpson and Wiggert 1969). Recent work using large-eddy simulations (LES) of ensembles of cumulus clouds have indicated that the values suggested by such methods are an order of magnitude smaller than found in shallow cumulus clouds (Siebesma and Cuijpers 1995). In fact this has been known since the 1970s, when analysis of data from the BOMEX and ATEX experiments‡ provided similar insights (e.g. Albrecht 1979).

Studies of the parametrization of shallow convection have suggested that increased entrainment rates, matching those diagnosed from LES models, improve simulated boundary-layer thermodynamic structure (Siebesma and Holtslag 1996). However, entrainment rate appears to be a highly variable quantity. Siebesma and Cuijpers (1995) suggest that the entrainment rate for convection in BOMEX varies with height, from $2.5 \times 10^{-3}$ m$^{-1}$ near cloud base to $1.5 \times 10^{-3}$ m$^{-1}$ near cloud top. For an ATEX case Bretherton and Pincus (1995) diagnosed a rate of $1.5 \times 10^{-3}$ m$^{-1}$ with little variation with height. Grant and Brown (1999) suggest that rates vary by a factor of two between shallow convection observed during BOMEX and a case of shallow convection observed over the North Sea (Smith and Jonas 1995). In a study of an ensemble of deep convective clouds simulated by a cloud-resolving model (CRM), Lin (1999) demonstrates a wide variation of entrainment rates for clouds of different depths. Clouds with lower cloud-top heights had the largest entrainment rates. Entrainment was also found to reduce with height through the depth of a cloud.

It is desirable for the formulation of entrainment used in convective parametrizations to reflect such variations, both in the vertical and from case to case. Siebesma (1997) suggested, on the basis of scaling arguments, that entrainment rate might vary inversely with height above cloud base. He also showed that the formulation of entrainment suggested by Nordeng (1994) for use in deep convective parametrization could be applied with some success to shallow convection. Grant and Brown (1999) considered

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† Here ‘entrainment rate’ is equivalent to ‘fractional entrainment rate’ defined by Arakawa and Schubert (1974). Detrainment rate is used similarly.
‡ Barbados Oceanographic and Meteorological EXperiment and Atlantic Tropical EXperiment, respectively.
several cases of shallow convection of differing depth, finding that the variation of entrainment rate between different cases could not be explained by considering the depth of the cloud alone. Using scaling arguments analogous to those applied to dry convective boundary layers, they suggest that entrainment rate can be estimated from the rate at which turbulent kinetic energy is generated by buoyancy in conjunction with the depth of the cloud layer and the cloud-base mass flux. Using statistical analysis of CRM simulations, Lin (1999) also links entrainment to parcel buoyancy, suggesting that, at any level within cloud entrainment is proportional to parcel buoyancy to the power of $-4/3$.

In the study described in this paper, the link between entrainment and the buoyant production of kinetic energy is further explored within the framework of the mass-flux approach to convection. Grant and Brown (1999) related entrainment to an undilute parcel ascent, but here the entrainment rate is estimated within the context of a dilute ascent. The resulting formulation is similar to that derived from a more turbulent viewpoint of convection and provides further understanding of how entrainment varies with height. The entrainment formulation is used in a mass-flux convection scheme in single-column model (SCM) simulations for cases of deep and shallow convection. Predicted entrainment rates are compared to estimates derived from parallel LES and CRM simulations. The entrainment parametrization is found to predict both the magnitude and vertical variation of entrainment between cases.

2. ESTIMATION OF ENTRAINMENT RATE

(a) Mass-flux form of cloud-mean vertical-velocity equation

The starting point for the derivation of the entrainment parametrization is the estimation of the mean in-cloud vertical velocity. The vertical-momentum equation (in two dimensions $(x, z)$, where $z$ is height, for brevity) is

$$\frac{\partial \rho w}{\partial t} + \frac{\partial \rho u w}{\partial x} + \frac{\partial \rho w w}{\partial z} + \frac{\partial p'}{\partial z} = \rho g \left( \frac{T_v'}{T_v} - l \right) + \rho g \frac{p'}{p} = 0, \quad (1)$$

where $u$ is horizontal velocity, $w$ is vertical velocity, $\rho$ is density, $p$ is pressure, $g$ is the acceleration due to gravity, $T_v$ is virtual temperature and $l$ is condensed water. Primed quantities are deviations from a hydrostatic basic state indicated by an overbar. Following Gregory and Miller (1989) and applying an average operator defined by

$$\bar{\phi}' = \frac{1}{A} \int_A \phi \, da = \sigma \bar{\phi}^c, \quad (2)$$

where $c$ denotes the whole cloud, $A$ is the domain area, $\sigma$ is the fractional cloud area (= $a_c/A$, with $a_c$ being the cloud area) and $\bar{\phi}^c$ is an average over the cloud, then the cloud-averaged vertical-momentum equation can be written as

$$\rho \sigma \frac{\partial \bar{w}^c}{\partial t} + \left\{ \rho \bar{w}^c - (\rho w)_b \right\} \frac{\partial \sigma}{\partial t} + \sigma \left( \frac{\partial \rho u w}{\partial x} \right)^c + \left( \frac{\partial \rho w^2}{\partial z} \right)^c - (\rho w^2)_b \frac{\partial \sigma}{\partial z} = \sigma \left( \frac{\partial p'}{\partial z} \right)^c - \rho \sigma g \left( \frac{T_v'}{T_v} - l \right)^c + \rho \sigma g \left( \frac{p'}{p} \right)^c = 0, \quad (3)$$

where $(\phi)_b$ refers to the value of $\phi$ at the cloud boundary.
Writing the vertical velocity at any point within a cloud as a combination of the mean cloud vertical velocity and a perturbation
\[ w = \bar{w}^c + w', \]
the cloud-averaged vertical-velocity equation can be rewritten as
\[
\rho \sigma \frac{\partial \bar{w}^c}{\partial t} + \left[ \rho \bar{w}^c \frac{\partial w'}{\partial t} + \sigma \left( \frac{\partial \rho u}{\partial x} \right)^c - \rho \bar{w}^2 \frac{\partial \sigma g}{\partial z} \right] + \left( \frac{\partial \rho \bar{w}^c}{\partial z} \right) = 0. \tag{5}
\]
Similarly the mass-continuity equation can be averaged over the cloudy part of the domain to give
\[
\left\{ \sigma \left( \frac{\partial \rho u}{\partial x} \right)^c + \rho \frac{\partial \sigma}{\partial t} - \rho \bar{w} \frac{\partial \sigma}{\partial z} \right\} + \left( \frac{\partial \rho \bar{w}^c}{\partial z} \right) = \rho \frac{\partial \sigma}{\partial t}. \tag{6}
\]
(This is similar to the form used by Gregory and Miller (1989) but there the time-dependent term on the left-hand side (l.h.s.) was neglected in error. Addition of the \( \rho \partial \sigma / \partial t \) term to both sides of the continuity equation gives a null change but allows the entrainment and detrainment terms to be defined similarly in the vertical-velocity and continuity equations.) In the mass-flux representation of convection the terms on the l.h.s. of Eqs. (5) and (6) concerning fluxes across the cloud boundary (rate of change of cloud area, horizontal flux divergence and the vertical variation of the cloud area with height) are represented by entrainment and detrainment fluxes
\[
-\rho \sigma \epsilon \bar{w}^c \bar{w} + \frac{\rho \sigma \delta \bar{w}^c \bar{w}^c}{\rho} + \frac{1}{\rho} \left( \frac{\partial \rho \bar{w}^c}{\partial z} \right) + \frac{1}{\rho} \left( \frac{\partial \rho \bar{w}^2}{\partial z} \right)
+ \frac{\sigma}{\rho} \left( \frac{\partial p'}{\partial z} \right) - \sigma g \left( \frac{T_v}{T_v} - l \right) + \sigma g \left( \frac{p'}{\rho} \right) = 0, \tag{7}
\]
\[-\rho \sigma \epsilon \bar{w}^c + \rho \sigma \delta \bar{w}^c + \left( \frac{\partial \rho \bar{w}^c}{\partial z} \right) = 0, \tag{8}
\]
where \( \epsilon \) is the entrainment rate and \( \delta \) is the detrainment rate. It is assumed that air entrained into the cloud has the properties of the cloud environment \( \bar{\varphi} \), and air detrained from the cloud has the mean properties of the cloud. Also time variation of in-cloud fields is neglected.

Combining Eqs. (7) and (8) to eliminate the detrainment rate and assuming that the vertical velocity within a cloud is much larger than in the cloud environment, the rate of change of mean cloud vertical velocity with height is
\[
\frac{1}{2} \frac{\partial \bar{w}^2}{\partial z} = g \left( \frac{T_v}{T_v} - l \right) - \frac{1}{\sigma} \left( \frac{\partial \rho \bar{w}^2}{\partial z} \right) - \frac{1}{\rho} \left( \frac{\partial p'}{\partial z} \right) - g \left( \frac{p'}{\rho} \right) - \bar{w}^2 \epsilon. \tag{9}
\]
Several processes are seen to affect the mean cloud vertical velocity—buoyancy, pressure perturbations, the growth of turbulent kinetic energy within the cloud and the entrainment of environmental air. Estimation of all these terms within the context of a
typical plume model used in mass-flux schemes is difficult. However, prior to the advent of CRMs several authors attempted to derive simplified versions of Eq. (9) in order to estimate vertical velocities in cumulus clouds. For example, Simpson and Wiggert (1969) suggested that the vertical variation of cloud-mean vertical velocity could be estimated by

$$\frac{1}{2} \frac{\partial \overline{w^c}^2}{\partial z} = \frac{g}{1 + \gamma} \left( \frac{T_v'}{T_v} - l \right)^c - \beta C_d \varepsilon \overline{w^c}^2 - \overline{w^c}^2 \varepsilon. \quad (10)$$

The effect of parcel buoyancy is reduced by a factor of $1/(1 + \gamma)$ to account for the effect of the growth of turbulence as a parcel ascends, Simpson and Wiggert (1969) suggesting a value of 0.5 for $\gamma$. The second term on the right-hand side (r.h.s.) is a drag term which can be interpreted as representing the effect of pressure perturbations, the third and fourth terms on the r.h.s. of Eq. (9). The drag is taken to be proportional to the entrainment rate $-\beta C_d \varepsilon$, with values for $\beta$ and the drag coefficient $C_d$ set to 1.875 and 0.506, respectively.

Here a similar approach is adopted, with Eq. (9) being approximated as

$$\frac{1}{2} \frac{\partial \overline{w^c}^2}{\partial z} = a g \left( \frac{T_v'}{T_v} - l \right)^c - b \delta \overline{w^c}^2 - \overline{w^c}^2 \varepsilon, \quad (11)$$

where $a$ and $b$ are coefficients.

As in Simpson and Wiggert (1969), buoyancy is reduced to account for turbulence within the updraught. The effects of the pressure perturbations within the cloud are also approximated by a drag term, although unlike Simpson and Wiggert’s formulation this is linked to detrainment rather than entrainment. This results from the different treatment of entrainment here as compared to that of the cloud model of Simpson and Wiggert. They assumed that for a given cloud type the entrainment rate was constant with height, while in this study (as will be demonstrated later) entrainment rate decreases through the cloud layer, becoming small in the upper troposphere in the case of deep convection. Using the entrainment rate to describe the pressure-perturbation terms results in small amounts of drag and the growth of excessive vertical velocities. Detrainment rates in the upper troposphere represent outflow from convective clouds, being larger than entrainment, and their use in representing the pressure terms in the vertical velocity equation provides greater ‘drag’ to offset acceleration due to positive buoyancy in the upper parts of deep convective clouds. Although ad hoc, some justification for such a treatment of the pressure terms can be drawn from List and Lozowski (1970) who noted that strong vertical gradients in pressure are associated with horizontal accelerations in the inflow and outflow regions of convective clouds, a process which entrainment and detrainment aim to represent. Coefficients $a$ and $b$ are chosen by comparing SCM simulations against available observations and LES/CRM studies for a number of cases. Values are $1/6$ and $1/2$ respectively.

(b) Entrainment-rate parametrization

From Eq. (11), entrainment acts to reduce acceleration due to positive buoyancy. In a similar manner to Simpson and Wiggert’s treatment of the effects of turbulent kinetic energy on buoyancy by a scaling parameter, it is assumed that entrainment acts to reduce the positive buoyancy of a parcel by a factor $C_e$, i.e.

$$\varepsilon \overline{w^c}^2 = C_e a g \left( \frac{T_v'}{T_v} - l \right)^c. \quad (12)$$
Dividing Eq. (12) by $\bar{w}^2$ provides an estimate of entrainment rate.

The physical basis of this formulation is seen by multiplying Eq. (12) by the cloud mass flux $M_c$ and integrating over the cloud depth, denoted by subscript cld, to give

$$\int_{cld} \epsilon M_c \bar{w}^2 \, dz = C_e ag \int_{cld} M_c \left( \frac{T_v'}{T_v} - 1 \right) \, dz. \quad (13)$$

The integral on the r.h.s. of Eq. (13) is proportional to the 'cloud work function' as defined by Arakawa and Schubert (1974), the rate at which kinetic energy is generated within the cloud layer by buoyancy. The l.h.s. is the rate at which energy is used by the process of entrainment. This is a 'kinematic' view of entrainment rather than the usual 'thermodynamic' view. Typically entrainment is viewed as a process which acts to bring unsaturated, relatively cold environmental air within the cloud, leading to reduced parcel buoyancy. However, if the air entrained is to become part of the cloud, as well as becoming saturated, it must be accelerated to the cloud vertical velocity. Although the nature of the forces involved in this process is not defined here, it is assumed that ultimately the energy is derived from the parcel buoyancy. The parameter $C_e$ must be defined and, by the above arguments, must be within the range of zero and unity. A value of zero leads to no entrainment, while unity implies that all of the buoyant energy is used in the entrainment processes, leading to termination of the ascent. It may be possible to specify $C_e$ from LES/CRM studies, but here it is specified by experiment using a version of the European Centre for Medium-Range Weather Forecasts (ECMWF) SCM; a value of 0.5 is used for shallow convection and 0.25 for deep convection.

(c) Comparison with alternative formulations

Several other authors (Nordeng 1994; Siebesma 1997; Grant and Brown 1999; Lin 1999) have suggested entrainment parametrizations based upon some form of the cloud kinetic-energy budget. It is informative to compare these with the analysis here.

(i) Grant and Brown (1999). Appealing to an energetic argument, Grant and Brown suggest that for shallow convection the characteristic value of entrainment for the whole cloud layer can be estimated as

$$\epsilon = C_E \frac{w^*}{M_b Z_{cld}}, \quad (14)$$

where $C_E$ is a coefficient which needs to be specified, $M_b$ is the cloud-base mass flux and $Z_{cld}$ is the cloud depth; $w^*$ is a velocity-scale for the cloud layer estimated from the 'cloud work function' for an undilute ascent, $A_U$:

$$\frac{w^{*3}}{Z_{cld}} = A_U = g \int_{cld} M_b \frac{T_v'}{T_v} \, dz, \quad (15)$$

where $T_v'$ is the temperature excess of an undilute parcel ascent over the environment temperature.

Grant and Brown demonstrate the validity of Eq. (14) for a case of cumulus over the North Sea (Smith and Jonas 1995) and shallow convection in BOMEX using the Met Office LES model, finding $C_E$ had a typical value of 0.03. The energy supplied to entrainment is 10 to 15% of the total undilute buoyant turbulent-kinetic-energy production rate. Although the derivation is for the whole depth of the cloud layer, LES show $C_E$ decreases with height through the cloud layer.
Equation (12) can be cast into the same form as Eq. (14). Multiplying by \( M_c \), and
defining the following relationships,
\[
M_c g \left( \frac{T_v'}{T_v} - 1 \right)^c = \alpha U = \alpha \frac{w^*}{Z_{cld}},
\]
\[
\bar{w}_c^{*} = \beta w^*,
\]
\[
M_c = \eta M_b,
\]
then for any level within the cloud layer
\[
\varepsilon = C_e a \frac{\alpha}{\beta^2 \eta \frac{M_b Z_{cld}}{M_b Z_{cld}}} = C_E \frac{w^*}{M_b Z_{cld}}.
\]

This is identical to the form derived by Grant and Brown (1999), but as it applies
to each level within the cloud provides a physical interpretation of the variation of
\( C_E \) with height and from case to case. The coefficient \( C_E \) varies with the amount of
dilution of the parcel, \( a \) (which relates to the difference between the temperature of
the dilute and undilute ascent), the growth of the vertical velocity with height, \( \beta \), and
the variation of cloud mass flux above cloud base, \( \eta \), as well as possible variations in
\( C_e \) and \( a \). For shallow convection, the values of these latter two parameters imply that
entrainment accounts for 10% of the buoyant production of kinetic energy within the
cloud, in agreement with estimates obtained by Grant and Brown.

(ii) **Siebesma (1997).** Following the ideas of Nordeng (1994), Siebesma suggests that
entrainment can be written as
\[
\varepsilon = \frac{1}{\bar{w}} \left( \frac{\partial \bar{w}}{\partial \bar{z}} \right),
\]
derived from the mass-continuity equation, ignoring variations in density and assuming
the cloud fraction is constant with height.

Using Eq. (12) to write the buoyancy term of Eq. (11) in terms of entrainment gives, after rearrangement,
\[
\varepsilon = \frac{C_e}{(1 - C_e)} \left( \frac{\partial \bar{w}}{\partial \bar{z}} + b \delta \right).
\]
The entrainment parametrization of Siebesma is a special case of that derived here,
with \( C_e \) set to 0.5 and neglecting the vertical cloud pressure gradient. The latter may be a
poor approximation as it has a similar magnitude as the entrainment term itself (Swann,
personal communication).

(iii) **Lin (1999).** Based upon statistical analysis of an ensemble of deep convective
clouds simulated by a CRM, Lin suggests that at any level of a cloud the entrainment
rate can be estimated by the empirical relation
\[
\varepsilon = \lambda \left( \frac{T_v^x}{T_v} \right)^{-4/3},
\]
where \( \lambda \) is a coefficient and \( T_v^x \) is the virtual-temperature excess of a cloud above its
environment, ignoring the effects of water loading but accounting for entrainment as the
parcel ascends through lower layers. The formulation is applicable to a single cloud type
(characterized by cloud-top height) rather than for average properties over the whole ensemble. Although empirical, Lin justifies the form of Eq. (20) by commenting that parcels with larger buoyancy are likely to be part of the protected core of a cloud and so have low entrainment rates. Conversely, parcels with low buoyancy are likely to have undergone substantial entrainment. Lin also notes entrainment rates decrease with height as buoyancy increases.

The form of Eq. (20) would appear to be different from the formulation derived before where entrainment is directly proportional to the local buoyancy of a parcel (Eq. (12)). However, comparison is complicated by inverse dependency upon \( \bar{m}c^2 \). At a given level within a cloud this is proportional to the vertical integral of parcel buoyancy (see Eq. (11)) leading to entrainment being inversely proportional to parcel buoyancy. It should be noted that inclusion of entrainment effects in the estimation of parcel buoyancy used in Eq. (20) also introduces a dependency upon the vertical integral of the temperature difference between cloud and environment.

The formulation suggested by Eq. (12) perhaps provides a more causal view of entrainment than Lin’s formulation. Parcels with larger vertical velocities, associated with larger buoyancy, will have the lowest entrainment rates. This results from entrainment accounting for a fixed proportion of the buoyant production of vertical kinetic energy, defined by the coefficient \( C_e \), although variation between different cloud types (shallow and deep) appears necessary in a similar manner to variations in \( \lambda \) between different cloud types in Lin’s study. As the integral of cloud buoyancy tends to increase with height, the entrainment rate suggested by Eq. (12) is expected to decrease with height as suggested by Lin (see section 3).

3. SINGLE-COLUMN MODEL TESTS

The entrainment parametrization has been incorporated into the ECMWF mass-flux convection scheme and used to carry out SCM simulations for several cases of deep and shallow convection. The SCM is based upon the atmospheric model of the ECMWF Integrated Forecasting System (IFS). Experiments here use either 31 or 40 levels (Teixeira 1999) in the vertical, with 5 or 13 levels, respectively, below 850 hPa. In the experiments described, the radiation scheme is inactive, as are the surface hydrology and orographic drag schemes (all cases being oceanic). Moist convection is represented by a bulk mass-flux scheme described later. The prognostic cloud scheme of Tiedtke (1993) is included, with water detrainment from convection acting as a source of cloud mass and cloud water. The vertical diffusion parametrization is based upon \( K \)-theory and for the unstable boundary layers, as in the convective cases considered here, a \( K \)-profile closure is used (Beljaars and Viterbo 1998).

The mass-flux convection scheme is a modified version of that described by Tiedtke (1989). Its is based upon a bulk cloud model approach with a single cloud type representing a cloud ensemble, although account is taken of the difference between shallow (cloud depths less than 2000 m) and deep convection (cloud depths greater than 2000 m). Changes to the cloud-base mass-flux closure have been described by Gregory et al. (2000), with a convectively available potential energy adjustment closure being used for deep convection while an assumption of the equilibrium of moist static energy in the subcloud layer is used for shallow convection.

The original version of the Tiedtke scheme used fixed entrainment and detrainment rates: \( 1 \times 10^{-4} \text{ m}^{-1} \) for deep convection and \( 3 \times 10^{-4} \text{ m}^{-1} \) for shallow convection. In more recent versions of the schemes used in the ECMWF IFS, these values were increased in the lowest part of the convective layer, by a factor of four at cloud base
decreasing linearly back to the original values 150 hPa above. Here the entrainment formulation is replaced by that described earlier. To estimate the vertical velocity within the cloud updraft, Eq. (11) is used. The vertical velocity at cloud base needs to be defined and is related to the subcloud-layer convective velocity-scale, \( w_{sc}^c = \left( \frac{g}{\theta_s} \frac{w'}{\theta'} z_{cb} \right)^{1/3} \), with \( z_{cb} \) being the height of cloud base. For shallow convection the vertical velocity at cloud base is set to \( \sqrt{2} w_{sc}^c \) while \( 2 w_{sc}^c \) is used for deep convection. Values larger than that suggested by \( w_{sc}^c \) are used to reflect that it is the ‘strongest’ plumes from the subcloud layer which develop into convective clouds. The value for cloud-base vertical velocity for deep convection may be underestimated as no account is taken of the mesoscale forcing due to the interactions of downdraught outflows with the mean flow.

Detrainment rates are also modified with a detrainment depth-scale of 500 m and 5000 m being used for shallow and deep convection, respectively, leading to detrainment rates of \( 2 \times 10^{-3} \) and \( 2 \times 10^{-4} \) m\(^{-1} \) respectively, which are applied throughout the convective layer. However, these fixed detrainment rates are replaced when vertical velocity decreases with height, usually near cloud top. If \( \overline{w}_k \) is the updraught vertical velocity in layer \( k-1 \) and \( \overline{w}_k \) that in the layer below, with \( \overline{w}_k < \overline{w}_{k-1} \), then the mass flux in layer \( k-1 \) is given by

\[
\frac{M_{c,k-1}}{M_c} = C_\delta \frac{\overline{w}_k}{\overline{w}_c},
\]

where \( M_c \) is the updraught mass flux. If the coefficient \( C_\delta \) is unity then Eq. (21) is obvious if density and cloud area are constant with height. However, by experiment \( C_\delta \) is set to 0.1 for shallow convection and 0.8 for deep convection. Assuming entrainment to be zero, using Eq. (21) in conjunction with the cloud-averaged continuity equation (Eq. (8)) allows a detrainment rate to be estimated.

(a) Shallow convection

Shallow convection in BOMEX and ATEX is considered, being intercomparison cases of the GEWEX* Cloud Systems Study Working Group 1 (Boundary Layer Clouds)†. Simulations using the Koninklijk Nederlands Meteorologisch Instituut LES model (Siebesma and Cuijpers 1995) are used to evaluate SCM simulations, being generally characteristic of results from many LES models which have simulated these two cases in recent years. The BOMEX simulation is six hours in length, while that for ATEX is seven hours long. Results presented from SCM and LES are averaged over the final four hours of the simulation for BOMEX and final five hours for ATEX. Emphasis is placed upon the performance of the entrainment parametrization rather than a detailed comparison of the original Tiedtke (1989) and revised convection schemes.

(i) BOMEX. Several quantities relevant to the current discussion from SCM and LES experiments are shown in Fig. 1. For the LES realizations ‘cloud core’ quantities are plotted representing averages over all points where the cloud condensate exceeds 0.1 g kg\(^{-1} \) which are positively buoyant. The ‘core’ of the cloud has been shown to be responsible for the majority of the eddy transport within the cloud layer (Siebesma and Cuijpers 1995).

* Global Energy and Water Cycle EXperiment.
† Details of these two intercomparison projects can be found at http://www.knmi.nl/~siebesma/gcss/bomex.html (1998) and http://www.asp.ucar.edu/~bstevens/atex (1998).
ENTRAINMENT IN CONVECTIVE CLOUDS

The parametrization underestimates the updraught vertical velocity (Fig. 1(a)), values being closer to that obtained by averaging all cloudy points in the LES simulations. Considering terms in the vertical-velocity equation (Eq. (11), (Fig. 1(b)), the entrainment and 'vertical pressure gradient' terms are of a similar order of magnitude and account for about half of the buoyancy forcing (which is calculated incorporating the effects of water loading). This is similar to results by Swann (personal communication) from LES simulations of this case. The mass flux (Fig. 1(c)) is in good agreement with that associated with the LES cloud cores, decreasing with height over the depth of the cloud layer, although this is in part due to the choice of the background detrainment rate.

The predicted entrainment rate has a similar vertical structure to that diagnosed from the LES simulations, with larger values near cloud base reducing over the lowest couple of 100 m of the cloud layer with a more gradual reduction above (Fig. 1(d)). However, the predicted value is only half that calculated from the LES results. It is possible to increase entrainment rate through a larger $C_e$ but this tends to reduce the depth of the convective ascent, giving a worse thermodynamics structure at the top of the boundary layer. This is a weakness of the bulk-cloud model. A single cloud is used to represent an ensemble of plumes which terminate at different heights. High rates of dilution in the lower part of the cloud layer will cause the temperature excess in the upper part to become small, with an underestimation of fluxes due to convection. Some compensation for this effect could be achieved by assuming that detraining air has the properties of the saturated-cloud environment rather than mean-cloud properties (for example, as in the scheme of Gregory and Rowntree 1990). However, it may not be desirable to match the large entrainment rates just above cloud base as these are associated with clouds which do not penetrate far into the cloud layer before their ascent terminates, indicated by the large detrainment rates in this region diagnosed from LES simulations (not shown). Such clouds are not well resolved by the model and it is likely that they are best represented by the boundary layer and cloud schemes.

As noted above, entrainment affects the excess of cloud parameters above those of the cloud environment, which together with cloud mass flux gives an estimate of the eddy transport due to convection. Figure 1(e) shows the virtual-potential-temperature excess (calculated incorporating water loading) of the parametrized updraught together with that of LES updraught (cloudy air with positive vertical velocity) and cloud core. The excess is too large in the parametrization at the lowest saturated layer above cloud base, partially because of the use of an undilute ascent from the surface, but the fact that this level is somewhat above the true cloud base, owing to model discretization, also contributes. In the main part of the cloud layer (600–1400 m) the parametrization is in reasonable agreement with the cloud core. The larger parcel excess values at cloud base may be necessary within the context of a bulk-cloud model to give realistic values in the upper part of the cloud layer as discussed previously. In the inversion layer (above 1400 m) the parcel excess in the convection scheme becomes negative while that of the LES cloud core remains positive (by definition). The virtual-potential-temperature excess of the LES updraught becomes negative at 1400 m, values above this being similar to those seen in the cumulus scheme, although the depth of penetration into the inversion in the LES is greater.

The eddy flux of virtual potential temperature (Fig. 1(f)) compares well with the LES results in the cloud and subcloud layer. However, near cloud base, negative values seen in the LES are absent. The vertical-diffusion scheme does provide a negative flux at cloud base owing to its boundary-layer top entrainment representation but this is more than offset by the positive flux of the convection scheme. As the negative values of eddy flux exist only over a shallow layer above cloud base (around 50 m deep) the SCM does
Figure 1. Single-column model (SCM) and large-eddy simulation (LES) results for simulation of BOMEX shallow convection: (a) updraught vertical velocity from SCM (solid line) and LES (dashed); (b) components of SCM updraught kinetic energy budget—buoyancy (solid), vertical pressure gradient (dotted), entrainment term (dashed); (c) updraught mass flux from SCM (solid) and LES (dashed); (d) updraught entrainment rates from SCM (solid) and LES (dashed); (e) updraught virtual-potential-temperature excess from SCM (solid), LES core updraught (dashed) and LES updraught (dotted); and (f) SCM total (solid), SCM vertical diffusion (short-dashed), SCM convection (dotted) and LES total eddy flux of virtual potential temperature (dashed). Averages are over 2-6 h.
Figure 1. Continued.
Figure 2. Single-column model (solid line) and large-eddy simulation (dashed) results for simulations of ASTEX shallow convection: (a) updraught vertical velocity, (b) mass flux, and (c) updraught entrainment rates. Averages are over 2–7 h.
not have adequate resolution to resolve this feature. In the inversion layer the LES eddy flux remains positive, that in the SCM becoming negative due to negative parcel excesses. While the LES updraught contribution to the eddy flux is negative in the inversion layer (not shown), it is offset by a positive downdraught eddy flux indicating that gravity-wave motion results from the overshoot of updraughts. The net flux due to cloudy motion in the inversion layer is near zero. The effects of downdraughts within the inversion layer are not explicitly accounted for in the convection scheme, although the use of the scaling factor $C_\delta$ in Eq. (20) increases the detrainment above that suggested by updraught vertical velocity alone, so taking some account of the mixture of upward and downward motion within the inversion layer. The positive value of the total eddy flux within the inversion layer in the LES results from motion within the cloud environment, a term usually neglected in the mass-flux approach to the parametrization of convection (see Gregory and Miller (1989) and Siebesma and Cuijpers (1995) for further discussion).

(ii) ATEX. The characteristics of the convection scheme in ATEX (Fig. 2) are similar to those seen in BOMEX. However, the mass flux (Fig. 2(b)) falls off more slowly with height than in BOMEX, a feature which is captured by the convection scheme, although the cloud-base mass flux is underestimated by the parametrization. Updraught vertical velocities (Fig. 2(a)) predicted by the convection scheme are more comparable with those from the LES, being smaller than for BOMEX in the upper part of the cloud layer. The high value of vertical velocity at 1500 m in the LES suggests that a few clouds penetrate into the base of the inversion layer with velocities considerably higher than the ensemble mean value. However, as the net mass flux in the LES is decreasing in the vicinity of this velocity increase, the implication is that such clouds must occupy an insignificantly small area. The better agreement between LES diagnosed and parametrized vertical velocity for ATEX than for BOMEX indicates the difficulty in predicting updraught vertical velocity across a range of different situations using the formulation discussed here.

Predicted entrainment rates (Fig. 2(c)) are again smaller than those diagnosed from the LES results, but the discrepancy in magnitude is not as large as for BOMEX. Once again large values at cloud base are not captured by the parametrization. The increase in entrainment rate towards cloud top in the LES is not captured by the entrainment parametrization. These larger values may be associated with the development of a stratiform cloud sheet in the upper part of the cloud layer, not accounted for by the
convection scheme. LES diagnosed entrainment rates are negative near cloud top partly because there are few grid points defined as cloud core at these levels. (Comments on negative entrainment rates derived from LES and CRMs data are made in the appendix.)

Above it was demonstrated that the entrainment parametrization has the same form as that derived by Grant and Brown (1999). This is demonstrated in Fig. 3 which shows the entrainment rates from the BOMEX and ATEX SCM simulations plotted against a normalized height (cloud base being at zero) scaled in the manner suggested by Grant and Brown (1999). Values of $w^*$ are 0.39 and 0.30 m s$^{-1}$ for BOMEX and ATEX, respectively (Grant, personal communication). The value of the coefficient is similar to that seen in LES simulations of Grant and Brown.

(b) Deep convection

The entrainment parametrization has also been tested in several cases of deep convection. Here deep convection in an idealized cold-air outbreak case, originally the subject of a study by Kershaw and Gregory (1997) and more recently by Grubisic and
Moncrieff (2000) is considered. The case is one of surface-forced convection (with sensible- and latent-heat flux of 123 and 492 W m$^{-2}$, respectively) growing under the influence of linear shear, with wind increasing from zero at the surface to 10 m s$^{-1}$ at 10 km. The simulations here are carried out using a 31-level version of the ECMWF SCM over a ten-hour period.

No average in-cloud vertical-velocity data are available for the CRM simulations of Kershaw and Gregory (1997), although they note peak values of 8 m s$^{-1}$. Grubisic and Moncrieff (2000) report an average peak updraught vertical velocity of 4 m s$^{-1}$ around 5 km. The parametrization scheme predicts peak vertical velocity in the middle of the cloud of 5 m s$^{-1}$ (Fig. 4(a)). As for shallow convection, the entrainment and vertical pressure-gradient terms of the mean updraught kinetic-energy budget oppose vertical acceleration due to buoyancy (Fig. 4(b)). The vertical pressure-gradient term increases from cloud base and peaks in the middle of the cloud layer, in a similar manner to other deep convective cases (Swann, personal communication), and in the lower troposphere reduces the buoyancy by up to 50% (at 5 km). The updraught mass flux predicted by the parametrization is in excellent agreement with that diagnosed from the CRM simulations (Fig. 4(c)). Unlike the shallow convective cases discussed previously,
a deep convective downdraught is present in this case, once again being captured by the convection scheme.

Updraught entrainment rate is diagnosed by the budget technique described by Siebesma and Cuijpers (1995) using the updraught momentum-flux data discussed by Kershaw and Gregory (1997)* and making a steady-state assumption for in-cloud properties (see appendix for further discussion). As indicated by Lin (1999), entrainment rates (Fig. 4(d)) are largest near the base of the cloud and reduced with height. Above 6 km the parametrized entrainment rate remains positive while that diagnosed from the CRM is negative. This may be a consequence of the neglect of the rate of change of in-updraught horizontal wind with time. Grubic and Moncrieff (2000) report that, although this term is small below 6 km, it is of comparable magnitude to horizontal advection and the cloud pressure-gradient term above this level and cannot be neglected. Accounting for time dependency of in-cloud fields, Grubic and Moncrieff diagnose entrainment from their simulations which is positive at all levels of the cloud, being largest near cloud base.

Although entrainment rates are not available for other cases, using the entrainment parametrization in simulations of deep convection in GARP† Atlantic Tropical Experiment and the Tropical Ocean and Global Atmosphere Programme, Coupled Ocean–Atmosphere Response Experiment leads to updraught mass fluxes in agreement with those diagnosed from CRM simulations and observations (for an example of such cases using an early version of the parametrization see Guichard and Gregory(1999) and Gregory and Guichard (1999)).

4. FURTHER DISCUSSION

A parametrization for lateral entrainment rate in convective clouds has been discussed. The magnitude of entrainment is linked to the rate of production of vertical kinetic energy by buoyancy, providing a bulk kinematic view of the process complementary to the more usual thermodynamic view that entrainment dilutes the parcel ascent. The parametrization has been included into a bulk mass-flux convection scheme, and in SCM simulations of shallow convection gives entrainment rates of the correct order of magnitude and with reasonable height variation, although values are generally less than those diagnosed from LES. The formulation also appears to work for deep convection, giving entrainment rates which are an order of magnitude lower than for shallow convection, and also decreasing with height as suggested by the recent study of entrainment in deep convection by Lin(1999). Scaling of the entrainment parameter \( C_e \) is necessary between cases of deep and shallow convection, implying that entrainment accounts for less of the buoyant production of vertical kinetic energy (the ‘cloud work function’) in deep convection than shallow convection. This perhaps reflects the different horizontal dimensions of deep and shallow convection compared to eddy length-scales. The scaling of the entrainment parameter alone is insufficient to account for the lower values of entrainment associated with deep convection, indicating that the parametrization responds to factors such as differing thermodynamic structures and microphysical processes in deep convective situations.

Comparison with other parametrizations suggested by Grant and Brown (1999) and Siebesma (1997) for shallow convection has been made. That of Siebesma (1997) is an approximate form of the parametrization discussed here, while that of Grant and

* Updraught is defined as an average over any grid point within a cloud with a condensed-water content greater than 0.1 g kg\(^{-1}\) and a vertical velocity exceeding 1 m s\(^{-1}\).
† Global Atmospheric Research Program.
Brown (1999), who couched their work in terms of turbulent quantities estimated for the whole cloud layer rather than the mean properties of a parcel ascent as here, is found to be equivalent. This is not surprising as both consider the effect of entrainment upon the cloud kinetic-energy budget. Although Grant and Brown emphasize the turbulent description of the shallow convection, their treatment of entrainment is only linked to this via the use of a ‘turbulent-velocity scale’ estimated from the rate of generation of turbulent vertical kinetic energy by buoyancy for an undilute parcel. This is related to the integral of the vertical eddy flux of virtual potential temperature through the cloud layer, which is well estimated by cloud-mean properties using the mass-flux approximation \( M_c(\bar{\theta}_v - \bar{\theta}_c) \) (Siebesma and Cuijpers 1995) and is effectively used in estimating the mean kinetic energy of the parcel in this study (Eq. (11)). The parcel approach, because it considers vertical variations rather than the whole cloud layer, provides an understanding of the vertical variation of the entrainment coefficient seen in the large-eddy simulation results of Grant and Brown (1999).

A crucial parameter to the parametrization is the vertical velocity within the cloud. It cannot be claimed that the approximate form used here has been fully justified on the basis of the current study, producing reasonable values for updraught vertical velocity in some cases but not in others. Aspects of this remain uncertain, especially the representation of vertical pressure gradients which play a large role in reducing parcel buoyancy. Use of large-eddy simulations of deep and shallow convection may provide insight into better approximations for estimating cloud vertical velocities, but given the large degree of scatter in this quantity seen between different LES realizations (Siebesma, personal communication) this may remain a difficulty in the parametrization approach followed here.

Although the parametrization has been implemented within the context of a ‘bulk’ mass-flux scheme it is equally valid to use it within a ‘spectral’ cloud-ensemble model of convection such as that discussed by Donner (1993). There a vertical-velocity equation was included in the cloud model but entrainment rate was related to cloud radius. Within the context of a ‘spectral’ approach, compromises noted previously associated with the use of a single-cloud model to describe an ensemble of cloud elements may be reduced and entrainment rates in better agreement with those diagnosed from LES and CRM data may result. Finally, while discussion has concentrated upon entrainment, further work is needed on an improved detrainment formulation for use in bulk mass-flux parametrization schemes. The spectral-mixing approach to entrainment suggested by Kain and Fritsch (1990) may provide a way forward here, allowing detrainment to be estimated via buoyancy sorting, with neutrally buoyant parcels leaving the convective plume. One weakness with such an approach has been the use of entrainment rates specified from water-tank experiments. Combination with the formulation of entrainment discussed here would appear to be straightforward. The focus upon the cloud kinetic-energy budget rather than the thermodynamic characteristics of plumes suggests that a further modification to the spectral-mixing approach might be appropriate: namely, rather than neutrally buoyant parcels detraining, a criterion on vertical velocity might be used.

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**Appendix**

**Comments on the estimation of entrainment rate from LES and CRM data**

For a general variable $\phi$, its value averaged over cloud air is given by

$$
\frac{\rho \sigma \partial \bar{\phi}^c}{\partial t} + \left[ (\rho \bar{\phi}^c - \langle \rho \phi \rangle_b) \frac{\partial \sigma}{\partial t} + \sigma \left( \frac{\partial \rho w \phi}{\partial x} \right)^c - \langle \rho w \phi \rangle_b \frac{\partial \sigma}{\partial z} \right]
+ \left( \frac{\partial \sigma \rho \bar{w}^c \bar{\phi}^c}{\partial z} \right) + \left( \frac{\partial \sigma \rho \bar{w}^c \bar{\phi}^c}{\partial z} \right) = \sigma \bar{S}^c, \quad (A.1)
$$

where $\bar{S}^c$ is the source of $\phi$ in cloudy air. Writing terms involving fluxes across the cloud boundary in terms of entrainment and detrainment fluxes, after substituting for detrainment rates from the mass-continuity equation (Eq. (8)), the entrainment rate is given by

$$
\epsilon = \frac{- (\rho \sigma \bar{w}^c \partial \bar{\phi}^c / \partial z) - \left( \partial \sigma \rho \bar{w}^c \bar{\phi}^c / \partial z \right) + \sigma \bar{S}^c - \rho \sigma \bar{\phi}^c / \partial t}{\rho \sigma \bar{w}^c (\bar{\phi}^c - \bar{\phi})}. \quad (A.2)
$$

The CRM derived entrainment rate in Fig. 3(d) is estimated from Eq. (A.2) using horizontal momentum, the source term being the horizontal pressure gradient, $-\sigma (\partial p'/\partial z)^c$. In-cloud winds were also taken to be invariant in time, an assumption which contributes to the negative entrainment rates seen in the upper troposphere.

Swann (personal communication) suggests that, even with the inclusion of the time variation of in-cloud fields, negative entrainment rates are still possible. This is due to assumptions made concerning the characteristics of the inflowing and outflowing air. Typically inflowing air is taken to have characteristics of the cloud environment while outflowing air has those of the mean cloud. It is possible to choose different values to these for entraining and detraining air, say $\bar{\phi}^l$ and $\bar{\phi}^O$. The rate of change of $\phi$ within cloud is then given by

$$
\rho \sigma \frac{\partial \bar{\phi}^c}{\partial t} + \rho \sigma \bar{w}^c \epsilon (\bar{\phi}^c - \bar{\phi}^l) - \rho \sigma \bar{w}^c \delta (\bar{\phi}^c - \bar{\phi}^O) + \frac{\partial \sigma \rho \bar{w}^c \bar{\phi}^c}{\partial z} + \rho \sigma \bar{w}^c \frac{\partial \bar{\phi}^c}{\partial z} = \sigma \bar{S}^c. \quad (A.3)
$$

Comparison with the previous derivation of the cloud-averaged equation for $\phi$ (Eq. (A.1)) shows that Eq. (A.2) only estimates an effective entrainment rate related to the true entrainment and detrainment rate by

$$
\epsilon_{\text{eff}} = \frac{\epsilon (\bar{\phi}^c - \bar{\phi}^l) - \delta (\bar{\phi}^c - \bar{\phi}^O)}{(\bar{\phi}^c - \bar{\phi})}. \quad (A.4)
$$

If the outflowing is colder than the mean over the cloud ensemble, the detrainment term on the denominator contributes to a negative effective entrainment rate. More accurate estimates of entrainment and detrainment rates might be obtained from CRM data through the simultaneous use of flux data from two independent variables (for
example, $u$ and $v$ or liquid-water temperature and total water) although this would require the determination of the characteristics of the inflowing and outflowing air.

Incorporating the generalized treatment of entrainment and detrainment into the derivation of an equation for cloud-mean vertical velocity gives

$$
\frac{1}{2} \frac{\partial \bar{w}^c}{\partial z} = g \left( \frac{T_v'}{T_v} - 1 \right) - \frac{1}{\sigma \rho} \left( \frac{\partial \bar{w}^c}{\partial z} \right) - \frac{1}{\rho} \left( \frac{\partial p'}{\partial z} \right) - g \left( \frac{p'}{p} \right) - \varepsilon (\bar{w}^c - \bar{w}_0^c) + \delta (\bar{w}^c - \bar{w}_0^c).
$$

Although the entrainment term is more complicated than in Eq. (9) it can be related to $\bar{w}^c$. The extra terms may play a considerable role in the mean vertical-velocity budget of the cloud. For example, if parcels detrain from the ensemble with near-zero vertical velocity ($\bar{w}^0 = 0$) rather than the updraft mean value, the final term on the r.h.s. of Eq. (A.5) acts to increase ensemble-mean vertical velocity with height. For a detrainment rate of $2 \times 10^{-3}$ m$^{-1}$ (typical of shallow convection) and an updraft velocity of 1 m s$^{-1}$, over a depth of 1 km a velocity increase of 2 m s$^{-1}$ would result. This is similar to the difference near cloud top between the value given by Eq. (11) and the cloud core value from the LES results for BOMEX (Fig. 1(a)). However, agreement of updraft vertical velocities for ATEX between the SCM and LES (Fig. 2(a)) without such a detrainment correction indicates that such an effect may not be universal and that incorporation of more general inflow and outflow characteristics into plume models may be difficult.

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