Rotational aspects of stratified gap flows and shallow föhn

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SUMMARY

Observations of föhn in the Alps and other mountainous regions suggest that the underlying dynamics is often affected by gap-like features in elongated ridge-like topography. To assess the dynamics of these flows, idealized numerical experiments are conducted with a hydrostatic numerical model, using $f$-plane geometry and a free-slip lower boundary condition. The topography is taken to be a two-dimensional ridge oriented in the west/east direction with a valley transect of depth $\Delta H$ across it. The upstream flow is westerly, with a constant wind speed $U$ and constant Brunt-Väisälä frequency $N$. The control parameters defined by this setting are a dimensionless gap depth $N\Delta H/U$, the ratio between ridge height and gap depth $H/\Delta H$, a Rossby number describing the south–north width of the ridge, and additional parameters associated with the shape of the gap. With intermediate Rossby numbers ($Ro \approx 1$) the setting resembles that of shallow Alpine south-föhn cases, which are characterized by a cross-Alpine flow essentially confined to valley transects. For small dimensionless gap depths and large Rossby numbers, the flow follows the predictions of linear theory and takes on an approximately symmetric pattern with respect to the ridge line. For $N\Delta H/U \gtrsim 1$, flow separation and splitting takes place upstream and downstream of the gap, respectively. The flow within the gap decouples from the flow aloft and is driven by the geostrophic south–north pressure gradient to yield a föhn-like flow. It is demonstrated that the limit $f \to 0$ is singular (i.e., the flow solution does not converge towards the symmetric $f = 0$ solution), and that there exist multiple stationary solutions for $f > 0$ (two with northerly and southerly flow across the gap, respectively, and one with north/south symmetry). The existence of these multiple steady states is related to a wake instability, yet vortex shedding is suppressed by the presence of the ridge downstream of the gap. Additional simulations are presented which demonstrate that a transient external forcing can induce transitions between the multiple flow solutions. The relationship of the idealized setting to Alpine shallow föhn is discussed, and additional experiments are conducted to assess the effects of surface friction and of an inversion present to the south of the ridge.

KEYWORDS: Alpine dynamics Downslope windstorms Föhn Gap winds Multiple flow solutions

1. INTRODUCTION

There is an extensive body of literature on the two-dimensional dynamics of downslope windstorms. This research has served to shed light upon the basic properties of gravity-wave generation and propagation (see the reviews of Smith (1979) and Durran (1990)), the role of nonlinear effects (Long 1955), the interaction of gravity waves with the vertical stratification profile (Klemp and Lilly 1975) and critical levels induced by breaking waves (Peltier and Clark 1979), the relationship between continuously stratified and hydraulic flows (Smith 1989; Durran 1990), and many more additional features of this fascinating flow problem. However, many of the observed cases of downslope windstorms cannot be explained by the classical two-dimensional theories. In the Alps, for instance, about 12% of the observed föhn cases are classified as ‘shallow’ (Seibert 1985). In contrast to the deep föhn, the cross-Alpine southerly flow is then essentially confined to the valley transects, whereas the mid-tropospheric flow can have a westerly or even a north-westerly direction. An impressive case of this type is the so-called Gueller föhn which occurred on 13 February 1976 (Gueller 1979). The upper-level synoptic chart for this case shows a north-westerly flow (see Fig. 1), yet surface observations in the upper Rhine valley (along the border between Austria and Switzerland) showed the occurrence of southerly föhn with its typical features (warm and dry conditions with relative humidity as low as 30% and strong southerly flow with peak gusts up to 20 m s$^{-1}$), yet the southerly flow did not extend past the Alpine ridge line but was confined to valleys. In the mid troposphere, north-westerly flow prevailed

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associated with an approaching cold front which ultimately terminated the föhn storm. Despite some similarity with other downslope windstorms, such as the Bora which is often associated with reversed flow at upper levels and concomitant critical levels (Smith 1987), shallow föhn and other gap flows cannot be idealized by two-dimensional geometry.

Three-dimensional gap flows are very common in many mountainous regions. One of the early detailed accounts is that of Scorer (1952) who studied the flow past the Straits of Gibraltar. Scorer noted that a strong stability below mountain-top level and a pronounced pressure gradient along the gap are conducive to strong winds. He stressed the importance of a cold-air pool underneath an inversion, and interpreted the pressure fall to the lee of the gap as being maintained by the descent of the inversion as the wind blows across the gap. Scorer also noted that 'the air converging towards the narrows follows fairly smooth streamlines' but when it emerges from the gap it 'does not readily fan out up a pressure gradient to a lower velocity but rather tends to emerge as a jet with large standing eddies which can be observed in the surface isobars, or smaller moving ones which are observed by aircraft as regions of very changeable wind, on the side of the jet.' Several subsequent studies attempted to interpret gap winds as a hydraulic phenomenon. Pettre (1982) studied the Mistral along the Rhône valley and compared it with results from a simple shallow-water model. He suggested that a lateral valley constriction near Valence was playing the role of a hydraulic control and promoting a transition to supercritical conditions, and he also detected a hydraulic-jump-like feature further downstream. Jackson and Steyn (1994a) carried out a detailed
field investigation of the gap flow in Howe Sound, British Columbia, and presented non-hydrostatic high-resolution integrations. In a companion paper (Jackson and Steyn 1994b) they subsequently employed a simple quasi-two-dimensional hydraulic model to interpret their observations. The model included an external pressure gradient and surface friction, and accounted for the variable cross-section of the valley along the gap axes. Jackson and Steyn concluded that the primary force balance for supercritical flow was between the external pressure gradient and surface friction, while for subcritical flow it was between the external and the height pressure gradients.

In all of the aforementioned studies consideration was given to events characterized by a pronounced inversion. However, even for continuously stratified upstream flow profiles, gap-induced transitions to supercritical flow were detected. Armi and Williams (1993) studied (using laboratory experiments and theoretical methods) the steady hydraulics of a continuously stratified fluid flowing from a stagnant reservoir through a horizontal constriction. Their analysis revealed a succession of transitions (with respect to individual wave modes) from subcritical to supercritical conditions as the flow accelerated through the constriction. This flow problem is related to the hydraulic theory of downslope windstorms, where similar transitions are induced by a topographic ridge (cf. Smith 1989; Durr 1990).

Most of the recent research on channelling and deflection of continuously stratified flows restricts attention to flow past three-dimensional isolated topographic obstacles with circular or elliptic shapes. Idealized numerical simulations have, in particular, demonstrated that the transition from the quasi-linear ‘flow-over’ to the nonlinear ‘flow-around’ regime shows some of the characteristics of a bifurcation (Smith and Gronas 1993) in which the dimensionless mountain height $\tilde{H} = NH/U$ acts as the control parameter. If the dimensionless mountain height is raised past some critical value, there is an abrupt transition to a regime where the impinging airstream experiences flow splitting and lateral deflection around the flanks of the obstacle, rather than being lifted over it. Such flows possess a wake with vortices and sometimes vortex shedding (Hunt and Snyder 1980; Smolarkiewicz and Rotunno 1989; Schär and Durr 1997).

More complex topography has also been considered. For instance, Saito (1993, 1994), Clark et al. (1994) and Zängl (1999) conducted numerical experiments of flow over two-dimensional or isolated mountain ridges transected by pass-like features, which were taken to represent the topography of the island of Shikoku, the Colorado front range, and the Inn valley in the Alps, respectively. All these studies detected an intensification of the fohn flow along the gap transecting the ridge, but attention was restricted to configurations with the incident flow approximately perpendicular to the ridge, thus not addressing cases of shallow föhn similar to the one alluded to above.

Beside the aforementioned effects, rotation may play some role as well. Theoretical studies suggest that rotational flow past obstacles should be balanced (i.e. qualitatively amenable to quasi-geostrophic dynamics) as long as $Ro\tilde{H} \lesssim 0.5$ for three-dimensional circular topography (Schär and Davies 1988; Trüb 1993), and $Ro\tilde{H} \lesssim 1$ for two-dimensional topography (Trüb and Davies 1995). Here $Ro = U/Lf$ is the Rossby number defined by the horizontal scale of the obstacle. Introducing typical Alpine values puts $Ro\tilde{H}$ in the range between 0.2 and 2, thus the flow belongs to the intermediate regime where rotational and gravity-wave effects are both important, but neither of the two dominates. According to recent studies, the critical mountain height is increased by rotational effects, and thereby makes flow-splitting less likely as compared with the flow past a smaller-scale obstacle of the same height (Trüb 1993; Olafsson and Bougeault 1997; Bauer 1997). In relation to the Alps there is, thus, a tendency for the flow to be
over the Alps on the scale of the obstacle, but around individual massifs and peaks (see Aebischer and Schär 1998).

In the present study consideration is given to an idealized Alpine shallow föhn setting. A schematic diagram illustrating this is presented in Fig. 2. The idealized Alps are shown as an elongated two-dimensional ridge intersected by a gap across it, representing a major Alpine pass such as the Gotthard or Brenner section in Switzerland and Austria, respectively. The ‘upstream’ flow is taken to be a westerly flow along the ridge line. The implied role played by rotation and flow-splitting effects is suggested by the balance of forces, as indicated in the figure. Upstream of the gap the flow is in geostrophic balance, i.e. the pressure gradient force acting towards the north is balanced by an equally large Coriolis force acting towards the south. This large-scale balance cannot be maintained within the gap if it is deep enough to suppress east–west motion of air parcels. If the flow is channelled, the Coriolis force in the direction of the valley axes disappears and there is a net acceleration of the flow towards north.

The flow depicted in Fig. 2 is closely related to the existence of counter currents in deep valleys such as the Rhine valley near Karlsruhe (Wippermann 1984; Kalthoff and Vogel 1992). As in Fig. 2, the geostrophic balance is disrupted by the decoupling of the flow within the valley from aloft, while the synoptic pressure gradient is hydrostatically transmitted into the valley, thereby inducing accelerations perpendicular (or even opposite) to the upper-level flow. However, in the aforementioned studies the valley bottom is quasi-horizontal and, thus, there are major differences from the shallow-föhn schematics illustrated in Fig. 2. Firstly, as a result of the vertical deflection of air parcels that transit along the gap over the ridge, gravity-wave generation and propagation will interact with the basic currents. Secondly, the complex three-dimensional structure of our setting can give rise to flow separation and splitting in the vicinity of the gap; these may induce anomalies of potential vorticity that are not present in the essentially two-dimensional setting alluded to by Wippermann (1984). The separation and splitting will induce wake
features to the lee of the gap. Such configurations have been investigated, for instance, by Pan and Smith (1998). They utilized non-rotating shallow-water dynamics and compared their results with synthetic-aperture radar data taken near Unimak Island in the Aleutian Chain.

The aforementioned hypothesis emphasizes the role of external pressure gradients. It is indeed interesting to note that the relevance of the south/north pressure gradient for Alpine föhn has long been recognized in operational forecasting practice. Most Alpine weather services use a föhn forecasting rule which gives ample weight to the pressure contrast between surface stations on either side of the main Alpine ridge. In Switzerland, for instance, a variant of the Widmer föhn test is used (Widmer 1966; Courvoisier and Gutermann 1971). The underlying rule implies that a south/north pressure contrast of more than 18.3 hPa between Venice and Tours is a fairly reliable precursor for the onset of southerly föhn within the next 36 hours. In this context it is worth noting that the classical two-dimensional downslope-windstorm theories would not yield a north–south pressure gradient unless the föhn flow is already established.

The specific goal of our study is to assess the mechanism and associated processes that result from the hypothesized schematics illustrated in Fig. 2. To this end, highly idealized simulations have been conducted using a hydrostatic numerical model with free-slip lower boundary conditions on a rotating $f$-plane. Particular objectives are to assess the nature of the flow response, to estimate the strength of the induced southerly cross-ridge flow, to study its sensitivity with respect to external control parameters, and to explore the role played by gravity waves and the flow separation/splitting effects.

The paper is organized as follows. Section 2 describes the numerical model used and its configuration for the current study. Section 3 contains an analysis of the quasi-linear regime associated with gap depths that are too small to induce proper channeling of the flow. In section 4 the nonlinear regime is assessed to test the basic hypothesis of the study. The sensitivity of the flow response and the existence of multiple flow solutions are then addressed in sections 5 and 6, respectively. Additional factors that make the flow problem more realistic, such as the presence of a pre-existing inversion to the south of the ridge and the effects of surface friction, are taken into consideration in section 7. Some concluding comments follow in section 8.

2. Definition of model problem

Consideration is given to the flow of a stratified stream of air along a two-dimensional mountain ridge of height $H$. To mimic a mountain pass the ridge is transected by a gap-like feature (Fig. 3). Both the ridge profile (in the $y$-direction), and the gap profiles (in the $x$-direction) are assigned a cosine shape with halfwidth $l$ given by

$$s(\chi, l) := \begin{cases} \cos^2(\pi \chi / 2l) & \text{if } |\chi| < l \\ 0 & \text{otherwise.} \end{cases}$$

(1)

Here $\chi$ stands for either $x$ or $y$, and $l$ denotes a scale parameter. The topography is specified as

$$h(x, y) := h_0(x)s(y, a_0(x)),$$

(2)

where the width $a_0(x)$ and height $h_0(x)$ of the ridge are specified as functions of the along-ridge coordinate according to

$$h_0(x) := H - \Delta H s(x, L)$$

(3)

$$a_0(x) := A[1 - \beta s(x, L)].$$

(4)
Figure 3. Idealized orography (contours every 400 m) (a) in the standard setting (A = 150 km, H = 3000 m, \(\Delta H = 900\) m, \(L = 75\) km, \(\beta = 0\)) and (b) with valleys to the north and south of the gap (A = 300 km, \(H = 3000\) m, \(\Delta H = 900\) m, \(L = 75\) km, \(\beta = 0.5\)). See text for further information.

Here the parameters \(H\) and \(A\) denote the height and width of the ridge away from the gap. The gap itself has a depth \(\Delta H\) and an along-stream width \(L\). The parameter \(\beta\) allows for the reduction of the width of the ridge in the vicinity of the gap (cf. Fig. 3(b)). In the standard setting, the parameters are assigned the dimensional values of \(A = 150\) km, \(H = 3000\) m, \(L = 75\) km, \(\Delta H = 900\) m and \(\beta = 0\). The associated orography is shown in Fig. 3(a). After assigning the so-defined orography to the numerical grid, a simple filter is applied in order to eliminate two-grid waves.

The topography parameters, as given above, are chosen so as to represent approximately the Alpine setting. Realistic passes across the Alps (such as the Gotthard or the Brenner section) are characterized by narrow valleys with scales down to a few kilometres. However, these passes are embedded in larger-scale dips in the Alpine ridge line, with horizontal scales not unlike the ones used above. Nevertheless, the poor representation of the narrow inner part of these cross-Alpine valleys is a limitation of our study and restricts direct applicability.

The upstream flow is of uniform stratification (Brunt–Väisälä frequency \(N\)) and wind speed \(U\), with the standard values of \(N = 10^{-2}\) s\(^{-1}\) and \(U = 10\) m s\(^{-1}\).

Consideration has been given to hydrostatic dynamics on an \(f\)-plane with a free-slip lower boundary condition. As a result of the hydrostatic scaling (Smith 1989; Smith and Gronas 1993) the model problem is then determined by four dimensionless parameters. These may be chosen to be a dimensionless measure of the gap depth (indicated by the hat symbol)

\[
\hat{\Delta}H = N\Delta H / U,
\]  

the ratio between ridge height and gap depth

\[
\delta = H / \Delta H,
\]

a zonal Rossby number based upon the gap width

\[
Ro^x = U / LF,
\]
and the aspect ratio
\[ \epsilon = \frac{L}{A}, \] (8)
which measures the ratio between the widths of the ridge and the gap. For the standard setting listed above and a mid-latitude Coriolis parameter of \( f_0 = 10^{-4} \text{ s}^{-1} \), the dimensionless parameters take the values \( \Delta H = 0.9, \delta = 3.33, Rho^x = 1.33, \) and \( \epsilon = 0.5 \).
Theoretical considerations suggest that the flow along the ridge is in good geostrophic equilibrium, while the flow across the gap is both affected by the background rotation and gravity-wave propagation \( (Rho^x = 1.33) \). The dimensionless depth of the gap \( (\Delta H = 0.9) \) is close to the critical dimensionless mountain height for flow past isolated bell-shaped topography (Smolarkiewicz and Rotunno 1989; Smith and Gronas 1993), and thus we expect substantial nonlinear effects. In addition, as a result of choosing a fairly broad gap, we expect the hydrostatic approximation to apply \( (U/NL = 0.013) \).

Numerical solutions are obtained using a hydrostatic model derived from the mesoscale numerical weather prediction model of the German Weather Service (DWD) and the Swiss Meteorological Institute. A detailed description of the model has been given by Majewski (1991), and information about its operational use can be found in quarterly DWD reports (Majewski and Schrodin 1999). The model is very flexible and was used in research mode for case studies of Alpine flows (Aebischer and Schär 1998), predictability studies (Fehlmann and Davies 1997), and as a regional climate model (e.g. Frei et al. 1998). The present simulations use an \( f \)-plane option and most of the parametrizations are switched off. The prognostic variables of the model are surface pressure, the horizontal wind components, and temperature. At the upper boundary a KDB-type gravity-wave absorber (Klemp and Durran 1983; Bougeault 1983) is used. Its implementation in pressure coordinates is due to Herzog (1995). The only parametrization package employed is a turbulence formulation with a second-order closure scheme of hierarchy level two (Mellor and Yamada 1974; see also Müller 1981). Most of the simulations (except for some simulations described in section 7) have been performed using free-slip lower boundary conditions. Consistent with the free-slip lower boundary condition, the vertical turbulent fluxes of heat and momentum are set to zero at the lower boundary, while they may take on non-zero values throughout the model atmosphere. At the lateral boundaries, the model is driven by an analytical background flow along the two-dimensional ridge using relaxation boundary conditions (Davies 1976).

The standard experiments are conducted using \( \Delta x = \Delta y = 5.6 \text{ km} \) and \( 121 \times 81 \) grid points in the horizontal. Most diagrams shown in this paper restrict attention to the inner part of the computational domain with \( 60 \times 40 \) grid points. In the vertical, 34 levels are used. Their density near the surface is somewhat increased in order to resolve better the dynamics near the surface. The lowermost level is approximately 50 m above ground. In the experiments with surface friction we used a one-way nesting technique. Details are presented in the relevant section of this paper.

To validate the model for our configuration, some initial tests were performed based upon the hydrostatic scaling laws. In a first test, we halved the values of \( f, U \) and \( N \) with respect to the standard setting. With these changes all the dimensionless parameters are unaffected. In a second test the scaling laws in the absence of rotation \( (f = 0) \) were tested by doubling \( A \) and \( L \). In this experiment, the horizontal grid spacing \( \Delta x \) was doubled as well, such that the gap was equally resolved in both directions. In both tests, the experiments yielded very similar results to those with the standard parameter setting, except for some minor differences that presumably originated from the turbulence scheme (which does not satisfy the hydrostatic scaling laws).
3. **LINEAR REGIME**

In this section we show that the shallow south föhn (discussed in more detail later in the paper) is a predominantly nonlinear phenomenon. This is checked by performing a simulation where the linear approximation applies, i.e. where all perturbation fields scale linearly with the gap depth $\Delta H$. Here we choose a quasi-linear value of $\Delta H = 300$ m, yielding a dimensionless gap depth $\overline{\Delta H}$ of 0.3.

The numerical integration is carried out to $t = 80$ h, which is well sufficient to approach steady state. Results are shown in Fig. 4. The patterns of the vertical wind and the potential temperature in the east/west cross-section running along the ridge crest and cutting across the gap (Fig. 4(a)) show the typical signature of vertically propagating hydrostatic gravity waves. The angle of ascent of the wave train is in accord with the dispersion relation for gravity waves, and is essentially determined by the zonal width of the gap. The vertical displacement of the potential-temperature contours tilt upstream with height, indicating an upward transport of energy and a downward transport of momentum. The low-level $\Theta$ contours follow the topography and there is no indication of either flow separation or flow splitting. The surface wind speed and wind vectors (Fig. 4(b)) reveal almost perfect south–north symmetry.

The most prominent feature in Fig. 4(b) is the wave train that appears downstream of the gap both on the northern and southern flank of the ridge. These waves have a vertical extension of $\approx 500$ m, a horizontal wavelength of $\approx 100$ km and they do not depend on the Coriolis parameter as is revealed by further simulations with different values of $f$ (not shown). The existence and properties of these waves can be explained by a simple parcel argument. Assuming an approximately undisturbed stratification, $N$, along the mountain's slopes, and a small meridional displacement, $\delta s$, along the mountain's slope from the upstream position, the equation of motion for an air parcel located directly on the topography is obtained from combining the vertical and meridional momentum equation to

$$\frac{D^2}{Dt^2}(\delta s) = -(N \sin \alpha)^2 \delta s,$$

(9)
where $\alpha$ denotes the slope angle of the northern or southern flank of the ridge. This is an oscillation equation with the oscillation period given by $2\pi/(N \sin \alpha)$. For an infinite slope ($\sin \alpha = 1$), $\delta s$ equals a vertical displacement, $\delta z$, and Eq. (9) describes the buoyancy oscillations in the free atmosphere. Multiplication by the undisturbed upstream velocity, $U$, yields the wavelength $\lambda = 2\pi U/(N \sin \alpha)$. If the angle $\alpha$ is small, $\sin \alpha$ may be replaced by the slope, $a$, to yield

$$\lambda = 2\pi U/(Na).$$

The appropriate values for $U$ and $N$ of our simulation are: $N = 10^{-2}$ s$^{-1}$, $U = 10$ m s$^{-1}$. The slope $a$ varies in the north/south direction, but can be reasonably taken to be the maximum value that appears on the northern and southern slopes of the ridge, i.e. $a = 0.06$. Inserting these values yields an estimate of $\lambda = 105$ km, which agrees very well with the observed order of magnitude for the wavelength ($\lambda \approx 100$ km). The scaling exhibited by (10) was validated using additional numerical experiments with modified slope and stability parameters. In an experiment with halved width $A$ of the ridge (and, thus, doubled slope $a$) $\lambda$ becomes approximately 57 km. Doubling $A$ leads to $\lambda \approx 170$ km, and halving the Alpine ridge height $H$ leads to $\lambda \approx 160$ km. These experiments show approximate scaling only, presumably owing to the large wavelength and the increased influence of the Coriolis force. Finally, doubling $U$ leads to $\lambda \approx 190$ km, which is in accordance with the simple scaling law (10).

The wave regime can, thus, be interpreted as a 'nearly trapped' gravity mode, where the terrain modifies the buoyancy force such as to yield a reduced oscillation frequency. Since the waves also appear in a run with $f = 0$ (see Fig. 11), the trapping mechanism cannot be attributed to a Doppler-shifted frequency less than $f$. Furthermore, these waves are distinct from topographic Rossby waves (cf. Pedlosky 1987) since they do not rely on background rotation. However, since the mode is shallow, one expects that boundary-layer processes (which are neglected in the current simulation and the above analysis) would be able to damp the mode effectively. This is confirmed in section 7. Similarly, we do not expect that these modes play a significant role in the Alpine setting, where complex small-scale topography inhibits the meridional displacement of air parcels on the mountain slopes.

4. NONLINEAR REGIME

In the present and subsequent sections we turn attention to the nonlinear regime. We choose a gap depth of 900 m, yielding a pass height of 2100 m. The dimensionless gap depth, which is a measure of the nonlinearity of the flow, takes the value $\overline{\Delta H} = 0.9$. The results are shown in Fig. 5.

In contrast to the linear regime (Fig. 4), a strong southerly wind through the gap can now be identified (Fig. 5(b)), and its maximum is attained on the northern downhll slope of the gap. The maximum wind speed has a remarkable amplitude of 16 m s$^{-1}$, which is not unlike the values obtained by two-dimensional downslope-wind theories. On the eastern flank of the gap the southerly flow extends up to the crest of the mountain ridge. A closer inspection of the winds within the gap (Fig. 6) also reveals flow splitting to the south-east and flow separation to the west of the gap, including a cyclonic vortex feature to the south, which results in a strong redirection of the low-level winds that includes a weak easterly component in the gap. Thus, the picture emerges that the strong southerly flow across the gap is closely related to flow-separation and flow-splitting processes induced by the presence of the gap.
The flow separation to the south-west of the gap results in a well-defined vortex-like flow feature, while the corresponding vortex to the north takes on the form of a pronounced shear line extending away from the topography which confines the zone of southerly flow to the north. The separation of the flow to the west of the gap is also evident in the isentropic distribution along the east/west cross-section (Fig. 5(a)). In contrast to the linear flow regime (Fig. 4(a)), the isentropes separate from the topography and exhibit a hydraulic-jump-like behaviour. To illustrate further the extent of the decoupling of the flow across the gap from aloft, the 301 K contour in Fig. 5(a), which runs along the ridge, is shown in bold.

The vertical wind velocity in Fig. 5(a) suggests the presence of two—rather than one—wave trains of vertically propagating gravity waves. The first emanates at the separation region near the western flank, and its steep angle of ascent is associated with the small horizontal extent of its source region. The second wave train is similar to that
observed in the linear case and has a smaller angle of ascent, and it appears that the gap as a whole acts as its source region.

For further analysis we compare in Fig. 7 the reduced surface pressure of the nonlinear and linear experiments. This is obtained by removing the hydrostatic part of the pressure distribution (determined from an undisturbed basic-state flow with stratification, $N$, and upstream velocity, $U$) such that only the geostrophic and dynamically-induced ageostrophic parts remain. In the nonlinear case (Fig. 7(b)) there is a well-defined pressure ridge at the south-eastern flank of the gap, which will be referred to as the 'fühln knee', a term borrowed from synoptic terminology. Note also how the north/south pressure gradient is much enhanced in the gap region as compared with the linear case. This enhancement is a result of gravity-wave propagation and is beyond the simple driving mechanism discussed in Fig. 2.

5. Sensitivity experiments

In the following subsection we present systematic sensitivity studies of the standard nonlinear solution with respect to (a) the topography’s shape and size, and (b) the upstream flow speed. The sensitivity of the flow with respect to background rotation is addressed in section 6.

(a) Sensitivity to the topography's shape and size

The first set of sensitivity experiments address the role of the topography’s scale in terms of the north/south ridge width $A$ and the west/east gap width $L$. Since a modification of both these parameters by the same factor changes the overall scaling of the topography, we also obtain information on the influence of the overall Rossby number. Results are shown in Fig. 8 for four experiments, and these can be compared with Fig. 5(b).

If the east/west extension of the gap is halved or doubled (Figs. 8(a) and (b)), the cross-Alpine southerly flow persists, but it becomes more concentrated near the eastern slope of the gap. If the gap width is doubled (Fig. 8(b)) the decoupling of the air in the gap from the flow aloft is reduced. This can be partly understood by the weakening of the flow splitting at the south-eastern flank of the gap, consistent with a decrease in
the zonal Rossby number. On the other hand, if the gap width is halved a Lagrangian argument applies; since there is some westerly flow component almost throughout the gap, a narrower gap implies that the air parcels stay only briefly within it, thus feeling the south/north pressure gradient only for a short period of time, and so reducing the net northward acceleration. To summarize, the zonal Rossby number $Ro^x = U/Lf$ is important for the splitting and redirection of the flow at the south-eastern slope of the gap. An intermediate east/west width of the gap seems optimal for inducing strong southerly winds across it.

The width of the Alpine ridge has a strong impact upon the character of the flow solution. Doubling of the width (see Fig. 8(d)) leads to substantially stronger southerly winds across the gap, and the wind maximum is more centred on the gap transect. Conversely, reducing the ridge width results in reduced southerly flow (cf. Fig. 8(c)). This reveals the importance of the meridional Rossby number $Ro^y = U/Af$, which is a dimensionless measure of the width $A$ of the ridge. If the air in the gap is decoupled from the westerly basic-state flow, and if an air parcel is driven by the external geostrophic pressure gradient force $-(1/\rho)\partial p/\partial y = fU$, simple Lagrangian
arguments imply that the air parcel stays in the gap for a time period of $t \sim O(\sqrt{Af/U})$, reaching a maximum meridional velocity of $v_{\max}/U \sim O(\sqrt{Af/U})$. The experiments with doubled and halved ridge width $A$, as well as the experiments with modified upstream velocity (see next subsection), conform well with the scaling rule provided that the gap flow is decoupled from aloft. This is shown in Fig. 9 where $v_{\max}/U$ is plotted against $\sqrt{Af/U}$.

Consistent with the arguments given in section 4, increasing the width (and thus decreasing the slope) of the Alpine ridge also implies an increase in the wavelength of the topographic waves (and vice versa), as can be noted in Figs. 8(c) and (d).

(b) Sensitivity to upstream flow speed

In this subsection, the sensitivity with respect to the upstream flow speed is addressed. Two associated numerical experiments are shown in Fig. 10, where we have modified the upstream flow speed to 5 m s$^{-1}$ and 20 m s$^{-1}$, respectively. The results illustrate that the pressure difference across the ridge cannot be taken as the sole control parameter. While the reduced geostrophic wind velocity (and thus pressure contrast) in Fig. 10(a) indeed reduces the southerly flow as compared with Fig. 5(b), a further increase of the wind velocity (cf. Fig. 10(b)) does not foster an amplification of the flow response. In contrast, Fig. 10(b) has the characteristics of the linear solution (cf. Fig. 4) and lacks a southerly flow across the gap. The absence of the nonlinear response is consistent with the reduced dimensionless mountain height, and might also be related to the increased overall Rossby numbers in the case with a 20 m s$^{-1}$ flow speed, implying that the ‘symmetry breaking’ Coriolis force loses its importance (but see also the following section). For a 20 m s$^{-1}$ upstream velocity, the Lagrangian scaling rule described at the end of section 5(a) fails (see point 5 in Fig. 9); this can, however, be attributed to the strongly diminished decoupling of the gap flow from the flow aloft.
The foregoing experiments demonstrate that a strong meridional pressure difference $\Delta p$ across the ridge is not a sufficient condition for the occurrence of a southerly wind across a gap transect. This is an interesting observation since many of the operationally used föhn forecasting rules in the Alps rely primarily upon $\Delta p$ (see the introduction). Our results suggest that it might in practice be useful to include, as a local measure of the dimensionless mountain height, the additional parameter $N/U$ into the underlying shallow föhn statistics. Here $N$ and $U$ denote some measure of the stratification below crest height, and of the westerly unperturbed wind velocity (say, at the 700 hPa level), respectively. If the $N/U$ parameter is large enough, the strength of the southerly flow scales approximately with $\Delta p$, while for small $N/U$ shallow föhn is absent irrespective of $\Delta p$. However, in practice, particularly for deep and narrow valleys such as in the Alps, it might also be that the flow is always within the nonlinear regime, such that consideration of $\Delta p$ is sufficient.

Finally, we note that in the case with smaller upstream velocity the topographic waves downstream of the gap disappear. In general, an increase of the ridge slope (see above), an increase of the stratification (see section 8), and a decrease of the upstream velocity (this section) inhibit the topographic waves. We believe that this behaviour relates to the different strengths of the flow separation and flow splitting in the vicinity of the gap.

6. Multiple flow solutions for $f = 0$

(a) Sensitivity of standard nonlinear solution for $f \to 0$

In this subsection we study the sensitivity of the standard nonlinear experiment in Fig. 5 with respect to background rotation. From the rules of hydrostatic scaling, this can also be taken to be the sensitivity of the hydrostatic flow with respect to the overall horizontal scale of the obstacle. We begin our analysis with a numerical experiment where the background rotation is switched off. If the Coriolis parameter is set to $f = 0$, the flow problem has perfect south/north symmetry with respect to the ridge axes. The associated numerical results (Fig. 11) confirm this expectation. The gravity-wave signature in the section running along the ridge (Fig. 11(a)) is very similar to the one of the standard experiment (Fig. 5(a)), yet the symmetry now precludes any flow through
the gap. Rather, the near surface flow solution in Fig. 11(b) is forced to look much like the linear solution (Fig. 4(b)), and shows the trapped gravity modes discussed in section 3.

Intuitively one would expect that, as a function of the Coriolis parameter, a continuous and smooth transition exists between the rotating flow solution (Fig. 5) and the non-rotating limit (Fig. 11), i.e. that the steady-state flow fields converge with decreasing Coriolis parameter $f$ towards the symmetric $f = 0$ state. However, numerical experiments show that this is (surprisingly) not the case. A systematic set of numerical experiments was conducted with the values of $f/f_0 = 1$, $1/3$, $1/6$ and $1/12$, where $f_0 = 10^{-4} \text{s}^{-1}$. All experiments were run up to time $t = 60 \text{ h}$, by which time a well-defined steady state had been obtained. A selection of the results is displayed in Fig. 12, in terms of the southerly flow component on the lowermost model level. The results demonstrate that the limit for $f \to 0$ does not converge towards the $f = 0$ state. Note, for instance, that there is an almost negligible change between the $f = f_0 / 6$ and $f = f_0 / 12$ experiments. Despite the weak rotational effects, there is a pronounced southerly flow across the gap, much unlike the symmetric $f = 0$ state shown in Fig. 12(d). An additional experiment with $f/f_0 = -1/12$ was also conducted to test the symmetry of the numerical code, and indeed has produced the same solution as that shown in Fig. 12(c), except for a change in sign. Therefore, we hypothesize that the flow problem in absence of rotation ($f = 0$) has multiple solutions: a symmetric one (which we have seen in Fig. 11) and two asymmetric ones (which are obtained in the limits $f \to 0^+$ and $f \to 0^-$.). Thus, several questions emerge: Do the asymmetric states also exist in the absence of rotation? Are they stable with respect to small-amplitude perturbations? Is it possible to induce a transition from one solution to another, i.e. what is the stability of the flow solutions with respect to finite-amplitude perturbations? These issues are addressed in the following subsections.

(b) Existence and stability of multiple solutions for $f = 0$

To test whether an asymmetric state can indeed exist in the non-rotating limit, the model was run with $f = 0$, but initialized with a flow-solution derived from the $f/f_0 = 1/12$ experiment by removing the geostrophic pressure gradient associated with the mean flow. The initial flow field had a southerly component across the gap, as
in Fig. 12(c). The purpose of the experiment was to test whether this asymmetry is able to persist within the symmetric non-rotating dynamical framework. The integration was carried out for 96 h with a horizontal resolution of $\Delta x = 5.6$ km, and clearly demonstrated the stability of the asymmetric flow solution, even in the non-rotating framework.

The stability of the symmetric state was tested by a similar 96 h long integration, and the flow was found to be perfectly stable at a resolution of $\Delta x = 5.6$ km.

Both the symmetric and asymmetric flow solutions can be classified as wake flows since they contain non-zero potential vorticity (PV). The PV is generated by dissipative processes associated with the flow separation on the upstream side of the gap and gravity-wave breaking over the gap region, similar to the numerical experiments of flow past isolated topography (Schär and Durran 1997). To the extent that the stability of wake flows also depends upon the Reynolds number (Batchelor 1967), and thus
upon the numerical resolution employed, the stability analysis described above does not guarantee stability of the flows at higher resolutions (or Reynolds numbers). We have, thus, repeated the stability experiments described above with an increased resolution of $\Delta x = 2.8$ km. As initial and lateral boundary conditions the steady fields of the 5.6 km runs were used.

In the case of the asymmetric state we find that the flow is stable, but in the corresponding run with the symmetric state the initially symmetric flow decays into the asymmetric one due to a wake instability. In stratified flows such instabilities are due to the presence of anomalies of PV in the wake region, and are associated with barotropic energy conversions (Schär and Smith 1993; Schär and Durrant 1997). Further details on the decay of the symmetric solution at high resolution are, thus, analysed in terms of the low-level distribution of PV. At early times in the simulation (Fig. 13(a)), the positive and negative PV anomalies resulting from the flow separation at the eastern edge of the gap show excellent north/south symmetry. The vortices are stationary (no vortex shedding) for two reasons: firstly, the induced flow field of each vortex helps the other to withstand the westerly basic-state flow and, secondly, the vortices are partly sheltered.
from the upstream flow by the topography. During the transition to the asymmetric state (Fig. 13(b)) the symmetric negative PV anomaly in the north intensifies, whereas the southern positive PV anomaly weakens. This is accompanied by a shift of the whole PV dipole towards the south. Again, in terms of vortex dynamics, we speculate that now the intensified northern vortex can be held stationary by a weaker southern vortex because its closer position to the pass increases the sheltering effect. In contrast, the southern vortex now feels more strongly the westerly basic-state flow, and so it needs an intensified northern vortex to maintain its position.

In our particular numerical experiment the northerly solution occurs, but this is merely a coincidence and depends upon minor asymmetries (such as rounding errors) in the initial fields. A schematic picture of the PV pattern corresponding to the three different flow solutions is depicted in Fig. 14.

In summary, whilst traditional wake instabilities in homogeneous flows past blunt bodies (Batchelor 1967; Hannemann and Oertel 1989), in shallow-water dynamics (Schär and Smith 1993) and in stratified flows past isolated smooth topography (Schär and Durran 1997) yield either vortex shedding or a transition into a turbulent flow regime, the initially symmetric flow along a ridge transected by a gap can decay and settle down to an asymmetric stationary flow. From the behaviour of the individual vortices, there is some evidence that vortex shedding is suppressed here by the sheltering effect of the underlying gap topography. The instability process thus provides a satisfactory explanation for the nature of the multiple flow solutions.

(c) Externally induced transition between multiple flow solutions

When multiple solutions of a nonlinear problem are stable with respect to small-amplitude perturbations, it is still likely that finite-amplitude perturbations are able to promote a transition between the different flow solutions. To test this hypothesis several experiments were conducted at a resolution of $\Delta x = \Delta y = 5.6$ km. With this resolution, all the three multiple solutions are stable with respect to small-amplitude perturbations (see previous subsection). In this subsection we describe an experiment where a transition from the symmetric to the southerly asymmetric flow solution is induced. Note that, in a high-resolution run with $\Delta x = \Delta y = 2.8$ km, this experiment would be meaningless since the symmetric solution is unstable (see previous subsection).

The experiment starts with the symmetric state at time $t = 0$ and is integrated over $t = 60$ h. Between $t = 12$ h and $t = 24$ h a northward directed pressure gradient is applied. The forcing starts at time $t = 12$ h, increases linearly from zero within the next 6 h to its peak value, and then decreases again until it vanishes at time $t = 24$ (see Fig. 15(b)). The peak value of the pressure gradient at time $t = 18$ h is given by $(\partial \phi / \partial y)_p = -0.0167 \text{ m km}^{-1}$, where $\phi$ denotes the geopotential height. The pressure
forcing is applied through the lateral boundary fields. Results in terms of the time traces of the wind components at the gap are shown by the full lines in Fig. 15(b). These show that the transient forcing indeed promotes a permanent transition into the asymmetric flow state.

Similar experiments were carried out to test the transition from the southerly to the northerly flow regime, and from the southerly to the symmetric one. The former of the two experiments was successful, but we were unable to find a suitable forcing that enabled the transition from an asymmetric into the symmetric regime. This is probably indicative of the weak stability of the symmetric solution with respect to finite-amplitude perturbations.

Other transitions studied included an experiment where the upstream velocity was linearly increased during 24 h from $U = 10 \text{ m s}^{-1}$ to $30 \text{ m s}^{-1}$, using the asymmetric flow solution as initial condition. The increase in upstream velocity reduces the nonlinearity parameter $\tilde{\Delta H} = N \Delta H / U$. As expected, a transition into the symmetric flow solution did occur, at a time when the upstream velocity had a value of about $U = 15 \text{ m s}^{-1}$.

7. More realistic föhn-like settings

The experiments discussed so far in this paper were highly idealized and omitted many of the typical characteristics of Alpine shallow föhn events. In the following subsection we present additional results where consideration is given in turn to the impact of (a) deep valleys leading to a gap-like pass, (b) surface friction, and (c) the presence of an inversion to the south of the ridge. All these aspects give rise to additional interesting features yet, since the parameter space spanned by these additional environmental settings is very large, our presentation restricts attention to a few selected simulations. Unless stated otherwise, the simulations follow (as closely as possible) the standard setting of the nonlinear experiments presented in section 5.

(a) Impact of deep valleys leading up to the gap-like pass

In section 5(a) it was shown that the width of the ridge has a major impact upon the southerly winds through the gap. However, the topography used prescribed identical
widths to both the ridge and gap transects. In contrast, the real Alpine topography is characterized by deep valleys that intrude from both north and south into the Alpine ridge, and join at major Alpine passes. In reality, the width of the gap transect is, thus, much narrower than the ridge itself. This might be relevant for the flow dynamics, since short gap transects reduce the importance of rotational effects and might, thus, be more susceptible to gravity-wave propagation.

To test this hypothesis we used the alternative topographic configuration shown in Fig. 3(b). Here, the parameter $\beta$ in Eq. (4) is set to 0.5. The gap has the same north/south width as in the standard case (cf. Fig. 5), while the ridge has the same width as in the wide-ridge case (cf. Fig. 8(d)). Keeping the meridional width of the gap the same as in the standard case ensures that the length scale for the meridional gravity waves in the gap is unchanged. Increasing the width of the ridge, on the other hand, implies a larger cross-ridge pressure contrast, and is thus conducive to stronger acceleration on the gap transect. Results are shown in Fig. 16 and demonstrate the importance of the valleys. From the three settings considered, Fig. 16(b) reveals the strongest southerly flow with values up to $\sim 18$ m s$^{-1}$, compared with $\sim 10$ m s$^{-1}$ and $\sim 13$ m s$^{-1}$ in Figs. 5 and 8(d), respectively. A narrow gap also concentrates the southerly wind into the centre of the gap transect. The isentropes penetrate only marginally into the gap (Fig. 16(a)), and the signatures of the flow separation and lee vortex to the south-west of the gap are much stronger, and a somewhat weaker vortex now appears to the north-east of the gap. The air in the gap is well decoupled from the flow aloft. Furthermore, as compared with the case in Fig. 8(d), the southerly wind maximum is displaced further north, consistent with the reduced rotational effects upon gravity-wave propagation. Rotational effects suppress gravity-wave propagation, and the wider the gap, the more symmetric is the wind distribution. The optimal geometry for a shallow föhn is, thus, one with a wide mountain ridge (which induces a strong pressure contrast) and a narrow gap width (which allows strong gravity-wave effects).

\textbf{(b) Impact of surface friction}

To study the effects of surface friction, several additional experiments were conducted using a full surface-layer formulation (Louis 1979) representing a no-slip lower
boundary condition. To this end, the roughness length for the whole domain was set to a constant value of $g z_0 = 0.1 \text{ m}^2\text{s}^{-2}$, a value that corresponds to arable land. The surface fluxes of heat were set to zero in these experiments. Otherwise the setting was the same as in the standard nonlinear case discussed in section 4.

The inclusion of surface friction invalidates the analytical solution of the basic-state uniform flow. Since an analytical solution including the effects of surface friction is no longer available, we used a one-way nesting technique to obtain the background flow fields in the presence of a two-dimensional ridge. To this end, at the boundaries of the outer domain (resolution $\Delta x = \Delta y = 11.2 \text{ km}$, domain size $N_x = 121$, $N_y = 81$ grid points) we specified a uniform flow and stratification, whereupon a well-defined boundary-layer structure developed that included, at low levels, a near-neutral stratification and some northward Ekman flow. The steady-state structure of this boundary-layer flow was then used to drive the simulation in the inner nest (resolution $\Delta x = \Delta y = 5.6 \text{ km}$, domain size $N_x = 121$, $N_y = 81$ grid points).

The solutions for the no-slip and free-slip solutions are compared in Fig. 17. Figures 17(a) and (b) show that, even with surface friction included, the southerly winds through
the gap persist, although they are substantially weaker (≈4 m s⁻¹) than in the case with a free-slip lower boundary condition (≈10 m s⁻¹). The isentropes in a west/east cross-section also reveal striking differences in the strength of the upper-level gravity wave. In the no-slip case (Fig. 17(a)) the isentropes above the ridge are only slightly displaced from their upstream height and do not penetrate into the gap. Also, the prominent hydraulic-jump-like feature of the free-slip simulation (Fig. 17(b)) is completely absent, and the southerly winds across the gap have an almost symmetric distribution with respect to the gap axes.

Figures 17(c) and (d) show the deviation from the mean zonal along-ridge flow speed ($U = 10$ m s⁻¹), together with the isentropic distribution in a south/north section running across the gap. In both cases, there is a pronounced deceleration within the gap. However, as indicated by the drastically different flow solution discussed earlier in the paper, the underlying physical mechanisms for the deceleration of the zonal flow are very different. In the free-slip case (Fig. 17(d)) the deceleration is largely associated with the hydraulic-jump-like feature evident in Fig. 17(b), while in the no-slip case (Fig. 17(c)) the deceleration is dominated by boundary-layer processes. The zonal flow shows some asymmetry with respect to the ridge axes, which includes an accelerated jet-like feature to the south, slightly below the height of the ridge line. These features are already present far upstream of the gap and are presumably associated with the inhibition of the Ekman southerly flow across the ridge.

An additional effect of the surface friction is to remove completely the stationary trapped gravity mode identified in the linear simulation. This is not surprising because the gravity-wave response is drastically reduced, and because the trapped gravity mode has a shallow vertical structure and is, thus, susceptible to damping by surface friction.

All of this shows that the boundary layer actively supports the decoupling of the gap flow from the upper-level westerly flow. For the $z_0$ values under consideration, surface friction indeed leads to a completely different flow solution where the upper-level flow is almost unaffected by the presence of the gap. While both flow solutions in Fig. 17 exhibit a southerly gap flow, the underlying physical mechanism are quite different. In the free-slip flow regime, the flow response is governed by a deep gravity-wave response, while in the no-slip regime it is dominated by a pronounced decoupling of the upper-level flow from the boundary layer. The selection of the flow regimes is likely to depend on a range of parameters, such as the upstream stratification and wind speed, the underlying $z_0$ value, and other factors. Additional experiments with increased surface friction (larger $z_0$) showed no qualitative differences from the one presented in Figs. 17(a) and (c). For experiments with reduced $z_0$ values we found that the flow situation approaches the free-slip solution in a continuous way. There is no abrupt transition from a no-slip flow solution to a free-slip solution.

Overall, the experiment gives support to previous studies of boundary-layer effects in stratified flow past topography (e.g. Miller and Durran 1991; Georgelin et al. 1994), which found that boundary-layer effects can have a major impact upon the strength of the upper-level gravity waves. Our study of these aspects have only covered a small portion of the parameter space, and the resulting effects demonstrate that this is an area that would merit further study.

In the non-rotating limit discussed in the previous section, we found that the flow problem has multiple solutions. Additional experiments in this limit, but with surface friction included, have indicated that boundary-layer processes (at least with the value of the roughness parameter that was selected) may completely suppress the multiple flow solution. Even with strong perturbations we were unable to promote a switch from the
symmetric to the asymmetric regime. This is not surprising since the detailed analysis in
the previous section reveals that the multiple flow solutions depend upon the presence
of gravity waves and flow-splitting processes, which are suppressed by boundary-layer
processes.

(c) Impact of an inversion

The presence of strong inversions to the south of the Alps is a common feature in
many observations of deep and shallow south föhn. The inversion to the south (and its
absence to the north) appears to be associated with low-level advection and orographic
blocking of cold air, and with diabatic cooling from evaporating precipitation. Depend-
ing on the situation, the inversion height may be located below the pass height, or in
between the pass and the ridge height. Each of these situations is expected to lead to a
particular flow response. Here we give consideration to an idealized setting where the
inversion is located in between the pass and the ridge height. Such an inversion at a
height of 2000 m was, for instance, noted in the ‘Alpine föhn of the century’ by Seibert
(1985) and Hoinka (1985). These studies also suggest that the inversion to the south
supports the blocking of the low-level air in the Po valley.

To mimic the presence of an inversion to the south of the ridge (above pass, but
below ridge height), an idealized two-layer atmosphere was specified. Our standard
configuration was enhanced by adding a lower more stably stratified layer to the south of
the ridge, which was assigned a stratification of $N = 0.02 \text{ s}^{-1}$. In the upper layer and to
the north of the ridge we specified $N = 0.01 \text{ s}^{-1}$, as in the previous experiments. In order
for the difference to become more clear, we chose a gap depth of 1500 m (instead of the
standard case with 900 m), yielding a pass height of 1500 m. The inversion itself was
located at a height of 2500 m, i.e. 1000 m above the gap saddle point. Since the ridge had
a height of 3000 m, the imbalance below 2500 m between northern and southern side
was ‘blocked’, except near the gap. A strong density-driven current could only occur in
the gap region.

Quasi-steady results using the free-slip lower boundary condition are shown after
80 h integration time in Fig. 18. Figures 18(a) and (b) show how the southerly flow
across the gap is heavily intensified by the presence of the inversion, resulting in a
pronounced downslope windstorm with wind speeds up to 22 m s$^{-1}$. Also, the jet of
southerly winds now extends much farther towards the north (Fig. 18(c)). The strong
gap winds of the inversion case are in accord with the increased meridional pressure
gradient, and the whole flow is now reminiscent of a ‘flowing off’, as in the hydraulic
theory. As a result, there is also a net warming near the gap exit resulting from the
adiabatic warming of air parcels descending from near the inversion level. Such a
warming (and drying) is a very common feature of föhn observations in the Alps. For
the simulations in the previous sections, however, it is only marginally present, since air
parcels that reach the lee slope of the gap originate from similar levels upstream.

The overall dynamics of the flow still appears comparable with the standard non-
linear case discussed in section 5, except for the gravity-driven enhancement resulting
from the mass imbalance on the two sides of the obstacle. For instance, the hydraulic-
jump-like feature on the western flank of the gap is present in all simulations, and there
is a clear indication of flow separation and flow splitting upstream and downstream of
the gap (not shown). Note also the absence of steepened or overturned gravity waves in
the south/north sections of Fig. 18(a). This feature is presumably related to the stronger
stability to the south which will limit the wave amplitude for northward pointing wave
vectors.
Additional experiments were conducted for slightly different configurations. One case studied included, in addition to an inversion to the south, a low-level cold-air pool to the north. As expected, the presence of this pool retards the northward progression of the warm föhn-like gap flow. In yet another case consideration was given to a two-layer structure with a step increase in stratification at a height of 3500 m. This configuration produced interesting interactions of gravity waves with the stratification profile, and appears to hinder flow-splitting in the lower layer.

8. CONCLUDING REMARKS

This study was motivated by the frequent occurrence of shallow föhn in the Alpine region. In contrast to deep föhn cases, the cross-Alpine southerly flow in shallow föhn cases is essentially confined to the valley transects, whereas the mid-tropospheric flow can have a westerly, or even a north-westerly, direction. Here, consideration was given to an idealized model problem: the flow of a rotating stratified stream of air along a two-dimensional ridge transected by a gap.

With intermediate Rossby numbers and a sufficiently deep gap, the flow within the gap decouples from the flow aloft and is driven by the geostrophic pressure gradient...
along the gap axes to yield a fohn-like flow. The associated dynamics is heavily affected by gravity-wave propagation, critical levels and nonlinear effects, and there is also some evidence that a transition to supercritical-like flow may occur. Factors that increase the strength of the pressure-driven flow across the gap include an optimal scaling of the ridge width (wide enough to feel the rotational effects, narrow enough to allow gravity-wave propagation), a deep gap to force the decoupling from the upper-level flow, and an inversion to the south of the Alpine-like topography which, in effect, increases the pressure gradient.

Of particular interest is the demonstration that the limit $f \to 0$ is singular, i.e. the flow solution does not converge towards the symmetric $f = 0$ solution. Rather, there exist multiple stationary solutions for $f = 0$ (two with northerly and southerly flow across the gap, respectively, and one with north/south symmetry). The existence of these multiple steady states is related to a wake instability, yet vortex shedding is suppressed by the presence of the ridge downstream of the gap. Their occurrence is of substantial conceptual and practical interest. To our knowledge it is the first demonstration that such multiple steady states can exist in simple mountain-flow problems. It also appears possible that the sudden onset and decay of a shallow south fohn may be related to the transition between multiple flow solutions, for instance when triggered by slow changes in the larger-scale flow. The current analysis of multiple steady states restricts attention to non-rotating flow with a free-slip lower boundary condition, but it is possible that similar multiple states may also exist in a more general framework.

The flow across the gap shows several of the characteristics of typical shallow Alpine south-fohn events. The realism of the solutions is further increased when an inversion on the right-hand flank of the ridge (looking along the ridge in the upper-level flow direction) is included. Such an inversion over the Po valley is often observed in Alpine foehn cases. The overall flow structure is highly complex, and shows several hydraulic-jump-like features. One such jump may occur on the gap axes downstream of the mountain pass (see Fig. 18(b)). Yet another one may occur in the long-ridge direction at the western flank of the gap (see Fig. 17(b)), and it appears to support the decoupling the near-surface southerly flow through the valley transect from the westerly flow aloft. At lower levels there is clear evidence for flow-separation and flow-splitting effects. Gravity-wave activity is responsible for a strong amplification of the imposed pressure gradient across the gap.

The direct applicability of the model problem to the shallow Alpine south fohn is clearly limited by its simplicity. In particular, sensitivity studies conducted in section 7 demonstrate that, beside rotational effects, the shallow fohn is sensitive to surface friction and to details in the stratification profile. Additional factors that might need consideration include moist dynamics, as well as a fully three-dimensional forcing (e.g. by a confined southerly flow ahead of an approaching cold front). Furthermore, the current study is limited to hydrostatic dynamics and is, thus, not applicable to narrow gaps with horizontal scales below a few kilometres. Despite these limitations, the adoption of an idealized flow configuration has served to reveal some of the basic dynamical properties of rotational flow past a gap, a flow problem that is very common in our atmosphere.

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