Reconsideration of the physical and empirical origins of $Z-R$ relations in radar meteorology

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SUMMARY

The rainfall rate, $R$, and the radar reflectivity factor, $Z$, are represented by a sum over a finite number of raindrops. It is shown here and in past work that these variables should be linearly related. Yet observations show that correlations between $R$ and $Z$ are often more appropriately described by nonlinear power laws. In the absence of measurement effects, why should this be so?

In order to justify this observation, there have been many attempts to create physical 'explanations' for power laws. However, the present work argues that, because correlations do not prove causation (an accepted fact in the statistical sciences), such explanations are suspect, particularly since the parametric fits are not unique and because they exhibit fundamental physical inconsistencies. So why, then, do so many correlations fit power laws when physical arguments show that $Z$ and $R$ should be related linearly?

It is shown in the present work that physically based, linear, relations between $Z$ and $R$ apply in statistically homogeneous rain. (Note that statistical homogeneity does not mean that the rain is spatially uniform.) In contrast, nonlinear power laws are empirical fits to correlated, but statistically inhomogeneous data. This conclusion is proven theoretically after developing a 'generalized' $Z-R$ relation based upon physical consideration of $R$ and $Z$ as random variables. This relation explicitly incorporates details of the drop microphysics as well as the variability in measurements of $Z$ and $R$. In statistically homogeneous rain, this generalized expression shows that the coefficient relating $Z$ and $R$ is a constant resulting in a linear $Z-R$ relation. In statistically inhomogeneous rain, however, the coefficient varies in an unknown fashion so that one must resort to statistical fits, often power laws, in order to relate the two quantities empirically over wide varying conditions. This conclusion is independently verified using Monte Carlo simulations of rain from earlier work and is also corroborated using disdrometer observations. Thus, the justification for nonlinear power-law $Z-R$ relations is not physical, but rather statistical, in that they provide convenient parametric fits for estimating mean $R$ from measured mean $Z$ in statistically inhomogeneous rain.

Finally, examples based upon disdrometer data suggest that such generalized relations between two variables defined by such sums are potentially useful over a wide range of remote-sensing problems and over a wide range of scales. The examples also offer hope that data collected over disparate sampling-volumes and sampling-frequencies can still be combined to yield meaningful estimates. Although additional testing is required, this allows us to write programs which combine estimates of $R$ using remote-sensing techniques with sparse but direct rainfall observations.

KEYWORDS: Disdrometer data Radar reflectivity Rainfall inhomogeneities Rainfall rate

1. ON THE NON-UNIQUENESS AND PHYSICAL INCONSISTENCIES OF NONLINEAR $Z-R$
   POWER-LAWS

A great deal of effort has been devoted to translating radar measurements of radar reflectivity factor, $Z$, into estimates of rainfall rate, $R$, beginning with the revolutionary papers of Laws and Parsons (1943) and Marshall and Palmer (1948) who noted that observed drop-size distributions could be fitted by an exponential function.

This finding led Marshall and Palmer (1948) to formulate a power-law relation of the form

$$Z = 296R^{1.47}, \quad (1)$$

where $Z$ has units of $\text{mm}^6\text{m}^{-3}$ and $R$ is in $\text{mm h}^{-1}$. Thus, while far from perfect, $Z$ became a useful tool for estimating rainfall. This classic relation between $R$ and $Z$ has been pursued essentially unchanged for the last 50 years with a few notable exceptions.

Most noteworthy is the linear relation between $Z$ and cloud water content, $W$, developed by Bartolome and Atlas (1951), as well as by Atlas and Bartolome (1953), for

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clouds, and applied to a wide variety of conditions including rain, as described by Atlas (1964). More relevant here, Atlas and Chmela (1957) extended this finding to include $Z$ and $R$ through the relation

$$Z = 5.4 \times 10^4 G \left( \frac{D_0^3}{\nu_0} \right) R,$$

(2)

where $D_0$ is the median volume diameter (i.e., the drop diameter dividing the liquid-water content into equal halves), $\nu_0$ is the terminal fall-speed of drops with diameter $D_0$, while $G$ and the bracketed term are functions only of the distribution of drop diameters.

List (1988) suggested that linear $Z$–$R$ relations apply only to equilibrium drop-size distributions. Such distributions are thought to be the result of a balance between drop break-up and coalescence leading to a drop-size distribution which is invariant in time. Consequently, increasing (decreasing) the drop concentration simply shifts such distributions up (down) on plots of concentration against drop size. Hence, regardless of drop concentration, $Z = CR$, where $C$ is a constant. It is doubtful, however, if such equilibrium distributions actually exist in natural conditions. (We believe they do not, in part since control volumes do not.) In particular, the linearity of a $Z$–$R$ relation does not prove that the drop-size distributions are necessarily in equilibrium. Instead, section 3 of the present work shows that linear relations, $Z = CR$, exist for more general reasons.

At the moment what is important, however, is that (2) is justified physically and theoretically whereas (1) is derived from observations. Clearly, the two expressions differ, and a natural question is, then, can theory (summarized in (2)) and observations (summarized in (1)) be made to agree? And, if so, how? The historic approach has been to create apparent physical explanations for (1) while ignoring (2). The rest of the present section will focus on how such an approach is inappropriate and leads to physical inconsistencies. That is, (1) and (2) cannot be reconciled using physical arguments. Later, in section 3, we offer, instead, a reconciliation of (1) and (2) based on considerations of the statistical physics of rain. However, before addressing that question, it is necessary first to reconsider the formulation of the drop-size distribution as originally discussed by Kostinski and Jameson (1999) and elaborated on in greater detail below.

Clear separation (decoupling) of number-density and size effects is needed in order to give a size distribution, $p(D)$, a proper definition. It is well accepted in other areas of atmospheric physics, for example, that $p(D)$ is a function only of size and not of the concentration, $n$. For example, Seinfeld and Pandis (1998, Eqs. (7.1) and (7.2)) elaborate clearly on the meaning of $p(D)$ as the probability of a particle picked at random having a diameter between given limits. Such a probability is an intensive not an extensive property. The very same point was made by Friedlander (1977) who emphasized that size distribution is independent of concentration. In other words, in many applications it is desirable for a statement like “30% of particles are contained between sizes $D_1$ and $2D_1$” to have a meaning independent of whether there are only tens or billions of particles per unit volume.

Therefore, let us view rain as a stochastic quantity and apply this same classical physics approach (e.g., as used in developing the Boltzmann distribution of molecular kinetic energies in an ideal gas) dating from well before the discovery of drop-size distributions. That is, let us regard the normalized part of the drop-size distribution as a probability density function (pdf) of drop diameter $D$. To that end, we begin by reconsidering the customary way of writing an exponential drop-size distribution, namely

$$N(D) = N_0 \exp(-\Lambda D).$$

(3)
First of all, multiplication by \( D \) and integration from 0 to \( \infty \) reveals that \( \Lambda = \overline{D}^{-1} \), where \( \overline{D} \) is the average drop-size, so that (3) becomes

\[
N(D) = N_0 \exp \left( -\frac{D}{\overline{D}} \right). \tag{4}
\]

Furthermore,

\[
\int_0^\infty N(D) \, dD = N_0 \overline{D} = n, \tag{5}
\]

where \( n \) is the total number of drops per unit volume. Consequently, (3) may finally be written as

\[
N(D) = n \left\{ \frac{1}{\overline{D}} \exp \left( -\frac{D}{\overline{D}} \right) \right\} = n \times p(D), \tag{6a}
\]

where the expression in the curly brackets is the *pdf of drop sizes*, \( p(D) \), that satisfies the necessary condition \( \int_0^\infty p(D) \, dD = 1 \). Then the average number of drops per unit volume within a size range \( (D, D + dD) \) is given by \( n \times p(D) \, dD \) in a manner completely analogous to the Boltzmann distribution of molecular energies.

There are three noteworthy points. Firstly, the meanings of \( N_0 \) and \( n \) are entirely different—as reflected in the units. The former has units of \( \text{mm}^{-1} \text{m}^{-3} \) while the latter is measured simply in inverse cubic metres with the physical meaning of number density (that is, the total number of raindrops per unit volume). Also note, using (5), that \( N_0 = n / \overline{D} \). Consequently, \( N_0 \) can increase for two very different reasons; firstly \( n \) increases (more raindrops per unit volume) and, secondly, \( \overline{D} \) decreases (smaller drops). Waldvogel (1974), for instance, reports a case where \( N_0 \) jumps by an order of magnitude while the rain rate \( R \) remains constant (see also Pruppacher and Klett 1997). Such a jump in \( N_0 \) can occur by shifting to more numerous smaller drops so that \( n \) increases, \( \overline{D} \) decreases, but \( R \) remains constant as already discussed by Kostinski and Jameson (1999). This might occur, for example, when, for some reason, drop breakup exceeds drop coalescence while the liquid-water content remains constant.

Secondly, (6) does not involve \( D_0 \), the so-called median drop-diameter or the size of drop that partitions the distribution of liquid-water content into two equal halves. One cannot, therefore, simply replace \( \overline{D} \) by \( D_0 \) in (6) since, as integration then shows, \( \overline{D} \) would have to equal \( D_0 \), only valid for monodisperse distributions. Thus, the expression of the pdf of \( D \) requires \( \overline{D} \). Use of \( \overline{D} \) has some advantages as well since, for example, over all drop sizes, (6) implies that \( W \propto n \overline{D}^3 \) while \( R \propto n \overline{D}^{3.67} \) for exponential drop-size distributions.

Thirdly, and most revealing, (6) shows that like all moments of the diameter distribution, \( \overline{D} \) is a function only of \( p(D) \) and is decoupled from the random variable \( n \). Conversely, \( n \) is a random variable decoupled from \( p(D) \) and, therefore, from \( \overline{D} \). The two random variables may or may not be weakly statistically correlated at times, but (6) at least allows us to address that issue.

Such a conclusion, however, seems to be contrary to the perceptions of many authors that drop-size distributions in heavier rain are different from those in lighter rain, thus implying that the distribution of drops depends upon the total number of drops, \( n \), in apparent conflict with the statement above. That is, \( p(D) \) should really be \( p(D, n) \).
Consider, for example, families of drop-size distributions such as those of Marshall and Palmer (1948) and Sekhon and Srivastava (1971) in which the slope, $\Lambda$, (which in our discussion $= \frac{1}{D}$) of the exponential decreases ($\bar{D}$ increases) with increasing rainfall rate. Moreover, the intercept for such families, $N_0$, (often incorrectly assumed to be some kind of proxy measurement of drop concentration), also increases as $R$ increases. Converting from $N_0$ to $n$, as described above, means that for an exponential distribution, $n \propto N_0 \bar{D} = \frac{N_0}{\Lambda}$ so that $n$ should also increase with increasing $R$. That is, for such families of distributions, there is usually positive correlation between increasing $n$ and increasing $\bar{D}$ (decreasing $\Lambda$). However, it is important to remember that not all size distributions belong to such families. It is possible, for example, to have significant rainfall rates associated with a very narrow distribution of drops (as occurs when the biggest drops descend first out of Florida cumulus clouds well ahead of the other smaller drops). On the other hand, at other times only light rainfall may come from stratiform clouds having relatively broad distributions of drops.

More importantly, such remarks are based upon a subtle but fundamental misapplication of the concept and meaning of correlation. As Blalock (1961) states so succinctly, “There are two distinct uses for regression equations, (1) as estimating equations, and (2) as causal models.” Yet according to Kotz and Johnson (1982) “That ‘correlation’ is not causation is perhaps the first thing that must be said.” And later on that same page, “It is not only true that correlation does not imply causation; it is also true that causation does not necessarily imply correlation . . .”

We, therefore, adopt this conservative view, consistent with many statisticians’ perspective, that the correlation relations of Marshall and Palmer and Sekhon and Srivastava are useful for providing estimates of mean values. It is neither necessarily correct nor clear that such equations can or should be used as the basis for developing physical, functional (causal) relations between $Z$ and $R$ unless one can explain precisely and physically how $n$ determines $p(D)$ or, conversely, how $p(D)$ determines $n$. Furthermore, not only is it doubtful if a general explanation of this problem exists, but it is also shown in section 3 that such nonlinear power-law $Z-R$ relations can arise entirely from statistical factors alone.

It also appears that this misinterpretation of correlation expressions as causal equations can also produce physical inconsistencies. For example, let us now ask the question: How do concentration and size factors enter the $Z-R$ relations of the form $Z = AR^b$? To address this, we begin with the definitions of $Z$, $W$ and $R$ for any drop-size distribution, namely

$$Z = n \times c_Z \int_0^\infty D^6 p(D) \, dD, \quad (7)$$

$$W = n \times c_W \int_0^\infty D^3 p(D) \, dD \quad (8)$$

and

$$R = n \times c_R \int_0^\infty V(D) D^3 p(D) \, dD, \quad (9)$$

where $V(D)$ is the terminal velocity of a drop with diameter $D$ and the various $c$’s are constants. Note that the number density $n$ appears in front of each integral.
In a $Z = AR^b$ (or $Z = AW^b$) relation, where $A$ is a constant, the two sides of the equation are measured by two different instruments (radar and rain-gauge or disdrometer) with vastly different resolution volumes. (Note that there is an inherent, fundamental ambiguity in trying to relate a measurement of a flux ($R$) across a surface to quantities that depend on the number of drops in a volume ($Z$, $W$, etc.) as discussed in the next section.) Let us imagine, therefore, an ideal radar whose sampling volume is small enough to equal that of a rain-gauge (i.e. ignore effects of scaling for now). Furthermore, for simplicity, let us just consider the relation $Z = AW^b$. Substituting (7) and (8) and allowing $A$ to include various powers of $c$'s, the power law implies that

$$n \times \int_0^\infty D^6 p(D) \, dD = A \times n^b \times \left( \int_0^\infty D^3 p(D) \, dD \right)^b,$$

(10)

or, dividing through by $n$,

$$\int_0^\infty D^6 p(D) \, dD = A \times n^{b-1} \times \left( \int_0^\infty D^3 p(D) \, dD \right)^b.$$

(11)

Equation (11) can be rewritten more simply as

$$\overline{D^5} = n^{b-1} \overline{D^3}^b,$$

(12)

where the overbar denotes averaging over the pdf of $D$.

Let us interpret (12) from two different viewpoints. Firstly, from the perspective of the strict functional independence between $n$ and $p(D)$ just described above, the appearance of $n$ on only one side of the equation but not the other requires $b = 1$ for all $N(D)$. On the other hand, those using correlations as substitute equations of causality would claim (since $A$ is a constant) that, for example,

$$n \propto \overline{D^3}^q,$$

(13)

where $q > 0$ so that in a relation such as $Z = AW^b$ as above, $b > 1$. But are such correlations unique and justifiable? The answer is no.

The functional dependence implied by (13) means that there should be positive correlation between $n$ and $\overline{D^3}$. Yet, based on (8), it is clear that $n$ and $\overline{D^3}$ can also be negatively correlated when $W$ is approximately constant. Hence, we conclude that there is no unique correlated behaviour between $n$ and $\overline{D^3}$ consistent with a functional relation such as (13). It is most reasonable, then, to set $b = 1$.

Finally, it is important to recall that because the $Z-R$ relations such as those of Marshall and Palmer are derived either directly or indirectly using correlations, the forms of such parametric equations remain somewhat arbitrary (for example, see Fig. 4(b)). Thus, any physical explanation used to justify one form is unlikely to be particularly useful in justifying another. Such non-uniqueness of explanations casts doubt on the validity of the entire approach of explaining correlations physically, at least not without clear delineation of physical mechanisms and without experimental results.

In a subsequent section, we show how the apparently irreconcilable differences between (1) and (2) may be resolved. However, in order to make the discussion up to now more concrete and in order to emphasize the truly stochastic nature of rain, we first make a minor diversion using video-disdrometer observations. A reader so inclined may skip directly to section 3. There we return to this discussion, using a precise
definition of stochastic rain to derive a generalized $Z$–$R$ relation that not only provides new insights into resolving the difference between (1) and (2) but also suggests some new and interesting applications.

2. AN OBSERVATIONAL EXAMPLE

(a) On the processing of disdrometer data

Disdrometers measure the flux of drops across a sample surface. Yet often the quantity of interest (such as $Z$) is a function of the number of drops in a sample volume. The flux, however, can be readily converted into a concentration after dividing by the terminal fall-speed at each drop size, and then adjusting units.

There are at least two difficulties with this procedure. Firstly, the counts of drops of different diameters in a sampled time interval refer to volumes that differ with drop diameter. Since the fall speed of raindrops increases with size, large drops falling through a horizontal surface in unit time come from a larger volume than do small drops. Consequently, the calculated concentrations do not occupy the same sampling volumes, volumes that often vary by factors of 1–10. Moreover, Kostinski and Jameson (1997) and Jameson and Kostinski (1998) have shown that the fluxes at different sizes are correlated amongst each other both temporally and spatially over many different scales. Thus, in general, rather than being distributed evenly, there is significant clustering or bunching of the rain. That is, regions richer in drops are interspersed with those where drops are scarcer. Given this clustered nature of rain, it may be quite presumptuous at times to assume that the small drop concentrations, occupying only a small fraction (about 10–20 per cent) of the sample volume filled by the larger drops, can be harmlessly combined to estimate drop-size distributions and concentrations.

More specifically, the sample surface of the video-disdrometer is 100 cm$^2$. In one second, then, the number of drops travelling at 6 m s$^{-1}$, will have come from a volume of 60 litres so that an estimate of the number of drops in a cubic metre requires multiplication by 16.67. On the other hand, drops falling at only 2 m s$^{-1}$ will have come from a volume of only 20 litres so that the estimate of the number of drops in a cubic metre requires multiplication by 50! (Note that long sample-times, such as one minute, may require division, but the assumptions of spatial homogeneity of concentration is still assumed for all sampled drop sizes, and the disparity in sample volumes still remains regardless of the duration of the sample periods.) Kostinski and Jameson (2000) showed that the multiplication (division) by a factor $\alpha$, where $\alpha > 1$, needed to estimate the drop concentrations in this way, distorts the count statistics since the mean increases (decreases) by $\alpha$, but the variance increases (decreases) by $\alpha^2$. One, therefore, should not use such concentrations for the study of the statistics of the spatial distribution of drop counts or for computing correlations among variables.

In order to avoid such potential difficulties, we instead begin by fixing the sample volume for all drops regardless of size. We then count the number of drops in a sequence of equal sample volumes. Since this volume is the area of the sensor times a length, this is accomplished by first dividing a distance into equal, incremental lengths, say 10 cm long. The sequential series of the arrival times of each drop across the sensor surface beginning at some initial starting time is then analysed such that an observed drop is placed into the nearest 'length' bin determined by the elapsed time multiplied by the nominal terminal fall-speed of the drop. This is then done for all the observed drops. Finally, a resample length, say 1 m or 10 m, is selected and a summation made over all the different drop-size bins over all the incremental distances. In other words, we place all the observed drops into a 'true' sample space and then spatially resample over
Figure 1. The rainfall rate, $R$, and radar reflectivity factor, $Z$, calculated for the spatially rebinned video-disdrometer data as described in the text for true sample volumes of 10 litres (lines) and 100 litres (symbols). The sample number corresponds to a vertical distance in metres.

a length (e.g. 1 m or 10 m) that will give us a reasonable sample volume such as 10–100 litres, for example.

Consequently, then, we can derive the ‘true’ $n$, $\bar{D}$, $Z$ and $W$ over a common sampling volume unaffected by the vagaries introduced through multiplication and division.

It is important to note, however, that this procedure limits the total space sampled to that corresponding to the product (sampling interval)×(fall speed of the smallest drop chosen for inclusion in the spatial resampling). For example, if the range of drop sizes implies a ratio of one to five between the smallest and largest fall speeds, then only 20 percent of the total interval will be available after spatial rebinning.

(b) An example and implications for new directions

Figure 1 is a plot of $R$ and $Z$ computed for the one-second spatially rebinned video-disdrometer-data at 0.01 m$^3$ (10 litre) and 0.1 m$^3$ (100 litre) volume resolutions. Obviously both quantities vary over a wide range during this period with the rainfall rate changing by a factor of three or four in the course of one or two successive samples, while $Z$ varies by factors of six or more. Why?

In Fig. 2, the distribution parameters $\bar{D}$ and $n$ are plotted. (Remember that for the exponential distribution (6), $1/\bar{D}$ is the ‘slope’ of the distribution.) It is clear that $n$ changes by factors of up to 200 or more, while $\bar{D}$ varies only by about a factor of about three. Also note that the coefficient $N_0$ introduced by Marshall and Palmer (and $= n/\bar{D}$ for an exponential distribution (Kostinski and Jameson (1999) and (6)), varies most of all. This occurs because, as Fig. 3 illustrates, $n$ and $\bar{D}$ are at most only weakly correlated. Thus, where $\bar{D}$ is small but $n$ is large, $N_0$ can become very large indeed, while the opposite is true when $\bar{D}$ is larger and $n$ smaller.

Furthermore, $n$ is not correlated with $R$ (as illustrated in Fig. 4(a) where the correlation coefficient $\rho = -0.15$). On the other hand, as discussed in the previous section, it is sometimes argued that $D_0$ (or in that the two are related equivalently, $\bar{D}$) is correlated with $R$. Indeed, Fig. 4(b) shows a correlation ($\rho = 0.78$). However, it is
Figure 2. The sample series of the mean drop-diameter $\bar{D}$, rain water content $W$, total drop number $n$ and Marshall-Palmer coefficient $N_0$ for the spatially resampled data at resolutions of 10 litres (lines) and 100 litres (circles). The sample number corresponds to a vertical distance in metres.

Figure 3. The scatter diagram of the total number of drops $n$, in 10 litres, versus the mean drop-diameter $\bar{D}$, using the spatially resampled video-disdrometer data at 10-litre resolution. Note the lack of correlation between the two variables.

It is important to remember that such correlation alone is not sufficient to deduce that the two are functionally (causally) related.

Interestingly, the correlation coefficient between $n$ and $\bar{D}$ is only $-0.23$ for these data (and only $-0.27$ for a resolution of 0.1 m$^3$). The slight negative correlation suggests that there is a very weak tendency in these data to conserve water. That is, if the number of drops increases, then the average diameter tends to decrease and vice
versa. Such behaviour is completely the opposite of expectations based upon power-law relations such as those of Marshall and Palmer. These power-law relations imply that, as increasing functions of $R$ (e.g., see expressions by Kostinski and Jameson 1999), both $\overline{D}$ and $n$ should increase together, i.e. they should be positively correlated. This is not so for the present data.

Furthermore, the ‘shape’ or ‘form’ of the drop-size distribution also varies rapidly (Jameson and Kostinski 2000) as is illustrated using the ratio of $\overline{D^3}$ and $\overline{D^2}$. This ratio is independent of drop concentration and, for an ‘infinite’ exponential distribution, should have a constant value of six. Obviously, as Fig. 5 illustrates, truncation effects and
distribution mixing (Jameson and Kostinski 2000) cause wide variations. Hence, the variability of $n$, $\bar{D}$ and drop-size distributions all contribute to produce great variability between $Z$ and $R$, a variability that acts to obscure any deterministic relation between the two variables.

This is a fundamental problem that is usually addressed by performing a 'fit' as Fig. 6 illustrates. However, neither a linear nor a power-law fit provide a particularly adequate description of how $Z$ and $R$ vary with respect to each other for these data. We explore the reason for this in the next section where we show that the 'explanation'
3. On the Deeper Meaning of Linear Z–R Relations

(a) Derivation of a generalized Z–R relation

The measurement of precipitation by radar is a multi-faceted, complex problem that depends upon details at the smallest microscale of individual scatterers up to large-scale meteorological structures all convolved with engineering characteristics of the measurement devices and observation procedures. While the latter concerns have been treated in many books and articles (e.g. Harrold et al. 1974; Collier et al. 1983; Zawadzki 1984; Joss and Waldvogel 1990) and while a great deal of attention is paid to the details of the microphysical scattering process as well, the stochastic structure of the precipitation itself is routinely neglected, in part because it is not well understood. Yet this stochastic nature of quantities like rain is responsible for much of the variability in both R and Z. Consequently, there is generally substantial statistical uncertainty surrounding all observed Z–R relations. We address this issue in this section.

To be more specific, R is defined by

\[ R = \text{const} \times \sum_{i=1}^{k} D_i^3 V_i, \]  

(14)

where \( D_i \) is the \( i \)th drop, \( V_i \) is the terminal velocity corresponding to \( D_i \), \( k \) is the instantaneous total random number of drops and the summation is over a unit sample-volume. We may then consider the quantity \( Y_i = V_i D_i^3 \) in (14) to be the random variable resulting from the transformation of \( D_i \) and \( R \) as the random sum described by a random total number of drops and random sizes.

Moreover, similar definitions apply to many other parameters in physical meteorology such as the rain water content \( W \) and radar reflectivity factor \( Z \) (in the Rayleigh scattering limit). That is,

\[ W = \text{const} \times \sum_{i=1}^{k} D_i^3, \]  

(15)

and

\[ Z = \text{const} \times \sum_{i=1}^{k} D_i^6. \]  

(16)

Consequently, assuming that \( k \) and \( p(D) \) are decoupled as argued above, it follows from (14) and (16) that the expected values, \( E \), of \( R \) and \( Z \) are given by

\[ E(R) \propto E(k)E(D^3 V) \]
\[ E(Z) \propto E(k)E(D^6), \]  

(17)

where \( k \) is the total number of drops. (Note that if \( k \) is too small, spurious correlations between \( k \) and the moments of \( p(D) \) can appear simply because of inadequate sampling across the entire \( p(D) \).)

However, in order to account for variability, it is necessary first to understand the variances of the quantities. For sums such as (14) and (16), the variance of such a
random variable can be expressed in terms of the conditional mean and variance (e.g., Ochi (1990), p. 65, Theorem 3.5) as

\[ \text{Var}(R) = E_k(\text{Var}(R|k)) + \text{Var}_k(E[R|k]), \]  

(18a)

where \( k \) is the total number of drops in one summation, the vertical bar denotes conditional probability, while \( E_k \) is the expectation value and \( \text{Var}_k \) is the variance with respect to random variable, \( k \). The second term on the right-hand side of (18a) can be rewritten (Ochi 1990, p. 67) as

\[ \text{Var}_k(E[R|k]) = \text{Var}[k](E[D^3V])^2. \]  

(18b)

We are then left with the first term. However, the variance of sums such as (14) and (16) is sensitive to the presence of correlations. For a given \( k \), the variance of \( R \) must be proportional to the variance of \( (D^3V) \). If each term in the sum \( R \) were statistically independent, the first term would be \( E[k]\text{Var}(D^3V) \) (Ochi 1990, p. 67). On the other hand, it has been determined that drops in the sum are not statistically independent (Kostinski and Jameson 1997; Jameson and Kostinski 1998). In fact, measurements show positive correlations at zero lags (Jameson et al. 1999) so that we may write

\[ E_k(\text{Var}(R|k)) = F_R E[k]\text{Var}(D^3V), \]  

(18c)

where, obviously, \( F_R \geq 1 \) where the equality holds when statistical independence applies.

By fixing \( k \) and determining \( D^3V \), observations indicate that the value of \( F_R \) (and that of the similar term \( F_Z \) for the radar reflectivity factor) generally falls in the range between one and four. We shall see later, though, that, in the end, the precise values are not critical.

Expression (18a) then becomes

\[ \text{Var}(R) = F_R E(k)\text{Var}(D^3V) + \text{Var}(k)E(D^3V)^2 \]  

(19)

and for \( Z \) the equivalent expression is

\[ \text{Var}(Z) = F_Z E(k)\text{Var}(D^6) + \text{Var}(k)E(D^6)^2, \]  

(20)

where \( F_Z = E_k(\text{Var}[Z|k])/(E[k]\text{Var}(D^6)) \) and \( F_R = E_k(\text{Var}[R|k])/(E[k]\text{Var}(D^3V)) \). Note that the variances of \( (D^3V) \) and \( D^6 \) are functions of the way the drop sizes are distributed and should be evaluated using each measured drop. However, at times this may be inconvenient or impossible, particularly if the data are unavailable. Instead, it may be necessary to estimate \( \text{Var}(D^3V) \) and \( \text{Var}(D^6) \) using an 'average' drop-size distribution measured over the interval of observations. Examples below show that such an approach often appears adequate. It is also worth mentioning here that the observed \( \text{Var}(R) \) and \( \text{Var}(Z) \) are affected by drop clustering as well as other factors such as sample volume-size.

After dividing by squares of the respective expected values given by (17), we write (19) and (20) in terms of the relative dispersions of each quantity, so that

\[ \frac{\sigma(R)}{E(R)} = \left\{ \frac{E(k)}{\sigma^2(k)} \frac{F_R \sigma^2(D^3V)}{E^2(D^3V)} + 1 \right\}^{1/2} \frac{\sigma(k)}{E(k)}. \]  

(21)

and

\[ \frac{\sigma(Z)}{E(Z)} = \left\{ \frac{E(k)}{\sigma^2(k)} \frac{F_Z \sigma^2(D^6)}{E^2(D^6)} + 1 \right\}^{1/2} \frac{\sigma(k)}{E(k)}. \]
These expressions may then be combined to yield the final result

\[
E(R) = \left\{ \frac{E(k) F_Z \sigma^2(D^6)}{\sigma^2(k) E^2(D^6)} + 1 \right\}^{1/2} \frac{\sigma_R}{\sigma_Z} E(Z),
\]

(22)

where \(\sigma_R\) and \(\sigma_Z\) represent the standard deviations of the rainfall rate and radar reflectivity factor, respectively, while \(\sigma^2\) denotes the variance. (Note that \(\sigma_Z\) only includes the intrinsic variability of \(Z\) after accounting for the effects of signal statistics as discussed in further detail later.) This turns out to be an extremely interesting equation that not only includes the microphysics through the pdfs of \(D^3V\), \(D^6\) and \(k\), but also incorporates the variability in \(R\) and \(Z\) as well. It is also noteworthy that the \(Z\) and \(R\) are still linearly related.

As discussed earlier, observations indicate that \(F_R\) and \(F_Z\) vary from near unity to around four. Moreover, since both are affected simultaneously by the same drop-correlations, they vary together in a highly correlated manner. Consequently, they tend to cancel each other in (22). Furthermore, as calculations and examples below show, they apparently often have little effect on the final value of the quantity in brackets. For convenience we will refer to the square root of the non-dimensional bracketed quantity as \(C\).

We begin exploring (22), firstly for the degenerate case of Poisson drop-counts and a monodisperse drop-size distribution. In this case, since the variances of \(D^6\) and \(D^3V\) are zero, \(C\) tends to 1. On the other hand, (19) and (20) imply that \(\sigma_R\) tends to \(\sigma_k E(D^3V)\), while \(\sigma_Z\) tends to \(\sigma_k E(D^6)\). Consequently, using (17), (22) reduces to the trivial example that \(E(R) = \text{const} \times E(k) \times D^3V\) which, of course, is the definition of the rainfall rate for a distribution of drops all having the same diameter.

Next, consider the somewhat more interesting case of a Poisson distribution of drops (i.e. all the drops are uncorrelated) but for a distribution of sizes. Then both \(F_R\) and \(F_Z = 1\) (i.e., there is no drop correlation), \(E(k)/\sigma^2(k) = 1\) (since the drop counts are Poissonian) so that

\[
C = \left\{ \frac{\sigma^2(D^6)}{E^2(D^6)} + 1 \right\}^{1/2}.
\]

(23)

For an ‘infinite’ exponential distribution such as (6), it is easy to show that the numerator equals 924 while the denominator is around 60 so that \(C \approx 3.9\). (Note that if both \(F_R\) and \(F_Z\) in (22) equal two, \(C\) could still be approximately 3.9.) The expression for \(E(R)\) no longer reduces to a trivial form, however, but using (19), (20) and (17), it is easy to deduce that \(E(R) \propto \text{r.m.s.} (D^3V) \times E(k) \times [E(D^6)/\text{r.m.s.} (D^6)]\) where r.m.s. denotes the root mean square.

As a final example, we consider the most realistic case of clustered (correlated) raindrops. Suppose the distribution of drop counts were geometric (so that \(E(k)/\sigma^2(k) \equiv 1/k\), e.g., Kostinski and Jameson (1997)) and that \(F_R = F_Z = 3\) (highly clustered, correlated rain). Then for \(k = 100\) and 1000 (remembering that counts have no units), \(C = 3.20\) and 1.79 respectively. Most values of \(C\) will, therefore, probably lie somewhere between, say, 1.2 and 3.7, while \(E(R)\) is given by (22).
Rain, however, is much more complex than suggested by these simple examples. We next consider the generalized $Z-R$ relation in light of the statistical properties of natural rain.

(b) The impact of statistical inhomogeneity on $Z-R$ relations

In a recent series of articles, Kostinski and Jameson (1997, 1999) and Jameson and Kostinski (1998, 2000) demonstrated that rain, rather than being uniformly distributed spatially as randomness allows (described by Poisson statistics), is, instead, clustered. That is, locations rich in drops are interspersed with relative ‘holes’, where rain is sparser, so that the rainfall rates themselves are clustered (for further discussion see Jameson and Kostinski (1999) pp. 3921 and 3931). Consequently, there is much greater variability in $R$ (and $Z$) than would occur if the raindrops obeyed Poissonian statistics (so-called ‘steady’ rain).

However, it is important to note here that statistical homogeneity can produce significant spatial variability because of the presence of correlated fluctuations (see the discussion by Jameson and Kostinski (2000) in their appendix). Statistical homogeneity means only that the mean and variance of the random variable are independent of the choice of origin. By contrast, in a statistically inhomogeneous process, the mean and variance of a random variable depend upon the origin. That is, they change throughout the set of data. Yet ‘systematic’ changes in the random variable alone over relatively short distances (i.e., distances much less than the correlation length) should not be construed as a sign of statistical inhomogeneity. Correlated fluctuations over scales less than the correlation length can also produce apparent ‘systematic’ changes even in a statistically homogeneous process as will be illustrated below.

In order to study the distribution of $R$, several different Monte Carlo, statistically homogeneous, realizations of ensembles of the rainfall rate were generated drop by drop, with each realization having different mean values of the rainfall rate (see Jameson and Kostinski (1999), pp. 3924, 3925 and 3930 for details). The results for one such 1000-value realization having the proper drop-to-drop correlations and cross-correlations as observed (see Jameson and Kostinski (1999) Figs. 9 and 10) for a mean rainfall rate of 12 mm h$^{-1}$ is illustrated in Fig. 7(a). Note that the maxima and minima in the value of $R$ are caused by correlated fluctuations. The methodology used to generate them guarantees that they are not the result of statistical inhomogeneities (see Jameson and Kostinski 1999). Moreover, the corresponding $Z-R$ correlation is plotted in Fig. 7(b). The $Z-R$ relation is linear with a very high degree of correlation ($r^2 = 0.996$) and a slope consistent with the calculated $\sigma_Z = 14.954$ mm$^6$m$^{-3}$, $\sigma_R = 22.3$ mm h$^{-1}$ and $C = 1.039$. Why is this?

The statistical explanation lies in (22). The bracketed term, $C$, as well as the ratio $\sigma_R/\sigma_Z$ are constant in statistically homogeneous rain because the variances remain fixed. Thus, for every ensemble of values derived using this statistically homogeneous process having an expected value $E(Z)$, there is an $E(R)$ such that $E(Z) = C E(R)$ where $C = C \times \sigma_R/\sigma_Z$.

There is also a physical explanation, however, that is based upon three observations. Firstly, the statistical homogeneity of the rain requires that the mean flux and mean number concentration at each drop size remain fixed. Consequently, the pdf of $D$, $p(D)$, is ‘steady’. (Note that ‘steadiness’ or ‘stationarity’ does not require nor imply that $p(D)$ is necessarily in equilibrium.) That is, there are fluctuations around $p(D)$, but ultimately, the mean distribution of observed diameters tends toward this ‘stationary’ function, $p(D)$. If not, then $E(R)$ and Var($R$) would change throughout the observation
volume. (It can be proven mathematically that these remarks apply to all drop size distributions described using exponential and gamma functions. For more complex drop size distributions, ‘steadiness’ requires that statistical homogeneity extend to higher moments of the distribution of \( R \) as well. In the limiting case of strict sense homogeneity, the entire distribution of \( R \) is invariant with respect to origin (Feller 1971), and there is always an accompanying steady drop size distribution.) Secondly, since for each observation of \( R \) and \( Z \) we have \( \sum_{i=1}^{k} D_i^3 V_i = k \bar{D}^3 V \) and \( \sum_{i=1}^{k} D_i^6 = k \bar{D}^6 \), the ratio \( Z/R \propto \bar{D}^3/\bar{D}^3 V \) is, then, a function only of \( p(D) \). However, because \( p(D) \) is ‘steady’, this ratio tends toward a constant value (with fluctuations) provided that the total number of drops, \( k \), is sufficient to represent (or span) \( p(D) \) adequately. Examination of the simulation used in Fig. 7 shows that this is indeed the case when \( R \geq 20 \) mm h\(^{-1}\). Moreover, this is true for any two zero-moments (i.e., average powers of diameter) of the drop distribution so that the linearity in (22) applies to many variables besides just \( Z \) and \( R \). But what happens when several different ensembles are combined to form a statistically inhomogeneous set of data?

Values of \( R \) and \( Z \) for six statistically homogeneous, drop-by-drop realizations having mean rainfall rates from 5 to 45 mm h\(^{-1}\) are combined into one scatter plot as illustrated in Fig. 8. Here, each realization can be thought of as being associated with its own unique \( Z-R \) line, but, much like real observations, these lines are then placed in different positions with respect to one another when combined to form a statistically inhomogeneous set of data. Consequently, the net \( Z-R \) relation is no longer linear, but is better fitted by a power law as given in Fig. 8 (\( \rho^2 = 0.947 \)). This particular power-law is not unusual (e.g., Atlas et al. 1990; Hirayama et al. 1997; Lecocq et al. 1997) so that we have essentially derived a realistic nonlinear correlation \( Z-R \) power-law just because of statistical inhomogeneity. So why is the fit to statistically inhomogeneous data nonlinear?

Because the expected values and variances of random variables are not constant for statistically inhomogeneous data, we see from (22) that \( C = C \times \sigma_R/\sigma_Z \) must also change in an unknown fashion throughout the data set. Hence, for every \( E(Z) \) we may write \( E(Z) = C E(R) \), where \( C \) is then no longer a constant but varies because the ensemble of \( (Z, R) \) pairs varies for each \( E(Z) \). Thus, power-law fits such as the
Figure 8. Scatter diagram of six Monte Carlo realizations of statistically homogeneous rain, each having different mean rainfall rates but all combined to form a statistically inhomogeneous data set. The power-law fit \((Z = 383R^{1.17})\) arises because of the statistical inhomogeneity.

Figure 9. As Fig. 6, with two lines added to indicate two likely subsets of statistically homogeneous data (see text).

one in Fig. 8 should be viewed only as statistical fits that attempt to account for this variability so that a mean \(R\) can be estimated from a mean \(Z\). That is, \(Z-R\) power laws (or any other correlation fits) deduced for statistically inhomogeneous data exist only as statistical entities. Any causal relations that may exist between \(R\) and \(Z\) cannot be deduced from such correlations nor can the scatter be entirely attributed to measurement error. While measurement errors can be important (e.g. Zawadzki 1984), the variability between \(Z\) and \(R\) also arises for more fundamental reasons.
Given this new perspective, let us return once again to Fig. 6. We now interpret the set of data as a mixture of at least three subsets. Two of these are statistically homogeneous, as indicated by the lines in Fig. 9. The remaining cloud of points most probably denote statistically inhomogeneous data, although some of them may also represent unresolved members of a statistically homogeneous set. The scatter about the fits plotted in Fig. 6, then, is caused not so much by measurement errors as by statistical inhomogeneities. Thus, while correlations in statistically homogeneous rain are linear and have precise physical meaning through (22), point-by-point values along such correlation fits in statistically inhomogeneous data are void of physical meaning. Whether a linear or power-law fit is more appropriate in Fig. 6 cannot be deduced scientifically and must be left as a matter of conjecture.

On the other hand, this does not mean that (22) has no role to play in the analysis of such statistically inhomogeneous data. Quite the contrary. Equation (22) appears to offer a new method of estimating the expected value of rainfall rate from a collection of radar-reflectivity-factor measurements without any knowledge of a specific $Z-R$ relation. This is illustrated in the examples below.

(c) Two examples using data

(i) Highly variable conditions. In order to avoid the potential hazards of flux data mentioned earlier, the spatially rebinned data discussed above and illustrated in Figs. 1 and 2 are used to compute $k$, $Z$ and $R$ for each of the 175 samples. Values of $E(k)$, $E(R)$, and $E(Z)$ as well as $\text{Var}(k)$, $\text{Var}(R)$ and $\text{Var}(Z)$ are then calculated. However, $E(D^6)$, $E(D^3 V)$ and their respective variances are computed for all 175 samples, using the overall drop-size pdf illustrated in Fig. 10. For these spatially rebinned data, it turns out that $C = 1.33$ while $E(R)$ using (22) is $15.3 \text{ mm h}^{-1}$ compared to the directly observed $14.7 \text{ mm h}^{-1}$. That is, the estimate is within four per cent of the observed value.

Next let us consider the same data, but this time use the flux measurements directly to estimate a flux pdf of $D$. (Such an approach is justified by Kostinski and Jameson.
(1999, appendix B). This pdf is then used to compute the variance and expected values of both $D^6$ and $D^3V$. On the other hand, $Z$ and $R$ are estimated after first converting the fluxes to drop concentrations in the classic manner by dividing by the terminal fall-speed of the drops as discussed earlier. As Fig. 11 illustrates, there are then significant differences (see Fig. 1) in these two approaches (spatial rebinning vs. concentrations from fluxes) in the resulting $R$ and $Z$ profiles. Nevertheless, in this example $C = 1.237$ compared to 1.33 (above) for the rebinned data, while $E(R) = 26.4 \text{ mm h}^{-1}$, a value within five per cent of the observed 27.6 mm h$^{-1}$. (Note that the means in these two examples differ because the results based on the fluxes use data acquired during the whole sampling period whereas the spatially rebinned data come from only a fraction of that period. Thus the average values need not be equal.)

Moreover, it appears better to use the pdf of $D$ based on fluxes rather than first to convert fluxes to concentrations in the classic manner using the terminal fall-speed. Specifically, in this latter example, using the pdf of $D$ based on concentrations rather than fluxes yields $C = 2.07$ and $E(R) = 44.2 \text{ mm h}^{-1}$ compared to the observed 27.6 mm h$^{-1}$. Hence, the processes of multiplication and division normally used to convert particle fluxes to concentrations and a pdf of $D$ can apparently sometimes lead to significant errors. This is not surprising given the discussion by Kostinski and Jameson (2000, appendix B).

(ii) *More tranquil conditions.* But what about applying (22) to a longer, more tranquil, time series? As an illustration, we consider a ten-hour (613 minute) sequence of one-minute Joss and Waldvogel (1967) disdrometer observations collected at Wallop's Island, Virginia, USA. The calculated values of $R$ and $Z$ are illustrated in Fig. 12. All the drop-flux data are combined to calculate a pdf of $D$ used to compute the expected values and variances of $D^6$ and $D^3V$. The result is that $C = 1.202$ and $E(R) = 2.27 \text{ mm h}^{-1}$ (observed average 2.39 mm h$^{-1}$). (It is worth mentioning that if $F_R = F_Z = 1.20$ in (22), these two values of $E(R)$ would agree completely.)
In the examples above, we have ensured that counts of drops all refer to the same volume. In reality, of course, different instruments sample over different volumes and at different intervals. While a complete study of how such differences affect (22) is worthy of its own research project and well beyond the scope of this current work, we can begin to probe the effects of different averaging and temporal sampling on (22) using these disdrometer data.

4. PRELIMINARY SIMPLE EXAMPLES OF THE EFFECTS OF AVERAGING AND SPARSE SAMPLING

Envisage a set of disdrometer measurements collected to characterize the rain distribution. Then imagine a radar that samples at a resolution equivalent to independent groups of, say, twenty of the disdrometer observations. What happens when both data sets are combined using (22)?

First consider the longest time-series of data, namely 613 min of Wallop's Island measurements described above. Values of $Z$ are computed for the drop-size distributions over 30 groups of twenty disdrometer-observations. Values of $\sigma_z$ and $E(Z)$ are then calculated and used in (22). The other quantities are computed as explained in section 3. The result is $E(R) = 2.60 \text{ mm h}^{-1}$ compared to the observed $2.39 \text{ mm h}^{-1}$. That is, even after a radar resolution reduction by a factor of twenty, the changes in $E(Z)$ and $\sigma_z$ compensate so that the estimated rainfall rate is still within nine per cent of the correct value. Of course, more variable conditions in other rain might lead to larger differences.

Consequently, we next use the spatially rebinned video-disdrometer data (Fig. 1). In order to preserve a reasonable number of samples (since there are only 175 in all), we average only over seven data-points (a resolution reduction of seven) to yield 25 samples of $Z$. The result is that $E(R) = 17.08 \text{ mm h}^{-1}$ compared to the observed $14.74 \text{ mm h}^{-1}$. So, even in these highly variable conditions, $\sigma_z$ and $E(Z)$ still compensate to within fifteen per cent of the correct values.

Next, imagine the same network of disdrometers but then consider an instrument such as a space-borne radar, for example, that sparsely samples the same rain. That is, instead of averaging over twenty disdrometer measurements, suppose only every
twentieth value of the Wallop's Island data is used to estimate $\sigma_Z$ and $E(Z)$. The result is that $E(R) = 2.63 \text{ mm h}^{-1}$ compared to the observed $2.39 \text{ mm h}^{-1}$. Once again, changes in $\sigma_Z$ and $E(Z)$ largely compensate to lead to a reasonable estimate of the rainfall rate.

In the more highly variable conditions of the spatially rebinned video-disdrometer data (Fig. 1) and using only every seventh value, $E(R) = 15.00 \text{ mm h}^{-1}$ compared to the observed $14.74 \text{ mm h}^{-1}$. That is, $\sigma_Z$ and $E(Z)$ compensate in this example to yield the correct estimate. Even more striking, even if the sample is only every 25th value, we find that $E(R) = 16.87 \text{ mm h}^{-1}$ so that to a large extent, $\sigma_Z$ and $E(Z)$ still compensate even though we are using only fourteen per cent of the $Z$ values!

Although there is obviously much research still to be done, these results are sufficient for us to take (22) seriously for a wide variety of applications. For example, the statistics (mean and variance) of both $Z$ and $R$ are computed as though each $Z$ and $R$ were independent, much like data from a sparse network of rain gauges and sampling by a space-borne radar. (Note that for $Z$, the variance is calculated by assuming an ensemble of unbiased mean values measured in such a way as to minimize any effects of non-Rayleigh statistics (Jameson and Kostinski 1996). The former can be readily calculated and accounted for so that the only relevant term computed from such a time series will be the variance of the measured mean $Z$ as used in the application of (22) in these examples.) Hence, even though up to now everything has been calculated from the disdrometer measurements, these examples and results, in our opinion, are sufficiently relevant to justify further research using radar, rain-gauge and disdrometer observations.

5. BRIEF REVIEW AND DISCUSSION

In the present paper the scaling of $Z-R$ relations is re-examined. It is shown, in agreement with past work, that based upon physical considerations, relationships between such variables should be linear. Yet $Z-R$ relations, as well as those between many other variables used in radar meteorology over the past 50 years, are usually expressed as power laws. Why? Because measured pairs of $Z$ and $R$ usually do not fall along straight lines. While some of this deviation from linearity is undoubtedly the result of errors in the observations, it is shown in this paper that nonlinear correlation-fits owe their existence to the stochastic nature of rain regardless of measurement effects. Specifically, a generalized $Z-R$ relation is developed from the characterization of the stochastic structure of rain that explicitly incorporates details of the drop microphysics as well as the variability in measurements of $Z$ and $R$. It is then shown that linearity will appear only in data which are homogeneous statistically, whereas nonlinearity will occur whenever the data are statistically inhomogeneous. Since almost all $Z-R$ relations are derived using statistically inhomogeneous data, the prevalence of power laws is not surprising.

What is surprising, however, is the number of attempts to explain such purely statistical correlation relations physically. We show here that such efforts are futile for at least three reasons. First, such parametric fits are not unique. Therefore, neither are there any unique physical justifications. Second, it is shown that the arguments usually invoked to justify power laws result in fundamental physical inconsistencies. Finally, and more importantly, such attempts assume that correlation is indicative of causation. That correlation is not the same as causation is well noted in the statistical sciences. Correlations alone cannot be used as a substitute for controlled experiments and well-founded theory. Most, if not all, $Z-R$ relations, therefore, are simply statistical correlation-fits.
The abundant presence of nonlinear $Z-R$ correlation-fits, however, does not preclude the use of the generalized $Z-R$ relation (22) for data analysis. Examples suggest that this and similar relations involving two variables defined by a sum such as (14) are potentially useful for a wide range of remote-sensing problems over a wide range of scales. Expression (22) and the examples offer great hope that data collected over widely disparate volumes and sampling frequencies can still be combined to yield meaningful estimates of $R$ from $Z$, for example. This is good news for space programs attempting to combine estimates of $R$ using remote sensing with ground-based observations. While the ground-based measurements using rain gauges and disdrometers yield the critical estimates of $\sigma_R$ as well as the statistics of $D^3V$ and $D^6$ at sparse but, one hopes, representative locations, the spacecraft fills in the voids, so to speak, by providing estimates of, say, $E(Z)$ and $\sigma_Z$ over locations that not only include the ground measurements but many other representative locations as well. While neither set of measurements alone is sufficient or complete, together they may offer a more powerful approach for making global-precipitation measurements. It is also important to remember that this discussion applies to many other variables used in remote sensing as well.

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