Combination theory and equilibrium evaporation

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SUMMARY

This paper is an analysis of equilibrium evaporation and its role in the energy balance of a terrestrial surface, as described by combination theory. Three themes are covered: first, a brief historical review identifies multiple definitions of the concept of equilibrium evaporation. Second, these are formalized by developing the basic principles of combination theory with minimum approximation. Several measures are utilized to do this: linearization is avoided, radiative and storage coupling are incorporated systematically, and actual and linearized saturation deficits are distinguished. The formalism is used to analyse several algebraically defined states and limits for the surface energy balance. Third, the thermodynamic foundation of equilibrium evaporation is analysed by studying surface-atmosphere feedbacks in arbitrary closed and open evaporating systems. It is shown that under steady energy supply any closed evaporating system evolves towards a quasi-steady state in which the Bowen ratio takes the equilibrium value $1/\varepsilon_0$, where $\varepsilon_0$ is the ratio of the latent- and sensible-heat contents of saturated air with temperature, evaluated at the volume-averaged temperature in the closed system. This applies whether the system is well-mixed or imperfectly mixed, and whatever the internal distribution of surface fluxes and surface and aerodynamic resistances. In contrast, open systems cannot reach such an equilibrium. This evolutionary definition of equilibrium evaporation differs from an alternative algebraic definition, the fully decoupled limit. The differences between the two definitions are identified, and the evolutionary definition is shown to be more fundamental. Thus, the correct temperature for evaluating $\varepsilon$ in determining equilibrium evaporation is the volume-averaged temperature in a closed region, which in the case of a convective boundary layer is well approximated by the mixed-layer temperature.

KEYWORDS: Combination theory  Convective boundary layer  Equilibrium evaporation  Penman–Monteith equation  Saturation humidity  Surface energy balance

1. INTRODUCTION

Combination theory describes the surface energy balance (SEB) of terrestrial surfaces, especially the land–atmosphere exchanges of energy through sensible- and latent-heat fluxes. One of its cornerstones is the Penman–Monteith (PM) equation. From its origin, widely recognized to be the paper of Penman (1948) on terrestrial evaporation, the development of combination theory can be traced over several decades through reviews by Slatyer and McIlroy (1961), Monteith (1973), Thom (1975), Monteith (1981), Brutsaert (1982), Monteith and Unsworth (1990) and McNaughton and Jarvis (1991).

Central to combination theory is the concept of equilibrium evaporation, the main subject of this paper. Equilibrium evaporation appears in the theory in several different ways and has a number of possible physical definitions. The broad goal here is to elucidate these definitions, identify which of them is fundamental, and connect them logically. To reach this goal, three themes are covered: (i) a brief historical review is undertaken to identify the multiple definitions of equilibrium evaporation; (ii) the basic

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principles of combination theory are developed formally in a way which avoids approximations at the level of principle; and (iii) the theory is used to analyse the thermodynamic foundations of equilibrium evaporation and identify the connections between the alternative definitions. Formal precision is necessary because some past confusion has arisen from a tendency to develop combination theory around convenient algebraic approximations, such as linearization of the dependence of saturation specific humidity on temperature. Here, such techniques are restricted to computational simplifications rather than structural components of the theory, so that the physics can be distinguished from the mathematical techniques.

The plan of the paper is as follows: after a review in section 2, section 3 summarizes basic principles and governing equations, leading to a discussion in section 4 of hydrometeorological states and limits. In section 5, combination theory is applied to explore the feedbacks between the SEB and the atmosphere closed and open systems, leading in the concluding section 6 to resolution of the issues raised above about equilibrium evaporation. A table of symbols is given in appendix A, and appendices B and C cover additional mathematical detail. The paper contains both review and new material, the main new contributions being the generality of the formalism in sections 3 and 4 and the thermodynamic analysis of equilibrium evaporation in section 5.

2. Review

Equilibrium evaporation occurs when the evaporative energy fraction \( \alpha = F_E / F_A \) takes the value \( \varepsilon / (\varepsilon + 1) \), and the Bowen ratio \( \beta = F_H / F_E \) takes the value \( 1 / \varepsilon \). Here \( F_H \) and \( F_E \) are the latent- and sensible-heat flux densities at the surface, \( F_A = F_E + F_H \) is the available energy flux density, \( \varepsilon = (\lambda / c_p) dQ_{\text{sat}} / dT \) is the ratio of the temperature variations in the latent- and sensible-heat contents of saturated air, \( Q_{\text{sat}}(T) \) is the saturation specific humidity at temperature \( T \), \( \lambda \) is the latent heat of vaporization of water, and \( c_p \) is the isobaric specific heat of air.

- **Early developments:** Schmidt (1915), in an important paper, was the first to show that the values \( \alpha = \varepsilon / (\varepsilon + 1) \) and \( \beta = 1 / \varepsilon \) arise when steady evaporation occurs into saturated air from a wet surface supplied with energy. He proposed that evaporation over the oceans is thermodynamically constrained to be \( \varepsilon / (\varepsilon + 1) \) times the supply of the "available energy" \( F_A \) (as he called this quantity). His essential argument was that: "Air in contact with water is warmed, but to accomplish this, not only the amount of heating to raise the temperature is required but considerably more. Since the lowest air layers (in contact with the water) are usually near the saturation point, it becomes necessary to add more moisture through evaporation for each rise in temperature in order to reach saturation pressure. The expenditure of energy for the simple temperature rise (designated by \( K \) for convection) is increased therefore by the heat of evaporation (designated by \( V \))." Thus \( V / K = \varepsilon \). Schmidt tabulated the ratio \( V / (V + K) = \varepsilon / (\varepsilon + 1) \) as a function of temperature, and used it with estimates of the zonally averaged surface radiation balance to estimate global ocean evaporation.

The development of combination theory occurred later. Penman (1948) combined the energetic and atmospheric diffusive controls on latent- and sensible-heat fluxes to predict the evaporation from wet natural surfaces in terms of forcing variables external to the surface, rather than internal variables such as the surface temperature. (A similar solution was published independently by Ferguson (1952) on the basis of work dating back to 1945, carried out in the employment of a South Australian company producing salt by the evaporation of sea water.) A major effort then ensued to include physiological limitations on evaporation imposed by vegetation. Penman
and Schofield (1951) introduced the idea of a physiological resistance, and Penman (1953) extended his equation to describe evaporation from a single leaf including a physiological resistance term. A related equation close to the modern form of the PM equation was given by Slatyer and McIlroy (1961), using the wet-bulb depression (the difference between dry-bulb and wet-bulb temperatures) as the measure of saturation deficit. Monteith (1965) identified the surface resistance as a bulk physiological property of a vegetated surface, and gave a means of measuring it; those crucial steps constitute one reason why the PM equation is appropriately named.

- **Meanings of equilibrium evaporation**: Throughout these and later developments, equilibrium evaporation \( \alpha = \varepsilon/(\varepsilon + 1) \) appeared in several ways. The first was evaporation from a wet surface into saturated air, as originally proposed by Schmidt (1915) and later by Priestley (1959; independently, in the last paragraph of his book) and Philip (1987b). This definition implies that equilibrium evaporation is an attribute of a wet surface. A second, more general meaning was indicated by Slatyer and McIlroy (1961, pages 3–73); in their hard-to-obtain but important monograph, \( \alpha = \varepsilon/(\varepsilon + 1) \) is the limiting evaporation at sufficiently large fetch over any surface, whether wet or not, under conditions of steady energy supply. In view of this meaning, they introduced the word 'equilibrium'. The idea of equilibrium evaporation as a general long-term or large-fetch limit was developed in the analysis of evaporation in an advective boundary layer by McNaughton (1976a,b), who showed that this result follows whenever the boundary layer is capped by a lid through which no exchange of air occurs. Perrier (1980) and McNaughton and Jarvis (1983) supported the same conclusion by analysing a simple 'closed-box' model of evaporation from a uniform surface under steady energy supply. Raupach (1991) showed that the result is independent of the spatial distribution of surface and aerodynamic resistances. Hence, this second meaning of equilibrium evaporation can be characterized as 'the long-term limit of evaporation into a closed system supplied steadily with energy'.

McNaughton and Jarvis (1983) also introduced a third, related meaning for the evaporation rate \( \alpha = \varepsilon/(\varepsilon + 1) \), by defining a 'decoupling factor' (a function of aerodynamic and surface resistances) quantifying the extent to which the surface is isolated from its environment. They showed from the PM equation that \( \alpha = \varepsilon/(\varepsilon + 1) \) is the evaporation rate in the limit of complete decoupling. Two other meanings (or systematic occurrences) for \( \alpha = \varepsilon/(\varepsilon + 1) \) had been identified earlier: the fourth (continuing the enumeration) is evaporation in the calm limit, or the limit of large aerodynamic resistance, as shown by Thom (1975) using algebraic manipulation of the PM equation. This has similarities to the decoupled-limit (third) meaning. The fifth, introduced by Monteith (1965) and Thom (1975), is the evaporation rate which is independent of aerodynamic resistance or the condition under which changing aerodynamic resistance does not change the SEB.

- **Field observations of evaporation**: In addition to its theoretical significance, equilibrium evaporation has been proposed by many researchers as a guide (even sometimes as a good approximation) for actual evaporation rates over well-watered (but not dry) vegetated surfaces, even though the relevant field conditions rarely conform with any of the above idealizations leading theoretically to the equilibrium SEB. Denmead and McIlroy (1970) were among the first to propose equilibrium evaporation as a guide to observed evaporation under well-watered conditions: for a well-watered wheat crop, they observed that \( \alpha(\varepsilon + 1)/\varepsilon \) (which is unity for equilibrium evaporation) was about 1.0 for moderate evaporation rates, and somewhat less than this at higher rates. Shuttleworth and Calder (1979) summarized data showing that \( \alpha(\varepsilon + 1)/\varepsilon \) is typically less than one for forests, unless the foliage is wet from intercepted rainfall and there is
scope for significant advection of dry air in which case very high values are possible. McAneney and Itier (1996) reviewed a substantial number of measurements which found $\alpha(\varepsilon + 1)/\varepsilon$ ranging from around 1 to 1.5 for well-watered pastures and crops. Valentini et al. (1999) tabulated the findings of 24 field experiments on crop, grass and forest surfaces, showing that $\alpha(\varepsilon + 1)/\varepsilon$ is typically in the range 1 to 1.5 for well-watered surfaces (surface resistance less than about 70 s m$^{-1}$), and reduces under the influence of water stress, broadly consistent with the observations of Shuttleworth and Calder (1979) and McAneney and Itier (1996).

- **Boundary-layer feedbacks and the Priestley–Taylor ratio:** Priestley and Taylor (1972) proposed that $\alpha(\varepsilon + 1)/\varepsilon$, now known as the Priestley–Taylor ratio, takes a nearly constant value of about 1.26 over a variety of moist natural surfaces, as a result both of surface and of entrainment processes. It was proposed by de Bruin (1983) and McNaughton and Spriggs (1986) that this occurs because the near-surface temperature and humidity are controlled by a feedback between the SEB and the entrainment of dry air into the daytime convective boundary layer (CBL), and that the essential dynamics can be captured in a simple slab model of the CBL combined with the PM equation to model the SEB. The stabilization provided by this feedback leads to a narrow range of quasi-equilibrium values of $\alpha(\varepsilon + 1)/\varepsilon$, found by Culf (1994) and Raupach (2000) to be typically 1.2 to 1.3 over vegetation in the absence of water stress, depending mainly on conditions above the CBL (see also McNaughton and Spriggs 1989; Raupach 1991; Betts 1994; McNaughton and Raupach 1996; Huntingford and Monteith 1998).

- **Feedbacks on the aerodynamic and surface conductances:** Aerodynamic and surface conductances ($g_a$ and $g_s$) or resistances ($r_a = 1/g_a$ and $r_s = 1/g_s$) play a key part in combination theory, as coefficients describing the transfers of energy and mass through atmospheric and surface pathways. Early (and even some recent) developments in combination theory were characterized by a tendency to treat $g_a$ and $g_s$ as prescribed surface properties independent of the SEB itself. However, it is now widely recognized that $g_a$ and $g_s$ are coupled to the SEB through several feedback processes, and are therefore state variables rather than external variables. Aerodynamic feedbacks couple $g_a$ with the SEB through thermal stability, increasing $g_a$ in unstable and decreasing it in stable conditions. This has been recognized for a long time but its implications have only recently been explored systematically (Jacobs and de Bruin 1992; Raupach 1998). Physiological feedbacks modify $g_s$ through its dependence on carbon assimilation, water supply, leaf-level saturation deficit, and leaf temperature at short time-scales, and through the dynamics of plant growth, leaf area index and nutrient cycling at longer time-scales. The present intention is not to review these processes, only to recognize them and the growing literature on their combined effects (Collatz et al. 1991; McNaughton and Jarvis 1991; Jacobs and de Bruin 1992, 1997; de Bruin and Jacobs 1993; Monteith 1995; Kim and Entekhabi 1998; Pielke et al. 1998; Raupach 1998).

- **Summary:** Equilibrium evaporation provides a physically robust, energy-bounded upper limit for the evaporative fraction $\alpha$ on terrestrial surfaces, such that (except in highly advective situations) $\alpha$ is not more than $ce/(\varepsilon + 1)$, with the Priestley–Taylor ratio $r$ near 1.26, the value originally suggested by Priestley and Taylor (1972). The reasons are to be found in the feedback interaction between the SEB and the daytime CBL. Therefore there is no lack of motivation for understanding and quantifying equilibrium evaporation. However, from a theoretical standpoint, there are no fewer than five alternative meanings of equilibrium evaporation. Hence, several questions arise. (i) Are all these meanings consistent? (ii) Is any one of them of fundamental precedence,
implying some or all of the others? (iii) What are the implications for the temperature at which \( \varepsilon \) should be evaluated? This last, apparently innocuous, question has practical implications because \( \varepsilon \) is strongly temperature dependent, roughly doubling with each 13 K rise in temperature. More significantly, it has theoretical implications because a sound answer goes a long way to answering the two preceding questions. The central goal of the present paper is to address these theoretical questions.

3. **Basic Principles**

(a) **Governing equations**

For a land surface element such as a uniform patch of vegetation or a single leaf, the SEB is:

\[
F_N - F_G = F_A = F_H + F_E,
\]

where \( F_N \) is the incoming net irradiance, \( F_G \) the heat flux into thermal storage, and \( F_H \) and \( F_E \) the sensible- and latent-heat fluxes into the air. ‘Flux’ is used throughout as shorthand for ‘flux density’, and a full list of symbols is given in appendix A. Raupach (1991, 1995), McNaughton (1994) and Raupach and Finnigan (1995) describe the areal averaging implicit in Eq. (1) and all that follows.

The fluxes \( F_N, F_H, F_E \) and \( F_G \) all depend on the surface temperature, \( T_s \). For \( F_N \), this dependence is given by the surface radiation balance:

\[
F_N = (1 - a_s)F_{S\downarrow} + e_s(F_{L\downarrow} - \sigma T_s^4),
\]

where \( F_{S\downarrow} \) and \( F_{L\downarrow} \) are the incoming global short-wave and long-wave irradiances, \( a_s \) is the surface albedo, \( e_s \) the surface emissivity, and \( \sigma \) the Stefan–Boltzmann constant. For \( F_H \) and \( F_E \), the dependence on \( T_s \) is given by the transfer parametrizations

\[
F_H = \rho c_p(T_s - T_a)/r_{aH},
\]

\[
F_E = \rho \lambda(Q_{sat}(T_s) - Q_a)/(r_{aE} + r_s),
\]

where \( T_a \) is the temperature and \( Q_a \) the specific humidity at an ambient reference point away from the surface, \( Q_{sat}(T) \) the saturation specific humidity, \( \rho \) the air density, \( r_{aH} \) and \( r_{aE} \) the aerodynamic resistances for heat and water vapour transfer, and \( r_s \) the stomatal or bulk surface resistance. Each resistance is the inverse of a conductance \( (g_{aH} = 1/r_{aH}, g_{aE} = 1/r_{aE}, g_s = 1/r_s) \), with conductances \((g)\) and resistances \((r)\) being used interchangeably according to convenience. The temperature, \( T \), is the potential temperature, so that \( c_p T \) is the specific entropy, and \( c_p T + \lambda Q \) the specific enthalpy (Iribarne and Godson 1973). The potential temperature is defined so that it equals the actual temperature at the ground level or at the evaporating surface, so that \( Q_{sat}(T) \) is a function of actual rather than potential temperature. The definitions of the resistances and the ambient reference point defining \( T_a \) and \( Q_a \) vary according to whether the equations describe a single leaf or a uniform patch of vegetation: for a single leaf, \( r_s \) is the leaf stomatal resistance, \( r_{aH} \) and \( r_{aE} \) are leaf boundary-layer resistances, and \( T_a \) and \( Q_a \) are defined in the canopy air space outside the leaf boundary layer; while for a uniform patch of vegetation the reference point for defining \( T_a \), \( Q_a \) and the aerodynamic resistances is at a height \( z_c \) in the horizontally homogeneous constant-flux layer or equilibrated internal boundary layer above the surface. Alternatively, the reference point can be in the well-mixed CBL above a heterogeneous or patchy land surface.

For the *storage heat flux*, \( F_G \), the dependence on \( T_s \) is given by the heat equation within the thermal store, a differential equation which imparts persistence or dependence
on prior states into the system. There are at least three ways to avoid this increase in complexity: assume \( F_G = 0 \), assume that \( F_G \) is proportional to \( F_N \), or replace the differential heat equation with the algebraic equation:

\[
F_G = k_g (T_s - T_g)/d_g,
\]

where \( T_g \) is a specified bulk temperature for the thermal store, \( k_g \) its thermal conductivity, and \( d_g \) a length-scale. This is a ‘one-layer’ model for the storage heat flux, and also the top-layer equation in a multi-layer model for \( F_G \) (see for example Deardorff 1977, 1978), \( T_g \) being the second-layer temperature.

(b) *Penman–Monteith equations*

To determine the SEB, Eqs. (1) to (5) are solved for the five unknowns \( F_N \), \( F_H \), \( F_E \), \( F_G \), and \( T_s \), taking as prescribed the irradiances \( F_{S1} \) and \( F_{L1} \), the radiative properties \( a_s \) and \( e_s \), the air temperature and humidity \( T_a \) and \( Q_a \), and the resistances \( r_s \), \( r_{ah} \) and \( r_{ae} \) (which may be prescribed as functions of other variables rather than as externally imposed values). The solution may be obtained in two ways, by algebraic elimination of \( T_s \) or by iteration.

The former route leads to the PM equations for \( F_E \) and \( F_H \). Algebraic elimination of \( T_s \) utilizes the slope of the saturation humidity \( Q_{sat}(T) \), which can be specified as the ratio \( \varepsilon \) of the changes in the latent- and sensible-heat contents of saturated air with temperature. Practical formulae for \( Q_{sat}(T) \) are summarized by Garratt (1992). The ratio \( \varepsilon \) can be defined either over a finite temperature interval (say \((T_a, T_s)\)) or at a single temperature (say \( T_a \)):

\[
\varepsilon_{sa} = \frac{Q_{sat}(T_s) - Q_{sat}(T_a)}{T_s - T_a}, \quad \varepsilon_a = \frac{\lambda}{c_p} \left[ \frac{dQ_{sat}}{dT} \right]_{T=T_a}.
\]

The simplest form of the PM equation is obtained by regarding \( F_N \) and \( F_G \) as specified (ignoring their dependence on \( T_s \)) and then eliminating \( T_s \) from Eqs. (1), (3) and (4) by introducing \( \varepsilon_{sa} \). The resulting famous equations for \( F_E \) and \( F_H \) are:

\[
F_E = \frac{\varepsilon_{sa} r_{ah} F_A + \rho \lambda D_a}{\varepsilon_{sa} r_{ah} + r_{ae} + r_s}, \quad F_H = \frac{(r_{ae} + r_s) F_A - \rho \lambda D_a}{\varepsilon_{sa} r_{ah} + r_{ae} + r_s},
\]

where \( D_a = Q_{sat}(T_a) - Q_a \) is the saturation deficit at the ambient reference point \( a \). Equations (7) are the exact solution of Eqs. (1), (3) and (4) when \( \varepsilon_{sa} \) is evaluated using the finite-difference form in Eq. (6). However, \( \varepsilon_{sa} \) depends on \( T_s \), so the right-hand side of Eqs. (7) retains an implicit dependence on \( T_s \). This dependence is sufficiently weak for it to be removed for many practical purposes, using the approximation \( \varepsilon_{sa} \approx \varepsilon_a \). It is important to note that this is a computational device, easily removed by procedures such as iteration, rather than an inherent structural component of the theory. In the present paper, the entire development recognizes the nonlinearity of \( Q_{sat}(T) \) and at no stage depends fundamentally on linearization. The temperature(s) at which \( \varepsilon \) is defined for exact (albeit weakly implicit) solution are always indicated by subscripts as in Eqs. (6).

It is also possible to incorporate the dependencies of \( F_N \) and \( F_G \) on \( T_s \), so that \( F_N \) and \( F_G \) become unknowns in Eqs. (1) to (5), along with \( F_H \), \( F_E \) and \( T_s \). This leads to the ‘isothermal’ or ‘radiatively coupled’ forms of the SEB and the PM equations for \( F_E \).
and $F_H$:

$$F_{A}^* = F_E + F_H/p$$

$$F_E = \frac{\varepsilon_s a r_{th} F_{A}^* + \rho \lambda D_a}{\varepsilon_s a + r_{th} + r_{IE}} = \frac{p\varepsilon_s F_{A}^* + \rho \lambda D_a g_a}{p\varepsilon_s + b + g_a / g_s}$$

$$F_H = p \frac{r_{th} F_{A}^* - \rho \lambda D_a}{\varepsilon_s a r_{th} + r_{IE}} = p \frac{(b + g_a / g_s) F_{A}^* - \rho \lambda D_a g_a}{p\varepsilon_s + b + g_a / g_s},$$

where $F_{A}^*$ is the isothermal available energy flux, $r_{th}$ and $r_{IE}$ are respectively the total resistances for heat and water vapour transfer, $r_a = r_{ah}$, and the ratios $p$ and $b$ are defined below in Eqs. (12). The isothermal available energy flux $F_{A}^*$ is equal to $F_A$ with $T_s$ replaced throughout by $T_a$:

$$F_{A}^* = F_{N}^* - F_{G}^*$$

$$F_{N}^* = (1 - a_s) F_{S} + e_s (F_{L} - \sigma T_{a}^4) = F_N + \rho c_p g_f (T_s - T_a)$$

$$F_{G}^* = k_g (T_a - T_g) / d_g = F_G - \rho c_p g_g (T_s - T_a),$$

where $g_f = e_s \sigma (T_s^4 - T_a^4) / (\rho c_p (T_s - T_a))$ is the radiative conductance (typically about $1/200 \, \text{m s}^{-1}$), $g_g = k_g / (\rho c_p d_g)$ is an analogous storage conductance, and asterisks denote isothermal fluxes (obtained by replacing $T_s$ with $T_a$, leaving all else unchanged). The conductance $g_f$ accounts for ‘radiative coupling’, the link between $T_s$ and outward long-wave radiation (Monteith 1973; Martin 1989). As with $\varepsilon$, a finite-difference expression for $g_f$ yields the exact solution of the SEB equations, but this cannot be evaluated without prior knowledge of $T_s$, so the approximation $g_f \approx 4e_g \sigma T_{a}^3 / (\rho c_p)$ is usually used. Similarly, the conductance $g_g$ accounts for ‘storage coupling’.

The total resistances and the ratios $p$ and $b$ in Eqs. (8) to (10) are given by

$$r_{th} = (g_{ah} + g_f + g_g)^{-1} = pr_a$$

$$r_{IE} = r_{as} + r_s = br_a + r_s$$

$$p = r_{th} / r_{ah} = g_a / (g_a + g_f + g_g)$$

$$b = r_{as} / r_{ah} = g_{ah} / g_{as},$$

where $g_a = g_{ah}$ and $r_a = r_{ah}$. Hence, $r_{th}$ is a parallel sum and $r_{IE}$ a series sum. The ratio $p$ (0 ≤ $p$ ≤ 1) quantifies the relative significance of aerodynamic, radiative and storage coupling: aerodynamic processes ($g_{ah}$) dominate as $p \to 1$, and radiative and storage processes ($g_f + g_g$) dominate as $p \to 0$. Equations (12) show that $g_g$ always acts in parallel with $g_f$ to form the composite conductance $g_f + g_g$, a result which enables analyses of radiative coupling to be extended easily to include storage coupling as well. Typically, $p$ is in the range 0.5 to 0.9, decreasing for smoother surfaces and lower wind speeds. The ratio $b$ quantifies differences between the aerodynamic transfers of heat and water vapour. For applications of combination theory to whole canopies under a ‘big-leaf’ approximation, $b$ can usually be taken as 1.0 in practice, though exceptions can arise for sparse canopies (Blyth 1995) and in strongly advective conditions (Lang et al. 1983) because of unsteadiness in the flow (Laubach et al. 2000). At leaf level, $b$ is about 0.9 for symmetrical leaves because of differences between the molecular diffusivities $\kappa_H$ and $\kappa_E$ for heat and water vapour, since $b = (\kappa_H / \kappa_E)^{2/3}$ for a flat-plate molecular boundary layer. For asymmetric (hypostomatus) leaves, $b$ can be up to twice this value.

Equations (8) to (10) will be described as the ‘radiatively coupled’ formulation, meaning ‘accounting for both radiative and storage coupling’, in contrast to the ‘radiatively uncoupled’ formulation, Eqs. (6). The radiatively coupled SEB, Eq. (8), contrasts
with its uncoupled counterpart \((F_A = F_E + F_H)\) through the inclusion of the factor \(1/p\) multiplying \(F_H\). Throughout, radiatively uncoupled equations are recovered from their coupled equivalents by replacing \(F_A^*\) with \(F_A\) and setting \(p = 1\) (or \(r_{alh} = r_{lh}\)). Hence, \(p\) encapsulates all the algebraic consequences of radiative and storage coupling. Both formulations are equally valid, the difference between them being that \(F_A^*\) is an externally imposed property of the system whereas \(F_A\) is a property of the SEB itself, through \(T_s\). Hence the radiatively coupled formulation gives a clearer picture of responses to external conditions.

The second main way of determining the SEB from Eqs. (1) to (5) is by iterative solution for \(F_N\), \(F_H\), \(F_E\), \(F_G\) and \(T_s\), with prescribed meteorological conditions at the ambient reference point and with \(g_a = 1/r_a\) and \(g_s = 1/r_s\) either prescribed or determined from other variables by additional equations. This is the approach usually used in the land-surface parametrizations in atmospheric circulation models. Such a solution naturally includes the nonlinearities in both \(Q_{sat}(T)\) and \(T_s^4\), but more importantly it can incorporate naturally the dependencies of \(g_a\) and \(g_s\) upon the SEB through aerodynamic and physiological feedbacks. Numerical methods for obtaining exact solutions have been outlined by numerous authors; for instance, Paw U (1987), Paw U and Gao (1988) and Milly (1991) approximated \(Q_{sat}(T)\) and \(T_s^4\) by polynomials in \(T_s\). The two most common iteration methods are Newton–Raphson iteration for \(T_s\) and iteration of the PM equation itself.

(c) Linearized saturation deficit

The saturation deficit, \(D_a = Q_{sat}(T_a) - Q_a\), is a nonlinear function of \(T_a\). It is useful to define a quantity similar to \(D_a\) which is linear in both \(T_a\) and \(Q_a\), and is therefore a conserved variable satisfying an identical linear conservation equation to \(T\) and \(Q\) (Priestley and Taylor 1972; McNaughton 1976a). A suitable variable is the linearized saturation deficit about an arbitrary reference temperature \(T_r\):

\[
\Delta_a^{(r)} = Q_{sat}(T_r) - Q_a + (c_p/\lambda)(\varepsilon_r - \varepsilon_{ra})(T_a - T_r).
\]

The linearized saturation deficit \(\Delta_a^{(r)}\) departs from the true saturation deficit \(D_a\) by an amount \(\delta_a^{(r)}\):

\[
\delta_a^{(r)} = D_a - \Delta_a^{(r)} = (c_p/\lambda)(\varepsilon_r - \varepsilon_{ra})(T_r - T_a) = (\varepsilon_r - \varepsilon_{ra})F_H/\rho \lambda g_a,
\]

where \(\varepsilon_r\) and \(\varepsilon_{ra}\) are defined by Eqs. (6). Figure 1 illustrates the connections between \(D_a\), \(\Delta_a^{(r)}\) and \(\delta_a^{(r)}\). Expanding \(Q_{sat}(T)\) about \(T_r\) in the derivation of the PM equation, Eqs. (13) and (14) yield

\[
F_E = \frac{p\varepsilon_r F_A^* + \rho \lambda g_a (\Delta_a^{(r)} + \delta_a^{(r)})}{p\varepsilon_r + b + g_a/g_s}.
\]

This more general form of the PM equation differs from Eq. (9) in that \(\varepsilon_{sa}\) is replaced by \(\varepsilon_r\) (defined at the arbitrary \(T_r\)) and \(D_a\) is replaced by the sum of the linearized air saturation deficit \(\Delta_a^{(r)}\) and the surface departure \(\delta_a^{(r)}\). As before, this form of the PM equation is exact but implicit in \(T_s\). Two special cases are important: first, if \(T_r = T_a\), then a little algebra recovers Eq. (9) exactly, the conventional PM equation with a nonlinear deficit \(D_a\). Second, if \(T_r = T_s\), then Eq. (15) becomes

\[
F_E = \frac{p\varepsilon_s F_A^* + \rho \lambda g_a \Delta_a^{(s)}}{p\varepsilon_s + b + g_a/g_s},
\]
in which ε is evaluated at the surface and the saturation deficit $D_a$ is replaced by its linearized form with no additive term (since $\delta_s^{(s)} = 0$). Hence, the additive deficit term $\delta_s^{(r)}$ in the general Eq. (15) disappears in the two main cases of practical interest: $r = a$, yielding Eq. (9), and $r = s$, yielding Eq. (16). Equation (9) will be called the ‘actual-deficit’ form, and Eq. (16) the ‘linearized-deficit’ form. The linearized-deficit form has the advantage of linearity in both $T$ and $Q$, at the expense of an implicit dependence on $T_s$ through both $\varepsilon_s$ and $\Delta_s^{(s)}$. The actual-deficit form is nonlinear in $T_a$ but is implicitly dependent on $T_s$ only through $\varepsilon_s$. Most of the following theory can be written exactly using either the actual-deficit or linearized-deficit forms of the PM equation.

**(d) Dimensionless equations**

Basic dimensionless ratios characterizing the state of the SEB are the Bowen ratio $\beta$, and the non-isothermal and isothermal evaporative fractions $\alpha$ and $\alpha^*$, defined by

$$\beta = \frac{F_H}{F_E}; \quad \alpha = \frac{F_E}{F_A}; \quad \alpha^* = \frac{F_E}{F_A^*}. \tag{17}$$

Respectively, $\alpha$ and $\alpha^*$ are the fractions of $F_A$ and $F_A^*$ used for evaporation. Only one of the three ratios is independent, since they are connected by the relationships

$$\beta = \frac{1 - \alpha}{\alpha} = p \left( \frac{1 - \alpha^*}{\alpha^*} \right); \quad \alpha^* = \frac{p\alpha}{1 + (p - 1)\alpha}; \quad \alpha = \frac{\alpha^*}{p + (1 - p)\alpha^*}. \tag{18}$$

The PM equation can be cast in a dimensionless form (Monteith 1973, Thom 1975, Huntingtonford 1995) by converting $F_E$ to an evaporative fraction and normalizing the aerodynamic and surface conductances $g_a$ and $g_s$ with the ‘climatological conductance’ $g_i$. This parameter combines the ‘climate’ variables of saturation deficit and available energy. The radiatively uncoupled and radiatively coupled forms of the normalized PM equation are

$$\alpha = \frac{\varepsilon + g_a/g_i}{\varepsilon + b + g_a/g_s}, \quad \alpha^* = \frac{p\varepsilon + g_a/g_i^*}{p\varepsilon + b + g_a/g_s}. \tag{19}$$
Figure 2. The normalized, radiatively uncoupled PM equation, Eq. (24a), on a roughness-wetness plane. (a) Evaporative fraction $\alpha = F_E/F_A$ plotted against $g_a/g_i$, with parametric variation of $g_s/g_i$; (b) $\alpha$ plotted against $g_s/g_i$, with parametric variation of $g_a/g_i$. Throughout, $\varepsilon$ is taken to be 2.20. The six hydrometeorological limits are indicated. See text for details.

where $g_i = F_A/(\rho \lambda D_a)$ and $g_i^* = F_A^*/(\rho \lambda D_a)$. Each of these equations can be written exactly in an actual-deficit form (where $\varepsilon = \varepsilon_{sa}$ and the deficit appearing in $g_i$ or $g_i^*$ is $D_a$) or a linearized-deficit form (with $\varepsilon = \varepsilon_s$ and deficit $\Delta_a^{(s)}$).

The first of Eqs. (19) shows that the radiatively uncoupled evaporative fraction $\alpha$ is specified entirely by $\varepsilon$, $b$ and two conductance ratios, say $g_a/g_i$ and $g_s/g_i$. Figure 2 shows the behaviour of $\alpha$ as a function of $g_a/g_i$ and $g_s/g_i$, by plotting $\alpha$ against $g_a/g_i$ with parametric variation of $g_s/g_i$ (Fig. 2(a)) and against $g_s/g_i$ with variation of $g_a/g_i$ (Fig. 2(b)). By contrast, the second of Eqs. (19) shows that the radiatively coupled $\alpha^*$ cannot be reduced to a function of just two conductance ratios, because of the
Figure 3. The normalized radiatively coupled PM equation, Eq. (24b), on a roughness–wetness plane. (a) Isothermal evaporative fraction $\alpha^* = F_E/F_A^*$ plotted against $g_a$ with parametric variation of $g_s$; (b) $\alpha^*$ plotted against $g_s$ with parametric variation of $g_a$. Both panels use the fixed values $g_A^* = 0.021 \text{ m s}^{-1}$ and $g_r + g_b = 0.0048 \text{ m s}^{-1}$, obtained when $F_{S4} = 700 \text{ W m}^{-2}$, $F_{L4} = 300 \text{ W m}^{-2}$, $a_s = 0.2$, $e_s = 1$, and $T_a = 20 \degree \text{C}$. Throughout, $e$ is taken to be 2.20. See text for details.

appearance of the factor $p = g_a/(g_a + g_r + g_b)$. Figure 3 shows the behaviour of $\alpha^*$ as a function of $g_a$ with parametric variation of $g_s$ (Fig. 3(a)) and as a function of $g_s$ with variation of $g_a$ (Fig. 3(b)), with fixed $g_A^*$ and $g_r + g_b$ throughout. The major differences between Figs. 2 and 3 are: first, the behaviour in the limit $g_a \to 0$, where $\alpha^* \to 0$ whereas $\alpha$ approaches the equilibrium value $e/(\epsilon + b)$; second, for the radiatively uncoupled $\alpha$, there is a specific $g_s/g_A$ (equal to $e/(\epsilon + b)$) at which $\alpha$ is independent of $g_a$ and equal to the equilibrium value $e/(\epsilon + b)$. This was noted by Monteith (1965),
Thom (1975) and Jacobs and de Bruin (1992), and is indicated by the 'focus point' in Fig. 2(b) through which all curves pass. One implication is that, neglecting radiative coupling, the energy balance over complex terrain (for instance an array of hills and valleys) is independent of $g_a$, and hence of the flow complications induced by the terrain, when $\alpha$ takes the equilibrium value (Raupach et al. 1992). However, in the radiatively coupled case (Fig. 3(b)), there is no equivalent value of $g_s/g_1^*$ at which $\alpha^*$ is independent of $g_a$.

4. ALGEBRAICALLY DEFINED HYDROMETEOROLOGICAL STATES AND LIMITS

From the PM equation or related equations it is possible to define several limiting states for the SEB, including: (i) steady, horizontally homogeneous evaporation from a wet surface into saturated air; (ii) the six hydrometeorological limits defined as any one of $g_a$, $g_s$ or $g_l$ become very small or very large; and (iii) the fully decoupled and fully coupled limits defined by McNaughton and Jarvis (1983). In all cases, these states are defined by allowing some quantity to approach an algebraic limit (zero or infinity) while holding all other quantities fixed. Although the concepts are well entrenched in the literature, it is clear that such states are hypothetical rather than physically realizable, because of feedback processes which mean that all other quantities cannot generally stay fixed as a selected quantity is varied. A key example is wet-potential evaporation, the evaporation rate that would occur if the surface were covered in free water. If a surface were physically wetted in this way, the ambient saturation deficit (among other things) would change by an amount dependent on the area wetted. Hence, these limits need to be interpreted as reference or scaling states, rather than as predictions about what would happen if the process in question were actually carried out.

(a) Evaporation from a wet surface into saturated air

This situation was the first basis for defining equilibrium evaporation (Schmidt 1915; Priestley 1959; Slatyer and McIlroy 1961) and its implications (Philip 1987b, 1989; Andreas 1989). The argument starts from the statement that, if the transfer processes and source–sink distributions for heat and water vapour are similar, then their flux and gradient ratios are proportional:

$$\beta = \frac{F_H}{F_E} = \frac{b c_p \partial T / \partial z}{\lambda \partial Q / \partial z}, \quad (20)$$

where the proportionality constant $b$ has been introduced consistent with Eqs. (12). This equation follows from a gradient-diffusion assumption ($F_H = K_h \rho c_p \partial T / \partial z$ and $F_E = K_e \rho \lambda \partial Q / \partial z$), where $K_h$ and $K_e$ are eddy diffusivities and $b = K_h / K_e$. In steady, horizontally homogeneous conditions, $F_H$ and $F_E$ are constant with height, so the gradient ($\lambda / c_p \partial Q / \partial T$) is also constant with height. If the air is saturated at and just above the surface, this gradient must equal $\varepsilon_s$, the value of $\varepsilon$ at the surface temperature $T_s$. Equations (20) and (18) then show that $\beta$, $\alpha$ and $\alpha^*$ take the values:

$$\beta_{(x)} = \frac{b}{\varepsilon_s}, \quad \alpha_{(x)} = \frac{\varepsilon_s}{\varepsilon_s + b}, \quad \alpha_{(x)}^* = \frac{\varepsilon_s}{\varepsilon_s + b}, \quad (21)$$

where subscript $(x)$ denotes the limit of evaporation from a wet surface into saturated air.

The argument leading to Eqs. (21) has a further outcome: in steady, horizontally homogeneous conditions, the air above the surface cannot be saturated except infinitesimally close to the surface, because of the curvature of $Q_{sat}(T)$ and the constancy
of \((\lambda/c_p)(\partial Q/\partial T)\) with height at the surface value \(\varepsilon_s\). Figure 4 shows that, to avoid supersaturation in horizontally homogeneous conditions, the actual deficit \(D_a\) and the linearized deficit \(\Delta_a^{(s)}\) must satisfy:

\[
D_a \geq D_a(\text{min}) = (c_p/\lambda)(\varepsilon_s - \varepsilon_{sa})(T_s - T_a) = (\varepsilon_s - \varepsilon_{sa})F_H/\rho\lambda g_a, \\
\Delta_a^{(s)} \geq 0. 
\]  

Hence, when taking the saturated limit, \(\Delta_a^{(s)}\) can approach zero but the actual deficit \(D_a\) can only go as low as \(D_a(\text{min})\) if supersaturation is to be avoided between the surface and the reference height \(z_o\).

(b) Six basic hydrometeorological limits

Six hydrometeorological limits can be defined by allowing each of \(g_a\), \(g_s\) and the saturation deficit to approach zero or infinity while holding other quantities fixed (recalling the qualification at the start of this section). The first systematic analysis of these limits was by Thom (1975).

(i) In the calm limit \((g_a \to 0)\), both \(F_E\) and \(F_H\) tend to zero and the SEB reduces to a balance between radiative and storage flux terms. This limit can be found using either the actual-deficit or linearized-deficit forms of the PM equation, Eq. (9) or (16) (noting that in the actual-deficit form, \(D_a\) becomes infinite as \(g_a \to 0\) because of Eq. (22)). The result is

\[
\beta \to \beta(x), \quad \alpha \to \alpha(x), \quad \alpha^* \to \alpha^*(x) \to 0. 
\]  

Hence the calm limit yields values of \(\beta\), \(\alpha\) and \(\alpha^*\) identical with those for steady evaporation from a wet surface into saturated air, Eq. (21). However, since \(g_a \to 0\), the limit also implies that \(p \to 0\), so \(\alpha^*(x)\) and \(\alpha^*\) approach zero (although \(\alpha\) and \(\beta\) stay non-zero). Also the surface temperature becomes very large because the surface is aerodynamically decoupled from its environment, so no sensible- or latent-heat
exchanges can occur and the only terms remaining in the SEB, Eq. (1), are the irradiance and storage fluxes.

(ii) In the rough limit \((g_a \to \infty)\), the surface and the surrounding atmosphere are tightly connected aerodynamically, and the surface temperature and humidity are close to air values. The latent-heat flux \(F_E\) approaches \(\rho \lambda g_s D_a = \rho \lambda g_s \Delta_a^{(s)}\), and the actual and linearized deficits become equal. The rough limit is often a good approximation for forests.

(iii) In the dry-surface limit \((g_s \to 0)\), \(\alpha\) and \(\alpha^*\) approach 0, \(\beta \to \infty\), and \(F_E\) is negligible in the SEB.

(iv) In the wet-surface limit \((g_s \to \infty)\), the surface is fully wet with free water. This limit is:

\[
F_E \to F_{E(w)} = \frac{p \varepsilon_{sa} F_A^* + \rho \lambda g_s D_a}{p \varepsilon_{sa} + b} = \frac{p \varepsilon_s F_A^* + \rho \lambda g_s \Delta_a^{(s)}}{p \varepsilon_s + b},
\]

where \(F_{E(w)}\) is a ‘wet-surface potential’ evaporation rate, an idealization for the maximum rate possible under a given saturation deficit and \(g_a\). This is a well-known case of a limit which is hypothetical rather than actual because of feedback on the deficit, which changes if an area is physically wetted by an amount dependent on the extent of the wetted region (McNaughton 1976a,b; Philip 1987a).

(v) In the dry-air or oasis limit \((D_a \to \infty\) or \(\Delta_a^{(s)} \to \infty)\), both \(\alpha\) and \(\alpha^* \to \infty\) and \(\beta \to -1\). The aerodynamic fluxes, \(F_H\) and \(F_E\), both become very large in magnitude, and the energy for evaporation is supplied by cooling the incident airflow. This limit is approached when a dry oncoming airstream encounters a moist surface. When the surface is wet \((g_s \to \infty)\), the surface corresponds to an ideal wet-bulb thermometer and \(T_s\) is equal to the wet-bulb temperature.

(vi) In the moist-air limit \((D_a \to D_{a(min)}\) or \(\Delta_a^{(s)} \to 0)\), \(\alpha\) and \(\alpha^*\) are bounded above by the calm limits, reaching these values when the surface is wet.

\[(c)\quad \text{The fully decoupled and fully coupled limits}\]

McNaughton and Jarvis (1983), Jarvis and McNaughton (1986) and Martin (1989) expressed \(F_E\) as a weighted sum of values in fully decoupled and fully coupled limits, defined so that:

\[
F_{E(x)} = \frac{p \varepsilon F_A^*}{p \varepsilon + b}, \quad F_{H(x)} = \frac{p b F_A^*}{p \varepsilon + b}, \quad \text{(fully decoupled)},
\]

\[
F_{E(y)} = \rho \lambda D_a g_s, \quad F_{H(y)} = p (F_A^* - \rho \lambda D_a g_s), \quad \text{(fully coupled)},
\]

where subscript \((x)\) denotes the fully decoupled limit and \((y)\) the fully coupled limit. Algebraically, the two limits respectively resemble the calm \((g_a \to 0)\) and rough \((g_a \to \infty)\) limits defined above. As elsewhere, the equations in this section can take either an actual-deficit form with \(\varepsilon = \varepsilon_{sa}\) and deficit \(D_a \geq D_{a(min)}\), or a linearized-deficit form with \(\varepsilon = \varepsilon_s\) and deficit \(\Delta_a^{(s)} \geq 0\). By rearranging the PM equation, \(F_E\) and \(F_H\) can be expressed as weighted sums of the fully decoupled and fully coupled limits:

\[
F_E = \Omega F_{E(x)} + (1 - \Omega) F_{E(y)} = \Omega \frac{p \varepsilon F_A^*}{p \varepsilon + b} + (1 - \Omega) \rho \lambda D_a g_s,
\]

\[
F_H = \Omega F_{H(x)} + (1 - \Omega) F_{H(y)} = \Omega \frac{p b F_A^*}{p \varepsilon + b} + (1 - \Omega) p (F_A^* - \rho \lambda D_a g_s),
\]

(26)
where the ‘decoupling factor’ \( \Omega \) is a weighting factor between 0 and 1, defined by:

\[
\Omega = \frac{\varepsilon r_{H} + r_{AE}}{\varepsilon r_{H} + r_{E}} = \frac{pe + b}{pe + b + g_{a}/g_{s}}.
\]  

(27)

Equations (26) are a second generic form of the combination equations for \( F_{E} \) and \( F_{H} \), the decoupling-factor or \( \Omega \)-factor form (the basic PM equations in section 3 constituted the first generic form). The factor \( \Omega \) can be interpreted in two ways: first, Eqs. (9), (16), (24) and (27) show that \( \Omega = F_{E}/F_{E(w)} \), so \( \Omega \) is the ratio of actual to wet-surface potential evaporation. Second, \( \Omega \) may be interpreted as a measure of the extent to which the surface saturation deficit \( (D_{s} = \Delta_{s}^{(s)}) \) is isolated from the ambient saturation deficit \( D_{a} \) or \( \Delta_{a}^{(s)} \). When \( \Omega \rightarrow 1 \), the deficit at the surface is fully decoupled from its surroundings and \( F_{E} \) approaches the flux \( F_{E(w)} \) controlled entirely by energy supply, whereas when \( \Omega \rightarrow 0 \), the surface is fully coupled with its surroundings and \( F_{E} \) approaches the flux \( F_{E(w)} \) controlled by the ambient saturation deficit and \( g_{s} \).

The second interpretation needs care. The fully decoupled limit \( \Omega \rightarrow 1 \) can be achieved physically in either of two ways, the calm limit \( g_{a} \rightarrow 0 \) or the wet-surface limit \( g_{s} \rightarrow \infty \). However, in the limit \( g_{a} \rightarrow 0 \), it also follows that \( p \rightarrow 0 \) and \( \alpha^{*} \rightarrow 0 \) (Eq. (23)), and the latent- and sensible-heat fluxes both tend to zero. In the limit \( g_{s} \rightarrow \infty \), we also have \((1 - \Omega)g_{s} \rightarrow g_{a}/(pe + b)\) and \( F_{E} \rightarrow F_{E(w)} \), so the second term in Eq. (27) vanishes only when the deficit is zero. Hence, the limits \( g_{a} \rightarrow 0 \) and \( g_{s} \rightarrow \infty \) both need to be augmented to provide a satisfactory definition of the decoupled latent-heat flux \( F_{E(x)} \). To do this, the fully decoupled and fully coupled limits can be defined formally as:

\[
\frac{\{g_{AH}, g_{AE}, g_{r} + g_{g}\}}{\{g_{s}, g_{i}^{*}\}} \rightarrow \begin{cases} 
0 & \text{(fully decoupled)} \\
\infty & \text{(fully coupled)}
\end{cases}
\]

(28)

where the ratios among the groups of conductances in braces are held constant. The numerator contains all coupling conductances via aerodynamic, radiative and storage fluxes, so the fully decoupled limit occurs when all these conductances together become very small (and conversely for the fully coupled limit). Equation (28) implies that when the fully decoupled limit is approached via the route \( g_{a} \rightarrow 0 \), all of \( g_{AH}, g_{AE} \) and \( g_{r} + g_{g} \) approach zero together and in proportion, so that \( p \) and \( b \) remain constant. Likewise, approach by the route \( g_{s} \rightarrow \infty \) also implies that \( g_{i}^{*} \rightarrow \infty \) and the deficit approaches its minimum value \( (D_{a} \rightarrow D_{a(min)} \) or \( \Delta_{a}^{(s)} \rightarrow 0) \), coinciding with wet-surface evaporation into saturated air.

Equation (28) formalises the reason why the SEB ratios \( \alpha, \alpha^{*} \) and \( \beta \) behave identically in the fully decoupled limit, the calm limit and for wet-surface evaporation into saturated air, approaching the values \( \alpha_{(x)}, \alpha_{(x)}^{*} \) and \( \beta_{(x)} \) given by Eqs. (21). The calm limit occurs when the numerator in Eq. (28) approaches zero, while wet-surface evaporation into saturated air corresponds to the denominator approaching infinity. These two cases cause decoupling for different physical reasons: decoupling in the calm limit occurs because the surface is isolated from the environment by large gradients in temperature and humidity; whereas it occurs in the wet-surface, saturated-air limit because the saturation deficit is always minimal and independent of external control.

(d) The evaporation rate independent of aerodynamic conductance

One meaning of equilibrium evaporation identified in section 2 is the evaporation rate independent of aerodynamic conductance (Monteith 1965; Thom 1975). By differentiating the PM equation with respect to \( g_{a} \) (holding all other quantities fixed) and
setting \( \partial F_E / \partial g_a = 0 \), it is found that:

\[
\alpha^*_a = \frac{p^2 \varepsilon g_a}{\varepsilon g_a + b}; \quad \beta_a = \frac{b}{p \varepsilon g_a},
\]

where the subscript \((a)\) distinguishes this condition. The result differs from condition \((x)\), Eqs. (21), in that \( p^2 \) replaces \( p \) and \( \varepsilon g_a \) replaces \( \varepsilon \). Both differences stem from the dependence of \( p \) on \( g_a \), and vanish in the radiatively uncoupled case, when Eqs. (29) reduce to \( \alpha = \varepsilon / (\varepsilon + b) \). This accounts for the 'focus point' in Fig. 2(b) but not in Fig. 3(b).

5. ATMOSPHERIC FEEDBACKS

\((a)\) Motivation and governing equations

All the theory reviewed to this point has concerned the behaviour of the SEB when forced by prescribed atmospheric conditions, without consideration of surface-atmosphere feedback. The essence of this feedback is that the ambient \( T_a \) and \( Q_a \) at a near-surface reference height are themselves influenced by the SEB, and hence are dependent rather than independent variables.

In general, the (potential) temperature \( T(x, t) \) and specific humidity \( Q(x, t) \) are governed by:

\[
\frac{\partial T}{\partial t} + U \cdot \nabla T = -\frac{\nabla \cdot F_H}{\rho c_p}, \quad \frac{\partial Q}{\partial t} + U \cdot \nabla Q = -\frac{\nabla \cdot F_E}{\rho \lambda},
\]

where \( U \) is the velocity vector and \( F_H(x) \) and \( F_E(x) \) are the vector fluxes for sensible and latent heat in the air at a point \( x \). Except very close to the surface, the fluxes are almost entirely due to turbulent transport. In contrast with the algebraic or diagnostic equations in sections 3 and 4, Eqs. (30) are a pair of differential or prognostic conservation equations. They can be solved over a control region \( V \) once initial and boundary conditions are specified (regarding \( U \) as specified for this purpose). The boundary conditions specify the fluxes \( UT + F_H / \rho c_p \) and \( UQ + F_H / \rho \lambda \) at bounding surfaces in the air, and \( F_H \) and \( F_E \) at the solid (ground) surfaces bounding \( V \). The latter fluxes are determined either by Eqs. (1) to (5) which govern the SEB, or (often more conveniently) by Eq. (8) and the PM Eq. (9) or (16), with the ambient conditions \( T_a \) and \( Q_a \) being at a reference point close to the surface. In this case \( T_a \) is eliminated as an extra dependent variable.

By volume integration, Eqs. (30) can be reduced, without approximation, to a pair of ordinary differential equations for the volume-averaged temperature \( T_v \) and humidity \( Q_v \) in the control region \( V \). Letting \( V \) be bounded at the ground by a surface \( S_g \) and in the air by a number of surfaces \( S_k \) (so that the total surface enclosing \( V \) is \( S_g + \Sigma S_k \)), and assuming for simplicity that advective fluxes dominate turbulent fluxes on the air surfaces \( S_k \), the volume-averaged equations can be written as:

\[
\frac{dT_v}{dt} = \frac{S_g}{V} \frac{F_H}{\rho c_p} + \sum_k \frac{S_k}{V} U_k(T_k - T_v), \quad (31)
\]

\[
\frac{dQ_v}{dt} = \frac{S_g}{V} \frac{F_E}{\rho \lambda} + \sum_k \frac{S_k}{V} U_k(Q_k - Q_v),
\]

where the fluxes \( F_H \) and \( F_E \) are averaged over \( S_g \), and \( U_k, T_k \) and \( Q_k \) denote surface-averaged quantities over each air surface component \( S_k \). These equations are derived in
Raupach (2000) for both fixed and moving regions, where the moving case includes the CBL.

Two important distinctions can now be drawn, the first being between thermodynamically closed and open systems. For closed systems there is no mass exchange with the external environment, so the mass-flow or advection terms disappear in Eqs. (31), while for open systems these terms are significant. The second distinction is between perfectly and imperfectly mixed systems, a perfectly mixed system being one for which the ambient conditions $T_a$ and $Q_a$ at all points are uniform and equal to the volume averages $T_v$ and $Q_v$. This section first treats closed systems, both well-mixed (section 5(b)) and imperfectly mixed (section 5(c)), followed by open systems (section 5(d)).

(b) Closed, well-mixed systems

The closed well-mixed system is the simplest case and a natural starting point. It is isomorphic with the well-known ‘box model’ for a convective boundary layer without entrainment (Perrier 1980; McNaughton and Jarvis 1983), since the region $V$ can be regarded as a well-mixed layer of depth $h = V/S_s$. Omitting advection terms, Eqs. (31) become:

$$\frac{dT_v}{dt} = F_H/(\rho c_p h); \quad \frac{dQ_v}{dt} = F_E/(\rho \lambda h).$$  \hspace{1cm} (32)

These equations must be closed by relating the volume averages $T_v$ and $Q_v$ to the fluxes $F_E$ and $F_H$. Because of the well-mixed condition, $T_v = T_a$ and $Q_v = Q_a$ for any ambient reference point $a$. Since $F_E$ and $F_H$ depend on $T_a$ and $Q_a$ through the saturation deficit, closure can be achieved by finding a rate equation for the volume-averaged deficit $D_v$ from those for $T_v$ and $Q_v$. For this purpose it is better to use the actual deficit $D_v = Q_{sat}(T_v) - Q_v$ rather than the linearized deficit $\Delta_v^{(s)}$, because $D_v$ is independent of surface temperature whereas $\Delta_v^{(s)}$ is not. A rate equation for $D_v$ can be obtained from the identity:

$$\frac{dD_v}{dt} = \frac{\partial D_v}{\partial T_v} \frac{dT_v}{dt} + \frac{\partial D_v}{\partial Q_v} \frac{dQ_v}{dt} = \left(\frac{c_p \varepsilon_v}{\lambda}\right) \frac{dT_v}{dt} - \frac{dQ_v}{dt},$$  \hspace{1cm} (33)

where $\varepsilon_v$ is evaluated at the volume-averaged temperature $T_v$ using Eqs. (6). Equations (32) and (33) yield the required equation:

$$\frac{dD_v}{dt} = \frac{F_D}{h}, \quad \text{with } F_D = \frac{\varepsilon_v F_H - F_E}{\rho \lambda}.$$  \hspace{1cm} (34)

The quantity $F_D$ is the flux of saturation deficit from the surface into the region $V$. Hence, despite the nonlinearity of the function $D_v(T_v, Q_v)$, an exact rate equation for $D_v$ can be constructed in the form $dD_v/dt = F_D/h$, where $F_D$ is a linear combination of $F_H$ and $F_E$ with coefficients dependent on $\varepsilon_v$ and thence $T_v$. (Betts (1994) used a similar approach to derive a rate equation for the saturation pressure in the well-mixed CBL, the pressure at which an air parcel becomes saturated on adiabatic ascent. This is a measure of the humidity deficit in the CBL, like the saturation deficit $D$ used here, but unlike $D$ it cannot be related easily to the surface boundary conditions on the fluxes $F_H$ and $F_E$.)

By using the PM Eqs. (9) and (10) for $F_E$ and $F_H$ with the reference point identified with the volume average, $F_D$ can be related to $D_v$ so that Eqs. (34) become closed in $D_v$:

$$F_D = g_q (D_q - D_v), \quad \frac{dD_v}{dt} = \frac{g_q (D_q - D_v)}{h},$$  \hspace{1cm} (35)
where $g_q$ is a conductance. $D_q$ is a deficit scale formed from $F^*_A$ and the resistances:

$$g_q = \frac{p \varepsilon_v + 1}{(p \varepsilon_v + b) r_\alpha + r_s} = \left( \frac{p \varepsilon_v + 1}{p \varepsilon_v + b} \right) \Omega g_a,$$

$$\rho \lambda D_q = \left( \frac{p \varepsilon_v F^*_A}{p \varepsilon_v + 1} \right) \left( r_s + r_\alpha \left( b - \frac{\varepsilon_s}{\varepsilon_v} \right) \right) \approx \left( \frac{p \varepsilon_v F^*_A}{p \varepsilon_v + 1} \right) r_s. \tag{37}$$

The last approximation applies when $b \approx 1$ and the difference between $\varepsilon_s$ and $\varepsilon_v$ is ignored.

The meanings of these scales are indicated by writing the solution for $D_v(t)$ in the simple case where $g_q$ and $D_q$ are constant. This solution is (Perrier 1980; McNaughton and Jarvis 1983):

$$D_v(t) = D_v(0) \exp(-t g_q/h) + D_q \{ 1 - \exp(-t g_q/h) \}, \tag{38}$$

tending to the steady state $D_v = D_q$, $F_D = 0$. Hence, $D_q$ is the final steady value of the volume-averaged saturation deficit in the closed system, and $h/g_q$ is the time constant for approach to the steady state. More generally, $g_q$ and $D_q$ vary in time, in response to variations in $r_\alpha$, $r_s$, $F^*_A$ and the temperature $T_v$ (which affects $\varepsilon_v$), and the solution of Eqs. (35) becomes:

$$D_v(t) = D_v(0) \exp(-s) + \int_0^s D_v(s - s_1) \exp(-s_1) \, ds_1, \tag{39}$$

where $s$ is a dimensionless time variable such that $ds = (g_q/h) \, dt$.

A number of conclusions emerge from this analysis.

(i) Equations (34) and (35) show that, in a closed, well-mixed region supplied steadily with energy, the steady-state SEB satisfies the condition $F_D = 0$, or $\varepsilon_v F_H - F_E = 0$ (because $dD_v/du = 0$ in the steady state). This condition defines the equilibrium energy partition:

$$\beta_0 = \frac{1}{\varepsilon_v}, \quad \alpha_0 = \frac{\varepsilon_v}{\varepsilon_v + 1}, \quad \alpha^*_0 = \frac{p \varepsilon_v}{p \varepsilon_v + 1}, \tag{40}$$

with corresponding equilibrium latent- and sensible-heat fluxes:

$$F_E(0) = \frac{p \varepsilon_v F^*_A}{p \varepsilon_v + 1}, \quad F_H(0) = \frac{p F^*_A}{p \varepsilon_v + 1}, \tag{41}$$

where the subscript $(q)$ denotes the long-term limit or steady state in a closed region supplied steadily with energy. Equations (40) are of similar form to, but quantitatively different from, Eqs. (21) for the fully decoupled limit, for reasons addressed in section 6(b).

(ii) The steady state of the SEB defined by Eqs. (40) and (41) is independent of physiological and aerodynamic feedbacks affecting the resistances $r_\alpha$ and $r_s$, because the steady-state deficit $D_q$ is a globally stable critical point or attractor for solutions to Eqs. (35), irrespective of possible dependencies of $g_q$ on the SEB through physiological or aerodynamic feedbacks. These affect the time scale $h/g_q$, but not the final equilibrium state (Raupach 2000).

(iii) Equations (40) and (41) are also independent of spatial heterogeneity in the energy fluxes within the closed system, because $F_E$ and $F_H$ are averages over all energy-exchanging surfaces.
(iv) In contrast with $D_v$, neither $T_v$ nor $Q_v$ reaches a steady state because of the continuous energy supply to the closed system. In the limit $t \to \infty$, $F_H$ and $F_E$ become nearly constant in time and $T_v$ and $Q_v$ both increase nearly linearly with $t$, consistent with Eqs. (32). Because of this slow warming, the final state is really a quasi-steady state in which $\varepsilon_v$ (and hence $D_g$, $F_H$ and $F_E$) all vary slowly. However, Eqs. (40) and (41) remain very good approximations, as shown in appendix C.

(v) A general property of a closed evaporating system, following from energy and mass conservation (Eqs. (32)), is that the inverse Bowen ratio $1/\beta = F_E/F_H$ is equal to $(\lambda/c_p)\,dQ_v/dT_v$ and therefore to the slope of the trajectory of the point $(\lambda Q_v(t),$
Numerical tests of the above five conclusions are shown in Figs. 5 to 7. Figure 5 shows trajectories of \((\lambda Q_v(t), c_p T_v(t))\) on the \((\lambda Q, c_p T)\) plane, found by solving Eqs. (32) numerically for \(T_v(t)\) and \(Q_v(t)\), with exact boundary conditions based on Eqs. (1) to (5). As the system approaches the quasi-steady state, the numerical calculations confirm that the slope of this line approaches \(\varepsilon_v\), so that the Bowen ratio approaches \(1/\varepsilon_v\) as required by Eqs. (40). Figures 6 and 7 compare the time evolution of \(\alpha^*\) in a closed evaporating system with that of \(\alpha^*_q\) (and \(\alpha^*_x\), discussed later in section 6(a)). These results were obtained from numerical solutions for \(T_v(t)\) and \(Q_v(t)\), as for Fig. 5, which were then used to find \(T_s(t)\), \(\alpha^*(t)\), \(\alpha^*_q(t)\) and \(\alpha^*_x(t)\), respectively from Eqs. (3), (19), (40) and (21). The calculations confirm that \(\alpha^*\) converges to \(\alpha^*_q\) from a range of initial conditions (Fig. 6) and for a wide range of values of \(b = r_{aE}/r_{aH}\) (Fig. 7). (vi) Figures 6 and 7 indicate that Eq. (40) is not exact, but its error is very small \((\alpha^* - \alpha^*_q)\) is typically about \(-0.002\) at \(tg_q/h = 5\), decreasing further in magnitude at larger times). This small error is a result of the slow time variation imposed by the steady warming of the system and the consequent increase in \(T_v\) and \(\varepsilon_v\). An approximate expression for the error is found in appendix C.
(vii) For later analysis of open systems, it is useful to express $F_E$ and $F_H$ as departures from the equilibrium (closed-system, steady-state) values $F_{E(q)}$ and $F_{H(q)}$. This leads to a third generic form of the combination equations for $F_E$ and $F_H$ (the others being the PM form and the $\Omega$ form). The resulting 'departure-from-equilibrium' combination equations, derived in appendix B, are:

$$
F_E = F_{E(q)} - \frac{\rho \lambda (D_q - D_v)}{(p e_{sv} + b) r_a + r_s}, \quad F_H = F_{H(q)} + p \left\{ \frac{\rho \lambda (D_q - D_v)}{(p e_{sv} + b) r_a + r_s} \right\},
$$

where $D_q$ is given by Eq. (37).

(c) Closed, imperfectly mixed systems

In a closed, imperfectly mixed region there is no mass exchange of air with the external environment, and the ambient $T_a$ and $Q_a$ vary with position $x$ rather than being uniform as in a well-mixed region. However, the volume-averaged $T_v$ and $Q_v$ still obey Eq. (32), and a volume-averaged saturation deficit $D_v = Q_{sat}(T_v) - Q_v$ can still be defined. Its rate equation is still given by Eq. (34), which for this purpose can be written as:

$$
dD_v = \frac{\varepsilon_v F_H - F_E}{\rho \lambda} = \frac{p \varepsilon_v F_A^* - (p \varepsilon_v + 1) F_E}{\rho \lambda}.
$$

To make this a closed equation in $D_v$, it is necessary to invoke a boundary condition specifying $F_E$ in terms of $D_v$, as in the closed, well-mixed case. However, the PM Eq. (9)
specifies \( F_E \) in terms of the deficit \( D_a \) at a specific reference point \( a \). Hence, we consider the difference:

\[
D_a - D_v = \frac{c_p \varepsilon_{av}}{\lambda} (T_a - T_v) - (Q_a - Q_v),
\]  

(44)

where \( \varepsilon_{av} \) is defined across the finite temperature interval \( T_a - T_v \), as in Eqs. (6). The increments \( T_a - T_v \) and \( Q_a - Q_v \) can be related to the fluxes \( F_H \) and \( F_E \) by defining an aerodynamic conductance \( g_{a(a \rightarrow v)} \) for the pathway from an arbitrary reference point \( (a) \) to a point \( (v) \) where \( T \) and \( Q \) take their volume-averaged values, such that:

\[
F_H = \rho c_p g_{a(a \rightarrow v)} [T_a - T_v], \quad F_E = \rho \lambda g_{a(a \rightarrow v)} [Q_a - Q_v].
\]  

(45)

The deficit difference \( D_a - D_v \) is then found to be:

\[
D_a - D_v = \frac{\varepsilon_{av} F_H - F_E}{g_{a(a \rightarrow v)}} = \frac{\varepsilon_{av} F_A^* - (\rho \varepsilon_{av} + 1) F_E}{g_{a(a \rightarrow v)}}.
\]  

(46)

Substituting this into the PM Eq. (9), we obtain a form of the PM equation in which \( F_E \) is related to the volume-averaged \( D_v \) rather than the deficit \( D_a \) at any specific reference point:

\[
F_E = \frac{p [\varepsilon_s + \varepsilon_{av}(g_{a(a \rightarrow s)}/g_{a(a \rightarrow v)})]}{p [\varepsilon_s + \varepsilon_{av}(g_{a(s \rightarrow a)}/g_{a(a \rightarrow v)})]} F_A^* + \rho \lambda g_{a(s \rightarrow a)} D_v \]

\[+ b + (r_s + r_{a(a \rightarrow v)})/r_{a(s \rightarrow a)}, \]

(47)

where \( g_{a(s \rightarrow a)} \) is the aerodynamic conductance from the surface \( (s) \) to an arbitrary reference point \( (a) \), that is, the conductance \( g_a \) throughout sections 3 and 4. The second line emphasizes that \( F_E \) is a linear function of \( D_v \), just as it is of \( D_a \) through Eq. (9). The equation for \( D_v \) is found from Eqs. (43) and (47) to be:

\[
\frac{dD_v}{dt} = \frac{p \varepsilon_{av} F_A^* - (\rho \varepsilon_{av} + 1)(c_1 + c_2 D_v)}{\rho \lambda},
\]  

(48)

which is a generalization of Eq. (34). It is not necessary to evaluate \( c_1 \) and \( c_2 \), but only to note that as \( t \to \infty \), the solution of Eq. (48) approaches a quasi-steady state in which \( dD_v/dt = 0 \) and \( (c_1 + c_2 D_v) = p \varepsilon_{av} F_A^*/(\rho \varepsilon_{av} + 1) \). With Eq. (47), this shows that the steady state exactly satisfies Eqs. (40) and (41) in imperfectly mixed systems, just as for well-mixed systems.

(d) Open systems

Open systems are considered in general in Raupach (2000). In an open system there is exchange of air between the system and its external environment, by flows across the bounding surfaces in the air. Examples are the growing convective boundary layer and the internal boundary layer formed downwind of a transition between surfaces with different energy and moisture availabilities. From Eqs. (31), the conservation equations for entropy and water vapour in a general open system are:

\[
\frac{dT_v}{dt} = \frac{F_H}{\rho c_p h} + \frac{g_e}{h} (T_e - T_v),
\]

\[
\frac{dQ_v}{dt} = \frac{F_E}{\rho \lambda h} + \frac{g_e}{h} (Q_e - Q_v),
\]  

(49)
where $h = V/S_e$ is a length-scale, $g_e = \Sigma(S_k U_k)/V$ is an ‘external conductance’, and $T_e$ and $Q_e$ are the average temperature and humidity of the air exchanged with the external environment, defined as $T_e = \Sigma(S_k U_k T_k)/\Sigma(S_k U_k)$ and likewise for $Q_e$. Equation (49) is a simple extension of Eq. (32) for a closed, well-mixed system, reverting to the closed case when $g_e \to 0$. The solution of Eqs. (49), with constant $g_e$ and with boundary conditions appropriate for an evaporating surface with constant $F_A^*$, $g_a$ and $g_s$, yields a quasi-steady SEB as $t \to \infty$, which can be written as a generalization of the corresponding quasi-steady solution for a closed region, Eqs. (40) and (41). With minor approximations that do not matter here, this solution is (Raupach 2000):

$$F_E = F_{E(q)} - \frac{\rho \lambda (D_q - D_e)}{(\rho \varepsilon_v + 1)(r_q + r_e)},$$  \hspace{1cm} (50)

where $D_e = Q_{sat}(T_e) - Q_e$ is the saturation deficit of the entrained air, $D_q$ is defined by Eq. (37), $r_q = 1/g_q$ is defined by Eq. (36), and $r_e = 1/g_e$. Equation (50) is of the form of the departure-from-equilibrium combination equation, Eq. (42). It shows that the SEB in a well-mixed region which is continuously entraining external air settles to a steady state in which $F_E$ differs from the equilibrium value $F_{E(q)}$ by an amount given by the second term in Eq. (50). Hence, in general, a well-mixed, open system can never settle to the thermodynamic equilibrium energy partition defined by Eqs. (40) and (41), unless $g_e \to 0$ or $r_e \to \infty$ (that is, unless the system becomes closed).

Raupach (2000) applied this general solution to the daytime convective boundary layer, in which case the entrainment rate $g_e$ and the surface buoyancy flux are linked through a feedback via the SEB. As shown there and in McNaughton and Raupach (1996), the resulting system has the properties that: (i) the equilibration rate is $1/[(r_e + r_q)h]$, substantially faster than the closed-system rate $1/(r_q h)$; and (ii) equilibrium energy partition is approached only when the entrainment rate $g_e$ becomes very small for dynamical reasons, so that the system approaches closure.

6. DISCUSSION AND CONCLUSIONS

(a) Alternative definitions of equilibrium evaporation

Three quantitatively different definitions of equilibrium evaporation have emerged. Definition (x), Eqs. (21), is steady evaporation in the fully decoupled limit, coinciding with the calm limit and with evaporation from a wet surface into saturated air, and all yielding the Bowen ratio $\beta_{(x)} = b/\varepsilon_v$. Definition (q), Eqs. (40), is the long-time limit of evaporation into a closed system (well-mixed or imperfectly mixed) supplied steadily with energy, yielding the Bowen ratio $\beta_{(q)} = 1/\varepsilon_v$. Definition (u), Eqs. (29), is the evaporation independent of aerodynamic conductance, yielding the Bowen ratio $\beta_{(u)} = b/\varepsilon_{sa}$. All three definitions coincide under the approximations which have hitherto been usual in combination theory: equal aerodynamic conductances for heat and water vapour ($b = 1$); neglect of radiative coupling ($p = 1$); and neglect of the exact temperatures at which $\varepsilon$ should be evaluated. These are often reasonable practical approximations, especially the first, but the concern here is with the conceptual differences.

Definition (u), the evaporation independent of aerodynamic conductance $g_a$, produces different results from either (x) or (q) because $g_a$ influences radiative coupling; that is, $p$ depends on $g_a$ as in Eqs. (12). Of more fundamental significance is the difference between (x) and (q), which is discussed in terms of the three questions posed in the introduction.
(i) Are definitions \((x)\) and \((q)\) consistent? They differ in the following way: while equilibrium energy partition is attained in both cases because gradients in humidity are constrained to track gradients in temperature through \(Q_{\text{sat}}(T)\), this constraint is imposed differently in the two cases. In definition \((x)\), \(Q\) and \(T\) track along spatial gradients in a direction of diffusion (usually vertical). This is demonstrated by the arguments leading to Eqs. (21), by the derivation of the PM equation itself and hence its calm and fully decoupled limits, and by the appearance of the factor \(b\), a property of diffusive transfer processes. On the other hand, definition \((q)\) arises when \(Q\) and \(T\) track along temporal gradients as a closed evaporating system approaches a quasi-steady state (temporal gradients may be replaced by spatial gradients along a direction of advection, under the transformation \(x = Ut\)). In this case \(T_o\) and \(Q_b\) increase together so that (as \(t \to \infty\)) the deficit flux \(\varepsilon_v F_H - F_E\) approaches zero and \(\beta_{(q)} \to 1/\varepsilon_v\), as shown in section 5(b) and Fig. 5. This occurs for any surface conductance \(g_s\), any amount of mixing (that is, any distribution of \(g_a\)) in the closed system, and any spatial distribution of surface fluxes which have average values \(F_H\) and \(F_E\).

(ii) Is either \((x)\) or \((q)\) of fundamental precedence? This question requires consideration of how the physical states required by both definitions are reached. The situation for definition \((q)\) is simple: supply a closed evaporating system with energy and wait for a quasi-steady state to develop. The approach to this state is governed by the differential Eqs. (32) and (34). Definition \((x)\), however, arises from the imposition of algebraic limits on algebraic equations which describe only the instantaneous properties of the system, not its evolution in time. These algebraic limits are not constrained to be realizable equilibrium states, that is, states to which the system evolves naturally. On this basis, definition \((q)\) is of fundamental physical precedence.

(iii) At what temperature should \(\varepsilon\) be evaluated? For many years the micrometeorological community has accepted that equilibrium evaporation should be defined using \(\varepsilon = \varepsilon_s\), evaluated at surface temperature. In contrast, the arguments of sections 5(b) and (c) show that the equilibrium energy partition in closed evaporating systems is \(\beta \to 1/\varepsilon_v\), evaluated at the average air temperature in the closed system (always a well-defined quantity). Two separate arguments have led to the widespread acceptance hitherto of \(\varepsilon = \varepsilon_s\): the first, summarized in section 4(b), proceeds from the definition of equilibrium as steady evaporation from a wet surface into saturated air. Priestley (1959) and later Philip (1987b) both used this approach to conclude that \(\varepsilon = \varepsilon_s\), considering only the case \(b = 1\). However, the problem is the identification of equilibrium with the condition of saturation at heights above the surface, which is an algebraic assumption about the equilibrium state rather than a deduction from the dynamics of the system (shown by considering \(b \neq 1\)). The second argument is that of McNaughton (1976a,b), who considered the dynamics of an advecting internal boundary layer and its approach to a quasi-steady state, using the differential Eqs. (30) with gradient-diffusion closure. He obtained solutions by linearizing the boundary conditions (Eqs. (1) to (5)) about a typical surface temperature, and then defining a pair of new dependent variables (the specific enthalpy \(c_p T + \lambda Q\) and the linearized saturation deficit \(\lambda A_g^{(s)}\)) to separate the boundary conditions. In the closed-system limit, these solutions led to equilibrium evaporation with \(\varepsilon = \varepsilon_s\), but this arose from the initial linearization about \(T_s\), rather than by allowing the value of \(\varepsilon\) at equilibrium to emerge from the physics.

The numerical results in Figs. 5, 6 and 7 confirm that the isothermal evaporative fraction \(\alpha^*\) converges to \(\alpha^*_{(q)}\) (Fig. 6(a)) much more rapidly than to \(\alpha^*_{(x)}\) (Fig. 6(b)) from a wide range of initial conditions. In fact, the convergence of \(\alpha^*\) to \(\alpha^*_{(x)}\) occurs only as a result of the steadily increasing temperature and decreasing heat flux in the
closed system, which causes $T_s - T_a$ and therefore $\varepsilon_s - \varepsilon_v$ to decrease continually. Similarly, for a wide range of values of $b = r_{AE}/r_{AH}$, the system evolves towards the energy partition $\alpha^{(a)}_q$ (Fig. 7(a)) as predicted by Eqs. (40), but the algebraically defined $\alpha^{(a)}_\varepsilon$ (Fig. 7(b)) is not even close to the eventual steady state SEB when $b$ is not equal to one. These results confirm that $\alpha^{(a)}_\varepsilon$ is not an equilibrium state.

(b) Concluding summary

Starting from an historical perspective, this paper has reviewed the description of the terrestrial surface energy balance (SEB) offered by combination theory, concentrating on the development of a theory which avoids approximations at the level of principle, and on the implications of surface–atmosphere feedbacks for the meaning of equilibrium evaporation.

Several measures have been taken to formulate the theory precisely. First, linearization has been avoided by tracking the temperatures at which $\varepsilon$ (the dimensionless slope of $Q_{sat}(T)$) must be evaluated for the theory to be exact. Second, the SEB and PM equations have been written in forms which account throughout for radiative and storage coupling. Third, careful distinction has been drawn between the actual saturation deficit $D = Q_{sat}(T) - Q$ and its linearized counterparts.

Two options for writing the PM equation have been identified. The actual-deficit form, Eq. (9), involves the ambient saturation deficit $D_a$ and $e_{sa}$, the finite-difference version of $\varepsilon$ evaluated between the surface temperature $T_s$ and the air temperature $T_a$ at an ambient reference point. This form has the advantage of analytical simplicity and use of a saturation deficit variable ($D_a$) which is independent of surface conditions. The linearized-deficit form, Eq. (16), uses a linearized saturation deficit $D_a^{(l)}$ and $e_s$ evaluated at $T_s$. This form is a natural lower-boundary condition for analyses with linearized variables, but the linearized deficit $D_a^{(l)}$ is not independent of surface temperature. The theory is exact under each option, but the equations remain weakly implicit because of the dependence of $\varepsilon$ upon surface temperature. For practical calculations, the dependence is handled either by approximating $\varepsilon$ with $e_a$ (evaluated at ambient air temperature $T_a$) or by calculating $T_s$ and $\varepsilon$ iteratively.

The theory has been used to analyse several hypothetical hydrometeorological states, defined by taking algebraic limits of various quantities while holding all else fixed. These include: steady evaporation from a wet surface into saturated air; the six limits defined by allowing the aerodynamic, surface and climatological conductances $g_a, g_s$ and $g_l$ to approach zero or infinity; and the fully decoupled and fully coupled limits of McNaughton and Jarvis (1983). A revised formal definition of the latter limits has been proposed which removes some inconsistencies and makes precise the well-known result that the SEB in the fully decoupled limit coincides with the calm limit ($g_a \rightarrow 0$) and the limit of evaporation from a wet surface into saturated air, all having the Bowen ratio $\beta_s = b_e / e_s$.

Feedbacks between the surface and the atmosphere have been analysed generically in both closed and open systems which contain evaporating surfaces supplied steadily with energy (the distinction being that airflow across the boundaries of the system is zero for closed but not for open systems). For all closed systems (whether well-mixed or imperfectly mixed, and whatever the distributions of fluxes and surface and aerodynamic conductances) the long-term limit is a quasi-steady state in which the deficit flux $\varepsilon_v F_H - F_E$ approaches zero and the Bowen ratio $\beta$ approaches the equilibrium value $\beta_{eq} = 1/\varepsilon_v$, where $\varepsilon_v$ is evaluated at the volume-averaged temperature $T_v$ (always a
well-defined quantity in a closed system). The time-scale for attainment of this quasi-steady state increases as \( g_s \) and \( g_a \) decrease. Open systems, including the convective boundary layer and the internal boundary layer formed downwind of a surface transition, do not reach the equilibrium energy partition \( \beta(q) = 1/\varepsilon_v \) unless they approach closure for dynamical reasons.

Two main alternative definitions of equilibrium energy partition have been highlighted: \( \beta(x) = b/\varepsilon_s \) (defined algebraically from combination theory as the fully decoupled limit, the calm limit or the limit of steady evaporation from a wet surface into saturated air, all of which are equal); and \( \beta(q) = 1/\varepsilon_v \) (defined dynamically as the long-time steady state in a closed evaporating system). The two definitions are not consistent, but the latter is the more fundamental as it is dynamically realizable whereas the former is not. Hence, the appropriate correct temperature for evaluating \( \varepsilon \) in determining equilibrium energy partition is the volume-averaged temperature in a closed region, which in the case of a convective boundary layer is well approximated by the mixed-layer temperature.

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APPENDIX A

List of symbols

- \( a_s \) surface albedo
- \( b = r_{aH}/r_{aH} \), ratio quantifying differences between aerodynamic transfers of heat and water vapour
- \( c_p \) isobaric specific heat of air
- \( D_a \) saturation deficit at location \( a \)
- \( D_{a(min)} \) minimum ambient saturation deficit in steady, horizontally homogeneous conditions, defined by Eq. (22)
- \( D_q \) saturation deficit defined by Eq. (37)
- \( D_v \) volume-averaged saturation deficit
- \( D_X \) saturation deficit defined by Eqs. (B.3)
- \( d_g \) length-scale specifying thermal store in Eq. (5)
- \( e_s \) surface emissivity
- \( F_X \) flux density of transferred entity \( X \), positive away from surface
- \( F_A^* \) isothermal available energy flux, defined by Eqs. (11)
- \( g_{yX} \) conductance, \( g = 1/r \), for transferred entity \( X \) over pathway \( y \)
- \( g_a \) (without specified entity) = \( r_{aH} \)
- \( g_g \) storage conductance, defined following Eqs. (11)
- \( g_{i1} = F_A^*/(\rho \lambda D_a) \), the climatological conductance
- \( g_{i2}^* = F_A^*/(\rho \lambda D_a) \), the isothermal climatological conductance
- \( g_q \) conductance defined by Eq. (36)
$g_r$  radiative conductance, defined following Eqs. (11)
$h$  height of enclosed, well-mixed region
$k_g$  thermal conductivity of thermal store
$p = r_H/r_{ah} = g_a/(g_a + g_r + g_g)$, a ratio between 0 and 1 quantifying relative
significance of aerodynamic, radiative and storage coupling between the
surface and its environment
$Q_a$  specific humidity at location $a$
$Q_v$  volume-averaged specific humidity
$Q_{sat}(T)$  saturation specific humidity at temperature $T$
$r_{yx}$  resistance, $r = 1/g$, for transferred entity $X$ over pathway $y$
$r_a$  (without specified entity) = $r_{ah}$
$r_i = 1/g_i = \rho \lambda D_a/F_A$, the climatological resistance
$r_i^* = 1/g_i = \rho \lambda D_a/F_A^*$, the isothermal climatological resistance
t  time
$T_a$  potential temperature at location $a$
$T_v$  volume-averaged potential temperature
$U_a$  flow velocity or wind speed at location $a$
x  streamwise coordinate
z  height or surface-normal coordinate
$z_a$  height of reference point for ambient conditions
$\alpha = F_E/F_A$, the evaporative fraction
$\alpha^* = F_E/F_A^*$, the isothermal evaporative fraction
$\beta = F_I/F_E$, the Bowen ratio
$\Delta_a^{(r)}$  linearized saturation deficit at location $a$ about temperature $T_r$, defined by
Eq. (13)
$\delta_a^{(r)} = D_a - \Delta_a^{(r)}$, the difference between the actual and linearized saturation
deficits at location $a$
$\varepsilon_a = (\lambda/c_p) \frac{dQ_{sat}}{dT}$, the ratio of the changes with temperature of the latent
and sensible contents of saturated air, evaluated at temperature $T_a$ at location $a$; see Eq. (6)
$\varepsilon_v$  value of $\varepsilon$ at volume-averaged potential temperature $T_v$
$\varepsilon_{sa} = (\lambda/c_p)[Q_{sat}(T_s) - Q_{sat}(T_a)]/(T_s - T_a)$, the finite-difference form of $\varepsilon$,
evaluated between temperatures $T_s$ and $T_a$ at locations $s$ and $a$; see Eqs. (6)
$\lambda$  latent heat of vaporization of water
$\rho$  air density
$\sigma$  Stefan–Boltzmann constant
$\Omega$  decoupling factor defined by Eq. (27)

Subscripts denoting transferred entity $X$ (applying to $F_X$, $g_{yx}$, $r_{yx}$)

A  available energy
D  saturation deficit
E  latent heat from surface into atmosphere
G  heat from surface into storage
H  sensible heat from surface into atmosphere
L $\downarrow$  incoming long-wave irradiance
L $\uparrow$  outgoing long-wave irradiance
S $\downarrow$  incoming short-wave irradiance
S $\uparrow$  outgoing short-wave irradiance
Subscripts denoting pathway (applying to $g_y X, r_y X$)

a  aerodynamic (surface to atmosphere)
e  external (external environment to well-mixed region)
g  storage (surface to thermal store)
r  radiative (surface to space by long-wave radiation excess arising from the surface-air temperature difference)
s  stomatal (intercellular cavity to leaf surface) or bulk surface
t  total (parallel sum of pathways a, r and g for sensible heat; series sum of pathways a and s for latent heat)

Subscripts denoting location (applying to $T, Q, D, \Delta, \delta, \varepsilon$)

a  ambient air at reference location
g  thermal store
r  arbitrary reference
s  surface
v  volume average in a closed or open system

Subscripts denoting particular states of the SEB

(w)  SEB for wet-surface potential evaporation
(q)  steady-state SEB in a closed evaporating system supplied with energy
(x)  SEB for a wet surface in saturated air, the calm limit and the fully decoupled limit
(u)  SEB when evaporation is independent of aerodynamic conductance
0  arbitrary reference SEB in Eqs. (B.1) and (B.2)

Note on dimensions: With unchanged algebra, the units of humidity can be either mass-specific (kg of water per kg of moist air) or mole-specific (moles per mole). In the mass-specific case, the following units apply: $Q$ and $D$ (kg-water kg-air$^{-1}$); $c_p$ (J kg-air$^{-1}$K$^{-1}$), $\lambda$ (J kg-water$^{-1}$); $\rho$ (kg-air m$^{-3}$). In the mole-specific case, the units are: $Q$ and $D$ (mol-water mol-air$^{-1}$), $c_p$ (J mol-air$^{-1}$K$^{-1}$), $\lambda$ (J mol-water$^{-1}$); $\rho$ (mol-air m$^{-3}$). In these units, ‘air’ is always moist air.

Appendix B

Departure-from-equilibrium form of the combination equation

For analysis of both closed and open systems, it is useful to formulate combination equations which express the latent- and sensible-heat fluxes as departures from equilibrium values. This can be done in a general way as follows. $F_{E}$ and $F_{H}$ are expressed as departures from arbitrary reference fluxes $F_{E0}$ and $F_{H0}$, chosen so that the SEB is satisfied. For any such $F_{E0}$ and $F_{H0}$, it follows from Eqs. (8) and (9) or (16) that:

$$F_{E} = F_{E0} - \frac{\rho \lambda (D_0 - D_a)}{(p \varepsilon + b) r_a + r_s}, \quad F_{H} = F_{H0} + \frac{\rho \lambda (D_0 - D_a)}{(p \varepsilon + b) r_a + r_s},$$

(B.1)

where $F_{E0}$, $F_{H0}$ and $D_0$ must satisfy:

$$F_{A}^x = F_{E0} + F_{H0}/p, \quad \rho \lambda D_0 = [(p \varepsilon + b) r_a + r_s] F_{E0} - p r_a F_{A}^x.$$

(B.2)

As these equations can be written exactly in either the actual-deficit form (with deficit $D_a$ and $\varepsilon = \varepsilon_a$) or the linearized-deficit form (replacing $D_a$ with $\Delta_a^{(s)}$ and setting $\varepsilon = \varepsilon_s$). Only one of $F_{E0}$, $F_{H0}$ and $D_0$ can be chosen independently, because of
the two constraints in Eq. (B.2). Three choices of this one variable are noteworthy: first, $D_0 = 0$ recovers the basic PM equations (9) or (16). Second, if $F_{E0}$ is chosen to be the fully decoupled limit $F_{E(x)} = p_{eq} F^*_A / (p_{eq} + b)$ defined in Eqs. (25), then $D_0$ is equal to a deficit $D_x$ (actual) or $\Delta_x^{(s)}$ (linearized) which exactly satisfies:

$$\rho \lambda D_x = r_s F_{E(x)} = r_s p_{eq} F^*_A / (p_{eq} + b) \quad \text{(actual-deficit form)},$$

$$\rho \lambda \Delta_x^{(s)} = r_s F_{E(x)} = r_s p_{eq} F^*_A / (p_{eq} + b) \quad \text{(linearised-deficit form).} \quad (B.3)$$

The equilibrium-departure equations for $F_E$ and $F_H$ are then:

$$F_E = F_{E(x)} - \rho \lambda \frac{(D_x - D_a)}{(p_{eq} + b) r_a + r_s}, \quad F_H = F_{H(x)} + \rho \lambda \frac{(D_x - D_a)}{(p_{eq} + b) r_a + r_s} \quad (B.4)$$

These express $F_E$ and $F_H$ as departures from the decoupled-limit fluxes $F_{E(x)}$ and $F_{H(x)}$. Finally, and most importantly for the present discussion, $F_{E0}$ and $F_{H0}$ can be chosen to be the equilibrium latent- and sensible-heat fluxes $F_{E(q)}$ and $F_{H(q)}$ in a closed region, defined by Eqs. (40) and (41). In this case, only the actual-deficit forms of the equations are meaningful because Eqs. (40) and (41) are derived from the condition that the actual deficit flux is zero. Taking the ambient reference conditions to coincide with volume-averaged conditions in the closed system, the equilibrium-departure forms of the combination equation become:

$$F_E = F_{E(q)} - \rho \lambda \frac{(D_q - D_v)}{(p_{eq} + b) r_a + r_s}, \quad F_H = F_{H(q)} + \rho \lambda \frac{(D_q - D_v)}{(p_{eq} + b) r_a + r_s} \quad (B.5)$$

where $D_q$ is the steady-state deficit in the closed region, defined by Eq. (37). These express $F_E$ and $F_H$ as departures from their eventual steady-state values $F_{E(q)}$ and $F_{H(q)}$ in a closed system supplied steadily with energy. The flux departures are proportional to the departure of the (actual) deficit $D_q$ from its steady-state value $D_q$.

Equations (B.4) and (B.5) are identical in form, but differ in the reference state from which a departure in $F_H$ and $F_E$ are taken: this state is the decoupled limit (x) in Eqs. (B.4) and the closed-system, steady-state limit in Eqs. (B.5). The difference between these two states implies a difference in the deficit $D_0$, which becomes $D_x$ in Eqs. (B.4) and $D_q$ in Eqs. (B.5). These two deficits become identical under the same approximations that make the (x) and (q) equilibrium definitions identical; that is, $b \approx 1$ and $\varepsilon_s \approx \varepsilon_v$.

APPENDIX C

The discrepancy caused by slow warming in a closed system

In section 5(b) and Figs. 6 and 7, it was indicated that for a closed evaporating system with constant $F^*_A$, $r_a$, $r_s$, and $h$, the isothermal evaporative fraction $\alpha^*(t)$ remains very slightly different from $\alpha_{eq}$ even at large times. This small discrepancy is caused by the continuous warming of the system and the consequent slow, continuous increases in the volume-averaged temperature $T_v$ and hence in $\varepsilon_v$. Here, the properties of the discrepancy are found by deriving an approximate expression for it.

Equation (39) is an exact solution of Eqs. (35) for the volume-averaged deficit $D_v$ in the closed region, even when the externally imposed quantities ($F^*_A$, $r_a$, $r_s$ and $h$) and the system properties ($T_v$, $Q_v$, and $\varepsilon_v$) are allowed to change. This solution, in terms of a dimensionless time $s$ such that $ds = dt (g_q / h)$, shows that $D_v(s)$ is the sum of a decaying contribution from the initial condition $D_v(0)$ (the first term in Eq. (39))
and an exponentially weighted average of the time-dependent $D_q$ over dimensionless times prior to the current time $s$ (the second term). When the time is large ($s \gg 1$), the first term in Eq. (39) is negligible, the initial condition is irrelevant, and $D_v(s)$ consists entirely of the second term. When $D_q$ is changing, as it is under the influence of slow warming and the associated change in $\varepsilon_v$, this second term and hence $D_v(s)$ is not equal to $D_q(s)$ at the current time $s$. Hence there is a non-zero discrepancy $D_q(s) - D_v(s)$ even at large times $s$, causing a (small) departure of $\alpha^*(s)$ from the slowly changing equilibrium value $\alpha^*_v(s)$. This is why Eqs. (40) are not exactly true.

To evaluate the deficit discrepancy $D_q(s) - D_v(s)$, the approximation $D_q(s) \approx \{ F^*_s / (s \lambda) \} \alpha^*_v(s)$ from Eq. (37) is used with Eq. (39). Ignoring the initial term because we are interested in large times only, Eq. (39) becomes:

$$D_q(s) - D_v(s) = \frac{F^*_s r_s}{\rho \lambda} \left\{ \alpha^*_v(s) - \int_0^s \alpha^*_v(s - s_1) \exp(-s - 1) \, ds \right\}. \quad \text{(C.1)}$$

The change with time of $\alpha^*_v$ is quite slow, so it is acceptable to approximate its local time variation by a linear relationship obtained from a Taylor expansion to first order:

$$\alpha^*_v(s - s_1) = \alpha^*_v(s) - s_1 \left\{ \frac{d\alpha^*_v}{dt} \right\}_s. \quad \text{(C.2)}$$

Putting this approximation into Eq. (C.1) and allowing the time integral to run to infinity (again because the interest is in large time values), it is found that:

$$D_q(s) - D_v(s) = \frac{F^*_s r_s}{\rho \lambda} \left\{ \frac{d\alpha^*_v}{dt} \right\}_s \int_0^\infty s_1 \exp(-s_1) \, ds_1 = \frac{F^*_s r_s}{\rho \lambda} \left\{ \frac{d\alpha^*_v}{ds} \right\}. \quad \text{(C.3)}$$

The derivative in Eqs. (C.2) and (C.3) can be written by considering the successive functional dependencies of $\alpha^*_v$ on $\varepsilon_v$, then on $T_v$, then on $t$, and then on $s$:

$$\frac{d\alpha^*_v}{ds} = \frac{d\alpha^*_v}{d\varepsilon_v} \frac{d\varepsilon_v}{dT_v} \frac{dT_v}{dt} \frac{dt}{ds}. \quad \text{(C.4)}$$

The four derivatives on the right-hand side are easily evaluated. First, Eqs. (40) show that:

$$\frac{d\alpha^*_v}{d\varepsilon_v} = \frac{p}{(p\varepsilon_v + 1)^2}. \quad \text{(C.5)}$$

The second derivative is evaluated by approximating $\varepsilon$ as an exponential function of $T$, so that:

$$\varepsilon(T) = \varepsilon(T_0) \exp[a_\varepsilon(T - T_0)], \quad \frac{d\varepsilon}{dT} = a_\varepsilon \varepsilon, \quad \text{(C.6)}$$

where $a_\varepsilon$ is about 0.06 K$^{-1}$. The third derivative is given by Eqs. (32), and the fourth by the definition of $s$, $ds = dt(g_q/h)$. Putting these together, it is found that:

$$\frac{d\alpha^*_v}{ds} = \frac{F^*_s a_\varepsilon}{\rho c_p g_q} \frac{p^2 \varepsilon_v}{(p\varepsilon_v + 1)^3}. \quad \text{(C.7)}$$

Finally, the discrepancy in evaporative fraction, $\alpha^*(s) - \alpha^*_v(s)$, can be related to the deficit discrepancy $D_q(s) - D_v(s)$ by Eqs. (42). Combining Eqs. (42), (C.3) and
(C.7) yields the sought result:
\[ \alpha^*(s) - \alpha^*_0(s) = - \frac{F_A^* r_s a_S}{\rho c_p} \frac{p^2 \varepsilon_v}{(p \varepsilon_v + 1)^4}. \] (C.8)

This shows that the discrepancy increases with both \( F_A^* \) and \( r_s \), and decreases as \( \varepsilon_v \) increases (roughly as \( 1/\varepsilon_v^2 \)). The estimate of the discrepancy given by Eq. (C.8) is in good agreement with numerical evaluations from calculations similar to those leading to Figs. 6 and 7.

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