Blocking as a local instability to zonally varying flows

By K. L. SWANSON*

University of Wisconsin–Milwaukee, USA

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SUMMARY

An idealized system consisting of a steady-state, axisymmetric vortex patch with topography forcing variations in the basic-state flow along the vortex edge is shown to possess local instabilities for a broad range of parameters. One branch of these instabilities corresponds in scale and structure to observed atmospheric blocks. The nonlinear equilibration of these blocking-type instabilities reproduces many aspects of observed blocking onsets; the potential for rapid non-modal growth and ready excitation of these local instabilities by transients suggests that this instability may lie at the root of observed blocking onsets.

KEYWORDS: Blocking Contour dynamics Local instability

1. INTRODUCTION

Due to their longevity and large amplitude, blocking flow configurations can cause prolonged anomalous weather situations over certain extratropical regions. For this reason, blocking has been a topic of heightened interest for synoptic and dynamic meteorologists for decades. Curiously, over this period there has been an uncomfortable tension regarding mechanisms instrumental in the onset and maintenance of blocking. While the importance of migratory, synoptic-scale transient eddies in blocking onsets has been emphasized in numerous observational and numerical studies (e.g. Berggren et al. (1949) and Rex (1950) for the former, Shufts (1983), Metz (1986), Haines and Marshall (1987), and Vautard and Legras (1988) for the latter), there exists substantial underlying evidence that low-frequency dynamics acting in isolation is important in blocking onsets. For example, Nakamura (1994) showed that amplification of the strongest blocking events observed over Europe is associated with a quasi-stationary Rossby wave train across the North Atlantic. He hypothesized that the local absorption of wave activity due to obstruction of Rossby wave propagation leads to the formation of strong blocking ridges over Europe. More explicitly, Nakamura et al. (1997) showed in a diagnostic study that low-pass filtered winds with synoptic-scale transient-eddy feedback removed at each step still yields blocking events over Europe.

In spite of these intriguing diagnostic hints, a precise understanding of how planetary-scale dynamics acting in isolation can yield blocking transitions remains elusive. While it is well known that idealized models containing only planetary-scale dynamics display blocking transitions as part of an overall spectra of nonlinear variability (Legras and Ghil 1985), the qualitative picture consistent with the emergence of global chaos in such systems (e.g. Palmer 1993) proves difficult to apply in detail to observed blocking onsets. At a certain level, split-flow blocking can be explained by resonant wave interactions (Loesch 1974; Colucci et al. 1981). However, it is rather difficult for planetary-scale waves to attain amplitudes resembling observed blocking events. Theories involving isolated structures, such as modons (McWilliams 1980) and solitons (Malanotte-Rizzoli and Malguzzi 1987) present appealing low-order pictures of how blocking events can be maintained for long periods in the presence of transients and dissipation, but again, forcing by synoptic eddies appears necessary to yield high planetary-wave amplitudes consistent with such structures. Given this situation, further

* Corresponding address: Department of Mathematical Sciences, University of Wisconsin–Milwaukee, Milwaukee, WI 53201, USA, e-mail: kswanson@csd.uwm.edu
insight into blocking mechanisms that are fundamentally local, involve only planetary-scale dynamics, and yet yield blocking transitions on synoptic time-scales is desirable.

If linear instability of the planetary-scale flow is to be a candidate for explaining blocking onsets, experience gained from attempts to understand the storm-track problem of localized synoptic eddy development in the atmosphere using linear normal-mode analysis (see Pierrehumbert and Swanson (1995) for a review) suggests that instability must satisfy a minimal set of criteria. First, it must be local in the sense of Pierrehumbert (1984), i.e. unstable modes must grow in situ after transients have left the area fostering the instability, in contrast with global modes that arise in a cyclic domain when a perturbation passes repetitively through the same unstable zone, acquiring more energy on each pass. Given that individual blocks mature and decay in place with no identifiable global propagating signature, this requirement appears reasonable. Secondly, the instability must be able to produce blocking events on synoptic time-scales. This requirement is in reality a finite amplitude statement; rapid growth without large disturbance equilibration amplitudes will have limited value in explaining observed blocking onsets. Finally, the physical setting of the instability must conform to the observed setting, which on synoptic scales typically involves locally sharp potential vorticity (PV) gradients associated with the tropopause.

In this study, we examine in detail an idealized system possessing an instability that satisfies these three requirements. The system consists of an axisymmetric vortex with homogeneous PV regions inside and outside the vortex separated by a single jump in PV at the vortex edge. The dynamics of wave propagation and the stability properties for this system were examined by Swanson (2000), where it was shown that accumulation of stationary-wave activity (or, in the terms of Nakamura (1994), Rossby wave obstruction) due to a vanishing group velocity can occur for realistic zonal variations in the flow along the vortex edge. Swanson (2000) also showed that this system possesses a spectrum of global linear instabilities, some of which in their nonlinear equilibration resemble blocking events. Closer examination of these instabilities reveals that they coexist as local instabilities, have growth rates relevant on synoptic time-scales, and in their nonlinear equilibration reproduce an impressive level of detail when compared to observed blocking onsets. As such, this system appears to provide a simple, physically consistent, linear-instability-based mechanism for the genesis of blocking events that requires only planetary-scale dynamics.

The outline of the paper is as follows: in the next section the relevant equations of motion are outlined. Section 3 explores local instabilities in this system, emphasizing regions of parameter space where such instabilities occur, as well as how these local instabilities differ from previously considered local instabilities to geophysical flows. Section 4 explores how the nonlinear equilibration of these local instabilities leads to the onset of blocking, while section 5 examines how mobile transient eddies influence blocking onset and maintenance in the presence of underlying local instabilities. Finally, the results are discussed and conclusions are drawn in section 6.

2. DYNAMICAL EQUATIONS

The framework in which we examine blocking as a local instability is equivalent barotropic dynamics on an $f$ plane, with topography forcing a zonally varying yet steady basic-state flow. The basic-state stream function $\Psi$ and PV, $Q$, are related by

$$Q = f_0 + \nabla^2 \Psi - L_D^{-2} \Psi + f_0 \frac{h}{H_{\text{ref}}},$$

(1)
where \( h(r, \theta) \) is the topography, \( r \) and \( \theta \) being radial and angular coordinates, respectively, \( H_{\text{ref}} \) is the reference depth of the fluid, \( f_0 \) the reference value of the Coriolis parameter, and \( L_D \) is the Rossby radius. This system differs from the more traditionally studied barotropic system by the addition of the stretching term \(-L_D^{-2} \Psi\), which incorporates the effects of stratification in a crude sense.

The basic state is chosen to be an axisymmetric single-contour vortex, with PV distribution

\[
Q = \begin{cases} 
Q_i, & \text{if } r < r_0, \\
Q_o, & \text{if } r > r_0;
\end{cases}
\]

(2)

i.e. \( Q_i \) and \( Q_o \) are the PV values inside and outside the vortex edge located at \( r = r_0 \). The velocity distribution corresponding to this PV distribution is

\[
V(r) = \begin{cases} 
\Delta L_D \alpha^{-1} I_1(r/L_D)K_1(r_0/L_D), & \text{if } r < r_0, \\
\Delta L_D \alpha^{-1} K_1(r/L_D)I_1(r_0/L_D), & \text{if } r > r_0,
\end{cases}
\]

(3)

where \( \Delta = Q_i - Q_o \) is assumed positive, \( I_n \) and \( K_n \) are the \( n \)-th-order modified Bessel functions, and

\[
\alpha \equiv I_0(r_0/L_D)K_1(r_0/L_D) + I_1(r_0/L_D)K_0(r_0/L_D)
\]

is a constant. The flow along the vortex edge associated with this PV distribution vanishes in the limit \( r_0/L_D \to 0 \). However, for values of \( r_0 \) larger than a few deformation radii the flow along the vortex edge approaches the limit \( \Delta L_D/2 \) of a purely zonal PV jump. In this limit, the angular velocity decays roughly exponentially away from the jump, with an e-folding characteristic length of \( L_D \).

Axisymmetry in the flow along the vortex edge is broken by introducing topography of the form

\[
h(r, \theta) = H_{\text{ref}} \gamma_0 \left[ 1 - (r/r_0)^2/2 \right] + H_{\text{ref}} \gamma_1 J_1(\kappa_1 r/r_0) \cos 2\theta,
\]

(4)

where \( \kappa_1 = 3.8317 \) is the first zero of the first-order Bessel function \( J_1 \), chosen so that \( r = r_0 \) remains a steady solution of the entire flow, and \( \gamma_0 \) and \( \gamma_1 \) are constants. This topography induces a forced stream-function component \( \psi_f \) (to be distinguished from the intrinsic stream function \( \Psi \) associated with the vortex itself) of the form

\[
\psi_f(r, \theta) = f_0 H_{\text{ref}} \left[ -L_D^{-2} \gamma_0 \left[ 1 - (r/r_0)^2/2 \right] + \frac{\gamma_1}{\kappa_1^2 + L_D^{-2}} J_1(\kappa_1 r/r_0) \cos 2\theta \right].
\]

(5)

The constant \( \gamma_0 \) allows independent control of the axisymmetric basic-state flow along the vortex edge, while \( \gamma_1 \) controls the wave-number-\( n \) azimuthal variations in that flow. The resulting basic states consist of regions of jet and diffuent flow that qualitatively resemble the flow in the extratropical upper troposphere.

Perturbations to this flow evolve according to

\[
\partial_t q + \partial_r(\psi, q) + \partial_{\theta}(\psi, Q) + \partial_{r\theta}(\psi, q) = 0,
\]

(6)

where \( \psi \) and \( q = \nabla^2 \psi - L_D^{-2} \psi \) are the perturbation stream function and PV, respectively, \( \partial_t \) (for example) denotes \( \partial/\partial t \), and \( \partial_{r\theta}(f, g) \equiv r^{-1} \partial_{\theta} f \partial_r g - r^{-1} \partial_{\theta} g \partial_r f \) is the two-dimensional Jacobian in cylindrical coordinates. Away from the vortex edge, the PV is constant and the disturbance PV vanishes, i.e.

\[
\nabla^2 \psi - L_D^{-2} \psi = 0.
\]

(7)
This fact can be exploited to derive the dynamical equation governing the time evolution of linear perturbations to the basic-state vortex patch, which can be written

\[
\frac{\partial}{\partial t}([\partial_r \psi]) + r_0^{-1} \frac{\partial}{\partial \theta}(U[\partial_r \psi]) + \Delta r_0^{-1} \partial_\theta \psi = 0. \tag{8}
\]

Here \( U = U(\theta) \) is the basic-state flow along the vortex edge and the square brackets denote the jump in the respective quantities across that edge. Interested readers are referred to Swanson et al. (1997) for an explicit derivation of this equation; the derivation for the equivalent barotropic case is identical to that for the barotropic case studied therein. The formulation of the problem is completed by requiring the perturbation stream function to be continuous across the edge and at \( r = 0 \), and to vanish as \( |r| \to \infty \).

For reference, as noted by Swanson et al. (1997), an infinitesimal displacement of the vortex edge, \( \eta \), normal to its basic-state position is related to the stream function by

\[
\eta = -\frac{[\partial_r \psi]}{\Delta}. \tag{9}
\]

Relating \( \Delta \) to a dimensional time-scale is an important, but necessarily ambiguous task. On middleworld isentropic surfaces that cut the tropopause, a typical isentropic PV jump across the tropopause is about \( \Delta_{IPV} = 4 \) PVU, where \( 1 \) PVU \( \equiv 10^{-6} \) K kg\(^{-1}\)s\(^{-1}\) (Hoskins et al. 1985). Under typical quasi-geostrophic scaling, this isentropic PV jump is equivalent to the dimensional quasi-geostrophic PV jump \( \Delta^* \) by

\[
\Delta_{IPV} \sim -g \left( \frac{d \theta_{ref}}{d p} \right) \Delta^* \sim \rho^{-1} \left( \frac{d \theta_{ref}}{d z} \right) \Delta^* \sim 0.02 \Delta^* \text{m}^2\text{kg}^{-1}, \tag{10}
\]

where \( \rho \approx 0.3 \) kg m\(^{-3}\) and \( d \theta_{ref}/dz \sim 6 \) K km\(^{-1}\) have values characteristic of the respective quantities in the vicinity of the extratropical tropopause. This suggests a dimensional quasi-geostrophic PV jump \( \Delta^* \sim 2 \times 10^{-4} \) s\(^{-1}\), with characteristic time-scale \( \Delta^{*{-1}} \sim (1/20) \) days. Coupled with a dimensional deformation radius \( L_D^* \sim 700 \) km, the intrinsic wind along the vortex has a dimensional value \( \Delta^* L_D^* / 2 \sim 70 \) m s\(^{-1}\).

Given the well-known inability of quasi-geostrophic theory to treat the large jump in static stability across the tropopause that occurs on isentropic surfaces, this scaling only provides a loose guide to the dimensional time-scales for our theoretical system. This should be kept in mind in interpreting the results below.

3. LOCAL INSTABILITY TO ZONALLY VARYING FLOWS

The concept of local versus global linear instability to streamwise varying flows was introduced by Pierrehumbert (1984) under the context of preferred regions of synoptic eddy development in the extra-tropical troposphere, the so-called storm-track problem. The criteria he derived relates the local instability of a flow slowly varying in the streamwise direction to whether local cross-sections of that flow are absolutely unstable (Merkine 1977). When a flow is absolutely unstable, the tendency for an initially localized disturbance pulse to amplify at any given location outweighs its tendency to propagate downstream; at large times, this results in disturbance growth even in locations far upstream of the initial pulse. For flows that vary slowly in the streamwise direction, Pierrehumbert showed that the growth rate for local instabilities depends upon the absolute growth rate (in the sense of Merkine) at the point of maximum supercriticality. As such, local instabilities behave in a similar way to absolute instabilities, with the caveat that streamwise variations in basic-state flow can prevent
disturbance growth from encompassing the entire domain. The instability character of local modes contrasts markedly with that of global modes, which rely on periodic boundary conditions and whose growth rate typically depends upon a domain-averaged measure of supercriticality.

The extension of Pierrehumbert’s analysis to the system studied here is not straightforward, as in the WKB* limit of a streamwise-invariant flow there is formally no instability. Rather, pulse dynamics are dominated by wave propagation; note, however, that accumulation of wave activity coincident with vanishing group velocity can occur if the flow is streamwise varying (Swanson 2000). This absence of instability flow suggests that local instabilities to this flow will differ from previously studied local instabilities to geophysical flows.

To test for local instability, we solve the full streamwise-varying linear stability problem (8) by using the Fourier transform along with the radial structure of the eigenmodes to relate $\psi$ to $[\partial_r \psi]$, as well as to calculate the $\theta$ derivatives. To distinguish local from global modes of instability, we solve the full one-dimensional stability problem with Newtonian relaxation of the disturbance particle positions $\eta$ in a localized portion of the domain (i.e. a sponge) to prevent recirculation of disturbances. Use of a sponge in this manner is typical experimental practice for the study of local instabilities to more complicated flows (e.g. DelSole and Farrell 1994). Equivalently, one could consider pulse development in a very long domain; linear instability experiments of this nature (not shown) confirm the results discussed below.

Figure 1(b) shows the local-mode growth rates for a vortex of dimensional radius $r_0 = 4200$ km, where the flow along the vortex edge varies as

$$U = U_0 + \epsilon \cos 2\theta,$$

i.e. two jets and two weak-flow regions with progression around the vortex edge. Local-mode growth rates are typically 50% of their global-mode counterparts, shown in Fig. 1(a). Local modes associated with blocking arise from the indicated instability branch; the gravest branch corresponds to a large-scale oscillation of the entire vortex.

* Wentzel–Kramers–Brillouin approximation.
while higher-order branches equilibrate at low amplitudes (Swanson 2000). The local-mode growth rates are relevant on synoptic time-scales, as dimensional e-folding times of a week or less are found for realistic zonal variation in the flow along the vortex edge. However, it is vital to note that regardless of the magnitude of the growth rate, the in situ amplification associated with local instability suggests these modes will be relevant to the generation of finite amplitude disturbances.

The dashed lines in Fig. 1 divide regions in \((U_0, \epsilon)\) space where stationary-wave accumulation is (above) and is not (below) predicted to occur for WKB slowly varying flows. As shown by Swanson (2000), accumulation only occurs for this system if the basic-state flow along the vortex edge makes a transition from a value greater than to a value smaller than \(U = \Delta L_D/2\) (dimensionally 70 m s\(^{-1}\)). The uniquely WKB concept of accumulation is still relevant to this non-WKB situation, as stationary-wave accumulation is a necessary condition for local instability. While globally unstable modes exist outside the range where accumulation occurs (Fig. 1(a)), local instability evidently requires some measure of trapping of the wave energy in the localized region that cannot occur unless the accumulation criterion is satisfied.

Figure 2 shows that the local-mode structure for \((U_0, \epsilon) = (0.6, 0.28)\Delta L_D\) (dimensionally \((80, 40)\) m s\(^{-1}\)) consists of a single wavelength disturbance localized to the weak-flow region centred on \(3\pi/2r_0\). Curiously, the instability is oscillatory with a period of approximately 14 dimensional days. Its highly unusual phase structure contains two pseudo nodes, one coinciding with the location of weakest flow, and the other with the downstream linear-theory accumulation point where \(U = 70\) m s\(^{-1}\). There is an inherent upstream–downstream asymmetry in the mode, with larger perturbation particle position deviations on the downstream side. Finally, the disturbance radiates a certain portion of energy downstream into the sponge; presumably this ‘loss’ of energy to the sponge results in the smaller local-mode growth rates compared with their global-mode counterparts.
The spatial scale of this disturbance, i.e. one ridge and trough within the weak-flow region, is consistent with observed spatial scale of blocks in the atmosphere; coupled with the fact it is a local instability, it appears to be a viable candidate for a linear-theory blocking mechanism. However, to confirm this, we must examine the nonlinear equilibration of these local unstable modes.

4. NONLINEAR EQUILIBRATION AND BLOCKING PARADIGMS

To study the nonlinear equilibration behaviour of these local instabilities, the numerical method of contour dynamics (CD) is used to obtain high-resolution solutions of (6) (Dritschel 1988, 1989). This method has been recently applied in a similar context by Polvani and Plumb (1992), Waugh et al. (1994), Nakamura and Plumb (1994) and Pieters and Waugh (1997) to study wave propagation and breaking on similar low-order contour models, and has been shown to reproduce some aspects of wave breaking in the atmosphere.

In CD, the velocity at a point \( x \) is given by

\[
u(x) = -\pi \sum_{j=1}^{N} \Delta_j \int_{C_j} G(|x - x_j|) \, d\mathbf{x}_j + \mathbf{u}_f(x),\]

where

\[G(r) = -K_0(r/L_D)\]

is the Green’s function for the modified Helmholtz equation in an unbounded domain, \( \mathbf{u}_f \) is the velocity calculated from the topographic forcing stream function (5), and \( C_j \) is the \( j \)th PV contour. The material conservation of PV ensures that it will remain piecewise constant, and the subsequent evolution of the contour is thus completely determined by the advection of the contours. The computational details of this procedure follow Dritschel (1988, 1989): briefly, each contour is numerically represented by a series of computational nodes that are advected by the velocity field (12). To preserve the resolution of the calculation, the positions of these nodes are continually adjusted, with nodes added in regions of high curvature. The algorithm used herein has the ability to perform surgery to eliminate filamentary structures; for longer time-scale integrations, surgery is required to keep the computational requirements reasonable. The equations of motion are solved in a domain that is unbounded in both \( x \) and \( y \), and the choice of parameters regarding node resolution, etc. are identical to those of Polvani and Plumb (1992). The vortex in the simulations shown later has a radius \( r_0 = 6L_D = 4200 \, \text{km} \), and the parameters \( \gamma_0 \) and \( \gamma_1 \) in (5) are adjusted so that the flow along the vortex edge has the structure \( U = U_0 + \epsilon \cos 2\theta \), where \( (U_0, \epsilon) = (0.6, 0.28)\Delta L_D \) (dimensionally (80, 40) \( \text{m s}^{-1} \)) as in the linear calculations of Fig. 2.

As the disturbance amplitude increases, nonlinear effects become increasingly important, and the equilibration of this oscillatory local instability leads to blocking formation. However, two qualitatively distinct flavours of blocking onsets emerge during this equilibration. Figure 3 shows the time evolution of an anticyclonic-type onset. Six dimensional days before blocking onset, the disturbance consists of a single ridge displaced slightly upstream from the region of weakest flow, located at the bottom of the vortex in the figure. That ridge slowly propagates upstream as it amplifies, ultimately spawning a substantial trough downstream three days before blocking onset. Note that, at this time, the disturbance resembles a meridionally propagating Rossby wave train, despite the obvious lack of a non-singular meridional PV gradient. After that point in
time, both the PV field and stream-function anomalies exhibit distinctive anticyclonic evolution that commences with the intrusion of low-PV air into the diffusent region. The resulting low-PV centre travels towards the centre of the unperturbed vortex, until it reaches the fringe of the weak-flow region in the vicinity of the vortex centre at saturation time. Meanwhile, a tongue of high-PV air intrudes below the blocking anticyclone. The mature block has a classical dipole (modon)-like structure, with well-defined anticyclonic and cyclonic centres (McWilliams 1980).

Anticyclonic movement of the low-PV centre has been observed in isentropic PV maps during a blocking episode over the North Sea (Shutts 1986), and was shown to be a characteristic of blocking evolution over Europe by Nakamura et al. (1997) and in the Euro-Atlantic region by Michelangeli and Vautard (1998). This rotation is consistent with the correlation between the PV and the radial wind changing sign during the blocking evolution; since this correlation is equivalent to the divergence of the extended Eliassen–Palm (EP) flux (Hoskins et al. 1983; Trenberth 1986), it is tempting to interpret the change in sign of this correlation as the convergence of wave-activity density flux associated with Rossby wave propagation into the amplifying blocking ridge. However, within the context of this idealized model simulation, there are no upstream disturbances and the amplification of the ridge only occurs through local instability. This highlights a problem with the EP flux as a diagnostic; the EP wave activity is not a conserved
quantity for streamwise-varying flows, e.g. Swanson et al. (1997). This is apparent in this context, as the equilibrating instability ‘generates’ substantial EP wave activity, or, more properly, following McIntyre and Shepherd (1987), pseudo-momentum.

Contrasting time evolution is observed for cyclonic-type blocking onset (Fig. 4). Six-dimensional days before onset, the disturbance has the form of a weak, streamwise extended trough displaced slightly upstream from the region of weakest flow. That trough remains in place and amplifies significantly, ultimately spawning a ridge slightly downstream of the region of weakest flow. This ridge amplifies rather slowly, ultimately pinching off and forming a mature block at the onset. Only very weak cyclonic evolution of the ridge is observed, in contrast with the anticyclonic case above where the blocking ridge and associated trough literally rotate about each other. In fact, the PV-radial wind correlation in the vicinity of the amplifying blocking ridge is small, consistent with the slower overall development of the cyclonic block compared to the anticyclonic block. Curiously, there appears to be a hint of downstream radiation, similar to that noted by Nakamura et al. (1997) for the Pacific blocking composites, witnessed by the weak trough downstream of the blocking ridge at the blocking onset. The mature block has the classical $\Omega$ structure, consisting of a well-defined anticyclone accompanied by a strong trough immediately upstream.

The resemblance of these anticyclonic and cyclonic blocking paradigms to the European and Pacific blocking composites of Nakamura et al. (1997) is uncanny, and lends credence to the possibility that an underlying local instability of the streamwise-varying flow similar to that described here plays an important role in overall structural
evolution of blocks. The intrinsic point that these simulations highlight is that the two
distinct flavours of blocking onset result from the same underlying unstable mode; the
distinction between the two flavours lies solely in the phase of the mode when nonlinear
effects begin to dominate.

Of course, it must be recognized that interactions with transients will be important
to both blocking formation and maintenance. In the next section, we examine several
experiments with active short-wave transient activity to assess how the presence of such
transients affects blocking onset and longevity.

5. EFFECTS OF SYNOPSIS-SCALE TRANSIENTS

From the larger perspective of constructing a ‘theory’ of blocking transitions, a
precise understanding of the role of synoptic-scale eddies in these transitions still re-
mains elusive. On one hand, there is a tendency to associate blocks with ‘exceptional’
individual synoptic eddies, an idea, epitomized by the result of Oortwijn and Barkemei-
jer (1995), that appropriately configured small-amplitude perturbations can be added to
virtually any flow and force a blocking transition. On the other hand, observational stud-
ies and simple numerical models point to the cumulative action of groups of synoptic
eddies, i.e. cyclone families, and their associated downgradient PV fluxes in blocking
transitions (Nakamura and Wallace 1990; 1993). These results suggest that the synoptic
eddies somehow act in concert to precondition the flow in some sense for the blocking
onset.

Within the context of the linear initial-value problem, the analysis of finite time-
period optimal disturbances initiated by Farrell (1989) provides a concrete tool with
which to examine super-normal growth that one might associate with rapid blocking
onsets. This approach is a straightforward extension of the linear stability analysis
outlined above. Suppose we schematically write that stability problem as

$$\partial_t \eta = L \eta,$$  \hspace{1cm} (14)

where \(L\) contains all relevant linear dynamics in (8), including the dissipation used to
localize the modes. For any time interval \(\delta t\), this system has the formal solution

$$\eta(\delta t) = \eta_0 \exp(\text{L}\delta t),$$  \hspace{1cm} (15)

where \(\eta_0\) is the disturbance structure at the initial time. As shown by Farrell, the
disturbance that has grown fastest over the interval \((0, \delta t)\) is the leading eigenmode
of the operator

$$M = \{\exp(\text{L}\delta t)|\exp(\text{L}\delta t)\}^T,$$  \hspace{1cm} (16)

while the initial disturbance structure that leads to that most rapidly growing disturbance
is the leading eigenmode of the operator

$$M^T = \{\exp(\text{L}\delta t)|^T\exp(\text{L}\delta t)\},$$  \hspace{1cm} (17)

where \[..|..|T\] denotes Hermitian transpose.

Figures 5(a) and (b) show the disturbance structures that respectively will and have
grown most rapidly over the optimization time intervals 1/2, 3, and 5 dimensional days,
respectively. In all cases, the evolved structures are localized to the weak-flow region,
and qualitatively resemble the linear-theory local mode of Fig. 2. The spatial scale of the
evolved disturbances (Fig. 5(b)) increases as the optimization time becomes larger; to
where the optimized six-day evolved structure is nearly indistinguishable from the local
mode itself. The phase selection is such that a full wavelength disturbance within the weak-flow region is preferred; significantly, this is the optimal configuration to generate a block. The maximum amplification curve associated with these disturbances is shown in Fig. 6; disturbance amplification is consistently much larger than the local-modal growth rate even for optimization periods greater than five days. As such, nonlinear wave–wave interactions exciting these optimal structures in principle should inspire blocking onsets on time-scales much shorter than the one week time-scale suggested by local instability normal-mode growth rates. Given the strong resemblance between the evolved optimal modes and the local modes, the nonlinear equilibration of optimal disturbance should resemble that of the local modes.

Curiously, the disturbance structures that will grow most rapidly (Fig. 5(a)) have larger spatial scales than their ‘evolved’ counterparts. This contrasts sharply with typical
Figure 7. Time slices of the vortex in the presence of a high-frequency transient wavemaker located at the left-hand side of the vortex. The dashed contour is the unperturbed vortex, while the solid contours and shading indicate the time evolution of the experiment, high-PV air originally within the vortex being shaded.

optimal-mode structures on geophysical flows, which are typically characterized by a substantial upscale cascade in their spatial structure as they mature (Farrell 1989). This feature, which is quite independent of the norm used to measure disturbance growth, highlights the radically different wave-accumulation mechanism underlying this instability, here compared with traditional critical layer-driven barotropic instability. As shown in Fig. 5(a), rapid growth is associated with the consistent displacement of the vortex edge upstream of the weak-flow region.

Some perspective on these rapidly amplifying modes can be obtained by examining nonlinear initial-value-problem experiments where short-wavelength transients are explicitly forced by a wavemaker at the upstream end of the weak-flow region. These experiments reveal that the short waves need not break in the weak-flow region for a blocking flow to be strongly forced; evidently, the downgradient PV flux associated with the increase in particle position amplitudes as the short waves propagate into the diffuent flow region (Swanson et al. 1997) is sufficient to inspire blocking transitions. Further, cyclonic-type blocking transitions appear to be preferred, both as an initial response to transient activity in the weak-flow region and as persistent features in longer-time-scale integrations, at least for the topography configuration used herein. Figure 7 shows a time series from one particular experiment. The wavemaker oscillates with a period of three dimensional days and generates synoptic time-scale transients on the left-hand side of the vortex; disturbances propagating out of the weak-flow region are absorbed in a sponge on the top of the vortex. This experiment shows the rapid formation of a cyclonic block between day $-3$ and the blocking onset associated with transient-eddy activity in the weak-flow region, and the subsequent reinforcement of that block between days $+1$ and $+2$ by a mobile cyclone that sweeps low-PV air into the blocking region itself. In this situation, the finite amplitude transients displace the basic-state contour outwards upstream of the weak-flow region (located at the bottom of the Fig. 7) at day $-3$. This outward push evidently excites an optimal mode with $\eta$ positive (inward) at the downstream end of the weak-flow region, the phase that favours cyclonic blocking onset.
6. Discussion and conclusions

The association of blocking transitions with local linear instabilities of the underlying flow as presented here has a number of conceptual advantages. In part, this is because any organizing principle behind blocking that rises beyond the intrinsically turbulent soup of individual synoptic eddies invariably present during blocking transitions is philosophically appealing, as it can (and should) be observationally tested. Ideally, such a test would take the guise of a ‘PV thinking’ approach (Hoskins et al. 1985), focusing on the dynamic tropopause in the period immediately preceding major blocking transitions, to determine the extent to which the large-scale flow differs from climatological conditions. Such a test, coupled with further theoretical work on the extension of the instability mechanism herein to more realistic models, will determine the ultimate value of this particular theory for blocking transitions.

There are several important limits to this model that must be emphasized. Perhaps the greatest shortcoming is the lack of the $\beta$ effect and consequent suppression of meridional radiation of Rossby waves. In the most extreme case, one might think that inclusion of $\beta$ may lead to intense meridional radiation of Rossby waves that would inhibit stationary-wave accumulation along the tropopause, and, as such, suppress the local instability. However, Rivest et al. (1992) argue that while the presence of $\beta$ does lead to the decay of tropopause edge waves, that decay will not exceed an exponential factor in a week, i.e. is at most similar in magnitude to the growth-rate time-scales of the instability studied herein. Further, they argue that, in general, decay rates will be much smaller than this, since observations of tropospheric meridional gradients of isentropic PV reveal those gradients to be very small (Hoskins et al. 1985). Hence, the existence of stationary-wave accumulation and local instability comes down to a question of time-scales, specifically whether the instability can generate stationary-wave activity faster than it leaks out of the local wave guide due to meridional propagation, which presumably depends strongly on the underlying meteorological situation.

In addition to concerns about the lack of $\beta$, of course, there remains the intrinsic limit imposed by quasi-geostrophic theory. In particular, as indicated in section 2, the use of a single deformation radius $L_\theta$ is intrinsically problematic, given the sharp contrast in static stability between the troposphere and stratosphere. As such, this instability must be pursued in more realistic models of the atmosphere, models that can more properly treat the contrast in static stability across the tropopause.

Even given these caveats, the level of detail that this simple system shares with observed blocking onsets is encouraging. The clear emergence of the cyclonic and anticyclonic paradigms for blocking onsets in the equilibration of two distinct phases of a single underlying local instability is particularly satisfying, as it suggests a single inclusive picture of blocking phenomenology may lie within our grasp, despite the obvious and important role of nonlinearity in such transitions. The potential of interpreting transients in the light of their kinematic group effect upon upper-tropospheric PV jumps and resultant excitation of optimal blocking-type modes is intriguing, as it expands inquiry in a new and interesting direction beyond the traditional focus on upper-tropospheric transient-eddy PV fluxes that has dominated thinking regarding the role of such transients in blocking transitions during the past two decades.

Above all, the fact that simple models can provide insight into blocking transitions, and more generally, into the generation of extratropical low-frequency variability is encouraging. For a healthy understanding of dynamical phenomena on these timescales, theory, simulation, and diagnostics must all advance together. The results of this study provide theoretical support for large-scale dynamical processes playing a vital
role in blocking transitions; the extent of that role must be better understood before the theory of blocking can be considered complete.

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