Deterministic prediction of the ensemble variance for a barotropic vorticity-equation model

By LARS PETER RIISHØJGAARD*

University of Maryland, USA

(Received 30 December 1999; revised 24 November 2000)

SUMMARY

The problem of estimating the error variance of a meteorological forecast is important both for data assimilation purposes and in order to increase the value of the forecast to the end user. Current data assimilation systems have no—or at best a severely limited—capability to estimate this quantity. In principle, the prognostic equation for the forecast error covariance is known from the Kalman filter. However, in its full form this equation is prohibitively expensive to implement and solve. In this article, the possibility of evolving a variance-only estimate is explored. It is shown that for a barotropic vorticity-equation model, an approximate equation for the error variance evolution can be written in a closed form under certain assumptions. The cost of solving this equation is comparable to that of solving the model equation itself. Through a demonstration experiment, it is shown that the approximate equation tracks the actual variance of a model run—represented by the variance over a randomly perturbed ensemble of forecasts generated by the model—remarkably well, qualitatively as well as quantitatively.

KEYWORDS: Data assimilation Forecast error variance Kalman filter

1. INTRODUCTION

In modern numerical weather prediction (NWP), providing an estimate of the uncertainty of a meteorological forecast is often considered nearly as important as providing the actual forecast itself. There are two reasons for this: one is that an uncertainty estimate significantly increases the value of the forecast to the users, the other is that the analysis algorithms of modern data assimilation systems rely on input information about the expected error statistics of the forecast as well as of the observations. Although these two areas of application share the need for error information, the exact nature of the information required differs between them. For the forecast user, an estimate of the forecast error variance—i.e. a measure of how large the error of a given meteorological quantity at a given point is likely to be—will normally suffice. For a data assimilation system, information is needed also about the forecast error correlations—i.e. the estimated spatial characteristics of the forecast error, and the expected relationship between forecast errors for different quantities.

In principle, the equation for the evolution of the forecast error covariance is known from the Kalman filter (e.g. Cohn 1997). However, solving this equation numerically in the context of a meteorological data assimilation system remains unfeasible—apart from the fact that no known strategy exists for specifying the staggering amount of information needed to initialize the covariance matrix at a given time (Dee 1991). For modern global models with a dimension of the state vector in the range of $10^7$, the forecast error covariance matrix contains on the order of $10^{14}$ elements, all presumably depending on the state of the model. In actual implementations, this matrix is often modelled based on time-averaged lagged forecast differences, involving a few parameters that are tuned so as to give a reasonable overall performance of the data assimilation system (e.g. Rabier et al. 1998). The propagation of forecast errors in a barotropic vorticity-equation model was studied by Bouttier (1993). He noted that forecast errors tend to be advected with the flow, and that rapid growth occurs in narrowly defined regions that appear to be.

* Corresponding address, present affiliation: Data Assimilation Office, Goddard Space Flight Center (Code 910.3), Greenbelt, MD 20771, USA.

1761
tied to features of the state itself. At the time of writing no operational NWP system explicitly evolves the forecast error covariance along with the forecast state itself.

Several of the possible strategies for evolving the forecast error covariance in time are based on treating variances and correlations separately. Such an approach does not imply any restriction on the form of the covariances, and it has the advantage that certain desirable features of the variance and/or correlation structures may be easier to impose in this framework. Some progress has been made on specifying state-dependent forecast error correlations, either through a coordinate transformation (e.g. Desroziers 1997), or through a direct local state-dependent modification of a standard isotropic correlation function (Riishøjgaard 1998). Both of these approaches leave the specification of the forecast error variance open. One known strategy for estimating this variance is to randomly perturb the initial state, run an ensemble of forecasts, and calculate the variance over this ensemble of forecasts. This is known as ensemble—or Monte Carlo—forecasting, and it can be used not only to forecast the variance or the spread, but also the ensemble mean, assuming that this would be a more likely forecast than any single ensemble member (e.g. Cohn 1997).

In the context of stochastic-dynamic prediction, Thompson (1986) proposed a set of equations to predict the evolution of the mean and the error variance of an ensemble of forecasts with a barotropic vorticity-equation model. The necessary closure to the variance prediction in particular was achieved through assumptions about the spatial structure of the error field at the initial time. The paper offers useful insight into the behaviour of the variance evolution, as well as an interpretation of the terms of the proposed closure equation. It does not include any experimental results, and the approach thus appears to have remained untested.

The present paper presents a strategy for predicting the forecast error variance of the same model system as the one discussed by Thompson. The approach here is somewhat different though. The point of departure is the Lagrangian character of the error variance for linear systems (Cohn 1993), and the main goal of the present paper is to derive and test a simple additional term in the prognostic equation for the error variance, that can capture the salient features of the error growth for a typical weakly nonlinear meteorologically relevant model. The approach taken here is thus less general than that of Thompson—in the sense that it focuses on growing errors exclusively, at the expense of basic conservation properties. On the other hand, it actually appears to be easier to generalize—in the sense that the approach carries over essentially as is, to a full primitive-equation model using, say, potential vorticity or potential temperature in lieu of vorticity.

The main result of the paper is a separate approximate stand-alone partial differential equation for predicting the evolution of the forecast error variance. Together with the companion article of Riishøjgaard (2000)—in which an algorithm for estimating the analysis error variance is proposed—this would provide an overall strategy for the forward propagation of error variance in a meteorological data assimilation system. When this is, in turn, coupled with the flow-dependent forecast error correlation model of Riishøjgaard (1998), the full error covariance information is evolved in a manner that is intimately coupled to the state of the atmosphere. This is similar in spirit to what is achieved by Kalman filtering; however, certain aspects of the approach are based on intuition and experimentation rather than on absolute rigour, and the accuracy of the variance estimate may, therefore, well be lower. On the other hand, the computational cost involved is several orders of magnitude smaller than that of the full Kalman filter, and it is likely that a full implementation, in an operational data assimilation system, of the approach described, would be within reach of current computers.
However, in this article we shall limit ourselves to showing that predicting the evolution of the forecast error variance is indeed possible in a barotropic vorticity-equation model framework. From a discrete Kalman filter viewpoint, this is equivalent to predicting the evolution of the diagonal of the forecast error covariance matrix separately from the rest of the matrix. From the continuum point of view, put forward by Cohn (1993), we are seeking an approximate closure to the variance prediction problem leading to an approximate partial differential equation on $R^2$, rather than attempting to solve the full covariance equation on $R^4$, where $R$ is the set of real numbers.

The benchmark for success of the variance prediction is the actual variance of the ensemble of forecasts carried out with the model from a set of randomly perturbed initial conditions.

2. Theory

Advection of (potential) vorticity is a governing process for the large-scale atmospheric dynamics, and it is well known that a barotropic vorticity-equation model can simulate the large-scale 500 hPa height field with some degree of success (see e.g. Bouttier 1993). The meteorological relevance of it, combined with the relatively modest computational cost and the freedom from having to consider diabatic effects, makes the barotropic vorticity-equation model very well suited as a test bench for the variance prediction problem. The numerical implementation used here is described by Risheiggaard et al. (1998); it is semi-Lagrangian, the horizontal resolution is $2.8^\circ \times 2.8^\circ$, and the field at the new time-level is found through bicubic interpolation to the departure point.

The basic equation of the model is

$$\frac{d\eta}{dt} = \frac{\partial \xi}{\partial t} + \mathbf{u} \cdot \nabla \eta = 0,$$

where $\mathbf{u}$ is the velocity, and $\eta$ is the absolute vorticity, as defined as the sum of the planetary vorticity $f$ and the relative vorticity $\xi = k \cdot \nabla \times \mathbf{u}$ where $k$ is the unit vector. The forecast error of $\delta \xi$ of Eq. (1) evolves according to the tangent-linear equation:

$$\frac{\partial \delta \xi}{\partial t} + \mathbf{u} \cdot \nabla \delta \xi + \delta \mathbf{u} \cdot \nabla \eta = 0,$$

where $\delta \mathbf{u} = k \times \nabla (\Delta^{-1} \xi)$. Note that we do not include a model error term on the right-hand side of Eq. (2). The reason for this is that we are concerned with predicting the evolution in time of the spread over an ensemble of forecasts, and since no link is made to actual atmospheric states through validation against observations, a model error is not naturally defined for these experiments. In the absence of model error, one may attempt to write down an equation for the covariance between the forecast errors at the horizontal locations $x_1$ and $x_2$, $P_{x_1 x_2} = \overline{\delta \xi(x_1) \delta \xi(x_2)}$, where the overbar denotes an average taken over a hypothetical ensemble of forecasts, of which the actual forecast is a member, by following the method of Cohn (1993):

$$\frac{\partial P_{x_1 x_2}}{\partial t} + \mathbf{u}(x_1) \cdot \nabla_1 P_{x_1 x_2} + \mathbf{u}(x_2) \cdot \nabla_2 P_{x_1 x_2} + \frac{\partial \delta \xi_1}{\partial t} \delta \mathbf{u}(x_1) \cdot \nabla_1 \eta(x_1) + \frac{\partial \delta \xi_2}{\partial t} \delta \mathbf{u}(x_2) \cdot \nabla_2 \eta(x_2) = 0.$$

Here $\nabla_1$ is the gradient with respect to the independent spatial variable at $x_1$. The space on which this equation is defined is the outer product of the model space with itself. In addition to the computational burden that this implies, the appearance of both vorticity
and velocity errors in the last two terms on the left-hand side of Eq. (3) poses a closure problem.

By setting \( x_1 = x_2 \) (and dropping \( x \) from the notation), we get the following equation for the forecast error variance \( P_{xx} = \delta \xi(x)^2 \equiv \sigma_f^2 \) (cf. Thompson 1986, Eq. (3a)):

\[
\frac{\partial \sigma_f^2}{\partial t} + \mathbf{u} \cdot \nabla \sigma_f^2 = -2\delta \xi \delta \mathbf{u} \cdot \nabla \eta.
\]

(4)

This equation is defined on the model space itself, and a numerical solution of it is thus within reach from a computational point of view. However, the closure problem of estimating the last term \( \Lambda \equiv -2\delta \xi \delta \mathbf{u} \cdot \nabla \eta \) remains to be addressed. This term can be positive, negative, or zero, depending on the sign of \( \delta \xi \) and the relative orientations of \( \delta \mathbf{u} \) and \( \nabla \eta \). For \( \Lambda = 0 \), the vorticity error variance is simply advected along with the vorticity itself. For \( \Lambda > 0 \), the material derivative is positive, corresponding to error growth along the trajectory, whereas for \( \Lambda < 0 \), the error is decaying.

Even though \( \Lambda \) can have any sign, arguably the most important forecast error variance evolution feature described by this term is the sudden growth of variance due to hydrodynamic instability. From a data assimilation point of view, it is preferable to overestimate rather than underestimate the forecast error variance, and a possible slow decay of the error, which is not captured by the variance prediction scheme, may therefore prove to be relatively unproblematic. In assessing forecast height errors from a similar model, Bouttier (1993) remarks that a decrease in error with time is rare and seems to be insignificant. With this in mind, we now proceed to seek a non-negative estimate of the material derivative, \( \Lambda \geq 0 \).

Note that in relative terms, the error growth along a material trajectory is most important when the second term on the left-hand side of Eq. (2) is significantly smaller than the third, that is when

\[
\frac{\partial \delta \xi}{\partial t} \approx -\delta \mathbf{u} \cdot \nabla \eta.
\]

(5)

In order for this approximate relation to hold, at least one of the following statements must be true:

(i) the wind speed is small (\( \| \mathbf{u} \| \approx 0 \)),
(ii) the spatial scale of the vorticity error is large (\( \| \nabla \delta \xi \| \approx 0 \)),
(iii) the gradient of the vorticity error is orthogonal to the flow.

Of these conditions, (i) can be verified explicitly within the forecast/data assimilation system, since \( \mathbf{u} \) is a prognostic model variable, whereas the vorticity error \( \delta \xi \) needed for (ii) and (iii) is not. However, since the aim here is to construct a system that predicts also the vorticity error variance \( \delta \xi^2 \), some information about the validity of (ii) and (iii) will be available in the system. We note that (ii) was identified also by Thompson (1986) as a condition that would tend to favour error growth.

Assuming now that \( \delta \mathbf{u} \) is constant over a short interval in time, a solution to Eq. (5) is

\[
\delta \xi \approx \delta \xi_0 - \delta \mathbf{u} \cdot \nabla \eta t.
\]

(6)

This assumption is valid only to the extent that the vorticity error locally changes more rapidly than the velocity error. Since vorticity is a higher-order derivative than velocity and thus accordingly exhibits a higher degree of small-scale structure, this may be a reasonable assumption. Note that in the limit of a spatially homogeneous velocity
error—a situation that by its very nature can only be transient—Eq. (6) states that the local vorticity error is simply due to a displacement \( \delta \mathbf{r} = \delta \mathbf{u} \mathbf{r} \) of the true vorticity field. In accordance with this we will use the term 'displacement error' to designate the error component described by the second term on the right-hand side of Eq. (6).

This term describes the local, non-advective growth of the vorticity error over a short time-span of length \( \tau \) for which one or more of the conditions (i) through (iii) are approximately fulfilled. Beyond this time Eq. (6) ceases to be meaningful. However, the growth episode associated with the flow configuration described by this equation will have modified the vorticity error, so that the new error is

\[
\hat{\delta} \zeta = \delta \zeta_0 - \delta \mathbf{u} \cdot \nabla \eta \tau.
\] (7)

Assuming that this equation describes the most important aspects of the growth, we now introduce the vorticity error of Eq. (7) in the expression for \( \Lambda \) in order to close Eq. (4):

\[
\Lambda = -2(\delta \zeta_0 - \delta \mathbf{u} \cdot \nabla \eta \tau) \delta \mathbf{u} \cdot \nabla \eta.
\] (8)

It is reasonable to assume that the non-growing, advective component of the error, \( \delta \zeta_0 \), is uncorrelated with the occurrence of flow configurations that lead to error growth, i.e. that \( \delta \zeta_0 \delta \mathbf{u} \cdot \nabla \eta = 0 \). Under this assumption, we have

\[
\Lambda = 2(\delta \mathbf{u} \cdot \nabla \eta)^2 \tau.
\] (9)

It may appear that by introducing a vorticity error of the form of Eq. (7) in the expression for \( \Lambda \), we have effectively assumed global validity of Eq. (5), even though we know this relation to be valid only locally. However, the parameter \( \tau \)—the time over which Eq. (5) approximately holds—appearing in Eq. (9) can act as a local scaling of this term. Bearing in mind conditions (i) and (ii) from the list above, one could for instance define

\[
\tau = L_\xi U^{-1},
\] (10)

where \( L_\xi \) is the length-scale of the vorticity error, and \( U \) is the typical advection speed. By letting \( \tau \) be defined in terms of local rather than global characteristics of the fields, the non-advective error growth described by Eq. (9) can thus be controlled by the local advective time-scale of the error. The actual expression for \( \tau \) used for the experiments to be described later is slightly different from Eq. (10), and will be given in the following section.

Next, we assume that the relative orientations of \( \delta \mathbf{u} \) and \( \nabla \eta \) are uncorrelated, so that

\[
(\delta \mathbf{u} \cdot \nabla \eta)^2 = \alpha \| \delta \mathbf{u} \|^2 \| \nabla \eta \|^2,
\] (11)

where \( \alpha \) is a constant. The final step in deriving the parametrization is to link the size of the velocity error to the size of the vorticity error. Note that \( \xi = \mathbf{k} \cdot \nabla \times \mathbf{u} \), and that therefore \( \delta \xi = \mathbf{k} \cdot \nabla \times \delta \mathbf{u} \). It thus appears reasonable to set

\[
\| \delta \mathbf{u} \|^2 = L_U^2 (\delta \xi)^2,
\] (12)

with \( L_U \) being a typical length-scale for the velocity field. From Eqs. (9), (11), and (12), the final closure for the variance equation (Eq. (4)) becomes

\[
\frac{\partial \delta \xi^2}{\partial t} + \mathbf{u}(x_1) \cdot \nabla \delta \xi^2 = 2\alpha L_U^2 \delta \xi^2 \| \nabla \eta \|^2 \tau.
\] (13)

Before we turn to the test results obtained with this equation, we briefly restate and comment on the basic assumptions that were used for deriving it:
substantial non-advective error growth implies that the advective time-scale for the vorticity error is large, i.e. that $\| u \cdot \nabla \delta \zeta \| \gg \| \delta u \cdot \nabla \eta \| $; this is used to get Eqs. (5) and (6),

(2) the orientations of $\delta u$ and $\nabla \eta$ are uncorrelated (Eq. (11)),

(3) the velocity error variance can be modelled in terms of the vorticity error variance (Eq. (12)).

Concerning (1), it is not clear that this is neither necessary nor sufficient for error growth to occur, although it is of course absolutely essential in the derivation of Eq. (13). One could, for instance, imagine a situation where the second and third term of Eq. (2) were both significant and roughly equal in magnitude, or a situation in which they would both vanish. Through the appearance of the local vorticity gradient and the variable $\tau$ in Eq. (13), the scheme does to some extent address these situations also, but the fact that they violate a basic assumption leading up to this equation means that its relevance to these contexts must be tested empirically.

Superficially, (2) may seem rather innocent in comparison, since it merely states that, on average, the correlation between a deterministic and a random variable is zero. Recall, however, that we are attempting to predict the random variable by a deterministic method, so in the present context we cannot a priori be sure that (2) is valid.

Assumption (3) basically states that the vorticity error is dominated by a particular region in spectral space. Since the basic goal of the scheme is to capture the contribution of synoptic-scale atmospheric dynamics to the error growth, this assumption is probably reasonable.

3. Experimental results

In this section we show results from experiments in which the parametrized variance equation (Eq. (13)) was used to predict the time evolution of the variance over an ensemble of forecasts. Only one example of the experiments carried out is described here. Other experiments showed essentially similar behaviour, and we therefore believe the results shown to be typical for this particular model. In order to set up the ensemble, the nonlinear model, Eq. (1), was integrated for 10 days from an initial state $\eta(t_0)$ being a January 500 hPa vorticity field from a three-dimensional climate-model simulation. The initial state is thus realistic, but generic. Over the course of the integration, the vorticity was written to file every $t_i, i = 1, \ldots, 10, t_i = t_0 + i \times 24$ h. From this model trajectory, all the possible combinations of differences were calculated

$$ \Delta \zeta_{ij} \equiv \zeta(t_i) - \zeta(t_j); \quad i = 1, \ldots, 10, \quad j = 1, \ldots, 10. $$

(14)

In order to generate a set of vorticity perturbations $\delta \zeta_{ij}$, each of these difference fields was then rescaled in order for its area-weighted $L^2$-norm—indicated by $\| \cdot \|$—to be 5% of the corresponding norm of the initial vorticity field:

$$ \| \delta \zeta_{ij} \| = 0.05 \times \frac{\| \Delta \zeta_{ij} \|}{\| \zeta(t_0) \|} \cdot \Delta \zeta_{ij}. $$

(15)

These perturbations were then added to the original initial state in order to generate a set of 90 perturbed initial states, from which an ensemble of 90 different 24-hour forecasts was carried out.

In Fig. 1, the ensemble mean relative vorticity is shown 6, 12, 18 and 24 hours into the simulations. The plots show the northern hemisphere north of 30° in polar stereographic projection. Regions of negative relative vorticity indicating cyclonic flow
are interspersed with regions of positive vorticity. In some cases the two regimes are separated by steep and fairly persistent gradients, e.g. in an elongated zone stretching from Iceland across northern Scandinavia into Siberia, and a similar zone extending from the Iberian peninsula out into the Atlantic Ocean.

In Fig. 2 the corresponding stream-function fields are shown with linearly spaced isolines. This is convenient for visualization of the flow, which here simply follows the isolines of the stream function with a speed that is proportional to the distance between them. It is interesting to note that of the vorticity-gradient features mentioned above, the one over northern Scandinavia is associated with a region of mostly moderate flow speeds, whereas the zone over the Iberian peninsula on the other hand is characterized by very large speeds.

In Fig. 3, the ensemble variance around the mean of the vorticity is shown. The isolines in this figure are logarithmically distributed, so that each additional line represents a doubling of the variance. As is evident from the amount of structure in the plots, the
Figure 2. As Fig. 1, but for the stream function. See text for further explanation.

Variance evolution is clearly non-uniform horizontally, which is at odds with the way the forecast error evolution is often prescribed for operational NWP systems. Aside from the amount of horizontal structure in the fields, it is also remarkable how explosive the error growth can be locally, even though there may be neighbouring regions where the error hardly grows at all over the 24-hour interval. The elongated vorticity-gradient feature over Scandinavia appears to be associated with very substantial error growth, whereas the corresponding feature over the Iberian peninsula only gives rise to modest growth. Other areas of error growth—e.g. over North America near the left edge of the plots, and over the Pacific near the top of the plots—also appear to be linked to gradients in the vorticity field.

There is clearly an advective component to the error evolution. However, there is also clear evidence of the phenomenon noted by Bouttier (1993) of forecast errors growing rapidly in narrow regions, whose exact location and extent both appear to be
state-dependent. The fact that the local gradient plays a role in determining the variance growth is to be expected from the discussion by Thompson (1986).

We now show an independent numerical prediction of the variance evolution depicted in Fig. 3, using the deterministic variance closure equation (Eq. 13) introduced in the previous section. The parameter $\tau$ of this equation—i.e. the length of the time interval over which Eq. (5) is a reasonable approximation—is parametrized on the local state of the model, so as to get an expression somewhat similar to the form of Eq. (10). First we define

$$L_\xi = \frac{|\delta \xi|}{\|\nabla \delta \xi\|}.$$  \hspace{1cm} (16)
Since \( \delta \xi \) is unknown, this definition is not immediately useful. However, using the chain rule \( \nabla (\delta \xi)^2 = 2 \delta \xi \nabla \delta \xi \), we have that

\[
L_{\xi} = \frac{2 \delta \xi^2}{\| \nabla \delta \xi^2 \|}.
\]  

(17)

Bearing in mind that \( \tau \) should be large when the scalar product \( \mathbf{u} \cdot \nabla \delta \xi \) is small (Eq. (5)), we now replace the product of vector norms \( \| \nabla \delta \xi^2 \| \| U \| \) in the denominator of Eq. (10) with the absolute value of the scalar product \( \mathbf{u} \cdot \nabla (\delta \xi)^2 \), so that we get

\[
\tau = \frac{\delta \xi^2}{| \mathbf{u} \cdot \nabla (\delta \xi)^2 |}.
\]  

(18)

Figure 4 shows the variance prediction using Eq. (13) with \( \alpha L_U^2 \) held constant at a value of \( 4 \times 10^{10} \) m², and \( \tau \) given by Eq. (18). The initial condition for \( (\delta \xi)^2 \) is the
ensemble variance at time $t = 0$, i.e. the variance of the set of initial perturbations, and the vorticity trajectory used for carrying it forward in time is a single member of the ensemble, namely the initial run used for generating the perturbations.

There is a great deal of similarity between Figs. 3 and 4, both in terms of the horizontal structure and in terms of the actual values. There are local discrepancies, and the ensemble variance fields generally appear to be somewhat noisier than the smoother fields generated with the forced advection Eq. (13). Since the former depicts the ensemble variance over an ensemble of supposedly randomly perturbed realizations, whereas the latter attempt to capture the essential features based on a single such realization, this sort of qualitative difference is to be expected. Overall the degree to which the error growth is captured by the simple Eq. (13) is remarkable.

It is worth emphasizing that the variance evolution tested here only contains one free parameter, $\alpha L_U^2$. The only ‘tuning’ involved in getting the results shown in Fig. 4 was thus carried out on this parameter.

4. DISCUSSION

The results shown in the previous section demonstrate the feasibility of predicting with very simple means the time evolution of the spread over an ensemble of randomly perturbed integrations of a barotropic vorticity-equation model. For the ensemble experiments shown here, the error growth over the integration period is highly inhomogeneous, and over certain areas also very rapid.

If the error growth of a real three-dimensional meteorological forecast exhibits this kind of inhomogeneity it would be highly desirable to capture it for the reasons given in the introduction to this article. At least intuitively, it appears likely that real forecast errors are indeed inhomogeneous, both horizontally and vertically. A typical weather map contains both relatively quiet regions and regions of rapidly developing and/or moving weather systems. It is reasonable to believe that the forecast error in the latter regions is both larger and more rapidly increasing than in the former. It is also reasonable to expect that a misplacement of the top of the boundary layer and/or the tropopause in a forecast would give rise to larger-than-normal forecast errors near these levels, because of the prevalence of strong vertical gradients in various quantities there.

Admittedly, the task of predicting the variance evolution for a full primitive-equation model is far more difficult than the task of doing it for a comparatively simple two-dimensional model like the one discussed here. However, the fundamental idea of a variance closure based on the actual prognostic equation of a Lagrangian variable—i.e. the reasoning leading from Eq. (1) to Eq. (2) and Eq. (5) to Eq. (13) can be expected to carry over to the general three-dimensional context of the primitive equations.

The obvious candidate variables to guide the variance evolution for a full NWP system are potential temperature and potential vorticity (PV). Of these, PV provides for the closest analogy to the experiments shown in this article, since it is one of the driving dynamic variables of the atmosphere. In reality it may not be the most appropriate choice, since the link between PV and observed variables such as wind components, heights, and temperature involves assumptions about the wind/mass balance, and solution of the associated balance equations. Potential temperature on the other hand is closely linked to traditionally analysed mass variables such as temperature, thickness, or height, and a prognostic variance scheme in this variable may therefore be easier to implement. Temperature error variances are straightforward to calculate from potential-temperature error variances. For integrated quantities such as heights, one would also need information about the vertical error correlation for potential temperature. It is worth mentioning
that a Lagrangian-based, anisotropic flow-dependent error correlation model is available for this quantity—a model that in conjunction with the dynamic variance evolution proposed here would provide a state-dependent evolution strategy for the full forecast error covariance matrix (Riishøjgaard 1998).

It is also worth noting that since potential temperature is conserved with the flow only under adiabatic conditions, the local non-advective tendencies provided by the model form a natural starting point for the parametrization of the model error due to uncertainties in the physics parametrizations, since this is the part of the model from which the tendencies themselves come.

5. SUMMARY AND CONCLUSIONS

A closure to the variance propagation problem for a barotropic vorticity-equation model has been proposed and tested. The closure amounts to parametrizing the nonlinear growth term for the vorticity error on the vorticity itself and its local gradient. The parametrization is easy and inexpensive to implement, and the computational cost of carrying the estimated error variance forward in time is comparable with that of solving the model equation itself.

In experiments where the variance thus predicted was compared with the actual variance of an ensemble of forecasts with the model, the parametrized equation was found to track the actual evolution of the ensemble variance with a great deal of realism.

In spite of the gross simplifications it involves (no vertical shear, no physics), the barotropic vorticity-equation model captures enough of the relevant large-scale dynamics to have some skill—albeit limited—in actual meteorological forecasting. It is a nonlinear model, and accordingly it depicts growth and decay of atmospheric waves in addition to mere advection. Because of this nonlinearity, the error-growth equation cannot be written in a closed form. It is, therefore, very encouraging that in spite of these difficulties, the error growth seems to be predictable with relatively simple means, as evidenced by the results presented here. The main ideas that are exploited for the simplified error evolution are that (i) over most of the domain, the error is dominated by advection, and (ii) locally, the error growth can be determined from the characteristics of the flow. This kind of Lagrangian reasoning has natural generalizations also to more realistic meteorological models. Work is currently underway towards testing whether the error in potential temperature or potential vorticity for a primitive-equation model can be simulated by an approach similar to the one described here.

ACKNOWLEDGEMENTS

The author wishes to thank his colleagues at the Goddard Space Flight Center, without whom this work would not have been possible. Thanks are due in particular to Dick Dee and Steve Cohn for fruitful discussions in the initial phase. Thanks also to the two anonymous reviewers for their comments and suggestions, and in particular to reviewer A for the reference to Thompson (1986).

REFERENCES

Bouttier, F. 1993 The dynamics of error covariances in a barotropic model. Tellus, 45A, 408–423


Dee, D.

Desroziers, G.


Riishøjgaard, L. P.

Riishøjgaard, L. P., Cohn, S. E., Li, Y. and Ménard, R.

Thompson, P. D.


