Possible links between cloud optical depth and effective radius in remote sensing observations

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(Received 17 October 2000; revised 7 June 2001)

SUMMARY

The theoretical relationship between cloud optical depth and effective radius is examined. The relationship between cloud optical depth and effective radius is shown to depend on (a) the relative variability of droplet concentration, cloud depth, mixing and precipitation, and (b) the correlation between the two most important parameters, namely cloud depth and droplet concentration. It is possible to obtain positive, negative or zero correlation between optical depth and effective radius for different values of (a) and (b). Of all these parameters, mixing appears to be the least important. If clouds are not affected by precipitation, negative correlations between optical depth and effective radius (the expected signature of the first indirect aerosol effect) can be caused by a small spread in cloud depths relative to that in droplet concentration, or (more importantly) by a positive correlation between droplet concentration and cloud depth.

Precipitation tends to reduce the correlation between optical depth and effective radius, because one of the main effects of precipitation is to increase the spread in droplet effective radius. If the effect of increases in aerosols is a reduction in precipitation, our results indicate that the correlation coefficient between optical depth and effective radius should increase, an effect that should be most clearly visible over the ocean. These findings illustrate the complexity of the physical processes that underpin the linkage of optical depth and effective radius.

KEYWORDS: Boundary layers Cloud microphysics Cloud optical properties Remote sensing Stratocumulus

1. INTRODUCTION

The radiative forcing due to the effects of anthropogenic aerosols on clouds—the indirect aerosol effect—is thought to be negative, but is highly uncertain. Two likely effects of aerosols on warm (liquid water) clouds have been identified. The first indirect effect refers to the radiative impact of a decrease in droplet effective radius that results from increases in aerosols (Twomey 1977). The second indirect effect refers to the radiative impact of a decrease in precipitation efficiency that results from increases in aerosols (Albrecht 1989). There is now strong observational support for the existence of the first indirect effect, but still only limited support for the second indirect effect (Penner et al. 2001).

Since the early demonstration by Twomey and Cocks (1982) of the possibility of simultaneously retrieving cloud optical depth and effective radius, there have been several attempts to obtain global coverage of these cloud optical properties using satellite data. The advantage of using satellite data is clear: they provide the opportunity to search for the signature of the indirect aerosol effect on a large-scale and thus supply the data necessary to track climate change and to validate climate models. The first large-scale analysis of satellite data was provided by Han et al. (1994), who gave broad confirmation of the idea that the effective radius is smaller in the northern hemisphere than in the southern hemisphere, and that the effective radius is larger over the oceans than over land.

Interestingly, the first attempt to correlate cloud optical depth and effective radius met with some unexpected results. The most obvious signature of the first indirect aerosol effect is that the correlation between optical depth and effective radius is expected to be negative. We will at times refer to this signature as the first indirect

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effect, even though, strictly speaking, the indirect effect refers to the radiative impact of these microphysical processes. Han et al. (1998), using a global satellite dataset, made a separation between clouds with large (>15) and small optical depth (<15). They concluded that for oceanic clouds with a small optical depth the correlation between optical depth and effective radius was positive, which they thought would probably be caused by high variations in cloud liquid water path (and hence) cloud depth. In all other cases the correlation was negative. Their conclusions are different from those of Brenguier et al. (2000) who showed from a more limited dataset obtained during the second Aerosol Characterization Experiment (ACE-2) that optical depth increased with effective radius for both unpolluted (i.e. oceanic origin) as well as polluted (continental origin) clouds. Austin et al. (1999) analysed four scenes of remote sensing data off the west coast of California, but only one showed in part the expected negative correlation, which further complicates the interpretation of the Han et al. (1998) results.

Lohmann et al. (2000) searched for confirmation of the Han et al. (1998) results in General Circulation Model (GCM) data, and suggested that precipitation affects the signatures of the correlation function. Unfortunately, at present, the comparison between the satellite results and the GCM data is fraught with difficulties. Due to their low vertical resolution, climate models cannot resolve the high variations in the vertical of effective radius that are commonly observed in aircraft in situ data, and which dominate the interpretation of remote sensing data. Another serious problem in comparing GCM and satellite data is the difference in horizontal resolution. A typical footprint of a satellite radiometer is 1 × 1 km, while for a GCM it is 100 × 100 km, two orders of magnitude difference in scale.

The present study is motivated by the need to shed further light on the expected link between cloud optical depth and effective radius, a link that will become more sharply defined as the research community continues to tap into the increasing reservoir of remote sensing data. This link is studied through theory, the roots of which were provided by Boers and Mitchell (1994) and Fairall et al. (1990), and were expanded by Brenguier et al. (2000). It will become apparent that, as expected, the four principal factors that determine the correlation between optical depth and effective radius are droplet concentration, cloud depth, mixing and precipitation. However, we find that a principal component in determining the correlation between optical depth and effective radius is the internal correlation between the most important controlling parameters, namely cloud depth and droplet concentration. One further important influence is precipitation, which has the uniform effect of reducing the correlation between optical depth and effective radius. However, the overall conclusion is that the observed link between the optical depth and effective radius is primarily a reflection of the underlying dominating physical processes, one of which is the first indirect aerosol effect, and perhaps not even the most important one. We will confine ourselves to horizontal scales of less than 40 × 40 km because, as will be shown below, at larger scales synoptic influences impact the organization of cloud systems.

2. Theory

The optical depth following Boers and Mitchell (1994) is defined as

$$\tau = \frac{3}{5} \bar{Q}_{\text{ext}} r^{\frac{1}{3}} \left( \frac{4 \beta \rho_w}{3 \rho_0} \right)^{-\frac{2}{3}} \Psi (1 - \beta)^{\frac{2}{3}} A^\frac{2}{3} N^\frac{1}{3} h^\frac{3}{3}. \quad (1)$$

Here, $\tau$ is the cloud optical depth, $\rho_w$ is the density of liquid water, $\rho_0$ is the density of dry air, $\beta$ is a parameter defining the departure of the liquid water from its adiabatic
state caused by mixing of the cloudy air with its environment (Betts 1983). $A_D$ is the vertical gradient of the liquid water mixing ratio under adiabatic conditions, which is temperature dependent (Betts and Harshvardhan 1987), $N$ is the droplet concentration, $Q_{\text{ext}}$ is the extinction factor, $h$ is cloud depth and $\Psi$ is a shape parameter, defined as $\Psi = [(\alpha + 1)(\alpha + 2)]/(\alpha + 3)^2]^{1/3}$. The parameter $\alpha$ is described below. The effects of mixing are accounted for by $\beta$. If there is no mixing then $\beta = 0$; the maximum value of $\beta = 1$ implies that the cloud disappears.

It is assumed that the density $n(r, z)$ of the droplet size distribution can be approximated by a gamma distribution:

$$n(r, z) = a(z)r(z)^{\alpha}\exp\{-b(z)r(z)\}. \tag{2}$$

Here, $r$ is the droplet radius, and $z$ is altitude above cloud base. This notation is exactly that of Boers and Mitchell (1994). The parameter $\alpha$ is assumed to be height-invariant, as this analysis does not provide enough restrictions to its value. However, a constant $\alpha$ is a good approximation under most circumstances (Boers et al. 2000). The importance of (1) is that the optical depth is dependent on the $(5/3)$-power of $h$ rather than the first power, which is due to the vertical variation in effective radius. Another important and often overlooked aspect of (1) is that it demonstrates the dependence of optical depth on mixing processes, which is taken into account by means of the factor $(1 - \beta)$.

If the values of $\alpha$ and $\beta$ are prescribed, and $N$ is assumed to be constant, then both the effective radius and optical depth can be calculated exactly. We set $\alpha = 7$, the same value as used in Boers and Mitchell (1994). Boers et al. (2000) subsequently found that this is an excellent value to use for continental stratocumulus; variations of 20% impact the uncertainty in the optical depth by around 3%. The extinction factor in (1) is approximated as $Q_{\text{ext}} \sim 2$, which is excellent for most water clouds.

Given these assumptions, the effective radius near cloud top, $R_{\text{max}}$, can be calculated in a straightforward manner as

$$R_{\text{max}} = \{(1 - \beta)A_Dh\}^{1/3} \left(\frac{4\pi \rho_w}{\rho_0}N\right)^{-1/3} \Psi^{-1}. \tag{3}$$

The liquid water path, $LWP_{\text{clld}}$, is defined as:

$$LWP_{\text{clld}} = \frac{1}{2}(1 - \beta)A_Dh^2, \tag{4}$$

so, the optical depth can be expressed as a function of the liquid water path:

$$\tau = 1.2\pi \frac{1}{2} \left(\frac{4\rho_w}{3\rho_0}\right)^{-3/2} \Psi(1 - \beta)^{-1/3}A_D^{-1/6}N^{1/3}2^\delta LWP_{\text{clld}}^{5/6}. \tag{5}$$

The present analysis makes no distinction between homogeneous and inhomogeneous mixing, two processes that are distinctly different (Paluch et al. 1996, and others). Each mixing process affects the shape of the vertical profiles of all relevant parameters, and hence affects the precise definition of cloud optical depth as per (1). Therefore, the factor $\beta$ encompasses the entire range of mixing processes available in the atmosphere, which is a necessary simplification in the present analysis. We note that the dependence of optical depth on $LWP_{\text{clld}}$ is not linear, which again is the result of the fact that the effective radius is not constant, but a strong function of height. Also, implicit in (5) is that there is no precipitation present. Equation (5) will be modified later on, when we consider precipitation. Equations (1) and (5) demonstrate that there are many factors
driving the variability in optical depth. They are in summary: (a) the droplet concentration, (b) the cloud depth, (c) mixing, (d) precipitation, (e) the shape of the droplet size distribution and (f) the temperature of the cloud. We will neglect (e) because of the small effect of the shape function on optical depth, and we also neglect (f). The temperature effect on $\tau$ through the parameter $A_D$ is actually large (Bettis and Harshvardhan 1987) but, provided each correlation study is limited to small geographical regions where the synoptic conditions are relatively uniform, this effect can be neglected as well.

3. Optical depth–effective radius link for non-precipitating clouds

(a) The first indirect aerosol effect

When the principal cause of variation in the optical depth with effective radius is a variation in droplet number concentration due to variations in the availability of cloud condensation nuclei, then eliminating $N$ from (1) and (3) and rewriting the optical depth yields

$$\tau = 1.2 \left( \frac{4 \rho_w}{3 \rho_0} \right)^{-1} (1 - \beta) A_D h^2 R_{\text{max}}^{-1}.$$  

(6)

The resulting correlation between the optical depth and effective radius is negative, but its value will, to a large extent, depend on the scatter in $N$ and in the depth of the cloud, $h$, both of which are entirely controlled by local conditions unknown to the investigator.

(b) Cloud depth variations

Although large local variations in droplet concentration are expected to occur over land, Boers et al. (1998) found that during wintertime over the Southern Ocean the droplet concentration varies within a narrow range between 30 and 60 cm$^{-3}$. Apart from sea-salt nuclei, the concentration of which varies largely with wind speed, highly localized sources of cloud condensation nuclei are only found near regions of high dimethylsulfide productivity, but even there the range of droplet concentration values observed is comparatively narrow (Boers and Krummel 1998). If variations in droplet concentration are small, variations in cloud depth and hence cloud liquid water may become more important in contributing to the correlation between optical depth and effective radius. Eliminating the cloud depth $h$ from (1) and (3) we arrive at

$$\tau = 1.2 \pi^2 \left( \frac{4 \rho_w}{3 \rho_0} \right) \Psi^6 \{(1 - \beta) A_D\}^{-1} N^2 R_{\text{max}}^5,$$

(7)

revealing a very strong $R_{\text{max}}^5$ dependence. This functional form also occurred to Lohmann et al. (2000), when considering the implications of comparing GCM output with remote sensing observations. The correlation between the optical depth and effective radius is now positive, but, again, its exact value will depend on the region chosen to illustrate the theory.

(c) Entrainment and mixing

Mixing processes are dependent on the strength of turbulence inside the cloud layer (primarily cloud top infrared cooling), and on the strength of the buoyancy discontinuity separating the cloud layer from its environment. Adiabatic clouds only occur if the cloud layer is well mixed, with little or no exchange with the environment. Although there are
some observations of adiabatic clouds (Albrecht et al. 1990), there is now a large body of evidence indicating the prevalence of non-adiabatic clouds (Boers et al. 1996, 1998; Boers and Krummel 1998; Brenguier et al. 2000; Boers et al. 2000). This means that the value of $\beta$ often departs from its adiabatic value of $\beta = 0$. Typical values of $\beta \sim 0.4$ have either been quoted (Betts and Albrecht 1987; Boers et al. 2000) or can be derived from relevant papers that do not explicitly calculate it from the liquid water content (Brenguier et al. 2000). Eliminating $\beta$ from (1) and (3) and rewriting (1) yields

$$\tau = 1.2\pi \Psi^3 Nh R_{\text{max}}^2.$$  \hspace{1cm} (8)

Hence, mixing would be responsible for a positive correlation between optical depth and effective radius, all other factors being equal. Again, though, this link is entirely different from the inverse $R_{\text{max}}$ law associated with the first indirect aerosol effect.

(d) Representation of remote sensing data in an optical depth–effective radius diagram

The underlying physical processes responsible for the horizontal distribution of optical depth are many and difficult to describe. They include boundary-layer evolution, surface energy transport and radiative transfer that shape the variability of cloud depth and liquid water path, and the distribution of sources and sinks of cloud condensation nuclei responsible for the droplet concentration near cloud base. Unfortunately, the variability of all the parameters controlling optical depth has rarely been measured simultaneously. So, there is little information that could guide us in establishing the $h, N$ correlation that could potentially influence the correlation between optical depth and effective radius through (1) and (3). Clearly, if the physical processes in the cloudy boundary layer favour a positive correlation between $h$ and $N$, then the $\tau, R_{\text{max}}$ correlation is different from when the $h, N$ correlation is negative.

In order to obtain credible correlation values and joint $\tau, R_{\text{max}}$ distributions, the parameters $h$ and $N$ must be simulated realistically, which can only be done by using a model that adequately represents the variance and variance change with scale size. Many geophysical parameters are self-scaling, which means that their energy spectra $E(k)$ are described by functions that have a negative power, $\lambda$, in wave number, $k$, i.e. $E(k) \sim k^{-\lambda}$.

The parameters exhibiting scaling behaviour include almost all of the parameters responsible for determining the value of the cloud optical depth, namely cloud top (Boers et al. 1988), liquid water (and hence droplet concentration; Davis et al. 1996; Marshak et al. 1997), boundary-layer depth (Boers et al. 1995), liquid water path (Cahalan and Snider 1989; Cahalan et al. 1994), cloud optical depth and cloud reflectance (Davis et al. 1996, 1997). To our knowledge there is, as yet, no evidence that cloud base is a scaling parameter. However, there is no reason to believe that cloud base variability would be structurally different from the rest of the parameters, as the physical processes that control cloud base are the same as those that control the other parameters.

Our model of choice is the bounded cascade model of Cahalan et al. (1994). This model simulates the negative power laws for individual parameters and, consequently, fulfills the requirement that the candidate model adequately represents the decreasing variance as a function of decreasing scale size. The distribution functions associated with the scaling parameters closely resemble (but are not equal to) log-normal distributions. Analysis of Landsat data (Davis et al. 1997) indicates that the scaling behaviour is valid roughly from 40 km down to 200 m. Our model is, therefore, restricted to these scales. The results are unlikely to extrapolate to larger scales, as synoptic-scale
Figure 1. The relation between optical depth and effective radius for a large positive correlation between cloud depth ($h$) and droplet concentration ($N$). See text for details.

Figure 2. As Fig. 1 but for a large negative correlation between $h$ and $N$. 
influences will dominate cloud systems. The most commonly used satellite technology permits the viewing of pixels of $30 \times 30$ m (Landsat) to $1$ km $\times$ $1$ km (AVHRR), although higher resolution will become available in the near future. This underlines the difficulty of comparing GCM results with remote sensing data, as the typical grid scale of a GCM (60–300 km) far exceeds the typical pixel size of a satellite radiometer. At smaller scale, three-dimensional radiative transfer will become important, invalidating our results as well.

Figures 1 and 2 show the joint $R_{\text{max}}$, $\tau$ distributions where the droplet concentration, and cloud depth are simulated as individual realizations of the bounded cascade model. The mean droplet concentration is $100$ cm$^{-3}$, while the mean cloud depth is $350$ m, but for Fig. 1 their correlation coefficient is $0.61$, while for Fig. 2 it is $-0.66$. The parameter $\beta = 0.4$ with an 8% variation imposed on it. Clearly, in Fig. 1 the $R_{\text{max}}$, $\tau$ correlation is different from its equivalent in Fig. 2. The isolines that were drawn on Figs. 1 and 2 represent lines of equal cloud depth (from upper left to lower right) and equal droplet concentration (from lower left to upper right). The comparison of Figs. 1 and 2 indicates that a negative $h$, $N$ correlation incurs a positive $R_{\text{max}}$, $\tau$ correlation, while the opposite is true when the $h$, $N$ correlation is positive. The essential point of this set of figures is that it is the internal correlation between the controlling parameters $h$ and $N$ that matters greatly in establishing the final $R_{\text{max}}$, $\tau$ correlation.

Figure 3 shows the correlation coefficient $r(R_{\text{max}}, \tau)$ plotted against $r(N, h)$. For the moment we only focus on the upper solid line in the plot, the broken line below it will be discussed later. As $r(h, N)$ increases from negative to positive there is a steady decline in $r(R_{\text{max}}, \tau)$, but $r(R_{\text{max}}, \tau)$ turns negative only when $r(h, N)$ exceeds $0.2$. For all simulations the standard deviation in droplet concentration $\sigma(N) = 26$ cm$^{-3}$, while $\sigma(h) = 45$ m. The value of $r(R_{\text{max}}, \tau)$ is also dependent on the comparative values of $\sigma(N)$ and $\sigma(h)$. If the variability of $h$ is doubled, i.e. $\sigma(h) = 90$ m as in Fig. 4, then
Figure 4. Optical depth–effective radius correlation coefficient as a function of cloud depth–droplet concentration correlation coefficient. The upper solid line represents a situation where the data are not affected by precipitation; the lower broken line represents data affected by precipitation. With respect to Fig. 3, the variance in cloud depth has been doubled.

$r(R_{\text{max}}, \tau)$ remains positive at all times. If $\sigma(h)$ is reduced to values smaller than 45 m, the variability in $N$ begins to dominate and $r(R_{\text{max}}, \tau)$ becomes more negative.

However, for the typical values used here, there is a clear propensity for the clouds to be associated with a positive $r(R_{\text{max}}, \tau)$ even if $\sigma(h)$ is as small as the value used in Fig. 3. Figure 3 demonstrates that in such a case a negative value of $r(R_{\text{max}}, \tau)$ can only be obtained if $r(h, N)$ is highly positive. If this is the preferred sign for $r(R_{\text{max}}, \tau)$ as seems to be the case based on the Han et al. (1998) data, then it is likely that an atmospheric process favours a positive $r(h, N)$. It is useful to speculate on the relevant process involved. If we imagine the cloudy boundary layer as a single closed circulating cell, then the upward moving branch containing unmixed moisture-laden air would impinge on the inversion and overshoot it. Mixing of these overshooting parcels with overlying warm and dry air would reduce the moisture content and increase the temperature, so that the liquid water content, cloud depth and droplet concentration would be reduced in the downward moving branch of the circulation. This implies that, on scales of the single cell circulation, $r(h, N)$ would be positive. It would be highly relevant if atmospheric data on their correlation could be obtained, a daunting task given the difficulty of measuring droplet concentration and cloud boundaries with remote sensing instruments. The only indirect evidence available so far is in the positive correlation between vertical velocity and droplet concentration observed in field studies (e.g. Vali et al. 1998).

The mean value of $\beta$ is kept constant at 0.4. As mentioned above, this value is reasonable given the available data, but turbulence on different scales will impose variations in this parameter as well, the clearest recent examples of which are provided by Brenguier et al. (2000). They found variations in liquid water that can be converted into values of $\beta$ ranging from 0.0 to 0.9. Although this variability is very large, optical depth is a height-integrated quantity and the variations in $\beta$ are therefore expected to
be much more modest. Figure 5 shows a $\tau$, $R_{\text{max}}$ plot with two filled circles. The lower of the two circles represents the mean of optical depth and effective radius for $\beta = 0.4$ for simulations such as performed earlier. Superimposed on the circle are bars indicating the standard deviation in optical depth and effective radius based on a specified variation in $N$ and $h$ (where variations in $N$ are independent of variations in $h$) as indicated in the upper left of the plot. The other filled circle represents the result of a translation of the mean $\tau$, $R_{\text{max}}$ point towards a position where $\beta = 0$. The position of this second point indicates that a change of $\beta$ over 40% of its entire possible range is equivalent to a change in either $h$ or $N$ of 20–25%. This implies that variability in mixing is less important in determining the $\tau$, $R_{\text{max}}$ correlation than variations in $h$ or $N$, which agrees with the conclusion of Brenguier et al. (2000). Clearly, our assumption of 8% variability superimposed on $\beta$ for the simulations above is questionable, and data are necessary to determine the variability in $\beta$ at high time resolution from vertically integrated parameters. This can be achieved by simultaneously measuring the depth and liquid water path of a cloud, i.e. through a collocation of a cloud radar, lidar and a microwave radiometer. Although Albrecht et al. (1990) and also Boers et al. (2000) were able to estimate $\beta$, only few data points were obtained in both studies and no data on its short-term variability could be derived.

4. THE ONSET OF PRECIPITATION

In a world where pollution increases the number of cloud condensation nuclei, a reduced size of cloud droplets would imply a reduction in precipitation and an increased lifetime of clouds (Albrecht 1989). In general terms, if the cloud droplets near cloud top exceed a certain size, collision-coalescence will mark the onset of precipitation. Precipitation affects the cloud in a complex manner. Pincus and Baker (1994) and Boers (1995) showed through model calculations that cloud depth is reduced
when precipitation occurs. However, both studies overlooked the fact that the removal of droplets by collision-coalescence reduces the optical depth of the cloud as well, regardless of its depth, a process thought to be responsible for the small optical depths observed during the Southern Ocean Experiment (Boers et al. 1996). Feingold et al. (1997) confirmed that this process is operational using large eddy simulation with explicit microphysics. Subsequently, Boers et al. (1998) calculated the fraction of liquid water confined in the drizzle mode to the total liquid water, and concluded that it was highly dependent on the value of the effective radius $R_{\text{max}}$ near cloud top. Precipitation was negligible for values of the effective radius smaller than 10 $\mu$m, with a rapid onset of drizzle as the effective radius increased beyond 12 $\mu$m, which supports the idea of a critical radius (Gerber 1996) controlling the occurrence of precipitation.

The difficulty is to quantify the influence of precipitation on optical depth, because it affects the optical depth in two separate ways, i.e. through cloud depth as well as through droplet concentration caused by the shift in microphysical structure of the cloud. As will be shown below, these two processes affect $R_{\text{max}}$ in different ways. We recall that $R_{\text{max}}$ designates the maximum effective radius of suspended cloud droplets, not the effective radius of the second (i.e. drizzle) mode of the combined drizzle/non-drizzle droplet spectrum. We suggest a practical method to parametrize the effect of drizzle on cloud optical depth based on the results of Boers et al. (1998). At any one point in time, the total liquid water $LWP_{\text{tot}}$ will consist of suspended cloud droplets $LWP_{\text{cl}}$ and drizzle droplets $LWP_{\text{drz}}$. Figure 6 illustrates the fraction of $LWP_{\text{cl}}$ over total liquid water path $LWP_{\text{tot}}$ as a function of the cloud top effective radius $R_{\text{max}}$ for all Southern Ocean case-studies discussed by Boers et al. (1998). The data suggest an ad-hoc functional form of the liquid water path as

$$
LWP_{\text{cl}} = LWP_{\text{tot}} f(R_{\text{max}}) = LWP_{\text{tot}} \frac{1}{2} \left\{ 1 - \tanh \left( \frac{R_{\text{max}} - R_{\text{crit}}}{R_{\text{rel}}} \right) \right\}. \quad (9)
$$

Here $R_{\text{crit}}$ is a critical effective radius marking the value at which 50% of liquid water is associated with drizzle droplets and $R_{\text{rel}}$ is a relaxation radius that determines the curvature of the tanh function. Other functional forms could have been chosen, but our choice is entirely guided by the need to obtain a mathematically continuous function at the high end of effective radius. In addition, it is clear that the function, which is plotted on Fig. 6, provides a reasonable fit to the available data. The critical value $R_{\text{crit}} = 14 \, \mu m$, and the relaxation value $R_{\text{rel}} = 3 \, \mu m$. The drizzle droplets were measured using a Particle Measurement System (PMS)-2-DC probe, while the suspended cloud droplets were measured using a PMS particle size spectrometer FSSP probe. The parameter $LWP_{\text{tot}}$ corresponds, to a first approximation, to the total amount of liquid water before the onset of precipitation. Hence it can be replaced by (4) and the optical depth can be rewritten as

$$
\tau = 1.2 \pi \left( \frac{4 \rho_w}{3 \rho_0} \right)^{-\frac{1}{3}} \Psi (1 - \beta)^{-\frac{1}{8}} A_D^{-\frac{1}{8}} N^{-\frac{3}{2}} 2^{\frac{3}{8}} LWP_{\text{tot}}^\delta \left\{ 1 - \tanh \left( \frac{R_{\text{max}} - R_{\text{crit}}}{R_{\text{rel}}} \right) \right\}^{\frac{5}{8}}. \quad (10)
$$

Next, (4) is used to reinsert the functional form of the liquid water path and then (3) is used to eliminate $N$.

$$
\tau = 1.2 \left( \frac{4 \rho_w}{3 \rho_0} \right)^{-1} (1 - \beta) A_D h^2 R_{\text{max}}^{-1} \left( \frac{1}{2} \right)^{\frac{3}{8}} \left\{ 1 - \tanh \left( \frac{R_{\text{max}} - R_{\text{crit}}}{R_{\text{rel}}} \right) \right\}^{\frac{5}{8}}. \quad (11)
$$
Essentially, (11) expresses the idea that drizzle droplets are not radiatively active in the visible optics regime, and thus do not contribute to the cloud optical depth. Equation (11) shows that a functional form of the optical depth for a precipitating cloud emerges that is captured in a simple power-law associated with the suggested signature of the indirect aerosol effect as before, but now multiplied by a tanh function associated with precipitation. Figure 7 shows a $\tau$, $R_{\text{max}}$ plot where the two curved isolines indicate the reduction in optical depth as the effective radius of the data approaches and exceeds the cut-off point near 10 $\mu$m (see also Fig. 6).

The three closed circles and asterisk on Fig. 7 illustrate the suggested reduction in cloud optical depth due to precipitation. The upper closed circle represents the mean $\tau$, $R_{\text{max}}$ point of a cloud without precipitation. For this point, $\tau = 28$, $N = 70$ cm$^{-3}$, $R_{\text{max}} = 15$ m, and $h = 700$ m. According to (11), the onset of precipitation would reduce its optical depth. The line connecting the upper filled circle to the asterisk indicates the amount of optical depth reduction. The asterisk is the intersection point of a vertical line through the upper filled circle (representing the non-precipitating cloud) and the curved broken line (which represents (11) and the reduction in liquid water path associated with precipitation), for an initial cloud depth of 700 m. The optical depth associated with this new precipitating cloud is $\tau = 10$. However, the indicated value of the effective radius at this point may be one of many possibilities, our functional form in (11) represents just one credible option.

If the liquid water path reduction is merely the result of a microphysical shift from numerous small droplets to a few large ones, and provided that the cloud depth is not affected, then a line along equal values of $h$ needs to be drawn through the upper filled circle until it intersects the horizontal line through the asterisk. The resulting point, the right lower circle, represents the lowest possible value, $N = 3.5$ cm$^{-3}$, and highest possible effective radius, $R_{\text{max}} = 41$ $\mu$m. Clearly, this is a somewhat extreme example,
Figure 7. The effect of precipitation on the relationship between optical depth ($\tau$) and effective radius ($R_{\text{max}}$). The upper filled circle represents the original data not contaminated by precipitation. The horizontal line connecting the two lower filled circles represents the entire range of possible values of the maximum observed value of the effective radius when precipitation occurs.

because at this end cloud droplets would be few and very large, and thus would become radiatively inactive in the optical regime. However, the simulations of Feingold et al. (1997) clearly demonstrate that an increase in $R_{\text{max}}$ at the onset of precipitation is a realistic option.

The other extreme is when the reduction in optical depth is caused entirely by a reduction in cloud depth (Pincus and Baker 1994; Boers 1995). The physical process associated with reduced cloud depth is where the evaporation of drizzle below the cloud suppresses turbulence and is responsible for decoupling the cloud layer from the surface. This restricts the upward transport of moisture. For situations such as these a line is drawn through the upper filled circle along lines of equal $N$ until the horizontal line through the asterisk is intersected. The resulting point, the lower left filled circle, represents the lowest value of the effective radius that can be reached, namely $R_{\text{max}} = 12.5 \mu m$.

Clearly, neither extreme is realistic, and the cloud evolving from a non-precipitating to a precipitating state will have an effective radius somewhere in between, as represented by our model. The essential point of this diagram is that the onset of precipitation reduces optical depth, but the effective radius $R_{\text{max}}$ of the precipitating cloud may no longer be single-valued. The reason is that the transitional pathway of the cloud effective radius from a non-drizzling to a drizzling cloud is largely uncertain, which will increase the variability of $R_{\text{max}}$, and thus reduce $r(R_{\text{max}}, \tau)$ as we will see below.

Unfortunately, no models exist that can educate us on the relative importance of either process. Using a typical value of $N = 100 \, \text{cm}^{-3}$, and cloud depth of 350 m as before, but now with somewhat larger standard deviations (see Fig. 8), we can tentatively illustrate the possible change in the $\tau, R_{\text{max}}$ point-cluster as the result of precipitation. Figure 8(a) represents the data without precipitation, while (b) represents the same data
Figure 8. The effect of precipitation on the correlation between optical depth and effective radius: (a) original uncontaminated data; (b) point cluster after the onset of precipitation. See text for details.

cluster but now where the data points with a large $R_{\text{max}}$, i.e. $R_{\text{max}} > 10\, \mu\text{m}$, are affected by precipitation as per (11). For the purpose of this simulation the precipitation-affected parcels are represented by values of $R_{\text{max}}$ randomly drawn from a normal distribution of $R_{\text{max}}$ values, centred on the value as represented in (11) but between the two extreme values of $R_{\text{max}}$ as indicated schematically in Fig. 7. Clearly, the original single point
cluster is considerably changed, with two lobes now present, one that is not affected by precipitation, while the other one is.

Interestingly, one of the Austin et al. (1999) panels (scene 9, their Fig. 2) bears a very strong resemblance to the point cluster simulated in Fig. 8. While they attributed the bi-modality of their scene to the presence of two regions, one with and the other without anthropogenic influences, it is possible that it represents a single point cluster after the onset of precipitation as modelled in this study.

We now re-examine Figs. 3 and 4 focusing on the broken lines in the graphs, which represent the correlation coefficient for the precipitation-affected data. When \( r(R_{\text{max}}, \tau) \) is most positive and \( r(h, N) \) is negative, the effect of precipitation is the largest, because parcels with the high values of \( h \) and \( R_{\text{max}} \) would incur the largest reduction in liquid water path (see (10) and (11)). Towards the other end, i.e. small values of \( r(R_{\text{max}}, \tau) \) at high values of \( r(h, N) \) the shape of the point clusters will be such that most parcels will not be affected by precipitation, so the change in \( r(R_{\text{max}}, \tau) \) is likely to be small.

One salient feature of Fig. 8 is the improbability of observing clouds with an effective radius exceeding about 16 or 17 \( \mu \text{m} \) near cloud top, because beyond those values clouds will disappear through precipitation. This appears to conform to the observations of Han et al. (1998). However, perhaps more importantly, precipitation has the almost universal effect of decreasing \( r(R_{\text{max}}, \tau) \).

5. CONTINENTAL AND OCEANIC CLOUDS

Han et al. (1998) made a separation between large and small optical depth (separation at \( \tau = 15 \)) and between continental and oceanic clouds. Accordingly, we attribute values of \( N = 50 \text{ cm}^{-3} \) for oceanic and \( N = 300 \text{ cm}^{-3} \) for continental clouds, and plot
four representative points on Fig. 9. They are indicated by large/small \( \tau \) and large (continental)/small (oceanic) \( N \). Figure 9 indicates that many continental clouds will not be affected by precipitation because their effective radius will not exceed the value beyond which coalescence will reduce the optical depth (i.e. beyond \( R_{\text{max}} = 10 \, \mu m \)). The point representing thick continental clouds has an \( R_{\text{max}} = 8.5 \, \mu m \), which is well below this critical value. However, a point cluster of the size shown in Figs. 1 and 2 is likely to contain many data points with \( R_{\text{max}} > 10 \, \mu m \). This means that, if our model of (11) is correct, it is conceivable that some point clusters representing thick continental clouds would also be affected by precipitation.

6. DISCUSSION AND CONCLUSION

We studied the effect of individual physical processes on the link between effective radius and cloud optical depth, the most important of which are mixing, precipitation and cloud depth. Due to the complex interplay between all these processes, when analysing selected regions of remote sensing observations it is unlikely that a uniform and constant relationship will be found anywhere. Indeed, it is somewhat surprising that Han et al. (1998) found consistent negative correlations of optical depth with effective radius for regions of high optical depth, and positive correlations for oceanic regions of low optical depth. Our findings indicate that it is not only the relative spread in droplet concentration and cloud depth that determines the correlation, but also the internal correlation between cloud depth and droplet concentration. If clouds are not dominated by precipitation, then the only two ways in which \( r(R_{\text{max}}, \tau) \) can become negative is when the distribution of cloud depths is comparatively narrow, or if \( r(h, N) \) is positive. If \( r(h, N) \) is positive, then the atmospheric process most likely to be responsible is the influence of mixing on downdraughts in the cloudy boundary layer represented by a single closed cell circulation.

Precipitation tends to reduce the correlation between optical depth and effective radius because one of the principal effects of precipitation is to widen the spread in droplet concentration. In fact, our calculations indicate that \( r(R_{\text{max}}, \tau) \) is negative for many situations where the spread in \( h \) is moderate, and the deepest clouds of low droplet-concentration are affected the most by a reduction in liquid water path through precipitation. If we further assign representative \( \tau, R_{\text{max}} \) points to oceanic or continental clouds of small or large optical depth, then it appears that deep continental clouds could be affected by precipitation in addition to the oceanic clouds. This may be the reason why the Han et al. (1998) study demonstrated large regions of negative \( r(\tau, R_{\text{max}}) \).

If precipitating clouds dominate \( r(R_{\text{max}}, \tau) \) then the interesting side effect of increased pollution could be a potential increase in \( r(R_{\text{max}}, \tau) \) in the future (the broken line in Figs. 3 and 4 would shift towards the solid line). This effect would be most clearly visible over the ocean because the sensitivity of oceanic clouds to increases in aerosols in a more polluted atmosphere is the largest.

Based on the present study it would seem highly relevant to obtain simultaneous data of cloud depth, mixing parameter, liquid water path and droplet concentration, because together they influence cloud optical depth. Since such data are regularly obtained at the Southern Great Plains Atmospheric Radiation Measurement (SGP-ARM) site, a re-analysis of data collected there seems warranted. The interplay between the various processes will undermine the occurrence of clearly identifiable functional relationships, although the relative positioning and shaping of point clusters on the optical depth-effective radius diagram will enhance our understanding of the mechanisms that underlie an important climatic process.
ACKNOWLEDGEMENT

This study was, in part, supported by the Australian Greenhouse Office.

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