Lightning initiation—conventional and runaway-breakdown hypotheses

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SUMMARY

We evaluate two candidate mechanisms for the onset of lightning in the relatively low electric fields measured inside thunderstorms. The first is conventional dielectric breakdown due to local enhancement of the electric field in the vicinity of hydrometeors. The second is runaway breakdown, due to extended acceleration of high-energy electrons (from cosmic rays or terrestrial sources of ionizing radiation) by the in-cloud electric field. We compare the electric fields required for lightning onset by each mechanism with those observed inside lightning-producing clouds, and we examine the sensitivity of the computed results to input parameters and assumptions. The conclusion of our analysis is that the conventional breakdown mechanism alone cannot trigger lightning while the runaway breakdown mechanism appears a more likely candidate. We identify the parameters on which each mechanism depends and emphasize the impact of observational uncertainties on our conclusions.

KEYWORDS: Atmospheric electricity Corona Cosmic-ray showers Drops Ice Lightning

1. INTRODUCTION

(a) Lightning initiation processes

Although lightning is arguably the most dramatic naturally occurring atmospheric phenomenon, there is still little understanding of the processes by which it is initiated. The commonly accepted picture of the evolution of a lightning stroke involves the following stages: (i) build up of in-cloud electric fields on large spatial scales via microphysical and dynamic processes; (ii) local enhancement of the electric field to produce regions in which ionizing collisions between accelerated electrons and air molecules result in propagating corona streamers; (iii) propagation of sufficient electric current through and beyond the region of very high local fields to form the hot, completely ionized lightning channel, or leader.

To explain or even analyse the ensemble of these processes is beyond the scope of this study. We are concerned here only with step (ii); that is, with the creation of propagating corona streamers in clouds. In laboratory studies the required electric field is denoted by the dielectric breakdown field, $E_{\text{breakdown}}$. The magnitude of $E_{\text{breakdown}}$ increases roughly linearly with pressure; at surface pressure in dry air, $p_0 \approx 1000$ mb, $E_{\text{breakdown}} \approx 2600$ kV m$^{-1}$.

(b) The puzzle

Historically, in-cloud electric-field profiles were inferred from ground-based measurements and occasional instrumented aircraft flights. Recently electric-field soundings have been collected by instrumented balloon (e.g. Stolzenburg et al. 1994; Marshall et al. 1995a,b), giving in situ field measurements during balloon ascents through thunderclouds. Although each of these methods is associated with important errors, some of which we discuss in section 5, all the measurements show that $E_{\text{max}}(p) < E_{\text{breakdown}}(p)$, where $E_{\text{max}}(p)$ is the highest measured field strength at pressure $p$. That is, the measured fields never equal the dielectric-breakdown field strength even in lightning-producing clouds; in fact, often $E_{\text{max}}(p)/E_{\text{breakdown}}(p) < 0.1$ in these clouds. Thus, the search for a lightning initiation process is the search for a process in which

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relatively weak macroscopic fields are locally enhanced to begin the initiation process described in step (ii), above.

(c) The initiation hypotheses

In this paper we discuss two hypothetical mechanisms by which the in-cloud electric field can be locally enhanced to produce propagating streamers even in relatively weak large-scale external fields. Both hypotheses begin with free electrons that are accelerated, producing electron avalanches through multiple ionizations. The first mechanism is conventional breakdown in the vicinity of hydrometeors; according to this theory the electric field is intensified sufficiently in the vicinity of small hydrometeors that avalanches of background electrons yield propagating corona streamers. The second hypothesis is runaway breakdown; high-energy electrons accelerate over long distances in the thundercloud electric fields, becoming ‘runaways’ that give rise to huge numbers of daughter, high-energy electrons that lead to eventual dielectric breakdown.

In section 2 we describe the laboratory and numerical analyses of conventional breakdown near drops and ice particles and conclude with calculations of the electric-field magnitude needed for production of streamers by this means. In section 3 and the appendix we describe the runaway-breakdown hypothesis (Gurevich et al. 1999), concluding with the in-cloud conditions required to achieve breakdown via this mechanism. In section 4 we discuss relaxation of the field by background electrons. We compare the required electric-field distributions for each mechanism with observed in-cloud electric-field profiles in section 5, and develop a model to simulate those fields. Finally, we discuss our results and their sensitivities in section 6.

One of the major purposes of this analysis is to identify the parameters on which the two hypotheses depend and, thus, to highlight the data and theoretical developments most needed for further progress in this field.

2. CONVENTIONAL BREAKDOWN NEAR HYDROMETEORS

The traditional explanation for the difference between $E_{\text{max}}$ and $E_{\text{breakdown}}$ is that the ambient electric field is greatly intensified and can reach $E_{\text{breakdown}}$ in the vicinity of a small hydrometeor even in the absence of large external sources of free electrons. This hypothesis has been investigated in laboratory settings as well as via numerical models of the discharge process.

Three basic types of discharge can occur in the vicinity of hydrometeors in an ambient electric field. These processes are sometimes collectively referred to as corona. The discharge that requires the lowest ambient electric field is the burst pulse discharge, which is intermittent. As the electric-field strength increases the burst pulse discharge may become a continuous glow discharge very close to the hydrometeor. Finally, at still larger electric-field values a streamer can form. Streamers can propagate away from the hydrometeor as filamentary channels. Creation of corona is a ‘cold’ process; thermodynamic equilibrium is not disturbed by it. If the current carried by the corona streamers reaches sufficient magnitude (Bondiou and Gallimberti 1994), then Joule heating destroys the equilibrium and a ‘leader’, or fully ionized channel can result. This process must require some process involving multiple streamers (Dawson and Winn 1965; Griffiths and Phelps 1976b; Castellani et al. 1998) and is outside the range of this work. We focus on the conditions for production of these propagating streamers, step (ii) in lightning initiation. For more details on the physics of the discharge process as it relates to hydrometeors see Schroeder et al. (1999).
(a) Background

Macky (1931) found that discharges could be initiated from falling water drops in uniform electric fields $\approx$800 kV m$^{-1}$. Discharges from drops were thought to require drop break-up until Loeb (1965) showed that discharges could initiate from the surfaces of drops that were highly distorted but not shattered by strong electric fields.

Dawson (1969) showed that surface distortion and corona were, in fact, competing processes. A set of experiments for both positive and negative discharges from water-drop surfaces showed that the transition from the surface-disruption mode to pure corona (distortion but no break-up) was pressure dependent. Richards and Dawson (1971) studied drop instabilities and discharge initiation from water drops falling at their terminal velocities in a vertical electric field. Their results showed that the large-scale applied electric fields necessary to initiate a discharge from water drops were still several times larger than the electric fields observed in thunderclouds.

Crabb and Latham (1974) showed that under certain circumstances a pair of colliding water drops could initiate positive discharges at electric fields as low as 250 kV m$^{-1}$. Griffiths and Latham (1974) looked at ice particles as a source for discharge initiation. They found that, at the low pressures where one expects to find ice particles in thunderstorms, the initiation electric fields were $\approx$250–400 kV m$^{-1}$.

Calculations by Coquillat and Chauzy (1994), Georgis et al. (1995) and Cooray et al. (1998) of discharge initiation from single raindrops and two closely spaced drops showed that the process required unrealistically high charges on the drops and/or unphysical drop configurations in order to achieve initiation electric fields comparable to those observed by Crabb and Latham (1974).

After a review of the literature on this subject Blyth et al. (1998) concluded that only two microphysical situations appeared capable of initiating a discharge in thundercloud conditions: warm colliding drops and individual ice particles. It would appear that the Blyth et al. (1998) conclusions still stand and that, if hydrometeors are to be considered the source of discharges in thunderclouds, the experimental work of Crabb and Latham (1974) and Griffiths and Latham (1974) offers the best insight.

(b) Discharge model

We present a brief summary of the model we have used for the discharge process. A detailed discussion can be found in Schroeder and Baker (1999). Following the earlier work of Griffiths and Phelps (1976a), Dawson and Winn (1965), Gallimberti (1979) and Bondiou and Gallimberti (1994) we modelled the positive discharge as a series of electron avalanches. Consider the electric field near the surface of a drop which is situated in an external field, $E_{\text{external}}$ (Fig. 1). Initially, the total field at a point a distance $r$ from the drop surface is

$$E(r) = E_g(r) = E_{\text{external}} + E_{\text{drop}}(r)$$

(1)

where $E_{\text{drop}}$ is the contribution due to charge induced on the drop and $E_g(r)$ is referred to as the geometric field.

Free electrons are created in the background atmosphere via photo-ionization and by the collisional detachment of negative oxygen ions. In the presence of an electric field, these free electrons are accelerated and undergo collisions with air molecules. At some radial distance from the hydrometeor $E$ is such that

$$\alpha(E/p) = \eta(E/p)$$

(2)
where $\alpha$ and $\eta$ are the ionization and attachment coefficients for electrons in air, respectively, and $p$ is the total air pressure (Harrison and Geballe 1953; Loeb 1965; Badaloni and Gallimberti 1972; Ibrahim and Singer 1982).

The surface defined by Eq. (2) is the ionization zone boundary. It is here that $E(r) = E_{\text{breakdown}}$; inside this boundary $\alpha > \eta$ and there is a net growth of free electrons. At surface pressure the ionization zone boundary is the surface along which $E \sim 2600$ kV m$^{-1}$.

Consider the events shown in Fig. 1. The point $r_1$ marks the intersection of the ionization zone boundary with the $r$-axis. When a free electron, starting at $r_1$, accelerates in the electric field towards the hydrometeor, the number of electrons grows exponentially with decreasing $r$. This is referred to as the primary electron avalanche. Due to the exponential nature of the growth, most of the ionizing collisions occur near the surface of the hydrometeor. The free electrons are then absorbed by the drop, leaving behind a region of low-mobility positive ions, modelled as a sphere with radius $r_0$ (Dawson and Winn 1965; Gallimberti 1979) and referred to as the streamer head. The total electric field is now a combination of the geometric field and the electric field due to the spherical charge concentration of the streamer head. Note that in order to simplify the problem, it is convenient to replace the three-dimensional problem by a one-dimensional one in which all avalanches occur along a line, denoted the $r$ axis (Dawson and Winn 1965; Gallimberti 1979).

In addition to ionization, collisions between the free electrons and air molecules also result in the excitation of the molecules, which then emit photons on decay. A certain fraction of these photons in turn have sufficient energy to ionize molecules that they encounter, creating photoelectrons. These photoelectrons then start a series of secondary avalanches which converge on the drop from all directions. This results in the formation of a second, new streamer head.

A burst pulse discharge is initiated if the number of photoelectrons created along the ionization zone boundary during the growth of the primary avalanche is equivalent
TABLE 1. EXPERIMENTAL AND MODEL RESULTS SHOWING THE ELECTRIC FIELDS \(E_{\text{init}}\) NECESSARY TO INITIATE A POSITIVE DISCHARGE AT \(p = 1000\) AND 500 mb

<table>
<thead>
<tr>
<th>Hydrometeor type</th>
<th>Source</th>
<th>Charge on hydrometeor?</th>
<th>(E_{\text{init}}) (kV m(^{-1}))</th>
<th>Init. type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Falling drops</td>
<td>M (experiment)</td>
<td>no</td>
<td>800</td>
<td>950</td>
</tr>
<tr>
<td>Falling drops</td>
<td>RD (experiment)</td>
<td>no</td>
<td>800</td>
<td>950</td>
</tr>
<tr>
<td>Colliding drops</td>
<td>CL (experiment)</td>
<td>yes</td>
<td>250</td>
<td>500</td>
</tr>
<tr>
<td>Ice, needle</td>
<td>GL (experiment)</td>
<td>no</td>
<td>1000</td>
<td>600</td>
</tr>
<tr>
<td>Ice, needle</td>
<td>GL (experiment)</td>
<td>no</td>
<td>100 pC</td>
<td>600</td>
</tr>
<tr>
<td>Ice, needle</td>
<td>SBB (model)</td>
<td>no</td>
<td>100 pC</td>
<td>600</td>
</tr>
<tr>
<td>Ice, needle</td>
<td>SBB (model)</td>
<td>no</td>
<td>100 pC</td>
<td>600</td>
</tr>
</tbody>
</table>


...to the number of photoelectrons that started the primary avalanche (commonly taken to be one) (Abdel-Salam et al. 1976).

A more stringent initiation condition exists for streamers. In this case the number of positive ions in the primary streamer head must be large enough to attract the secondary avalanches to the streamer-head surface. This is achieved when the radial electric field around the streamer head is of the order of the geometric field \(E_{g}\) (Eq. (1)) (Abdel-Salam et al. 1976). In addition:

(a) The number of positive ions in the streamer head that results from the secondary avalanches, must equal \(N_{1}\), the number of positive ions created by the primary avalanche, and

(b) the radius of the secondary streamer head must equal the radius of the primary streamer head (Dawson and Winn 1965).

These conditions ensure that the initial streamer-head charge density is reproduced in the second streamer head. Continued reproduction of the streamer head in subsequent steps results in propagation of the positive streamer away from the drop surface. The distance that the streamer will propagate depends on the magnitude of the applied electric field.

The electric field necessary to sustain stable streamer propagation, \(E_{\text{propagation}}(p)\), was measured by Griffiths and Phelps (1976a) as a function of air pressure, \(p\), and absolute humidity. Streamers, once initiated, will continue to propagate provided \(E \geq E_{\text{propagation}}\). Griffiths and Phelps (1976a) found that \(E_{\text{propagation}} \sim 400\) kV m\(^{-1}\) for dry air at \(p = 1000\) mb and that \(E_{\text{propagation}}(p) \propto p^{1.5}\).

(c) Results: Requirements for streamer formation via conventional breakdown

We have constructed a model to calculate the values of \(E_{\text{init}}\) needed to initiate a discharge in the vicinity of different kinds of hydrometeors (Schroeder et al. 1999). Table 1 shows \(E_{\text{init}}\) as observed in a number of experiments and calculated from the model.

Note that whereas temperature plays essentially no role in corona initiation from liquid drops, the surface conductivity of ice drops dramatically below about \(-18^\circ\)C,
as does its ability to play the role of 'electrode' in streamer formation (Griffiths and Latham 1974). Thus the ice entries in Table 1 refer to temperatures above this threshold.

Hydrometeor shape is an important factor in determining $E_{\text{init}}$ and leads to a large amount of scatter in $E_{\text{init}}$ values (Schroeder and Baker 1999). The model results presented in Table 1 should thus be considered 'best case' values.

The model results shown here assume that the initial electron avalanche begins from a single free electron on the ionization boundary. We investigated the effect that multiple 'seeds' would have on $E_{\text{init}}$, and found that $E_{\text{init}}$ decreased by $\sim 25\%$ as the seed electron number was increased from 1 to 1000.

Positive discharges are initiated at lower electric fields than negative discharges and as a result most of the hydrometeor investigations (including our own) have focussed on positive discharges (Gallimberti 1979; Bondiou and Gallimberti 1994). These would occur at the lower end of hydrometeors located above the negative charge centre in convective clouds with classical bipolar charge structure. Hydrometeors located below the negative charge centre have negatively charged lower ends. Investigation of this situation will require the modelling of negative streamers (Castellani et al. 1998). No attempt has been made in this study to model these negative processes.

3. RUNAWAY BREAKDOWN

(a) Background: Sources of high-energy electrons

The presence of fast electrons in thunderclouds is typically detected by the bremsstrahlung radiation (X-rays) that they produce. Early attempts, between 1930 and 1980, to detect the presence of these fast electrons produced mixed results. More recent studies by Parks et al. (1981) and McCarthy and Parks (1985), making use of aircraft-borne X-ray spectrometers in thunderstorms, clearly show sharp increases in the X-ray flux prior to lightning strikes.

There are several sources of high-energy electrons in the atmosphere. Chief among these are anthropogenic and natural in situ radioactivity (radon, radium and their decay products), and cosmic-ray fluxes. Israelsson et al. (1987) found a dramatic increase in lightning over Sweden during 1986 accompanying fallout from the Chernobyl nuclear-power-plant accident. In this instance, the increase in ionizing radiation was attributed to the decay of caesium 134, which gives rise to 0.6 MeV electrons. Smaller localized sources of radioactivity may be found near certain industrial centres. These do not contribute significantly to world lightning.

The naturally occurring concentration of radon in clear air near the surface is $\approx 1 \times 10^{-10}$ Cu kg$^{-1}$ or 4 decays sec$^{-1}$kg$^{-1}$, producing a gamma-ray flux of approximately 10 cm$^{-2}$s$^{-1}$, of which about 15% have energies over 1 MeV (Beck 1974). Over oceans at sea level the concentration drops to $\approx 10^{-12}$ Cu kg$^{-1}$ or 0.04 decays sec$^{-1}$kg$^{-1}$. At 6 km the background radon concentration is approximately $2.8 \times 10^{-12}$ Cu kg$^{-1}$ or 0.1 decays sec$^{-1}$kg$^{-1}$, which corresponds to an ion source strength approximately 1/100 times that due to cosmic rays. It is interesting to note that the measured radon concentration aloft in New Mexico clouds was $\approx 1.75 \times 10^{-11}$ Cu kg$^{-1}$ or 0.65 decays sec$^{-1}$kg$^{-1}$, (Wilkening 1970), suggesting that radon near the ground, where the concentration is large, can be transported to higher altitudes in clouds. If this process were occasionally to bring much higher concentrations of radon or its daughters aloft, these might produce large quantities of seed electrons.

In general, however, cosmic rays provide by far the largest natural source of free electrons in the troposphere. The measured fluxes of very high-energy cosmic particles
into our atmosphere are summarized in Table 2 (Gurevich et al. 1999, hereafter GR99). At these high energies the flux is independent of latitude.

The worldwide average lightning frequency is about 12–16 flashes per second, rising to a maximum of around 55 s\(^{-1}\) in the northern-hemisphere summer (Christian et al. 1999). An individual convective thundercloud of area 10 km\(^2\) produces lightning at a typical rate of 1–10 flashes per minute; i.e. 0.01 (km\(^2\)s\(^{-1}\)). The numbers in Table 2 show the flux of very high-energy cosmic-ray particles into the troposphere is significantly greater than these observed lightning frequencies.

The very high-energy cosmic-ray particles undergo collisions with nuclei of atmospheric gases, producing high-energy electrons that lose energy through ionization and bremsstrahlung radiation and gain energy by moving through an electric field. If the field is greater than a threshold ‘break even’ value they become ‘runaway’ electrons. GR99 suggested that these can produce huge electron avalanches and can trigger lightning in a process they term ‘cosmic-ray shower–runaway breakdown’, referred to hereafter as CRSRB. We present a simplified version of their model in the appendix; its results are summarized here.

\( (b) \) The break-even electric field and runaway electrons

Consider an electron (with charge \( e \)) at altitude \( z \) with kinetic energy \( K(z) \) moving in an electric field \( E(z) \) (positive downward). The energy loss due to collisions with air molecules, \( I(K) \), produces an effective frictional force \( F(K, z) \), proportional to the local air density \( \rho(z) \): that is,

\[
\frac{dK}{dz} = eE(z) - I(K)\rho(z) \tag{3}
\]

\[
eE(z) - F(K, z). \tag{4}
\]

\( I(K) \) decreases with increasing \( K \) for \( K < 1 \text{ MeV} \), and reaches a minimum value \( I(K \approx 1 \text{ MeV}) = I_{\text{min}} \); thus, at any altitude \( z \) we can define the frictional force for 1 MeV electrons, \( F_{\text{min}}(z) = I_{\text{min}}\rho(z) \). From Eq. (4) we see that for electric fields such that

\[
E(z) \geq \frac{I_{\text{min}}\rho(z)}{e} = E_{\text{be}}(z) \tag{5}
\]

electrons of \( K = 1 \text{ MeV} \) gain kinetic energy at an increasing rate even while losing energy via ionization and bremsstrahlung radiation. Such electrons are termed \textit{runaways} and \( E_{\text{be}}(z) \) is called the \textit{break-even} electric field.

The critical break-even field for runaway behaviour is

\[
E_{\text{be}}(z) \approx \pm 167\rho(z) \approx \pm 200p(z) \tag{6}
\]

(Marshall et al. 1995a,b), where \( E_{\text{be}} \) is in kV m\(^{-1}\), \( p(z) \) is in atmospheres and we have assumed \( \rho(z) \) (kg m\(^{-3}\)) = 1.208p(z) (atm). In calculations presented below we assume \( p(z) = p(0)\exp(-z/H) \), \( H = 8.4 \text{ km} \).
Figure 2. Sketch of electron multiplication processes in the region of a thunderstorm in which $E(z) \geq E_{be}$ (Eq. (5)) in the runaway breakdown model. This region is assumed to be of depth $L_{be}$, the distance over which the electric field exceeds $E_{breakdown}$, centred around $z_{max}$, the altitude above the surface at which the field reaches its maximum value, $E_{max}$. See text for further explanation.

(c) Evolution of energetic-electron distribution

Figure 2 illustrates the runaway breakdown process. Point A1 shows the process described by GR99, who considered the case that a very high-energy cosmic-ray particle is the source of the initial high-energy electrons. The particle collides with atmospheric nuclei. This collision produces a myriad of particles. Of particular interest are neutral pions that decay, producing two gamma rays that in turn produce the initial high-energy electrons (point ‘B’), sometimes referred to as ‘cosmic-ray secondaries’. Point A2 illustrates the possibility that high-energy decay products emitted from atmospheric radio nuclides produce the high-energy electrons.

The next steps are independent of electron source. (Note that Fig. 2 shows electrons moving downward only. However, the fast electrons would travel upward if subject to electric fields in the opposite sense, such as those typically created by gravitational separation of hydrometeors in convective thunderstorms; they would move horizontally if accelerated by lateral fields in stratiform electrified clouds.)
The in-cloud electric-field magnitude \( E(z) \) is greater than \( E_{\text{be}}(z) \) over a region of length \( L_{\text{be}} \). If \( L_{\text{be}} \) is sufficiently large, the electrons newly produced by ionization can also become runaways. A cascade, or avalanche, of fast electrons ensues and is referred to as runaway breakdown (point 'C' in Fig. 2).

Also of consequence are the even more numerous slower electrons created in the collisions, with energies 1 keV < \( K < 1 \) MeV (point 'D') that do not accelerate enough to become runaway electrons. The maximum concentration of slow electrons, \( N_{\text{se}} \), is produced at the end of the high-field region by the avalanche of runaway electrons.

\( (d) \quad \text{Creation of breakdown electric field} \)

To follow the evolution of the electrons and the accompanying electric fields with high accuracy it would be necessary to use a full simulation of all the particle–particle interactions. We have chosen instead to use an approach based on highly simplified conservation equations, because the analytic nature of the solutions allows us to easily test their sensitivities to parameter variations and to estimate the role of this process in atmospheric clouds. In the appendix we derive the conservation equations for the fast and slow electrons, and use Poisson's Equation to calculate the electric field they produce.

We show (Eq. (A.14)) that if \( N_{\text{se}} \) reaches a critical value in some small region, the electrical conductivity in that region becomes high enough, \( \sigma \approx 10^{-4} \) S m\(^{-1} \), to form a plasma. Because of the high conductivity in the plasma the region quickly polarizes: the electric field within it drops nearly to zero and at the edges \( E > E_{\text{breakdown}} \).

Figure 3 shows the calculated electric-field profile on the axis of a cylindrical shower for a case in which the critical electron density is reached at the lower end of the high-field region. The spikes in the field at the boundaries are the regions in which super-breakdown fields could initiate lightning. The very high field can persist over distances of the order of 10 m. This distance is more than sufficient for a streamer to make the transition between 'cold' processes to a 'hot' leader process (Mrázek et al. 1998;
Bondiou, personal communication). Thus, the requirement for production of lightning by the runaway mechanism is that $N_{sc} \geq N_{crit}$ somewhere in the core of the electron avalanche.

The attractiveness of the runaway mechanism for lightning initiation lies in the fact that at any altitude $E_{be}(z)$, the field required to generate runaway electrons with $K \approx 1$ MeV, is more than an order of magnitude less than $E_{breakdown}$, the local field required to produce conventional breakdown via an avalanche of free, low-energy electrons of energies $K \approx 30$ eV. Thus, in thunderstorms it appears that an avalanche of high-energy electrons is more likely than an avalanche of the more plentiful, low-energy electrons (Marshall et al. 1995a).

(e) Results: Requirements for lightning initiation via runaway breakdown

The plasma, and accompanying breakdown electric fields, can only be formed if $L_{be}$ is sufficiently large. We define $L_{crit}$ as the minimum distance over which $E_{ext}$ must be instantaneously greater than $E_{be}$ in order to form a conducting plasma, in this essentially one-dimensional treatment. Equation (A.15) shows that $L_{crit} = \lambda_i f(z, \delta, \ldots)$, where $\lambda_i$ is the distance between those ionization events in which a runaway electron generates a second runaway. At any altitude $z$, $\lambda_i$ is a function of the air density $\rho(z)$ and of $\delta = E_{max}/E_{be}$. The factor $f$ depends on the original electron source and is a function of the other parameters of the system. We have assumed in this calculation that the drift velocity of the runaway electrons is $c$, the speed of light. Since (Eq. (A.15)) $L_{crit}$ is proportional to the drift velocity ($v_z$), the critical length would decrease if a smaller value were assigned to $v_z$.

Figure 4 shows approximate values of $L_{crit}(z, \delta)$, calculated for a range of values of $\delta$ and $z_{max}$. In these calculations we assume $E_{be}$ remains roughly constant over the vertical distance $-0.5L_{be} \leq (z - z_{max}) \leq 0.5L_{be}$. Figure 4 shows that $L_{crit}$ is quite
sensitive to $z_{\text{max}}$, due largely to decreasing density with altitude which increases the distance between ionization events. Interestingly, $L_{\text{crit}}$ is relatively insensitive to $\delta$. This is significant in that large values of $\delta$ are not observed in thunderstorms. (See section 5.)

Figure 4 shows that for the runaway-breakdown hypothesis to explain lightning initiation at $\delta = 1.2$, and $z_{\text{max}} = 6 \text{ km}$, $E$ must exceed $E_{\text{be}}$ over approximately 2 km. At lower altitudes, the required distance, $L_{\text{crit}}$, drops below 1 km. Thus, in relatively shallow winter storms in regions like Japan, the depth of the high-field region can be relatively small. Note, however, (see Eq. (6)), that $E_{\text{be}}$ increases significantly as $z_{\text{max}}$ decreases. It is important to note also that because of approximations used in our calculation these values of $L_{\text{crit}}$ may be overestimated, as discussed in section 6.

Before we compare these constraints on the electric field with observed fields we discuss the relaxation process in the atmosphere that acts to diminish the electric field in any region where $E > E_{\text{be}}$.

4. DISSIPATION OF THE FIELD BY BACKGROUND HIGH-ENERGY ELECTRONS

As our discussion has shown, the relatively rare very high-energy cosmic-ray particles and high concentrations of radioactive nuclides can lead to avalanches of runaway electrons in regions where the electric field $E(z) > E_{\text{be}}(z)$. On the other hand, on average, there is always a constant, horizontally uniform background source of runaway electrons (produced by background radioactivity and cosmic rays) that create ions at a steady rate. In the absence of accelerating high electric fields this background ion source is balanced by ion recombination, resulting in a steady-state background ion density and the background atmospheric conductivity. In the presence of electric fields $E \geq E_{\text{be}}$ these background electrons become accelerated and produce more electrons.

To investigate this phenomenon we follow McCarthy and Parks (personal communication). Imagine that at $t = 0$ hydrometeor interactions have created an in-cloud electric field $E(z)$ that surpasses $E_{\text{be}}(z)$ somewhere in the cloud. The background runaway electrons accelerate, producing slow electrons and eventually ions. The concentration of these ions begins to increase and thus the electrical conductivity in the cloud also increases. The time rate of change of the electric field due to currents in a medium of electrical conductivity $\sigma$, where $\varepsilon_0$ is the permittivity of a vacuum, is given by

$$\frac{\partial E(z, t)}{\partial t} + \frac{\sigma(z, t)E(z, t)}{\varepsilon_0} = 0.$$  \hspace{1cm} (7)

An increase of the conductivity accelerates the field modification: the background electrons interact with high electric fields to decrease those fields, eventually cutting off the conditions for an avalanche.

In the absence of a cosmic-ray shower the atmospheric conductivity $\sigma$ is due only to the positive and negative ions. Initially, (i.e. just after establishment of the high electric fields in the cloud) the time rate of change of conductivity in the high-field region is given by $\dot{\sigma} = e\mu_{\text{ion}}\dot{N}_{\text{ion}}$, where $\mu_{\text{ion}}$ is the ion mobility and $\dot{N}_{\text{ion}} = (\text{rate of production of background runaway electrons (m}^3\text{s}^{-1}) \times (\text{number of daughter 1 MeV electrons created per background runaway}) \times (\text{number of ions created per 1 MeV electron})$. From Eq. (A.11) we find that in the plasma region

$$\dot{\sigma} = e\mu_{\text{ion}} \times Q \times \left\{\exp(L_{\text{be}}/\lambda_i)\right\} \times \left(\frac{\lambda_i}{\lambda_s}\right)$$  \hspace{1cm} (8)

where the background electron source $Q \approx 10 (\text{m}^3\text{s}^{-1})$ (Daniel and Stephens 1974) and $\lambda_s$ is the ionization length for slow electrons. At a short time $t$ after creation
of the high fields, $\sigma(t) \approx \sigma(0) + \dot{\sigma} t$. Thus, $\tau_{\text{relax}} = (\dot{\sigma}/2e_0)^{-0.5}$. Inserting the values $L_{\text{be}} = 2 \text{ km}$, $\lambda_i = 200 \text{ m}$ into Eq. (8) yields $\tau_{\text{relax}} \approx 0.2 \text{ s}$. As the field strength decreases, the conductivity also decreases and the rate of change of the field becomes more complicated.

Note that according to this simple treatment, the conductivity due to the background flux of electrons varies as $\exp(L_{\text{be}}/\lambda_i)$ and $\tau_{\text{relax}} \propto \exp(-L_{\text{be}}/2\lambda_i)$. Thus, the relaxation time becomes longer if the region of high field strength is shorter. For $L_{\text{be}} = 600 \text{ m}$, for example, $\tau_{\text{relax}} \approx 7 \text{ s}$. For $L_{\text{be}} \ll \lambda_i$ no/insufficient ionization events can take place and the conductivity is essentially unchanged by the field.

This calculation shows two important features of the time dependence of the electric field. First, the field relaxation time $\tau_{\text{relax}} \approx 0.2 \text{ s}$ is much greater than $\tau_{\text{av}} \approx L_{\text{be}}/c$, the time for an avalanche to develop from one high-energy electron burst. Thus if a very high-energy cosmic-ray particle enters the cloud the avalanche it produces can easily be completed before the runaways cause the redistribution of the in-cloud electric field.

The second point is that $\tau_{\text{relax}}$ can be comparable to the resolution time of the balloon measuring systems, $\tau_{\text{balloon}} \sim 0.1 \text{ s}$. Thus, balloon soundings may well miss the high-field excursions.

5. IN-CLOUD ELECTRIC-FIELD PROFILES

We now briefly examine observations of the magnitude and vertical extent of the electric field within thunderstorms, and we compare the observed fields with those required by each of the lightning initiation mechanisms we have discussed. According to the conventional breakdown–hydrometeor hypothesis (Table 1), electric-field magnitudes must reach more than $200 \text{ kV m}^{-1}$ locally to produce streamers, while Fig. 4 shows that according to the runaway-breakdown hypothesis, lightning initiation requires that $E > E_{\text{be}}$ for distances of the order of 1 to 3 km.

(a) The observations

Figure 5 (Marshall et al. 1995a) shows several examples of electric-field soundings made by instruments attached to balloons ascending through Oklahoma and New Mexico storms. Note that only the vertical component of the field is shown, whereas the total magnitude of the field is involved in any breakdown mechanism. According to these and earlier measurements (Stolzenburg et al. 1994) the maximum observed electric fields within thunderclouds rarely exceed $150 \text{ kV m}^{-1}$ and are thus generally substantially smaller than $E_{\text{init}}$, the field needed to generate corona at the surfaces of hydrometeors (see Table 1).

All reported field magnitudes are far below $E_{\text{breakdown}}$. The soundings show that peak fields reach and even slightly exceed $E_{\text{be}}$, although they do not exceed $E_{\text{be}}$ over large distances. Lightning events seem to occur only for $E > E_{\text{be}}$.

In order to interpret these findings we look at the manner in which the measurements are made and at their limitations.

(b) Limitations of balloon soundings

The Lagrangian nature of balloon field soundings means they do not provide an instantaneous profile of the electric field. Moreover, lightning events can alter the electric field, making it difficult to truly know the vertical extent over which $E > E_{\text{be}}$ at any instant.

The balloon soundings are incapable (due to coarse resolution) of resolving intense very local fields such as those shown in Fig. 3. Moreover, the time resolution of the
Figure 5. Balloon soundings of the vertical component of electric field through six different thunderstorms. Each ‘L’ indicates a lightning event and the curves on either side of the sounding give the breakdown electric field ($E_{bc}$) at that altitude. Reproduced from Marshall et al. (1995a).

balloon measurements may not be adequate to capture the very high fields; as we now show, these are of extremely short duration.

(c) Simulation of in-cloud electric fields

We utilize a very crude ‘toy’ model to examine the effects of both background electrons and bursts of high-energy electrons on the development of the electric field in an idealized cloud. The cloud is assumed to be cylindrical with a radius of 1500 m and contains two charging zones in the form of charged regions of equal depth carrying charge of equal magnitude and opposite sign. The negative charge region is centred at 2 km and the positive at 5 km. We assume a constant charge separation, or generating current $I_{gen} = 0.6$ amps. The charging is counteracted, for $E > E_{be}$, by a discharge current due to background electrons. When the electric field exceeds $E_{be}$ at any point a test is performed to see if the distance $L_{be}$ over which $E > E_{be}$ is greater than $L_{crit}$, the depth required to initiate a lightning discharge (see Fig. 4). If this requirement is not met, charge that is separated via the runaway mechanism described in section 4 is distributed along the region where $E > E_{bc}$, and it is diminished by the discharge current, with time constant $\tau_{relax}$. If $L_{be} > L_{crit}$, we simulate a lightning strike by reducing charge by 20% everywhere.
Figure 6. Vertical profile of the modelled electric field: (a) just prior to the moment when $E > E_{be}$, (b) some time after the electric field has exceeded the break-even field but before $L_{be} > L_{crit}$, the requirement for lightning initiation has been met, (c) just prior to a lightning flash and (d) just after lightning.

Figure 7. (a) Vertical profile of the electric field of Fig. 6 as it would be recorded by an instrumented balloon with an ascent rate of 10 m s$^{-1}$. We have assumed an instrumental time constant of $\tau_{balloon} = 0.2$ s. (b) Balloon sounding taken from Ada OK, 1984. See Fig. 5.

In a cloud of area $A$ the recharging, or charge generation time is $\tau_{gen} = E_{be} \varepsilon_0 (A/l_{gen}) \approx 10$ s in the calculations presented here. If $\tau_{gen} \ll \tau_{relax}$ the requirement for initiating lightning is never met because the background flux of high-energy electrons continually prevents the electric field from exceeding $E_{be}$. On the other hand, as the charging rate increases, the local value of $\delta$ increases as the background electrons are not able to limit the field to values near $E_{be}$. Figure 6 shows profiles of the vertical electric field computed using this model.

The flux of background, high-energy electrons effectively prevents the vertical electric field from exceeding $E_{be}$ over very large distances. However, with continued
growth of the main charge regions, the distance over which \( E > E_{\text{be}} \) for short periods of time exceeds \( L_{\text{crit}} \), resulting in a lightning flash. The charge is everywhere reduced and the process continues.

Figure 7 utilizes the same simulation as Fig. 6 but gives the vertical electric-field profile that would be observed by a balloon ascending at a rate of 10 m s\(^{-1}\), starting at the ground 230 s before the first lightning event.

The computed electric-field profile in Fig. 7, is qualitatively similar to observations, shown in Fig. 5. The excursions in the electric field are caused both by lightning events and the background runaways.

6. DISCUSSION AND CONCLUSIONS

The major results of the last three sections can be summarized as follows.

- Conventional breakdown near hydrometeors

  1. Positive corona streamers from ice particles require ambient fields of about 600 kV m\(^{-1}\) at 500 mb and temperatures greater than about \(-18^\circ\text{C}\).
  2. Positive corona streamers from colliding raindrops can occur in electric fields as low as 200 kV m\(^{-1}\) at 500 mb.
  3. Once initiated, corona streamers can continue to propagate as long as the ambient field is over \(\approx200\) kV m\(^{-1}\) at 500 mb.
  4. These results are most sensitive to hydrometeor shape and air pressure.

- Runaway breakdown

  1. Observed frequencies of fluxes of very high-energy cosmic-ray particles into the atmosphere exceed observed lightning frequencies. \textit{In situ} natural radioactivity, due to convergence and lofting of radionuclides from the boundary layer, does not seem adequate to provide the necessary source of high-energy electrons aloft but radioactive pollution events may serve in this capacity.
  2. Production of breakdown electric fields by the runaway mechanism requires instantaneous electric fields of greater than break-even magnitude over distances of up to several kilometers. The length of the required high-field zone increases and the break-even field decreases with increasing altitude.
  3. The charge relaxation time \(\tau_{\text{relax}}\) is comparable to the microphysical charging time \(\tau_{\text{gen}}\); both are generally of the order of a few seconds. Thus, regions in which \( E > E_{\text{be}} \), the break-even field, cannot persist for times greater than this.
  4. These results are dependent on the magnitude of the exponential factor in Eq. (A.15), and thus on \( \lambda_i \), the distance scale for the production of runaway electrons. \( \lambda_i \) is itself a function of air density (and thus the location of the high-field region) and \( \delta = E_{\text{max}}/E_{\text{be}} \). The results also depend strongly on the fast-electron drift velocity, which we have set equal to \( c \). This simplification means that our estimates of \( L_{\text{crit}} \) may be too large by a factor of as much as two or three in some cases.

- Comparison of the observations with the predictions

  1. Balloon measurements of electric-field sounding are compromised by the Lagrangian character of the measurements, the small volume of space sampled and coarse/mixed-phase spatial and temporal resolution. The observations are qualitatively similar to those produced by our toy model (see Figs. 5 and 7), suggesting that runaway breakdown may be involved in lightning initiation.
2. The critical field for conventional breakdown, $E_{\text{init}}$, is several times larger than observed fields. However, given the problems discussed above with balloon soundings, and the fact that $t_{\text{balloon}} \approx t_{\text{relax}}$, the fact that such large electric fields are not observed does not preclude their short-lived existence in small, localized regions of a thundercloud. Conventional breakdown for step (ii) of lightning initiation, therefore, is not absolutely ruled out by the balloon observations. It would appear that this mechanism is most likely when the hydrometeors are drops and, therefore, it might be relevant in the mixed phase of clouds, near the negative charge centre.

3. Recent observations using new lightning locating systems (Defer et al. 2000) show a great deal of lightning-like activity in short bursts high in clouds where the temperatures are likely to fall below $-18\, ^{\circ}\text{C}$. These are more likely to be due to short runaway breakdown events of length $L_{\text{be}} < L_{\text{crit}}$ than to streamers generated by hydrometeors.

4. Satellite observations (e.g. Christian and Latham 1998) show that in the northern hemisphere values of lightning-stroke radiance tend to be greater in winter than summer. In winter, lightning tends to be triggered from lower altitudes than in summer. Both conventional and runaway breakdown critical fields increase as altitude decreases. Therefore, assuming the higher lightning radiance observed in winter results from higher lightning-initiation electric fields, the observed seasonal variation in radiance is consistent with either trigger mechanism.

5. Atmospheric radioactivity is less over the oceans than over continents, while cosmic-ray fluxes of high-energy particles are independent of geography. Thus, the fact that lightning frequency over continents is higher than over oceans might be in part related to lack of initiating high-energy electrons, if in situ sources are important.

It appears that the conventional breakdown mechanism alone is not a satisfactory explanation of lightning triggering in most cases, while the runaway-breakdown mechanism appears more likely. However, current uncertainties in the theory and observations prevent more definite conclusions.

(a) Directions for future work

In order to make further progress, the following data would be most useful:

1. Instantaneous small-scale electric and microphysical measurements. Gurevich (personal communication) and colleagues have performed and wish to conduct further experiments in the Tien-Shan mountains correlating cosmic-ray-shower electrons, muons, neutrons, etc. with X-ray and $\gamma$-ray emissions. They will attempt to locate the electromagnetic emissions generated by the polarization currents which arise in the CRSRB process. If they can also locate lightning strokes during these measurements, important correlations predicted by the CRSRB mechanism can be studied.

2. Data on the microphysical parameters of the model, particularly $\lambda_3$ and the collision and attachment frequencies, for the pressures and gases found in atmospheric clouds and relevant ranges of electron energies for both conventional and runaway breakdown.

Moreover, the theoretical treatment presented here is highly schematic. A more accurate treatment would include:

1. Three-dimensional particle simulations in the spatially varying cloud atmosphere, with a range of electron energies.
2. Relativistic effects.
3. Multiple sources of high-energy electrons. In our numerical work we have assumed one initial cosmic-ray particle of energy around $10^{15}$ eV is the source of the high-energy electrons. However, if several triggering events were to follow each other at time intervals of the order of the shower formation time, intense local fields can be created even for much smaller values of $L_{\text{crit}}$ than we have calculated here.

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APPENDIX

Runaway-breakdown model equations

Here we present our application of the GR99 model for cosmic-ray shower–runaway breakdown. For further detail that paper should be consulted.

(b) Fast-electron concentration

We assume that prior to the cosmic-ray shower the in-cloud electric field $E(z)$ exceeds $E_{\text{be}}$ over a depth $L_{\text{be}}$. Somewhere in this region the field reaches a maximum value $E_{\text{max}} = E_{\text{be}}\delta$. It is convenient to assume a parabolic profile:

$$E(z) = E_{\text{be}} \left[ 1 + (\delta - 1) \left( 1 - \frac{4z^2}{L_{\text{be}}^2} \right) \right]$$

(A.1)

where $z' \equiv z_{\text{max}} - z$. We refer to the region $-0.5L_{\text{be}} \leq z' \leq 0.5L_{\text{be}}$ as the high-field region. We assume this region is sufficiently short that the break-even field strength does not vary greatly within it.

In the case that the source of high-energy electrons is a cosmic-ray particle of energy $\varepsilon_{\text{cr}}$, the density of fast electrons (called ‘secondary’ electrons) produced by the interaction between cosmic rays and air nuclei is given by the NKG (Nishimura, Kamata, Greisen) empirical formula (Nishimura, personal communication). For altitudes between about 3 and 10 km in air the density of secondary electrons ($\bar{\rho}_e$) near the axis in the plane orthogonal to the axis is

$$\bar{\rho}_e(\varepsilon)(\text{m}^{-2}) \approx 2.4 \times 10^{-3}(\varepsilon_{\text{cr}}/\beta)/\sqrt{\ln(\varepsilon_{\text{cr}}/\beta)}.$$

(A.2)

For air, $\beta \approx 72$ MeV.

The mean energy of the secondaries, initially $\approx 30$ MeV, decays to about 1 MeV over a distance $L_0(m) \approx 30$ MeV/$eE_{\text{be}}$. Thus, the cosmic-ray particle gives rise to a cylindrical volume of runaway ($K \approx 1$ MeV) electrons of density

$$N_{e,\text{initial}}(\text{m}^{-3}) = \frac{\bar{\rho}_e(\varepsilon_{\text{cr}})}{L_0}$$

(A.3)

which proceed downward at velocity $c$. They pass through the high-field region so quickly that their effect is that of a spatially uniform, almost instantaneous source of fast electrons everywhere in the high-field region. As they pass through the region they
ionize air molecules, creating daughter electrons that diffuse radially and drift vertically in the region of high electric field, and in turn create new electrons via ionization. Thus, the number of new fast electrons grows with distance travelled in the high-field region. Neglecting the small lateral diffusion, the density of high-energy electrons obeys the conservation equation

\[ \frac{\partial N_{fe}}{\partial t} + \frac{\partial}{\partial z}(v_z N_{fe}) = \frac{e N_{fe}}{\lambda_i}. \] (A.4)

The mean distance between ionizing events giving rise to new 1 MeV electrons is \( \lambda_i \):

\[ \lambda_i(z) = \frac{m c^2 u(\delta) a}{2e E_{be}(z)}, \] (A.5)

where \( m \) is the mass of the electron. The factor \( a \approx 10 \) is the log of the ratio of the electron energy (here, about 1 MeV) and the average ionization energy in air. \( u(\delta)mc^2/2 \) is the minimum kinetic energy a newly created electron can have if it is to accelerate to 1 MeV in an electric field of magnitude \( E = E_{be} \delta \). The function \( u(\delta) \approx (1/\delta)(1 - 0.1 \ln(\delta)) \) for \( \delta \gg 1 \); a power series for this function can be derived from the results in Gurevich et al. (1992).

(Note that the characteristic distance between events creating runaway electrons is actually larger than the distance between ionizing collisions because some of the electrons produced by ionization must be accelerated up to 1 MeV before they join the flux of runaways. However, it is easy to show that the acceleration distance is small compared with \( \lambda_i \), so the approximation is valid within the spirit of this argument.)

We assume the initial fast electrons travel the full length \( L_{be} \) of the high-field region. Then solution of Eq. (A.4) with initial condition Eq. (A.3) shows that the maximum fast-electron density is found at the lower end of this region, where

\[ N_{fe}^{\text{max}} = \frac{\bar{\rho}_{e}}{L_0} \exp \left( \frac{L_{be} c}{v_z \lambda_i} \right). \] (A.6)

The drift velocity of the fast electrons is \( v_z \) and this solution holds for times of the order of, but somewhat greater than, \( L_{be}/v_z \). Note that we have assumed in Eq. (A.6) that the drift velocity and the ionization length \( \lambda_i \) are independent of \( z \) in this region. These approximations are appropriate since in typical thunderstorms the electric field does not vary much in the high-field region.

\( v_z \) has been measured for slow electrons and parametrized as a decreasing function of \( E/p \) (Badaloni and Gallimberti 1972), but its value for fast electrons must be estimated from calculated frequencies of collisions of 1 MeV electrons with neutral air molecules; \( v_z(K, z) = I(K)\rho(z)/(m v_{\text{coll,fe}}) \). Taking \( v_{\text{coll,fe}} \approx 10^8 \text{ s}^{-1} \) (GR99) gives \( v_z(1 \text{ MeV}, z) \approx 0.5c \).

The maximum density of fast electrons occurs in the case that the original cosmic-ray particle collision occurred near one end of the high-field zone. In this case, at the other end of the high-field zone the density rises to a maximum at \( t \approx L_{be}/c \) and decays again. The maximum concentration is approximately

\[ N_{fe}^{\text{max}} \approx \frac{\bar{\rho}_{e}}{L_0} \exp \left( \frac{L_{be}}{\lambda_i} \right). \] (A.7)

We use this density as a starting point for our calculation of the slow-electron density, below.
(c) Production of slow electrons

In addition to the fast electrons produced under runaway conditions, a large number of slow electrons of energies $1 \text{ keV} \leq K \leq 1 \text{ MeV}$ are produced in the high-field region. Their density, $N_{se} (\text{m}^{-3})$ increases at a rate

$$\frac{\partial N_{se}}{\partial z} = \frac{N_{fe}}{\lambda_s} \tag{A.8}$$

where the ionization length for the slow electrons, $\lambda_s$, is about $6 \times 10^{-6} \text{ m}$.

The initial density of the slow electrons at its maximum is, therefore,

$$N_{se}^{\text{max}} \approx \frac{\lambda_i}{\lambda_s} N_{fe}^{\text{max}} = \frac{\bar{\rho}_e(e_{cr}) \lambda_i}{L_0} \frac{\lambda_i}{\lambda_s} \exp \left( \frac{L_{be}/\lambda_i}{L_0} \right). \tag{A.9}$$

(d) Development of high local electric fields in the avalanche

We now consider the modification of the high electric field by the slow electrons during a shower. The time rate of change of the electric field is given by Eq. (7) and its spatial variation is given by Poisson’s Equation:

$$\vec{\nabla} \cdot \vec{E} = \frac{\sigma}{\varepsilon_0} \left( N_+ - N_- - N_{se} \right) \tag{A.10}$$

where $N_+$ and $N_-$ are the concentrations of positive and negative ions, respectively. After creation, the concentration of slow electrons declines due to attachment to oxygen molecules, at rate $\nu_a (\text{s}^{-1})$, and to motion in the electric field:

$$\frac{\partial N_{se}(z, t)}{\partial t} = -\vec{\nabla} \cdot \frac{\sigma(z, t) \vec{E}(z, t)}{\varepsilon_0} - \nu_a N_{se}(z, t). \tag{A.11}$$

The attachment frequency $\nu_a \sim \eta / v_z$ (see Eq. 2) where $v_z$ is the drift velocity of a slow electron, estimated from Badaloni and Gallimberti (1972) as a function of electric field. In the simplified treatment presented here, we use a mean value; $\nu_a = 10^7 \text{ s}^{-1}$ (Morrow 1996; GR99).

During an avalanche the ions may be considered stationary, so the electrical conductivity is due only to the electrons.

$$\sigma(z, t) = N_{se}(z, t) e \mu_{se} = \frac{N_{se} e^2}{m v_{\text{coll,se}}}, \tag{A.12}$$

where $\mu_{se}$ is the mobility of slow electrons and $v_{\text{coll,se}} (\text{s}^{-1})$ is the rate of collisions of slow electrons with neutral molecules. We use a mean value; $v_{\text{coll,se}} = 8 \times 10^{10} \text{ s}^{-1}$ (Berezin et al. 1989; GR99).

From Eq. (A.11) we can define two regions in the shower. In the outer part of the shower, where the density of slow electrons is relatively low, the electron concentration (and, therefore, the electrical conductivity and, hence, the electric field) decreases due to attachment to air molecules. On the other hand, in regions where the local electron density is high, charge motion redistributes the charge more quickly than electron attachment, and a plasma patch is created. The interface between these regions is defined by the condition that

$$(\nu_a)^{-1} = \frac{\sigma(z_{\text{boundary}}, \tau_{\text{av}})}{\varepsilon_0}. \tag{A.13}$$
From Eq. (A.12) this defines the critical slow-electron density,

$$N_{\text{crit}} \approx \frac{m_e v_{\text{sc}} c \rho_{\text{coll,se}}}{e^2}. \quad (A.14)$$

Under typical circumstances, this yields $N_{\text{crit}} \approx 3 \times 10^{14}$ m$^{-3}$. The charge flows to the boundary of the plasma region and produces intense electric fields in its vicinity.

In order to create the intense fields the depth $L_{\text{be}}$ must be large enough to create $N_{\text{crit}}$ electrons m$^{-3}$ over a small region. That is, from Eq. (A.9), the criterion for runaway breakdown is approximately

$$L_{\text{crit}} = L_{\text{be}} = \lambda_i \ln \left( \frac{N_{\text{crit}} c L_0}{\rho_e v_z \lambda_i} \right). \quad (A.15)$$

The results shown in Fig. (3) were calculated assuming $v_z = c$ using a Runge–Kutta scheme to follow the time evolution of the charged species, whose conservation equations are: Eq. (A.11) for the slow electrons, and

$$\frac{\partial N_-}{\partial t} = v_a N_{\text{se}} \quad (A.16)$$

$$\frac{\partial N_+}{\partial t} = 0 \quad (A.17)$$

with initial conditions $E(z, 0)$ given by Eq. (A.1), $N_{\text{se}}(z, 0) = N_{\text{se}}^{\text{max}}$ (Eq. (A.9)),

$$N_-(z, 0) = 0 \quad (A.18)$$

$$N_+(z, 0) = N_{\text{se}}(z, 0). \quad (A.19)$$

At the beginning of each time step the conductivity is calculated for each level and the resulting charge concentrations at the end of the time step are used to evaluate the electric field (see Fig. 3) and to re-evaluate the conductivity. The time step $\Delta t$ is chosen such that $N_{\text{se}} \Delta t^{-1}$ is greater than any term on the right-hand side of Eq. (A.11).

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