Modelling the generation of gravity waves by a maritime continent thunderstorm

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SUMMARY

Observed wind and temperature profiles are used to initialize a model calculation of a maritime continent thunderstorm, and the numerical solution is used to explore the effect of tropospheric shear on the gravity-wave generation. The resultant convective system is qualitatively similar to that observed. The modelled gravity waves propagate away from the cloud with wave-fronts that are approximately circular, implying that the convective clouds do not generate waves which propagate in a preferred direction.

The gravity-wave generation is related to the oscillation of the convective updraughts about their level of neutral buoyancy. While the general features of the gravity waves are similar in most respects to a previous study which used an idealized wind profile, the frequency of the gravity waves is Doppler-shifted by the tropospheric wind shear. The result is a much broader power spectrum in comparison to the idealized cases.

KEYWORDS: Cloud-resolving models Convection Gravity waves MCTEX

1. INTRODUCTION

In general, internal gravity waves have length-scales that cannot be resolved properly by current general-circulation models (GCMs). However, when the vertical flux of horizontal momentum associated with vertically propagating gravity waves is ignored, GCMs produce middle-atmosphere circulations that are too close to radiative equilibrium. In particular, they do not reproduce the observed mesospheric temperature reversal (see Holton (1982) for details), nor do they accurately simulate the middle-atmospheric winds. For these reasons virtually all GCMs now include, albeit crudely, simple parametrizations of gravity-wave drag. These parametrizations have led to improvements in the modelled large-scale circulation, although there remain serious errors. (For further details see Hamilton (1997) and the papers therein.)

Most work on parametrizing gravity-wave drag in GCMs has focused on stationary, orographically generated waves (e.g. Palmer et al. 1986; Lott and Miller 1997), and there have been few attempts to parametrize convectively generated waves even though their effects may be relatively large (the main exceptions being Rind et al. (1988), Kershaw (1995), Chun and Baik (1998) and Chun et al. (2001)). The central problem in parametrizing convectively generated waves is that the mechanisms responsible have not been well understood. In the literature, there are three opposing views on the mechanism by which clouds generate gravity waves.

The first of these is the so-called obstacle effect, in which waves are generated by the drag induced when cloud tops impinge on a shear layer. These waves are said to behave like steady mountain waves, implying that they only propagate upstream. The obstacle effect is at the heart of Kershaw’s (1995) parametrization of the drag induced by convectively generated gravity waves. While the steady-state assumption allows analytic progress, observations (e.g. Pfister et al. 1993) and modelling studies (e.g. Piani et al. 2000; Lane et al. 2001) have shown that deep convection generates gravity waves that propagate in all directions away from the cloud, including downstream. For this reason, some studies (e.g. Pfister et al. 1993) have likened the convection to a transient mountain that generates waves as it displaces the surrounding isentropes.

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The second mechanism is the time-varying convective heating within a cloud, which is often assumed to be the source of the gravity waves (e.g. Bretherton and Smolarkiewicz 1989). As a test of this mechanism, Pandya and Alexander (1999) used the convective heating produced in a reasonably complete numerical cloud model to force a linear model. Among other things, they showed that, although such a forcing produced a spectrum of waves with the same shape as a fully nonlinear model, the waves amplitudes were grossly overestimated. Chun and Baik (1998) examined the waves generated by a steady, externally imposed heat source and applied their analysis to a parametrization of convectively generated waves. This parametrization was later implemented in a GCM by Chun et al. (2001). Dynamically, the gravity waves generated by this mechanism are similar to those generated by the steady obstacle effect.

The third mechanism by which gravity waves are generated is the oscillation of convective updraughts about their level of neutral buoyancy; this mechanism was originally proposed by Pierce and Coroniti (1966) and modelled by Fovell et al. (1992). This mechanism is essentially the same as that discussed by Lane et al. (2001) (hereafter referred to as LRC).

All three mechanisms may play roles in generating waves. However, using a cloud-resolving model of deep convection, LRC found that the largest-amplitude waves are non-hydrostatic and are generated by the oscillation of convective updraughts about their level of neutral buoyancy. This is probably the case for deep-convective-cloud populations, but the mechanisms may be different for shallow or more organized systems.

LRC studied a maritime continent thunderstorm over the Tiwi Islands, Australia. These convective systems, known locally as Hectors, occur regularly in the pre-monsoon period and were the focus of the recent Maritime Continent Thunderstorm Experiment (MCTEX). (For further details see Keenan et al. (1990, 2000).) The primary aim of LRC was to clarify the basic dynamical process by which individual convective clouds generate gravity waves, and for this reason they used an idealized wind profile to define the environmental flow. Like LRC, all previous studies that have modelled the generation of waves by convection have done so in a highly idealized environment. One of the key aims of the present paper is to examine how these results are modified by a more realistic environmental wind profile, focusing especially on the role played by middle-tropospheric wind shear. It will be shown that shear in the middle troposphere plays an important role in attenuating the power in parts of the wave spectrum through critical-level dissipation, while broadening the remaining part.

The aim of this paper is to determine the effect of tropospheric wind shear on the generation of gravity waves by convective clouds. It will be shown that, while the wave-generation mechanism is consistent with that described by LRC, tropospheric shear has a profound influence on the subsequent evolution of the power spectrum. It will be shown that shear in the middle troposphere plays an important role in attenuating parts of the wave spectrum through critical-layer dissipation and by Doppler-shifting high-frequency, downstream-propagating waves to intrinsic frequencies beyond the local Brunt–Väisälä frequency. The middle-level shear plays a key role also in broadening remaining parts of the wave spectrum.

The remainder of this paper is organized as follows. Section 2 describes the model configuration and its initialization. In section 3, the evolution of the convective system is outlined and compared briefly to observations taken during MCTEX. The general features of the wave field are discussed in section 4. In section 5, power spectra of the modelled gravity waves are calculated, and compared to a spectrum derived from LRC's model calculation. In section 6, a simple representation of the gravity-wave source,
incorporating the effects of tropospheric shear, is introduced. The idealized spectrum is compared to the spectrum derived from the model calculations. Finally, in section 7 the results and conclusions are summarized.

2. Model Description

The numerical model was originally developed by Clark (1977). In brief, the model is three-dimensional, non-hydrostatic and anelastic. It is formulated in terms of terrain-following height coordinates, and the finite-difference approximations are second order in both space and time. The model includes parametrizations of the boundary-layer physics, warm rain processes, ice microphysics (Koenig and Murray 1976), and subgrid-scale turbulence (Lilly 1962; Smagorinsky 1963). The model geometry will be discussed in height coordinates, with \( x, y, \) and \( z \) representing the zonal, meridional, and vertical directions, respectively.

The present configuration is the same as that used in LRC (their experiment E2). The model has two computational domains. The first is Domain 1, which has a horizontal resolution of 2 km, encompasses the Tiwi Islands and surrounding ocean, and uses open lateral boundaries. The second, Domain 2, is nested both horizontally and vertically within Domain 1. It has 1 km horizontal resolution, and covers mainly the land mass associated with the Tiwi Islands. Both domains have 162 and 92 grid points in the zonal and meridional directions, respectively. In the vertical, Domain 1 (Domain 2) extends to 50 km (19 km), and the grid length varies from 50 m (25 m) at the surface, to 400 m (200 m) in the middle troposphere, and finally 800 m (400 m) in the stratosphere. To absorb vertically propagating waves, the uppermost 10 km of Domain 1 includes a Rayleigh-friction sponge layer. While the model incorporates the Tiwi Islands' topology, the topography from the mainland that encroaches on Domain 1 is removed for this calculation. For simplicity, the calculation presented here assumes an \( f \)-plane approximation, and neglects internal absorption and scattering of solar radiation. Over land, the calculation assumes the Bowen ratio is 1 and the albedo is 0.2. For further details of the accuracy of the nesting and vertical grid refinement see Clark and Farley (1984) and Clark and Hall (1996), respectively.

The numerical calculation is initialized with a single sounding taken during MCTEX at 0400 Local Standard Time (LST) 27 November 1995 (LST = UTC + 9.5 hours). The procedure for initializing the model with a single sounding is discussed more fully by Lane et al. (2000). The radiosonde sounding on this day reached a height of 24 km. Above this height, isothermal and dry initial conditions are assumed. The tropopause is located at a height of 16 km, and the moisture profile is conditionally unstable. As the water-vapour mixing ratio near the surface is relatively large (approximately 20 g kg\(^{-1}\)), the lifting condensation level is only 700 m above ground level. The level of free convection ranges from 1.5–3.0 km, depending on the origin of the lifted parcel; likewise the level of neutral buoyancy (LNB) ranges from 10–14 km. The wind profile used to initialize the model integration is based on this observed sounding and is shown in Fig. 1. The wind profile consists of a lower-tropospheric easterly jet (\(-8\) m s\(^{-1}\)), which changes slowly to an upper-tropospheric westerly jet (10 m s\(^{-1}\)). There is a shear layer just below the tropopause where the flow changes from westerly (7 m s\(^{-1}\)) to easterly again (\(-5\) m s\(^{-1}\)). The meridional flow is almost zero with a slight northerly bias (approximately \(-2\) m s\(^{-1}\)) in the lower troposphere. Carbone et al. (2000) suggest that the lower-tropospheric easterly jet plays a fundamental role in organizing the convection.
Figure 1. The (a) zonal ($U$) and (b) meridional ($V$) components of the wind profile from the 0400 LST 27 November 1995 sounding during MCTEX (solid line). Also shown are the approximations to these profiles that are used to initialize the model calculation (dashed).

This study is primarily concerned with the gravity waves generated, and how the inclusion of tropospheric shear affects the propagation of these waves. Lower-tropospheric shear will control the organization of the convection, whereas upper-tropospheric shear will have little effect on the convective organization. With this in mind, and to simplify the analysis of the gravity waves, the wind profile is simplified above 10 km in height. Nevertheless, the resultant profile is not significantly different from the observations. To further simplify the analysis, the stratospheric wind is chosen to have zero shear. Stratospheric shear has no effect on the generation of the waves, and its inclusion only obscures the source mechanism by changing the medium through which the gravity waves propagate, possibly introducing critical levels. The wind profile used to initialize the model is also shown in Fig. 1.

The model integration begins at 0700 LST with only Domain 1. At 1000 LST Domain 2 is initialized by interpolating the solution from Domain 1. Both Domain 1 and Domain 2 are integrated until 1700 LST with numerical time steps of 10 and 5 seconds, respectively.

3. Evolution of the Convective Clouds

The modelled convection evolves in a similar way to that in the idealized work described by LRC. The depth and intensity of the convection are almost identical as the same thermodynamic profile is used to initialize both numerical integrations. Consequently, the sea-breeze development and the amount of work done to overcome the convective inhibition are very similar in each case. The major differences between the two cases lie in the level of organization of the convective clouds, and the orientation of the cloud bands. The reason for these differences is discussed in more detail later in the section.
A plan view of the vertically integrated total cloud water (condensed water plus ice) at 1330 LST is shown in Fig. 2. At 1200 LST, the sea-breeze convergence is sufficiently strong to initiate a number of deep convective cells at the southern end of the islands. By 1330 LST (Fig. 2), some of these cells have merged to form a zonally oriented convective line at approximately \( y = 65 \) km. A second line forms parallel and approximately 30 km north of the first line. Each cloud line is approximately 100 km long, although the second line is less contiguous than the first, consisting principally of two or three very large convective cells. The maximum convective updraught (approximately 30 m s\(^{-1}\)) occurs around 1300–1400 LST, after which time the convection is dominated by evaporatively driven downdraughts.

During MCTEX, a number of observational instruments were deployed to construct a detailed picture of the convection. One such instrument was a polarimetric weather radar. In general, reflectivities from such a radar gives information about rainwater and ice particles suspended within the cloud; this information is usually converted to an estimation of rainfall rates using empirical formulae. In this paper, the fine-scale details of the convective system are not the primary concern, and the radar data are used as a general guide to the orientation and development of the mature convective system.

The radar reflectivity at 1501 LST is shown in Fig. 3. The image shows that the convection is dominant in the south-western part of the island with two east–west oriented convective lines evident. The more northward line consists of larger, more intense structures, which dominate the island convection. The radar reflectivity suggests that the orientation and general structure of the convection are reproduced by the model (c.f. Fig. 2). However, the modelled convective system develops at least an hour earlier than that observed. Nevertheless, the level of agreement between the observations and the numerical model is satisfactory to address the central aim of this study. For more detailed studies concerning the simulation of these systems, see Golding (1993), Saito et al. (2001) and Crook (2001).

In comparison with the model run described here, a number of shorter, less organized cloud bands developed in the model run of LRC. This change is most probably due to
the different low-level (<1 km) winds in the initial profiles. In particular, the low-level flow in LRC was weak and easterly, while here the low-level flow is dominated by a slightly stronger (approximately 3 m s\(^{-1}\)) northerly flow (in addition to the easterly flow). The sea breezes, which propagate in the opposite direction to the low-level sheared flow, promote lifting at the sea-breeze front, generating a line of convective cells. This explains the tendency for preferred convective development at the southern part of the island here, and the western part of the island in LRC. Nonetheless, the convergence caused by both sea breezes, which propagate in opposing directions, is probably the most important mechanism. In addition to the change in orientation of the cloud bands, the convection here is more organized than that modelled by LRC. This organization is probably also due to the interaction of gust fronts with the increased low-level shear included here.

4. GENERAL FEATURES OF THE WAVE FIELD

Throughout the model integration, gravity waves are prominent features of both the modelled troposphere and the modelled stratosphere. The wave amplitude, which increases as the strength of the convection increases, has a maximum in the stratosphere at about 1400 LST, shortly after the maximum convective activity. Zonal and meridional cross-sections of vertical velocity at 1300 LST (Fig. 4) show that the stratospheric waves have amplitudes of about 0.3 m s\(^{-1}\), horizontal wavelengths of approximately 15–20 km, and vertical wavelengths of approximately 4–6 km.

A horizontal cross-section of the vertical velocity through \(z = 40\) km (Fig. 5) shows families of concentric rings centred over individual convective clouds. These rings are the wave-fronts, which propagate radially away (and upward) from the clouds. Note that the vertical velocities are generally larger on the eastern side of the rings. This asymmetry is apparent also in the zonal cross-section of the vertical velocity (Fig. 4(a)), and is due to the refraction of the waves as they propagate upwards through
Figure 4. (a) Zonal cross-section of vertical velocity through $y = 100$ km for Domain 1. (b) Meridional cross-section of vertical velocity through $x = 100$ km for Domain 1. Vertical velocity is contoured at 0.1 m s$^{-1}$ intervals, with the negative values dashed. Both plots are valid at 1300 LST. Note that (b) has a different horizontal scale from (a).

Figure 5. Horizontal cross-section of vertical velocity at $z = 40$ km in Domain 1 at 1330 LST.
the shear layer in the upper troposphere. (See LRC for a more detailed discussion.) The wave sources implied in Fig. 5 are highly localized, and their position and number are highly variable.

5. SPECTRAL ANALYSIS

Spectral analysis is probably the technique most commonly used to identify gravity waves in both observations and model calculations. However, when the wave source is highly variable, such analyses can be difficult to interpret. The aim of the following section is to explore the relationship between the most energetic peaks in the power spectrum and the modelled wave field.

In a steady, horizontally homogeneous, background flow, the frequency and horizontal wave number of linear gravity waves are conserved following wave-packet trajectories (see Bretherton (1966) for details). In contrast, the vertical wave number is not conserved and may vary strongly due to vertical changes in stratification and background-flow velocity. Therefore, frequency and horizontal wave-number spectra are less sensitive than vertical wave-number spectra to variations in background flow, and for this reason they will be used here.

The power spectra are calculated from a two-dimensional (horizontal) space–time section of vertical velocity. At a constant height, it is assumed that the vertical velocity, \( w \), is of the form

\[
w(x, t) = \text{Re} \left\{ \int_{k} \int_{\omega} A(k, \omega) \exp[i(kx - \omega t)] \, d\omega \, dk \right\},
\]

where the frequency \( \omega \) is non-negative, the zonal wave number \( k \) can be either positive or negative, \( A \) is the spectral amplitude, and \( \text{Re} \{ \} \) represents the real part of the term enclosed in braces. The horizontal phase speed (more correctly, the trace speed) \( c \) is defined as

\[
c = \frac{\omega}{k},
\]

and therefore, \( k > 0 \) (\( k < 0 \)) represents waves propagating in the positive (negative) \( x \)-direction. If the meridional wave number \( l \) replaces \( k \) and \( y \) replaces \( x \), Eqs. (1) and (2) describe the formulation in the meridional direction. These two-dimensional spectra are calculated from a space–time section of vertical velocity taken at a height of 20 km for both the zonal and the meridional cases separately. The absolute value of the derived spectrum is squared (to give the power) then smoothed over five adjacent frequency/wave-number bins. The sampling height of 20 km is chosen for two reasons. The first and most important of these is that the analysis is conducted close to the source, yet far enough away that convective updraughts do not directly influence the analysis. Second, 20 km is above any wind shear and consequently well away from the regions where wave refraction or critical-level dissipation affect the spectra.

The vertical velocity from Domain 1 is stored at 2 minute intervals for the entire model run (0700–1700 LST) at the zonal cross-sections \( y = 60 \), 90 and 120 km and at the meridional cross-sections \( x = 110 \), 160 and 210 km. A power spectrum is calculated for each individual cross-section, and the respective spectra are averaged to form one zonal and one meridional spectrum.
Figure 6. Power spectrum of vertical velocity for (a) the meridional direction and (b) the zonal direction, calculated at height $z = 20$ km from Domain 1 (see text). For clarity, the spectra are normalized with logarithmic contours and only the largest 1.5 orders of magnitude are shown ($10^{-15}$–1 in 15 contours), with dark shading representing high spectral power. Also shown are contours of horizontal phase speed $c$ at 5 m s$^{-1}$ intervals.

(a) General features

Figures 6(a) and (b) show the power spectrum of the vertical velocity in the meridional and zonal directions, respectively, for the entire time integration. For clarity, only the largest 1.5 orders of magnitude are shown (in 15 contours). The horizontal phase speed $c$ is plotted also.

First consider the meridional spectrum (Fig. 6(a)). The power is largest for frequencies around 0.005–0.006 s$^{-1}$ (periods of approximately 20–17 minutes), and for wave numbers in the range $2–4 \times 10^{-4}$ m$^{-1}$ (wavelengths from 30–15 km) in both propagation directions. The spectrum is roughly symmetric about $l = 0$, although waves propagating to the south ($l < 0$) have larger amplitudes and are biased slightly towards higher frequencies. Waves with phase speeds between $-5$ and $5$ m s$^{-1}$ contribute little to the spectral power. This may be due to critical-level dissipation, as a consequence of the relatively weak meridional winds in the troposphere (bounded between $\pm 3$ m s$^{-1}$),
or it may be a property of the wave source. There is little spectral power at frequencies greater than 0.008 s\(^{-1}\), which is close to the upper-tropospheric Brunt–Väisälä frequency. As discussed by Pandya and Alexander (1999), waves with intrinsic frequencies greater than the upper-tropospheric Brunt–Väisälä frequency are evanescent when they propagate vertically through such a region. (The intrinsic frequency, \(\tilde{\omega} = \omega - U k - V \dot{l}\), where \(U\) and \(V\) are the zonal and meridional wind, respectively, is the frequency viewed in a reference frame moving with the local wind.)

Now consider the zonal spectrum (Fig. 6(b)). The maxima in the zonal spectrum are located in much the same regions as the maximum in the meridional spectrum. However, the zonal spectrum is less smooth than the meridional spectrum and the dominant frequencies are not as well defined. Furthermore, the zonal spectrum is highly asymmetric about \(k = 0\), with higher-frequency eastward-propagating \((k > 0)\) waves more prevalent. Waves with phase speeds in the range \(-5 \leq c \leq 5 \) m s\(^{-1}\) are absorbed by critical-level dissipation within the shear layer centred below the tropopause, and this process is reflected in the shape of the spectrum. At 20 km the zonal wind is easterly with a magnitude of 5 m s\(^{-1}\) (Fig. 1) and, consequently, waves generated in the troposphere will be refracted as they propagate into the stratosphere (as described in LRC). Waves with \(k < 0\) propagate downstream and their phase lines are refracted towards the horizontal. Thus, their vertical-velocity signature is reduced. This reduction is evident as a lack of spectral power of the lower phase-speed waves \((-5 \geq c \geq -10\) m s\(^{-1}\)), which suffer the greatest refraction to the horizontal.

While both spectra in Fig. 6 are biased, the bias in the zonal spectrum towards eastward-propagating waves is more pronounced than the bias in the meridional spectrum towards southward-propagating waves. The bias in the spectra can be explained as follows. Suppose a cloud emits a pulse of gravity waves at the LNB, and that the waves propagate in both the positive and negative directions with a wave number of given magnitude and a given source frequency. (The source frequency \(\tilde{\omega}_{\text{source}}\), is the frequency of the waves viewed in a frame of reference moving with the wind at the LNB.) The ground-based frequency is related to the source frequency through the expression

\[
\omega = \tilde{\omega}_{\text{source}} + U_{\text{source}} k + V_{\text{source}} \dot{l},
\]

where \(U_{\text{source}}\) and \(V_{\text{source}}\) are the zonal and meridional components of the wind at the LNB. Equation (3) implies that, if the flow at the LNB is positive, waves with positive (negative) horizontal wave numbers will have larger (smaller) ground-based frequencies than their source frequency. As the horizontal wave number is constant along a ray, a larger (smaller) ground-based phase speed corresponds to a larger (smaller) ground-based frequency. Thus, a bias towards higher-frequency, eastward propagating waves (in the zonal spectrum) suggests that the waves are generated in a region where the flow is positive, implying that the most energetic waves are generated in the height range 10 < z < 14 km (see Fig. 1). Likewise, the slight bias in the meridional spectrum towards higher-frequency, southward-propagating waves suggests that these waves are generated in a region where the meridional flow is negative, which is anywhere between 5–10 km (see Fig. 1). Since the meridional spectrum is broadly symmetric, the waves are probably generated in the same region as the zonally propagating waves (where the meridional flow is zero). Note that in the horizontal plane the phase lines are approximately circular, implying that the most energetic waves in the zonal and meridional directions are part of the same wave packet. Of course, not every cloud in the cluster has the same depth, and consequently the shift in frequency will depend on the velocity of the wind at the LNB of each individual cloud. This point will be discussed in more detail in section 6.
(b) *Comparison with the spectrum from LRC*

Although the waves in the stratosphere appear relatively monochromatic (e.g. Fig. 4), the spectra are relatively broad (e.g. Fig. 6). LRC considered a similar numerical model integration using an idealized wind profile. Their tropospheric wind profile had zero meridional flow, and was essentially \(-5 \text{ m s}^{-1}\) in the zonal direction. Therefore, the primary difference between LRC and the case considered here is the tropospheric wind shear. The effect of wind shear on the gravity-wave spectrum is determined by comparing the \(\omega-k\) spectrum (Fig. 6(b)) with an equivalent spectrum from LRC’s model integration.

In the model run considered here, tropospheric shear affects the power spectra in two ways. First, the gravity waves are absorbed at critical levels, and second, the moving wave source causes the waves to be Doppler shifted with respect to the ground. In LRC, there is no tropospheric shear, and the wave source moves at a constant wind velocity \((-5 \text{ m s}^{-1} \text{ in the zonal direction and } 0 \text{ m s}^{-1} \text{ in the meridional direction})\) regardless of the clouds’ depth. Therefore, the shear, which complicates the spectra here, is not present in LRC, and consequently their spectra should be simpler.

Using the data from LRC, a power spectrum of vertical velocity in the zonal direction is calculated for their experiment E2 and is shown in Fig. 7. This spectrum is calculated using the same method used to calculate the spectrum shown in Fig. 6(b), except that the model output is available only from 1200–1400 LST. Reducing the length of the time series reduces the resolution in Fig. 7 in comparison with Fig. 6; this point will be discussed in more detail later.

Consider the zonal spectrum (Fig. 7) generated in a zero-shear environment. While the spectrum has similar horizontal wave-number maxima to those in Fig. 6, the frequency of these maxima are very different. The spectrum from LRC is shifted towards lower frequencies for eastward-propagating waves \((k > 0)\), and towards higher frequencies for westward-propagating waves \((k < 0)\). In their case, the zonal flow in the troposphere is constant, and therefore the convective gravity-wave source is also moving at this velocity. Recall, the ground-based phase speed of the gravity waves equals the source phase speed plus the background-flow speed at the LNB. Or in terms of frequency,

\[
\omega = \hat{\omega}_{\text{source}} + U_{\text{source}}k, \tag{4}
\]

where \(U_{\text{source}} = -5 \text{ m s}^{-1}\) is the zonal wind velocity in the troposphere in LRC’s calculation. If a moving source emits a spectrum of gravity waves, the ground-based frequency will be subsequently Doppler shifted. In this case with a negative tropospheric flow, the ground-based frequency of eastward-propagating waves \((k > 0)\) will be decreased, while the ground-based frequency of westward-propagating waves will be increased. Using Eq. (4) and assuming that the dominant source frequency is approximately \(0.005 \text{ s}^{-1}\) (estimated to pass through the absolute peak in the spectrum), then the ground-based frequency is given by

\[
\omega = 0.005 - 5k.
\]

This line is also plotted in Fig. 7, and agrees well with the slope of the power spectrum. As discussed earlier, refraction increases the amplitudes of the eastward-propagating waves in Fig. 7 while decreasing the amplitudes of those propagating in the opposite direction. (The spectrum is calculated at 20 km where the zonal flow is \(-10 \text{ m s}^{-1}\).)
Figure 7. Same as Fig. 6(b), except using data from experiment E2 from LRC. The line $\omega = 0.005 - 5k$ is plotted also (thick line).

(c) Uncertainty and consistency?

In LRC, a mechanistic description of the gravity-wave generation was presented. This discussion attributed the generation of the dominant gravity waves to the oscillation of convective updraughts about their LNB, and accordingly the source frequency of the waves was estimated to be $0.007 \text{ s}^{-1}$. This estimate was consistent with the peak in their wave-number spectrum, which corresponded to horizontal and vertical wavelengths of approximately 17 and 5 km, respectively. (Note that this measure of consistency relies on the assumption that the gravity waves in the stratosphere are small amplitude.) However, in the discussion above, the source frequency of their waves was estimated as $0.005 \text{ s}^{-1}$, an underestimate of approximately $0.002 \text{ s}^{-1}$. The two possible reasons for this underestimate are discussed now.

The first reason concerns the resolution of the calculated power spectra (Figs. 6 and 7). In Fig. 6 the lowest resolved, non-zero frequency in the spectra is approximately $2 \times 10^{-4} \text{ s}^{-1}$. However, due to the shorter time series used to calculate the spectrum from LRC, the lowest resolved, non-zero frequency in Fig. 7 is approximately $1 \times 10^{-3} \text{ s}^{-1}$. Therefore, the uncertainty in the frequency and the estimate of the source frequency for LRC's case is relatively large.

The second, and possibly the more important reason concerns the temporal variability of the gravity-wave generation. Individual convective cells within the modelled cloud cluster develop and dissipate on time-scales from 30 minutes to 1 hour. Consequently, the gravity waves are not generated with a constant amplitude; the amplitudes are probably largest during the growth stages of the cloud. Furthermore, the modelled gravity-wave motions are not truly sinusoidal, and are slightly damped by model diffusion and parameterized dissipation. Thus, in the growth (decay) stage of the convection, the gravity waves are likely to show a bias towards upward (downward) vertical velocity. Such a life cycle will cause the gravity waves to be amplitude modulated with a period similar to the convective lifetime. By way of illustration, consider the one-dimensional approximation to this process:

$$\cos(\omega_m t) \cos(\omega_d t) = \frac{1}{2} [\cos((\omega_d + \omega_m) t) + \cos((\omega_d - \omega_m) t)]$$,
where $\omega_m$ and $\omega_d$ are the modulation and dominant frequencies, respectively. Here $2\pi/\omega_m$ is the cloud lifetime. The key point to note is that modulating the amplitude of a monochromatic wave eliminates the spectral power of that wave and adds power to two side bands. This is, of course, an extreme example because the modulated time series is infinitely long. Nonetheless, it illustrates the essential idea.

Suppose $\omega_d = 0.007$ s$^{-1}$ and $\omega_m = 0.002$ s$^{-1}$ (a cloud lifetime of 52 minutes, which is typical of the deep convective cells simulated). Then, the upper side band ($\omega_d + \omega_m = 0.009$ s$^{-1}$) will be evanescent and the spectral peak will not be present in the stratosphere, whereas the lower side band ($\omega_d - \omega_m = 0.005$ s$^{-1}$) will contain the maximum power in the spectrum as in Fig. 7. These results suggest that care must be taken when interpreting power spectra, because the maximum power in the spectrum may reflect the convective life cycle rather than the frequency of the strongest gravity wave. Furthermore, the convective life cycle is strongly correlated with the convective heating, and therefore, the incorrect interpretation of the spectrum may appear to lend support to the hypothesis that convective heating is the primary wave generator.

Piani et al. (2000) modelled a highly organized three-dimensional squall line, and analysed the stratospheric gravity waves using spectral analysis. The peak in their power spectrum (their Fig. 9) was at $(\omega, k) \approx (0.002$ s$^{-1}$, $1 \times 10^{-4}$ m$^{-1}$) (corresponding to a period and wavelength of 45 minutes and 50 km, respectively). However, a zonal cross-section of the vertical velocity (their Fig. 4(a)) shows that the horizontal and vertical wavelengths are approximately 33 km and 9 km, respectively (for eastward-propagating waves). Using the dispersion relation, their stratospheric zonal velocity ($-10$ m s$^{-1}$), and their Brunt–Väisälä frequency (0.022 s$^{-1}$), these wavelengths correspond to $(\omega, k) \approx (0.004$ s$^{-1}$, $2 \times 10^{-4}$ m$^{-1}$). Thus, in our view it is likely that their spectral peak represents the modulation of the dominant signal by the convective lifetime.

In the power spectra considered herein (Figs. 6 and 7), the maximum in the power spectrum at $k = 0$ is at a lower frequency (approximately 0.002–0.004 s$^{-1}$) than the non-zero wave numbers. This may be due to large-scale circulations generated by the cloud ensemble. Such a circulation has a (large) horizontal scale, like that of the cloud ensemble ($k \approx 0$), and a period comparable to the lifetime of the convective clouds. It is difficult, if not impossible, to distinguish between such a circulation and gravity waves.

6. REPRESENTATION AND EVOLUTION OF THE SOURCE SPECTRUM

The model solutions discussed above support the hypothesis that the largest-amplitude gravity waves are generated by the convective updraughts as they oscillate about their LNB with frequencies close to the local Brunt–Väisälä frequency. Such waves have no preferred direction of propagation and their spectrum is initially symmetric in a frame of reference moving with the LNB of the cloud. A constant tropospheric flow affects the ground-based frequency spectrum by Doppler-shifting the waves, whereas a sheared flow also broadens the frequency spectrum (cf. Figs. 6 and 7). In this section, the mechanism that broadens the spectra is described and a simple representation of the gravity-wave source spectrum is developed.

As the convective system develops during the day, the height of the cloud tops increases. However, at any particular time there is a spread of cloud-top heights within the population, and the updraughts within each of these convective clouds have a different LNB. Furthermore, each cloud emits gravity waves from its own LNB with amplitudes that depend on the intensity of each convective cell. In cases with no
tropospheric shear (as in LRC), the height of the gravity-wave generation is virtually irrelevant because all wave sources move at the same speed, and consequently generate approximately the same ground-based frequency and wave number in the spectrum. In the case with tropospheric shear (the case considered here), each convective cloud generates a symmetric (intrinsic) spectrum of gravity waves at its LNB. Due to the tropospheric shear, the gravity-wave sources (each individual convective cloud) move at different wind velocities. Thus, the waves generated at different depths will have different ground-based frequencies and the total wave spectrum will be broadened. This is the case for the zonal spectrum (Fig. 6(b)) where the spectrum is broadened towards higher frequencies for eastward waves and towards lower frequencies for westward waves. Unlike LRC (Fig. 7), the tropospheric flow is not constant, and consequently the ground-based frequency of the waves will not lie on a single line in $\omega-k$ space.

Within a cloud, the height of the maximum updraught velocity coincides with the LNB of that cloud. With that point in mind, the probability distribution of the LNB for the model calculation is estimated. At each point in horizontal space in Domain 2, and for each time between 1030 and 1700 LST (at 30-minute intervals), the height of the maximum vertical velocity is identified. If this vertical velocity corresponds to a point which is in cloud (condensed-water mixing ratio greater than 0.1 g kg$^{-1}$) and it is a deep cloud (ice mixing ratio greater than 0.1 g kg$^{-1}$), then this height is added to 1 km height bins to form the probability distribution. (Note, the ice condition is imposed to remove the skewness of the distribution caused by boundary-layer cumulus which probably contribute little to the total wave spectrum.) The probability distribution is shown in Fig. 8. The distribution is not an exact representation of that of the LNB, because it contains heights which correspond to developing clouds as well as mature clouds. Furthermore, it also contains multiple contributions by the same cloud. Figure 8 shows that there is a distinction between the deepest clouds whose LNB are contained within the 10–14 km bins, and middle-tropospheric clouds whose LNB are contained within the 5–9 km bins. It is probable that the deeper the cloud the more intense its updraughts and, therefore, the larger the wave amplitude. However, there are more middle-tropospheric clouds than deep clouds, so it may be true that the shallower clouds are equally important to the spectrum as deep clouds. The relative importance of shallow and deep convective clouds in generating gravity waves requires further investigation.
Nevertheless, considering the large variation in depth of the clouds (in time and space), it is likely that the gravity-wave generation consists of contributions from different heights, and therefore different source velocities.

A simple calculation is now performed to test our hypotheses concerning the shape and evolution of the wave spectrum. The aim of this calculation is to determine whether the emission of a symmetric (intrinsic) spectrum of gravity waves in a sheared tropospheric flow evolves the characteristic features of the spectra from the model run. The zonal case (Fig. 6(b)) is considered because this is the case that shows the greatest asymmetry in the spectrum. The release of a symmetric (intrinsic) spectrum of gravity waves at the zonal wind speed is simulated. The release is at five heights, which are \( z = 8, 9, 10, 11 \) and 12 km, encompassing part of the distribution of the LNB of the clouds. (The zonal velocities corresponding to these heights are 1.2, –1.0, 5.1, 5.0 and 5.0 m s\(^{-1}\), respectively.) The spectrum emitted is an approximation to the spectrum from LRC (Fig. 7) in the absence of a background flow, and is defined by

\[
{\log_{10} A_b(k, \omega)}^{1/2} = \exp\left[-(k - k_0)^2/\sigma_{k0}^2 - (\omega - \omega_0)^2/\sigma_\omega^2\right] + \exp\left[-(k + k_0)^2/\sigma_{k0}^2 - (\omega - \omega_0)^2/\sigma_\omega^2\right] + \frac{1}{2} \exp\left[-k^2/\sigma_{k1}^2 - (\omega - \omega_1)^2/\sigma_\omega^2\right],
\]

where \( k_0 = 4 \times 10^{-4} \) m\(^{-1}\), \( \omega_0 = 6 \times 10^{-3} \) s\(^{-1}\), \( \omega_1 = 3.5 \times 10^{-3} \) s\(^{-1}\), \( \sigma_{k0} = 4 \times 10^{-4} \) m\(^{-1}\), \( \sigma_{k1} = 3 \times 10^{-4} \) m\(^{-1}\) and \( \sigma_\omega = 2 \times 10^{-3} \) s\(^{-1}\); \( A_b(k, \omega) \) will be called the base amplitude spectrum, and Fig. 9(a) shows the normalized square of \( A_b \) (with logarithmic contours). In constructing the spectrum it was assumed that the gravity waves are centred at \( \hat{\omega}_{\text{source}} = 0.006 \) s\(^{-1}\). This frequency is larger than that determined in section 5(b), but justified considering the discussion in section 5(c). The spectrum is relatively broad in \( k \) with maxima corresponding to horizontal wavelengths of approximately 17 km. Also included in the base spectrum is power at lower frequencies and \( k = 0 \). The power at these lower frequencies was discussed in the previous section and is probably due to cloud-induced circulations which do not necessarily behave like gravity waves. The power in this part of the spectrum is not essential to the calculation, but is included to aid in the comparison with the modelled spectra. At each release height, the source frequency of each component wave is Doppler shifted using Eq. (3) to form the respective ground-based frequency spectrum for each wave number \( k \). The five amplitude spectra are averaged, and this average represents the total spectrum of gravity waves generated by this ensemble of clouds with different depths (and different source velocities). In order to compare the idealized spectrum and the zonal spectrum taken from the model, the effects of important physical processes are included (simply) into the calculation as follows:

(i) **Critical-level dissipation.** When there is a critical level between the respective release heights and 20 km, the appropriate part of the \( \omega-k \) spectrum is set to zero, representing total dissipation of these modes.

(ii) **Changing wave signatures.** The waves will be refracted in the shear layer near the tropopause where the flow changes from westerly to easterly. Hence, the vertical velocity signature of the westward-propagating waves \( (k < 0) \) will be reduced in comparison to the eastward-propagating waves \( (k > 0) \). As described in LRC, the square of the vertical-velocity amplitude of a transient wave packet scales like the local intrinsic frequency cubed, \((\omega - Uk)^3\). To include this effect, the base power is divided by the corresponding value of \( \hat{\omega}_{\text{source}}^3 \), forming a normalized wave-action spectrum. The wave action of a wave packet is unaffected by wave refraction. The final wave-action
spectrum is multiplied by \((\omega - U_{\text{strat}}k)^3\), where \(U_{\text{strat}} = -5 \text{ m s}^{-1}\) is the stratospheric flow, to form the power; this incorporates the effect of wave refraction on the vertical-velocity amplitude. (Note, this adjustment does not take into account the vertical change in density between the individual release heights and 20 km. However, the release heights span approximately half of one density scale height, and therefore the effect of density is probably small. Also, the final spectrum is normalized and it is unnecessary to incorporate the density at 20 km into the adjustment.)

(iii) Evanescent. As waves propagate vertically from their release heights, some parts of the gravity-wave spectrum become evanescent. The frequency at which evanescence occurs is equal to \(N + Uk\), where \(N\) is the Brunt–Väisälä frequency. In the case considered here, the region where this has most effect is in the upper troposphere, above the shear layer. Without solving the Taylor–Goldstein equation, the attenuation of these modes cannot be accurately described. Moreover, were the simple calculation described here to be used as the basis of a parametrization for a GCM, it would be far too time-consuming to explicitly solve the Taylor–Goldstein equation. However, by applying a
low-pass filter $M(k, \omega)$ to the power spectrum, the effect of this attenuation can be approximated. The parametrized spectrum is multiplied by $M(k, \omega)$, which is defined as

$$M(k, \omega) = \begin{cases} 
1, & \text{for } \omega - U_{straf}k < 0.007 \text{ s}^{-1}, \\
\frac{1}{2}(1 + \cos[(\omega - (0.007 + U_{straf}k)\pi/0.005)], & 0.007 < \omega - U_{straf}k < 0.012 \text{ s}^{-1}, \\
0, & \text{for } \omega - U_{straf}k > 0.012 \text{ s}^{-1},
\end{cases}$$

where the interval [0.007, 0.012] is chosen because it spans the range from the minimum to the maximum tropospheric Brunt–Väisälä frequencies.

The calculations (in the absence of critical levels) can be summarized as follows,

$$A_p^2(k, \omega) = M(k, \omega)(\omega - U_{straf}k)^3 \left\{ \sum_{Z_e} \frac{A_b(k, \omega - U(Z_e)k)^2}{(\omega - U(Z_e)k)^{3/2}} \right\}, \quad (5)$$

where $A_p$ is the predicted spectrum, and $Z_e$ are the release heights. After performing the above calculations, the spectrum is smoothed over five adjacent frequency and wave-number bins. The predicted spectrum is shown in Fig. 9(b).

The shape of the spectrum predicted by this simple calculation has much in common with the zonal spectrum from the numerical model (Fig. 6(b)). In particular, the predicted spectrum possesses a bias towards higher frequencies for eastward-propagating waves $k > 0$, and the respective maxima in both propagation directions approximately coincide with those in Fig. 6(b). However, the amplitudes of the westward-propagating waves are slightly underestimated. Also, one difference between the predicted spectrum and the numerical model spectrum (Fig. 6(b)) is that individual peaks of local maxima are not reproduced. These maxima are not reproduced because we have chosen a very smooth base spectrum. However, in the model, gravity waves are generated at a range of heights determined by the heights of the clouds that make up the population, and with a range of amplitudes determined by the strength of the updraught. For these reasons one would not expect the detailed structure to be reproduced but, instead, only the broadest features of the spectrum.

By making a number of simple assumptions about the gravity-wave source, the broad features of the wave spectrum for a convective system with complicated tropospheric shear has been reproduced. This result supports the idea that, in the model, gravity waves are generated principally when convective updraughts oscillate about their LNB.

7. SUMMARY AND CONCLUSIONS

In this study, observed wind and thermodynamic profiles were used to initialize a model calculation of a Hector thunderstorm over the Tiwi Islands. The convection developed into two zonally oriented cloud lines. The structure of the cloud lines agreed reasonably well with radar observations for the same day, although the modelled convection developed about an hour earlier than observed.

Gravity waves were prominent features of the modelled troposphere and stratosphere. The waves had amplitudes of approximately 0.3 m s$^{-1}$, horizontal wavelengths of approximately 17 km and vertical wavelengths of approximately 5 km. In the horizontal plane, the waves had roughly circular fronts, implying that waves were emitted in all horizontal directions.
Power spectra of the vertical velocity were calculated along zonal and meridional cross-sections. The meridional spectrum was approximately symmetric about \( l = 0 \), with a slight bias towards southward-propagating waves. The zonal spectrum showed strong asymmetry about \( k = 0 \), with a bias towards higher-frequency eastward-propagating waves (\( k > 0 \)). This bias was consistent with a source moving in the positive zonal direction, which suggested that the strongest source of the waves was in the height range from 10–14 km.

The analysis showed that, although the general features of the gravity waves were not strongly affected by tropospheric shear, the details of the power spectra were.

A power spectrum of the gravity waves for experiment E2 from LRC was also calculated. This spectrum illustrated the effect of a constant tropospheric flow on the gravity waves. The waves suffered a systematic shift towards lower ground-based frequencies for upstream-propagating waves, and towards higher ground-based frequencies for downstream-propagating waves. In comparison, tropospheric shear broadened the power spectrum. This broadening was attributed to wave generation at different depths, and therefore different source velocities. These waves were Doppler-shifted with respect to the ground, and therefore each source contributed to a different part of the frequency spectrum.

Another important effect of tropospheric shear was to cause critical-level dissipation of the gravity waves. This dissipation had a pronounced influence on the shape of the spectrum in the stratosphere, removing all but the fastest waves.

It was shown that the absolute peak in the power spectrum need not provide a reliable way to identify the frequency of the waves. The reason for this is that the amplitude modulation of the waves by the convective life cycle is imprinted on the spectrum. This highlighted the danger in interpreting individual peaks in power spectra as the dominant gravity waves.

Following LRC, it was argued that convective clouds generate gravity waves when the updraught rapidly decelerates after passing its LNB. A simple calculation was performed to test whether this hypothesis was consistent with the wave spectrum in the stratosphere produced in the numerical model. The main ideas behind this calculation were that:

(i) In a convective cloud cluster, the clouds have a variety of depths, and consequently generate waves at different heights;
(ii) each cloud generates waves with the same symmetric source spectrum; however, the waves from each cloud are Doppler-shifted with respect to the ground by the sheared flow;
(iii) the result is a broad spectrum of gravity waves which depends strongly on the tropospheric flow.

Given an initial spectrum consistent with the hypothesized generation mechanism, and the background flow used to initialized the numerical model, the subsequent spectrum in the stratosphere was calculated. The predicted spectrum agreed qualitatively with the zonal spectrum calculated from the model. These results suggest a simple representation of the normalized source spectrum suitable for use with the gravity-wave drag parametrizations currently used in GCMs. However, before implementation of such a scheme is possible, further work is required to determine typical base spectra, and spectral amplitude as a function of cloud and environmental properties.

Finally, the analysis in this paper has been consistent with the idea that oscillating convective updraughts generate the dominant gravity waves. Furthermore, although tropospheric shear does not seem to play a role in their generation, it is important in
Doppler-shifting the waves, and in broadening and creating a bias in the ground-based frequency spectrum. Nevertheless, before such a scheme could be implemented in a GCM, detailed analysis of clouds as a gravity-wave source is required. This analysis should determine the relationship between the shape and intensity of the base spectrum, and cloud intensity and organization. However, these calculations have not described the generation of large-scale inertia gravity waves by convective cloud clusters, which may also make an important contribution to the momentum budget of the middle atmosphere.

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