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ABSTRACT

The possibility of estimating fundamental parameters common in single-moment ice microphysics schemes using radar observations is investigated, for a model-simulated supercell storm, by examining parameter sensitivity and identifiability. These parameters include the intercept parameters for rain, snow and hail/graupel, and the bulk densities of snow and hail/graupel. These parameters are closely involved in the definition of drop/particle size distributions of microphysical species but often assume highly uncertain specified values.

The sensitivity of pure model forecast as well as model state estimation to the parameter values, and the solution uniqueness of the estimation problem are examined. The ensemble square-root filter (EnSRF) is employed for model state estimation. Both forecast and assimilation sensitivity experiments show that the errors in the microphysical parameters have a larger impact on model microphysical fields than on wind fields; radar reflectivity observations are therefore preferred over those of radial velocity for microphysical parameter estimation. Among the three intercept parameters, the pure forecast is most (least) sensitive to rain (snow) intercept while the sensitivity to hail density is generally larger than that to snow density. The analyzed model state in the assimilation sensitivity experiments is, however, found to be most (least) sensitive to hail (rain) intercept, and there is a larger sensitivity to hail density than to snow density.

The time scales of analysis response to errors in individual parameters are also investigated. The results suggest that a successful estimation of the parameters can be expected within the typical lengths of assimilation window needed for state estimation. The response functions calculated for the forecast as well as assimilation sensitivity experiments for all five individual parameters show concave shapes, with unique minima occurring at or very close to the true values; therefore true values of these parameters can be retrieved at least in these cases where only one parameter contains error at a time.

The identifiability of multiple parameters together is evaluated from their correlations with forecast reflectivity. Significant levels of correlations are found that can be interpreted physically. As the number of uncertain parameters increases, both the level and the area coverage of significant correlations decrease, implying increased difficulties with multiple-parameter estimation. The details of the estimation procedure and the results of a complete set of estimation experiments will be presented in Part II of this paper.

1. Introduction

The accuracy of numerical weather prediction (NWP) depends very much on the accuracy of the initial state estimation and the accuracy of the prediction model. Various advanced data assimilation techniques have been developed in the recent decades that improve the estimation of model initial conditions. Among these methods are the four-dimensional variational assimilation (4DVAR) (Le Dimet and Talagrand 1986; Courtier and Talagrand 1987) and the ensemble-based assimilation methods (Evensen 1994; Evensen and Leeuwen 1996; Burgers et al. 1998; Houtekamer and Mitchell 1998; Anderson 2001; Bishop et al. 2001; Whitaker and Hamill 2002; Evensen 2003; Tippett et al. 2003), which have the advantage of closely involving a numerical
For convective-scale NWP, explicit microphysics schemes are used to predict the evolution of clouds and precipitation. Most microphysics schemes use the 'bulk' approach of parameterization, in which the particle or droplet size distributions (DSDs) are parameterized in functional forms. Often, significant uncertainties exist with the treatment of the microphysical processes and the microphysical parameters. Previous sensitivity studies (e.g. McGuffie et al. 1991; Ferrier et al. 1995; Gilmore et al. 2004; van den Heever and Cotton 2004) demonstrate that the structure and evolution of simulated convective systems are very sensitive to microphysical parameterizations. Variations in microphysical parameters, such as collection efficiencies, DSD parameters and particle densities, have profound effects upon the characteristics of precipitation systems and their associated dynamical processes.

Because of many assumptions involved, the microphysical parameterization can be an important source of model error for convective-scale data assimilation and prediction. Parameter estimation is a common approach to dealing with model error associated with uncertain parameters. The inverse problem of parameter estimation concerns with the optimal determination of the parameter by observing the dependent variable(s) collected in the spatial and time domains (Yeh 1986). Various methods have been used for parameter estimation, among which variational parameter estimation with an adjoint model is popular in the literature of meteorology and oceanography (Navon 1998). The ensemble Kalman filter method (hereafter EnKF) has recently been tested successfully for the atmospheric state estimation at the convective scale with simulated (Snyder and Zhang 2003; Zhang et al. 2004; Tong and Xue 2005; Xue et al. 2006) and real (Dowell et al. 2004) radar data. The results with simulated data, under the perfect model assumption, have been excellent, while the quality of state estimation with real data, when model error inevitably exists, is generally not as good. More recently, Aksoy et al. (2006) used EnKF for the simultaneous estimation of state variables and model parameters in a relatively simple two-dimensional sea-breeze model with encouraging success.

In this study, we set out to investigate the ability of the EnKF in correcting the errors in some of the fundamental parameters in model microphysics, where complex process interactions and high nonlinearities usually exist. In the framework of EnKF, parameter estimation is realized by treating the uncertain parameters as independent model variables and using the covariance information sampled from the ensemble to estimate the parameters given available observations (Anderson 2001). Such a technique is often referred to as state vector augmentation.

The well-posedness as well as parameter identifiability are the main issues that are directly related to the possibility of successful parameter estimation, no matter what technique is used. The inverse problem for parameter estimation is often ill-posed (Chavent 1974; Yakowitz and Duckstein 1980). As was reviewed by Yeh (1986), the ill-posedness is generally characterized by the non-uniqueness and instability of the identified parameters. In the case of non-uniqueness, the estimated value often depends on its initial guess and is not guaranteed to be close to the “true” value. The instability of the inverse solution stems from the fact that small errors in the observations will cause serious errors in the identified parameters. Yakowitz and Duckstein (1980) demonstrated that a small sensitivity of the model output in terms of observations to the change of unknown parameters (parameters to be estimated) implies identification instability. The problem is that a larger difference in the parameter may be manifested by only very small changes in the model output of observations, which may be smaller than anticipated measurement error.

As the first part of this study, we investigate the possibility of retrieving some microphysical parameters with the EnKF method through a detailed sensitivity analysis. The issue of parameter identifiability will be addressed. The results will guide our design of the parameter estimation experiments and also help us understand the estimation results. The microphysical parameters to be estimated are the intercept parameters of rain, snow, and hail/graupel size distributions, and the bulk densities of hail/graupel and snow. These parameters have been shown by the sensitivity studies referenced earlier to have significant effect on the precipitation processes and dynamics of convective storms. Other model parameters are assumed to be correct.

This paper is organized as follows. In section 2, we briefly describe the microphysics scheme and its limitations, which partly motivate this study. The uncertainties of the chosen microphysical parameters based on previous observational studies will also be discussed. Section 3 briefly describes the numerical model, the simulation configuration for a supercell thunderstorm and the experimental setup for sensitivity analysis. Section 4 discusses the results of sensitivity analysis. The parameter identifiability issue is addressed in section 5. Summary and conclusions are given in section 6. Results of the parameter estimation experiments will be presented in Tong and Xue (2006, Part II hereafter), Part II of this paper series.
2. Microphysics description

a. Microphysics scheme

The microphysics scheme in the ARPS (Xue et al. 2000; Xue et al. 2001; Xue et al. 2003) model used by this study is a 5-class (cloud water, rain, cloud ice, snow and hail/graupel) single-moment bulk scheme after Lin et al. (1983, hereafter LFO83). The scheme assumes that the drop size distributions (DSD) of rain, snow and hail/graupel have an exponential form:

\[ n_x(D) = n_0x \exp(-\lambda_x D), \]  

where \( x \) represents \( r \) (rain), \( s \) (snow) or \( h \) (hail), for particular hydrometeor species. The DSD is assumed monodisperse for non-precipitating cloud water and cloud ice. \( n_x(D)\delta D \) in Eq. (1) is the number of drops per unit volume between diameters \( D \) and \( D + \delta D \) and \( n_0x \) is the so-called intercept parameter, which is the value of \( n_0x \) for \( D = 0 \). The slope parameter, which equals to the inverse of the mean size diameter of each distribution, is diagnosed as:

\[ \lambda_x = \left( \frac{\pi D^2 n_0x}{\rho q_x} \right)^{0.25}, \]  

where \( \rho \) is the constant particle bulk density, \( \rho \) is the air density and \( q_x \) is the hydrometeor mixing ratio.

With single-moment bulk microphysics schemes, only one moment of the DSD functions is predicted. In the LFO83 scheme, as well as almost all single-moment schemes, the mixing ratio of each hydrometeor, which is proportional to the third moment of the DSD function, is predicted and the intercept parameter \( n_0x \) is a prescribed constant. It can be seen from Eqs. (1) and (2) that the DSD is a function of two adjustable parameters \( n_0x \) and \( \rho \). For a given mixing ratio \( q_x \), the larger is the intercept parameter or the density, the more the hydrometeor spectrum is weighted towards smaller drops (Fig. 1). For model simulations, adjusting these parameters can directly impact the bulk terminal velocity and the number concentration of species, which can result in the change of the trajectories of the hydrometeors within the cloud and the particle growth rates. These changes in the microphysical processes will affect the water budgets within the cloud and hence the latent heating and hydrometeor loading, which in turn lead to the changes of the buoyancy and subsequent updraft and downdraft patterns.

With the use of prescribed parameters, typical single-moment microphysics schemes generally cannot adequately represent convective clouds of various types of precipitation systems. For example, the parameterization of the LFO83 scheme is formulated for the intense continental storms with the presence of high-density hails while the somewhat similar scheme of Rutledge and Hobbs (1983; 1984) is more suitable for oceanic systems. The differences come from either the treatment of the microphysical processes and/or the use of different parameters, such as those of hydrometeor density and DSD intercept.

Fig. 1. (a) Number concentration per mm diameter size and (b) mass-weighted mean terminal velocity of rain (for \( N_{0r87}: n_{0r} = 8 \times 10^7 m^{-4}, N_{0r86}: n_{0r} = 8 \times 10^6 m^{-4} \) and \( N_{0r36}: n_{0r} = 3 \times 10^6 m^{-4} \)), snow (for \( N_{0s37}: n_{0s} = 3 \times 10^7 m^{-4}, N_{0s36}: n_{0s} = 3 \times 10^6 m^{-4} \) and \( N_{0s16}: n_{0s} = 1.19 \times 10^6 m^{-4} \)) and hail/graupel (for \( N_{0h45}: n_{0h} = 4 \times 10^5 m^{-4}, N_{0h14}: n_{0h} = 1.59 \times 10^4 m^{-4} \) and \( N_{0h44}: n_{0h} = 4 \times 10^4 m^{-4} \)). The terminal velocities are calculated for an air density of 1.0 kg m^{-3}. The default values of the microphysical parameters are \( n_{0r} = 8 \times 10^6, n_{0s} = 3 \times 10^6 m^{-4}, n_{0h} = 4 \times 10^4 m^{-4} \), \( \rho = 913 \ kg m^{-3} \) and \( \rho = 100 \ kg m^{-3} \) unless otherwise indicated by the curve legends.
as one moment of the distribution function and/or dividing the hydrometeors into more categories. By predicting two moments (Ziegler 1985; Murakami 1990; Ferrier 1994; Meyers et al. 1997; Cohard and Printy 2000) or three moments (Milbrandt and Yau 2005) of the distribution function, the DSD parameters are effectively treated as prognostic variables rather than being prescribed as constants. More recently, Straka and Mansell (2005) developed a single-moment bulk microphysics scheme with ten ice categories, which allows for a range of particle densities and fall velocities for simulating a variety of convective storms with less need for parameter tuning.

Although sophisticated microphysical schemes are attractive and represent the future direction of convective-scale modeling and NWP, they are expensive and much research is still needed on the treatment of processes involving the additional moments before they can be widely used. The increased number of prognostic variables in the model also poses a larger problem for state estimation or model initialization. The single-moment bulk schemes are widely used in current research and operational models; the ultimate goal of our current line of study is therefore to overcome, to the extent possible, the shortcomings of such single-moment schemes by constraining uncertain microphysical parameters using data, i.e., by estimating the parameters as well as the model state variables using radar observations of the convective storms.

### b. Uncertainties in the microphysical parameters

The parameters selected for this study are the intercept parameters of rain, snow and hail/graupel DSDs, and the densities of snow and hail. Observational and sensitivity studies indicate that the coefficients associated with the formula for hydrometeor fall speeds and the collection efficiency parameters are also uncertain and can affect the microphysical processes significantly. In this study, we focus on the density and intercept parameters, because they are more fundamental and directly affect a large number of processes in the microphysics parameterization.

As pointed out earlier, with the LFO83 single-moment bulk microphysics scheme, the intercept parameters and the bulk densities of snow and hail are assumed to be constant in space and time. The default values of the intercept parameters for rain, snow and hail size distributions in the ARPS model are $8 \times 10^5 \text{ m}^4$, $3 \times 10^3 \text{ m}^4$ and $4 \times 10^4 \text{ m}^4$, respectively, following LFO83. The densities of rainwater, snow and hail are specified to be $1000 \text{ kg m}^{-3}$, $100 \text{ kg m}^{-3}$ and $913 \text{ kg m}^{-3}$, respectively (see Table 1).

A number of observational studies indicate that the intercept parameters of hydrometeor distributions can vary widely among precipitation systems occurring in different large-scale environments. Also, within the same precipitation system the intercept parameters can vary spatially and with the evolution of the system. The observed hail/graupel intercept parameter, $n_{0h}$, varies from $10^2 \text{ m}^{-4}$ to greater than $10^8 \text{ m}^{-4}$. Observed snow intercept parameter, $n_{0s}$, varies from $10^5 \text{ m}^{-4}$ to $10^8 \text{ m}^{-4}$ (Gunn and Marshall 1958; Passarelli 1978; Houze et al. 1979; Houze et al. 1980; Lo and Jr. 1982; Mitchell 1988; Braham 1990). Joss and Waldvogel (1969) found that $n_{0h}$ varies between $10^6 \text{ m}^{-4}$ and $10^8 \text{ m}^{-4}$. A number of studies have shown a systematic decrease in $n_{0h}$ as precipitation changed from convective to stratiform (e.g., Waldvogel 1974; Tokay et al. 1995; Tokay and Short 1996; Cifelli et al. 2000).

In the LFO83 scheme, the term hail is used loosely to represent high-density graupel, ice pellets, frozen rain and hailstones. According to Pruppacher and Klett (1978), the bulk density of hail has been found to vary between 700 kg m$^{-3}$ and 900 kg m$^{-3}$ and the observed density of graupel ranges from 50 kg m$^{-3}$ to 890 kg m$^{-3}$. The term snow in the LFO83 scheme is used to represent snow crystals, snowflakes and low-density graupel particles. Snow density varies greatly from one snow event to the next. The density of freshly fallen snow observed in literature varies from 10 kg m$^{-3}$ to approximately 350 kg m$^{-3}$ (Judson and Doesken 2000).

All these indicate that there exist great uncertainties with the values of the intercept and density parameters, and assuming same values for all precipitation events can lead to significant errors in the prediction model. Estimating their values for specific events using data is likely to significantly reduce such errors or uncertainties.

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**Table 1. A summary of the uncertainty ranges, defined by the lower bound $p_i$ and upper bound $\bar{p}_i$, and the control values for intercept parameters $n_{0h}, n_{0s}, n_{0r}$, and the density of snow and hail $\rho_h$.**

<table>
<thead>
<tr>
<th>Parameter $p_i$</th>
<th>$\underline{p}_i$</th>
<th>$\bar{p}_i$</th>
<th>Control values of parameters, $p_{ic}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hail/graupel intercept $n_{0h}$ (m$^{-4}$)</td>
<td>$4 \times 10^2$</td>
<td>$4 \times 10^6$</td>
<td>$4 \times 10^7$</td>
</tr>
<tr>
<td>Snow intercept $n_{0s}$ (m$^{-4}$)</td>
<td>$5 \times 10^2$</td>
<td>$1 \times 10^8$</td>
<td>$3 \times 10^5$</td>
</tr>
<tr>
<td>Rain intercept $n_{0r}$ (m$^{-4}$)</td>
<td>$3 \times 10^2$</td>
<td>$8 \times 10^7$</td>
<td>$8 \times 10^6$</td>
</tr>
<tr>
<td>Density of hail/graupel $\rho_h$ (kg m$^{-3}$)</td>
<td>400</td>
<td>913</td>
<td>913</td>
</tr>
<tr>
<td>Density of snow $\rho_s$ (kg m$^{-3}$)</td>
<td>20</td>
<td>400</td>
<td>100</td>
</tr>
</tbody>
</table>
3. Model and experimental settings

a. The prediction model and truth simulation

The forecast model used in this study and the truth simulation are inherited from Tong and Xue (2005, hereafter TX05). Briefly, the ARPS (Xue et al. 2000; Xue et al. 2001; Xue et al. 2003), a fully compressible and nonhydrostatic atmospheric prediction system is used. The truth simulation is for the May 20, 1977 Del City, Oklahoma supercell storm case (Ray et al. 1981; Xue et al. 2001). The model configurations for the truth simulation are exactly the same as those in TX05. The default values of the microphysical parameters in the model are used in the truth simulation (see Table 1).

b. Experimental design for sensitivity analysis

In this study, the forward method is used for sensitivity analysis (Crook 1996). For each parameter, we perform a set of sensitivity experiments, within which only the parameter considered is varied within its range of uncertainty while all other model parameters are set to be their true values. The true values of the microphysical parameters are used in the control experiment (CNTL). We first examine the influence of each of the five parameters on the model simulation. Then we estimate the influence of the parameter values on the model state estimation via the ensemble square-root filter (EnSRF, Whitaker and Hamill 2002) algorithm and deduce the limits within which the model parameters may be estimated. The EnSRF is a variation of the standard EnKF, which does not require perturbing the observations. The particular configurations of radar data assimilation using the EnSRF algorithm are described in Part II.

Suppose \( \mathbf{p} = (p_1, p_2, \ldots, p_n)^T \) is the vector of the uncertain microphysical parameters. An admissible set \( P_{ad} \) of \( \mathbf{p} \) based on the parameter range can be defined as

\[
P_{ad} = \{ \mathbf{p} | p_i \leq \bar{p}_i \leq \bar{p}_i, i = 1, 2, \ldots, n \},
\]

where \( p_i \) and \( \bar{p}_i \) are the lower and upper bounds of the \( i \)th parameter. The values of \( p_i \) and \( \bar{p}_i \) applied in this study can be found in Table 1. The admissible set \( P_{ad} \) of \( \mathbf{p} \) given in Table 1 may not span all observed parameter values that might have appeared in the literature. For example, for hail \( q_h \), we do not allow values of \( n_{oh} \) that are larger than 4x10^6 m^-3 (corresponding to small graupel cases). For snow \( q_s \), we do not allow it to represent high-density snow or graupel therefore snow density \( \rho_s \) is no larger than 400 kg m^-3.

The parameter estimation problem consists of finding an estimated value \( \hat{\mathbf{p}} \) of \( \mathbf{p} \) from information taken from the observations, the parameter-to-observation mapping, and the prior information about the parameters. The problem can often be constructed as finding \( \hat{\mathbf{p}} \in P_{ad} \), such that \( J(\hat{\mathbf{p}}) \leq J(\mathbf{p}) \) \( \forall \mathbf{p} \in P_{ad} \). Here \( J(\hat{\mathbf{p}}) \) is the output criterion, which is generally the minimization of a “norm” of the difference between the observations and the model output of observations. Therefore, in this study, we are especially interested in the sensitivity of the model output, in the forms of observations, to the microphysical parameters. The response function for the sensitivity analysis is therefore defined as

\[
J_{\mathbf{p}}(\mathbf{p}) = \frac{1}{\sigma_p} \sum_{i=1}^{M} (\eta_i(\mathbf{p}) - \eta_i^*)^2,
\]

where \( \eta_i(\mathbf{p}) \) and \( \eta_i^* \) are, respectively, the model solution in the form of observation and the corresponding observation. The observations in the current case contain the simulated observations of radial velocity \( V_r \), and/or reflectivity \( Z \). The forward observation operators that project the model state to the observations will be described in Part II. The observation errors are included by adding random errors to the ‘error-free’ observations to give

\[
\mathbf{V}^s = \mathbf{V}(\mathbf{x}', \mathbf{p}') + \sigma_v \mathbf{v},
\]

\[
\mathbf{Z}^s = \mathbf{Z}(\mathbf{x}', \mathbf{p}') + \sigma_z \mathbf{z},
\]

where \( \mathbf{v} \) represents the Gaussian random variable vector with zero mean and unit standard deviation, and \( \sigma_v \) and \( \sigma_z \) are the standard deviations of the observation errors added to \( V_r \) and \( Z \), respectively. In this study, the relative sensitivity of the model output of \( V_r \) and \( Z \) with respect to the five parameters will be compared, which will help us decide the right data to use for parameter estimation. To facilitate the comparison, the difference between the model output of \( V_r \) or \( Z \) and the simulated observation is normalized by the estimated observation \( rms \) error \( \sigma_{V_r} \) or \( \sigma_Z \). The summation in Eq. (4) is over the data points where reflectivity is greater than 0 dBZ.

The response function defined in Eq. (4) is calculated for each sensitivity experiment and for the control experiment. For data assimilation sensitivity experiments, the ensemble mean of the analyses at each analysis time is used to calculate \( \eta_i(\mathbf{p}) \). The actual response function presented for analysis is

\[
\Delta J_{\mathbf{p}}(\mathbf{p}) = J_{\mathbf{p}}(\mathbf{p}) - J_{\mathbf{p}}(\hat{\mathbf{p}}),
\]

where \( J_{\mathbf{p}}(\mathbf{p}) \) is the response function calculated from the control data assimilation experiment.

4. Results of Sensitivity Analysis

In this section, we examine the sensitivities of the pure model prediction and of the EnSRF state estimation to the microphysical parameters that we intend to estimate. As discussed earlier, a sufficient level of sensitivity is necessary for successful parameter estimation.

a. Sensitivity of pure forecast to microphysical parameters

Fig. 2 shows the variations of \( J_{\mathbf{p}} \) and \( J_{\mathbf{p}} \) against the deviation of the parameters from their true values. The response functions are calculated from the output of pure model forecast (without data assimilation) every 5 minutes and averaged over the expected data assimilation window, i.e., from 25 min to 100 min. These forecasts start from a true atmospheric state at 20 minutes with parameter perturbations introduced at the same
time. The observational data used in calculating the response functions are extracted from the truth simulation and are ‘error-free’, i.e., they are \( V(x^*,p^*) \) and \( Z(x^*,p^*) \) defined in section 3b. The microphysical parameters are expressed in logarithmic form since most of them can vary by more than an order of magnitude. The symbols on each curve represent parameter values sampled from \( P_{ad} \). It is stated that \( V \) or \( Z \) is more sensitivity to one parameter than the other if the same amount of change in the parameter value causes more change in the response function from that of the truth simulation.

As can be seen from Fig. 2, model reflectivity shows a much stronger sensitivity to all five parameters than model radial velocity. We note here that in this study, the reflectivity formulation is assumed to be perfect and is similar to that used in TX05. The formulation will be further described in Part II.

For the three intercept parameters, at \( \Delta 10 \log(n_{0h}) = 10 \), or when the intercept parameters are an order of magnitude larger than their control values, \( J_z \) is larger than \( J_v \) by a factor of 2 for \( n_{0h} \), a factor of 3 for \( n_{0r} \) and a factor of 8 for \( n_{0s} \) (Fig. 2a, b). At \( \Delta 10 \log(\rho_h) = -2 \), \( J_z \) is larger than \( J_v \) by a factor of 3.2 for \( \rho_h \) and a factor of 2.7 for \( \rho_s \). This is not surprising because the microphysical fields are more directly affected by microphysical parameterization than the velocity field. The larger sensitivity of \( Z \) to the microphysical parameters suggests that \( Z \) data should be more useful for microphysical parameter estimation.

With respect to the three intercept parameters, both radial velocity and reflectivity show the largest sensitivity to the intercept of rain and the smallest sensitivity to the intercept of snow. With a change of +5 in the three logarithmic-form intercept parameters, which corresponds to \( n_{0r} = 2.53 \times 10^4 \text{ m}^{-4} \), \( n_{0r} = 9.49 \times 10^4 \text{ m}^{-4} \) and \( n_{0h} = 1.26 \times 10^5 \text{ m}^{-4} \), the variation of \( J_z \) due to the change in \( n_{0r} \) is more than twice as large as that due to the change in \( n_{0h} \) (Fig. 2b) and the corresponding variation of \( J_v \) due to the change in \( n_{0h} \) or decrease in \( n_{0h} \) is more than 4 times larger than that due to the change in \( n_{0h} \) (Fig. 2a). Both radial velocity and reflectivity show comparable sensitivity to \( \rho_h \) and \( \rho_s \), when \( -1 < \Delta 10 \log(\rho) \leq 0 \), and smaller sensitivity to \( \rho_s \) when \( \Delta 10 \log(\rho) \leq -1 \) (Fig. 2c and d).

b. Sensitivity of the distribution of hydrometeors to the microphysical parameters

To better understand how the model simulated supercell storm is influenced by varying the microphysical parameters and how such changes affect reflectivity, which is observed, we now examine the variation of the microphysical fields due to the change of the parameters. In addition to the control simulation, two sensitivity experiments will be presented for each of the five parameters. For each of three intercept parameters, the two values are chosen such that \( \Delta 10 \log(n_{0h}) = 10 \) and \( = -4 \), respectively. For the densities of hail and snow, one sensitivity experiment has \( \Delta 10 \log(\rho) = -2 \), and the other has the lower bound value of \( \rho_h \) and the upper bound value of \( \rho_s \), respectively (Table 2). Fig. 3 shows the vertical profiles of the time-averaged hydrometeor mixing ratios for the ten forecast sensitivity experiments plus those of the control simulation.

As the hail intercept \( n_{0h} \) increases (decreases) or the hail density \( \rho_h \) decreases (increases), the hydrometeor species have a similar trend of variations (Fig. 3a, d, f and i). The variation in the hydrometeors due to the increase by one order of magnitude in \( n_{0h} \) is generally larger than that caused by the decrease of \( \rho_h \) from its upper bound to the lower bound. Among the five hydrometeor species, \( q_h \) and \( q_h \) have the largest sensitivity to hail parameters \( n_{0h} \) and \( \rho_h \). Larger hail intercept or smaller hail density results in more \( q_h \) aloft and less \( q_h \) in the anvil of the storm (Fig. 3a, d and Fig. 4e, n).

Fig. 1 shows that increasing \( n_{0h} \) or decreasing \( \rho_h \) results in higher number concentrations and smaller terminal fall speed of hail. Reduced terminal velocity results in more \( q_h \) being suspended aloft, as indicated by the higher level at which the maximum \( q_h \) centers are located (Fig. 4b, e, n). We can also see that out of the updraft core, more \( q_h \) are horizontally advected into the anvil region. Therefore, larger reflectivity is found in the updraft core and adjacent anvil region (Fig. 4c, f, o). The longer residence time of \( q_h \) aloft results in more collection of snow in the updraft region and lower amounts of snow being transported to the anvil region, which leads to smaller reflectivity in upper level anvil apart from the updraft core. Because of the slower fall speed, less \( q_h \) and \( q_r \) within the convective core reach the ground after the storm becomes mature. The increase in \( q_r \) at the low levels (Fig. 3a, d) is mainly due to the increased \( q_h \) in the anvil precipitation region and more hails/graupels are converted to raindrops after they fall below melting level from the anvil.

It can be seen from Fig. 1a that as \( n_{0h} \) or \( \rho_h \) increases (decreases), the snow size distribution is more heavily weighted toward smaller (larger) particles. However, for a given \( q_h \), a larger snow intercept results in a larger number concentration, while a larger snow density results in a smaller number concentration of snow. The terminal fall speed of snow is not very sensitive to the change in snow intercept or snow density (Fig. 1b).

The species most sensitive to the change in snow parameters are \( q_r \) and \( q_s \) (Fig. 3g, j, note the different x-axis scale of Fig. 3j from those of Fig. 3f, h and i). Three production terms of snow in the LFO83 scheme, namely, the accretion of cloud water by snow, accretion of cloud ice by snow and deposition growth of snow, are proportional to the snow intercept parameter and inversely proportional to the snow density. When the number concentration of snow increases due to the increase in \( n_{0h} \) or decrease in \( \rho_h \), more cloud water and cloud ice are depleted by the accretion and deposition growth of snow and less \( q_i \) is advected to the anvil re-
When \( n_{0s} \) or \( \rho_s \) increases, in the anvil precipitation region close to the eastern boundary of the domain, the reflectivity below 6 km becomes weaker (Fig. 4c, i, r). This is mainly caused by the decrease in the amount of \( q_h \) between 3 km and 6 km and the decrease in \( q_r \) below 3 km in that region, which can be seen more clearly by using smaller contour intervals (not shown). Note that the reflectivity formulation used here is a function of precipitating hydrometeors, \( q_r, q_s, \) and \( q_h \), only. Even though \( q_i \) and \( q_c \) are very sensitive to the changes in \( n_{0s} \) and \( \rho_s \), they do not contribute to the reflectivity change. The accretion of snow by hail/graupel is inversely proportional to both \( n_{0s} \) and \( \rho_s \) in the LFO83 scheme; therefore, as \( n_{0s} \) or \( \rho_s \) increases, less \( q_h \) is produced by accreting snow. Less \( q_h \) in the anvil results in less \( q_h \) melting to become \( q_r \) as hail falls below the freezing level. Also note that the reflectivity in the anvil region above 7 km becomes stronger as \( n_{0s} \) increases, which is primarily due to the increase in \( q_i \) there. The reflectivity in the upper level anvil is less sensitive to the change in snow density because \( q_i \) is less sensitive to \( \rho_s \) than to \( n_{0s} \).

Larger (smaller) \( n_{0s} \) results in higher (lower) number concentration of rain and the distribution is more heavily weighted toward drizzle (larger raindrops) (Fig. 1a). Increasing \( n_{0r} \) enhances \( q_r, q_h \) and reduces \( q_s \) in the convective region (Fig. 4b and Fig. 4k). Therefore, larger \( n_{0r} \) results in higher reflectivity within and below the convective core (Fig. 4l and Fig. 4c). The reflectivity in the anvil precipitation region does not reach the ground when \( n_{0r} \) increases. This is because less \( q_r \) is found below 2 km in that region (not shown). Fig. 1b shows that the terminal fall speed of raindrops decreases (increases) as \( n_{0r} \) increases (decreases). As \( n_{0r} \) increases, both the increase in the number of small raindrops and the decrease in the terminal fall speed enhance the evaporation rate for raindrops at the low levels, which leads to less rain reaching the ground below the anvil and a stronger cold pool. Among all the five DSD parameters, the minimum temperature below 2 km is most sensitive to the intercept parameter of rain (not shown).
c. Sensitivity of the EnSRF analyses to microphysical parameters

The sensitivity analyses based on the pure-forecast experiments reveal how much the numerical prediction of storms could be affected by the errors in the microphysical parameters. Our main goal here is to apply the EnSRF method to simultaneously estimate the microphysical parameters and the model state variables. Since the numerical model is closely involved in the EnSRF data assimilation process, it is important to know how much the model state estimation will be affected by the errors in the microphysical parameters. The possibility of estimating these parameters through data assimilation depends on how sensitive the analyses of the model state variables are to these parameters when data assimilation cycles are performed, although the sensitivity to the model simulation is a prerequisite.

The details of data assimilation and parameter estimation procedure are described in Part II. The initial forecast ensemble is initialized at 20 min, with smoothed random perturbations added to a horizontally homogeneous ensemble mean defined by the environmental sounding. The data are assimilated every 5 min starting at 25 min and the assimilation window ends at 100 min. Fig. 5 shows the time-averaged response function $\Delta J_\eta$ of each assimilation sensitivity experiment, plotted against the deviations of the five parameters from their true values. For the control assimilation experiment, $\Delta J_\eta$ equal to 0 according to Eq. (7). The response functions shown in Fig. 5 are calculated using observations that are assimilated into the model. The standard deviations of observation errors for $V_r$ and $Z$ are assumed to be 1 m s$^{-1}$ and 3 dBZ, respectively.

Similar to what was found in the forecast sensitivity experiments, the analyzed radial velocity is less sensitive than the analyzed reflectivity to each of the five microphysical parameters. This indicates again that the reflectivity data is more useful for estimating the microphysical parameters. Also, the same trends of change in the hydrometeor species and in the reflectivity fields due to the changes in the parameter values (Fig. 3 and Fig. 4) are found in the analyzed model fields from the assimilation sensitivity experiments (Fig. 6 and Fig. 7).

The relative sensitivity of the analyzed $V_r$ or $Z$ to different parameters appears to be different from that in the simulation experiments. First, the relative sensitivities of the analysis to the three intercept parameter are less symmetric than that of pure forecast. For example, the sensitivities of the analyzed $Z$ to the three intercept parameters can be listed at an increasing order as $\Delta J_z (n_{0r}) < \Delta J_z (n_{0h}) < \Delta J_z (n_{0s})$ and $\Delta J_z (n_{0r}) \approx \Delta J_z (n_{0h}) < \Delta J_z (n_{0s})$, when the intercept parameters have positive and negative deviations, respectively (Fig. 5b). Second, the relative sensitivities of radial velocity...
with respect to different intercept parameters are not always consistent with those of reflectivity. For example, Fig. 5a shows that the analyzed $V_r$ is more sensitive to $n_{0h}$ than to $n_{0s}$ as the two parameters positively deviate from their true values, while with the same kind of deviation in the two parameters, the analyzed $Z$ is somewhat more sensitive to $n_{0s}$ than to $n_{0h}$. Different from what was found in the simulation sensitivity experiments, the analysis generally has the smallest sensitivity to the intercept parameter of rain (Fig. 5a and b). The analysis is apparently less sensitive to $s_\rho$ than to $h_\rho$ (Fig. 5c and d) when both $s_\rho$ and $h_\rho$ have negative deviations.

Fig. 7 shows the analyzed reflectivity from the assimilation sensitivity experiments, which have the same changes in the microphysical parameters as the forecast experiments shown in Fig. 4. The differences in $Z$ at low level anvil precipitation region (below 3 km) between the control experiment (Fig. 7a) and sensitivity experiments (Fig. 7b, c, d, e and f) are smaller than those in Fig. 4. It can be seen from Fig. 3 and Fig. 6 that the estimated $q_r$, $q_s$, and therefore $q_w$ below 4 km in almost all cases are less sensitive to the changes in the parameters than the corresponding species in the forecast experiments (dash and dash-dot curves in Fig. 6a-e and Fig. 3a-e). That is why we see smaller sensitivity of analyzed $Z$ at the low levels. Among the five assimilation sensitivity experiments, the analyzed $Z$ from experiment N0r87 (Fig. 7d) looks most like that of the CNTL. Fig. 4l shows that in the absence of data assimilation, the error in $n_{0r}$ result in much weaker reflectivity and even no reflectivity at the low-level anvil precipitation region. Probably because of the more effective correction to $q_r$ and $q_s$ during the data assimilation process, the analyzed $Z$ shows the smallest sensitivity to $n_{0r}$.

Fig. 4. Vertical cross sections of mixing ratios (g kg$^{-1}$) of hydrometeors and radar reflectivity (dBZ) through the maximum updraft at $t = 70$ min for control simulation (a-c); N0h45 (d-f); N0s37 (g-i); N0r87 (j-l); $\rho_s 400$ (m-o) and $\rho_h 400$ (p-r).
Fig. 5 shows only the temporally averaged sensitivity of the EnSRF analysis in terms of the observed quantities, $Z$ and $V_r$, to the five parameters. In fact, the time scale of the system’s response in terms of measured quantity to the parameters, i.e., how fast the model responds to the changes in the parameters, is an important factor that will affect the parameter estimation. For sequential data assimilation methods, if the response is slow, then it will take a long time and many analysis cycles to correct the parameters. For methods with assimilation windows of a limited length, such as the four-dimensional variational data assimilation method, the parameter estimation may fail completely if the response is weak within the given assimilation window.

The time scale of the system’s response to individual parameters is examined through assimilation sensitivity experiments, with one parameter perturbed in each of them. To facilitate the comparison among different parameters, the intercept parameters are chosen to be an order of magnitude larger than their true values. The hail and snow densities are chosen to have their lower and upper bounds, respectively. The five experiments are N0h45, N0s37, N0s87, $\rho_h400$ and $\rho_s400$ (Table 2).
Fig. 5. The response functions $\Delta I_{J_r}$ (a and c) and $\Delta I_{J_z}$ (b and d) of the assimilation sensitivity experiments against the logarithmic-form deviation of the parameters from their true values.

Fig. 6. The same as Fig. 3, but for data assimilation sensitivity experiments $N_{0h}45$ (thick black) and $N_{0h}14$ (thick gray) (a and f); $N_{0s}37$ (thick black) and $N_{0s}16$ (thick gray) (b and g); $N_{0r}87$ (thick black) and $N_{0r}36$ (thick gray) (c and h); $\rho_{h}400$ (thick black) and $\rho_{h}576$ (thick gray) (d and i); and $\rho_{s}400$ (thick black) and $\rho_{s}63$ (thick gray) (e and j).
Fig. 7. Vertical cross sections of the ensemble mean of radar reflectivity (dBZ) through the maximum updraft at $t = 70$ min for control data assimilation experiment (a), assimilation sensitivity experiments N0h45 (b), N0s37 (c), N0r87 (d), $\rho_s$400 (e) and $\rho_s$400 (f).

Table 2. List of the parameters values, $p_i$, and the logarithmical deviations of these parameters from their control values, $10\log_{10}(p_i)$, in the simulation and assimilation sensitivity experiments. The parameter that is changed from its control value is listed for the corresponding sensitivity experiment while other parameters used the control values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Larger deviation</th>
<th>Smaller deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>$p_i$</td>
</tr>
<tr>
<td>$n_0$ ($m^{-5}$)</td>
<td>N0h45</td>
<td>$4 \times 10^5$</td>
</tr>
<tr>
<td>$n_0$ ($m^{-3}$)</td>
<td>N0s37</td>
<td>$3 \times 10^7$</td>
</tr>
<tr>
<td>$n_0$ ($m^{-3}$)</td>
<td>N0r87</td>
<td>$8 \times 10^7$</td>
</tr>
<tr>
<td>$\rho_s$ (kg m$^{-3}$)</td>
<td>$\rho_s$400</td>
<td>400</td>
</tr>
<tr>
<td>$\rho_s$ (kg m$^{-3}$)</td>
<td>$\rho_s$400</td>
<td>400</td>
</tr>
</tbody>
</table>

Fig. 8 shows the time series of the response functions, $\Delta J_V$, and $\Delta J_Z$, at each analysis time from the five experiments. Since the analyzed $Z$ is much more sensitive to the changes in the parameters than the analyzed $V$, we are more interested in the system response time in terms of $Z$. As can be seen from Fig. 8b, in most cases, either the response functions $\Delta J_Z$ are close to or below 0 in the first two assimilation cycles, or $\Delta J_Z$ decreases to be close to or below 0 from the first to the second cycle. Significant and continuous increase in $\Delta J_Z$ usually occurs after the first two assimilation cycles. The negative values are also found when the response functions are calculated using error-free observation data (not shown). Reasonable system response to the errors in the parameters should results in positive response functions. The negative values during the initial cycles are probably a reflection that the model state estimation is poor at this stage therefore the system response is not reliable.

It is not easy to tell exactly how large the system response in terms of $\Delta J_Z$ needs to be to allow successful parameter identification from the observations. The straight line in Fig. 8b is our estimation of the minimum
threshold for reliable system response. This minimum threshold is chosen such that the unreliable responses as indicated by the negative values of $\Delta J_z$ in the first two assimilation cycles are no larger than this threshold.

Table 3 lists our estimation of the system response time scales for the five assimilation sensitivity experiments. Among the five assimilation sensitivity experiments examined, the shortest response time is 15 min or 3 assimilation cycles, and the longest time is 25 min or 5 assimilation cycles. The time scales of the model response to the same amount of change in the intercept parameter has a unique minimum in a given region and if the minimization is continuously dependent on the measurement errors.

Based on the minimum threshold of reliable response given in Fig. 8, the system response time scale can be defined as the time for $\Delta J_z$ to increase over and remain above the minimum threshold. Parameters can be listed in an increasing order as $T_{\rho_{h400}} \leq T_{\rho_{s400}} \leq T_{\rho_{sr}}$. The system also responds more quickly to the change in $\rho_s$ as it decreases from its upper bound to its lower bound values than to the change in $\rho_h$ as it increases from the control value to its upper bound, although the time averaged response function $\Delta J_{\rho}$ of $\rho_{400}$ has almost the same value as that of $\rho_{400}$ (Fig. 5d).

Table 3. List of the time scales of the system response to the change in one of the microphysical parameters in terms of analysis reflectivity for assimilation sensitivity experiments N0h45, N0s37, N0r87, $\rho_{400}$ and $\rho_{400}$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>time</th>
<th>Number of cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>N0h45</td>
<td>15 min</td>
<td>3</td>
</tr>
<tr>
<td>N0s37</td>
<td>20 min</td>
<td>4</td>
</tr>
<tr>
<td>N0r87</td>
<td>25 min</td>
<td>5</td>
</tr>
<tr>
<td>$\rho_{400}$</td>
<td>15 min</td>
<td>3</td>
</tr>
<tr>
<td>$\rho_{500}$</td>
<td>25 min</td>
<td>5</td>
</tr>
</tbody>
</table>

It can be seen from Fig. 8b that the times that $\Delta J_z$ curves cross the minimum threshold are also the times at which $\Delta J_z$ has a large growth rate from the earlier cycles. Table 3. After the first significant increase, the response function grows more moderately in N0r87, $\rho_{400}$ and $\rho_{400}$. A large growth in the response function occurs again during the later assimilation cycles of N0h45 and N0s37.

5. Parameter identifiability

a. Parameter identifiability as revealed by the response function

An important issue associated with parameter estimation is the parameter identifiability. The concept of identifiability addresses the question of whether it is at all possible to obtain unique solutions of the inverse problem for unknown parameters of interest in a model from data collected in the spatial and time domains (Navon 1998). Various definitions of parameter identifiability can be found in the literature (Kitamura and Nakagiri 1977; Chavent 1979; Sun and Yeh 1990). A definition suitable for the estimation process using the output least square error criterion was given by Chavent (1979). A parameter is said to be least-square identifiable if the least squares performance function for identifying the parameter has a unique minimum in a given region and if the minimization is continuously dependent on the measurement errors.

The response function defined by Eq. (4) is actually the performance function that is to be minimized if the inverse problem is solved by using the output least square error criterion. As shown in Fig. 2, the response functions of both $V_r$ and $Z$ against the variations of all five parameters in the simulation sensitivity experiments all have a concave shape, and there is a unique minimum for each case. This is an indication of a unique mapping between the parameters and the model solution in terms of radar observations, even though the microphysical process and the observation operators are highly nonlinear.

The response functions $\Delta J_z$ from the assimilation sensitivity experiments also show concave shapes (Fig. 5), but the curves have more gentle slopes and flatter bottoms than the corresponding functions of simulation sensitivity experiments (c.f., Fig. 2). The minimum of the response functions is not always located exactly at the zero deviation point as was found in the simulation sensitivity case, but is always very close to that of the control experiment and the minimum is very close to 0. The smaller gradient of the analysis response function indicates that the signal of model error is weaker when data are used to constraint the model state evolution. The concave shape of the response functions and their single minimum indicate a high probability of finding the true value. These results suggest that the five parameters considered can be identified, at least individually, from radar data with a certain degree of accuracy, even when they are estimated simultaneously with the model state variables.

From the flatness of the response functions near the bottom of their curves (Fig. 5), we can estimate the limit
of the accuracy that the parameters can be estimated. Fig. 5a and Fig. 5b show that if the three intercept parameters vary within the range of $\Delta \log(n_{0r}) = \pm 1$, the analysis will not sense the errors in the intercept parameters much in terms of $V_r$ and $Z$. In another word, if the error of the estimated intercept parameters is within $\Delta \log(n_{0r}) = \pm 10\log(\rho_h) = \pm 0.5$ for the snow and hail/graupel densities according to Fig. 5c and Fig. 5d.

Fig. 8. Time evolution of (a) $\Delta J_{V_r}$ and (b) $\Delta J_Z$ from assimilation sensitivity experiments N0h45, N0s37, N0r87, $\rho_h$400 and $\rho_s$400 (c.f., Table 2).
Fig. 9. Correlation coefficients calculated from the ensemble members of single-parameter ensemble forecasts at $t = 70$ min. The correlation [thick solid (dash) contours represent positive (negative) values at intervals of 0.2] between forecast $Z$ and (a) $n_{0h}$ at 1.5°, (b) $n_{0s}$ at 1.5°, (c) $n_{0r}$ at 0.5°, (d) $\rho_h$ at 1.5°, (e) $\rho_s$ at 0.5° at 1.5°, (f) $n_{0h}$ at 5.3°, (g) $n_{0s}$ at 5.3°, (h) $n_{0r}$ at 4.3°, (i) $\rho_h$ at 5.3°, (j) $\rho_s$ at 6.2° elevation levels. The shading and thin solid contours represent $Z$ from simulations N0h45 (a and f); N0s37 (b and g); N0r87 (c and h); $\rho_h$400 (d and i); and $\rho_s$400 (e and j).

Fig. 10. As in Fig. 9, but the correlation coefficients are estimated from the members of 5-parameter forecast ensemble at $t = 70$ min. The shading and thin solid contours represent $Z$ from control simulation.
It can be seen from Fig. 9 that in certain regions of the storm, the microphysical parameters are highly correlated with reflectivity. The maximum values of $\rho_{\text{nh}}(\text{,})$ for the five single-parameter cases are all greater than 0.9. A large part of the storm is dominated by regions where $\rho_{\text{nh}}(\text{,}) \geq 0.6$. This confirms again that these five individual parameters can be identified from the reflectivity data.

Comparing the reflectivity fields from simulation sensitivity experiments in Fig. 9 with that of control simulation in Fig. 10, we can see that the locations of the positive and negative correlation regions are consistent with the variation in reflectivity due to the change in parameter value. For example, as $n_{0h}$ increases (experiment N0h45), stronger reflectivity is found in the low level anvil precipitation region (gray shades in Fig. 9a, Fig. 10a), therefore $n_{0h}$ is positively correlated with Z in that region, as indicated by positive thick contours in Fig. 9a. The reflectivity in Fig. 9f and Fig. 9i represents precipitation between 3 km and 7 km, given that the radar is located at the southwest corner of the domain. The further the reflectivity echo is away from the radar, the higher is the displayed hydrometeors located. As $n_{0h}$ increases (as in N0h45) or $\rho_{h}$ decreases (as in R400), more $q_{h}$ is transported aloft and less $q_{s}$ exists in the anvil, resulting in lager reflectivity at middle levels and smaller reflectivity at higher levels. This is why the reflectivity closer to the radar is positively (negatively) correlated with $n_{0h}$ ( $\rho_{h}$ ) and the reflectivity further away from the radar is negatively (positively) correlated with $n_{0h}$ ( $\rho_{h}$ ) in Fig. 9f and Fig. 9i.

Fig. 10 shows that as the number of adjustable parameters increases to five, the maximum value of $\rho_{\text{nh}}(\text{,})$ decreases for all parameters and the area with significant correlation also decreases. This indicates that as the number of uncertain parameter increases, the identifiability of each parameter decreases. From Fig. 9 we can see some common correlation regions for different parameters, e.g., the negative correlation region at lower level anvil precipitation region for $n_{0s}$, $n_{0r}$ and $\rho_{s}$ (Fig. 9b, c and e), and the positive-negative correlation pattern at higher level anvil region for $n_{0t}$, $\rho_{h}$ and $\rho_{s}$ (Fig. 9g, i and j). When all the parameters vary simultaneously, the correlations in those common regions become very weak (Fig. 10b, c, e, g, h, i and j), which implies that the contribution of a particular parameter to the change of reflectivity is hard identify if its contribution is similar to those of other parameters.

Comparing the spatial correlation patterns in Fig. 10 with those in Fig. 9, we can see that the positive and negative correlation coefficients estimated from the 5-parameter experiment are at similar locations as those estimated from single-parameter experiments, even though in certain regions of the storm, significant values of correlation coefficient are missing in the 5-parameter case. This suggests that it may still be possible to estimate five parameters simultaneously, because the information contained in the correlation coefficients remains correct. However, because the correlations for some parameters become weaker, whether errors in the observations will cause instability in parameter estimation is uncertain.

Table 4 shows the spatially and temporally averaged absolute correlation coefficient between $\eta$ and parameter $P_{i}$, i.e., $\rho_{\text{nh}}(\text{,})$, calculated from single-parameter and multiple-parameter forecast ensembles. The correlation coefficients for radial velocity are always smaller than the corresponding ones for reflectivity. The correlations generally decrease as the number of uncertain parameter increases. For the three-intercept parameter case, the most significant decrease in the correlation level occurs to $n_{0s}$. The decrease in the correlation level becomes smaller as the number of uncertain parameters increases from 3 to 5. The additional parameters, $\rho_{h}$ and/or $\rho_{s}$, have relatively low correlations with reflectivity.

6. Summary and conclusions

The possibility of estimating five fundamental microphysical parameters from radar observations is investigated by addressing issues associated with pa-
parameter sensitivity and identifiability. These five parameters are the intercept parameters for rain, snow and hail/graupele, and the bulk densities of hail/graupele and snow, which are usually pre-specified constants in single-moment bulk microphysics schemes and are involved in the definition of drop/particle size distributions. The identifiability of individual parameters is examined from two aspects: the sensitivity of the model forecast or model state estimation in terms of the observed quantities to the changes in the parameter values and the uniqueness of the inverse problem solution for parameter estimation.

Sensitivity analyses were carried out based on pure-forecast and data assimilation sensitivity experiments for a supercell thunderstorm case. Within these experiments, the microphysical parameters are varied within their observed ranges of uncertainty individually. A response function, which measures the difference between the observations and the corresponding model state subjecting to the parameter perturbations, was calculated for each of the sensitivity experiments.

Both forecast and assimilation sensitivity experiments show that the errors in the microphysical parameters have larger impact on model microphysical fields than on wind fields. The model radar reflectivity is more sensitive to the microphysical parameters than the model radial velocity is. Generally, the larger the sensitivity is, the higher is the likelihood of correct parameter identification. Therefore, radar reflectivity is preferred over radial velocity for microphysical parameter estimation. Among the three intercept parameters, the pure forecast, in terms of $V_r$ or $Z$, is most sensitive to $n_{0r}$ and least sensitive to $n_{0s}$. It is generally more sensitivity to $\rho_v$ than to $\rho_s$. The relative sensitivities of the estimated or analyzed model state in terms of $V_r$ and $Z$ to different parameters are somewhat different from the case of forecast sensitivity. Among the three intercept parameters, the analyzed $V_r$ and $Z$ are generally most sensitive to $n_{0r}$ and least sensitive to $n_{0s}$. The possible reasons of the difference in the responses between the two cases require further investigation. It will require an understanding of the nonlinear dynamics of the data assimilation process and its interaction with the parameter errors.

Another factor that affects parameter estimation is the time scale of the system response to the parameter changes. The shorter the response time, the faster the parameters can be corrected through data assimilation cycles. The time series of the response functions were presented for five sensitivity experiments, which include an increase in the three intercept parameters individually by an order of magnitude and the use of the upper and lower bound values for the snow and hail/graupele densities, respectively. According to these experiments, the five parameters can be listed in an increasing order of the response time as $T_{n_{0r}} = T_{\rho_v} < T_{n_{0s}} < T_{\rho_s} <= T_{\rho_s}$. The response times are found to be between 15 to 25 minutes or 3 to 5 assimilation cycles. For this particular radar data assimilation problem, when the model is perfect, at least 8 assimilation cycles (40 minutes) are needed to arrive at a good state estimation. Within such a time window, successful estimation of these parameters can therefore be expected, although in practice, too small initial parameter errors can slow down the estimation process, as will be discussed in Part II.

The response functions calculated for the forecast sensitivity experiments for all five individual parameters show concave shapes and have unique minima equaling those of truth simulation. The response functions obtained from assimilation sensitivity experiments also show concave shapes, but the gradients of the response functions with respect to the parameters become smaller, especially, when the parameters are very close to their true values. It is most likely that each of the five microphysical parameters can be estimated individually from radar reflectivity data. From the assimilation sensitivity experiments, we can also estimate the likely accuracy of the estimated parameters, which is limited by the flatness of the response functions near their minimum. The likely accuracy limit in the logarithmic unit is about 1 for intercept parameters or about 0.5 for particle densities. The errors in the parameters smaller than the limits should have negligible impact on the model state estimation.

The identifiability of the microphysical parameters, especially when they vary simultaneously, was also evaluated from their correlations with the model output of radar observations based on the ensemble forecasts. For single-parameter cases, all five parameters are highly correlated with radar reflectivity, in terms of the maximum values and area coverage of the significant correlations. The correlations with radial velocity are lower. The physical meanings of the correlations between the microphysical parameters and radar reflectivity can be explained by the hydrometeor changes caused by the changes in the parameters. As the number of uncertain parameters increases, both the level and the area coverage of significant correlations decrease, which implies that the degree of difficulties will be higher with multiple-parameter estimation.

In Part II of this paper, the details of the simultaneous estimation of the microphysical parameters and model state variables using the EnSRF algorithm from radar data will be presented. The sensitivity analysis and parameter identifiability discussed here will guide us with the experiment design and help us understand the results of estimation. The parameter identifiability issue will be further discussed based on the estimation results.

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